# Quantum discord plays no distinguished role in characterization of complete positivity : Robustness of the traditional scheme

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The traditional scheme for realizing open-system quantum dynamics takes the initial state of the system-bath composite as a product[1]. Recently, however, the influence of possible system-bath initial correlations on the reduced dynamics has been attracting considerable interest[2–4]. The influential work of Shabani and Lidar[4] famously relates this issue to quantum discord[5, 6], a concept which has in recent years occupied the centre-stage of quantum information theory and has led to several fundamental results[7]. They suggest that reduced dynamics is completely positive if and only if the initial system-bath correlations have vanishing quantum discord. Here we show that there is, within the Shabani-Lidar framework, no scope for any distinguished role for quantum discord in respect of complete positivity of reduced dynamics. Since most applications of quantum theory to real systems rests on the traditional scheme, its robustness thus demonstrated could be of far-reaching significance.

The traditional (folklore) scheme is contrasted with the Shabani-Lidar (SL hereafter) scheme [4] in Fig. 1, using the same notation as SL, and showing in what sense the latter is a generalization of the former. This generalized formulation of quantum dynamical process (QDP) allows SL to transcribe the fundamental issue of complete positivity (CP) of reduced dynamics (QDP) to the following precise question: What are the necessary and sufficient conditions on the SL collection $\Omega^{SB}$ of possibly correlated system-bath initial states so that the induced QDP (Fig. 1, right)

$$\rho_S(0) \to \rho_S(t) \tag{1}$$

is well defined and guaranteed to be CP *for every* joint unitary $U_{SB}(t)$? Inspired by the work of Rodriguez-Rosario et al. [3], and highlighting it as *'a recent breakthrough'*, SL advance the following resolution :

**Theorem 1** (SL)*: The QDP in equation (1) is CP for all joint unitaries* $U_{SB}(t)$ *if and only*

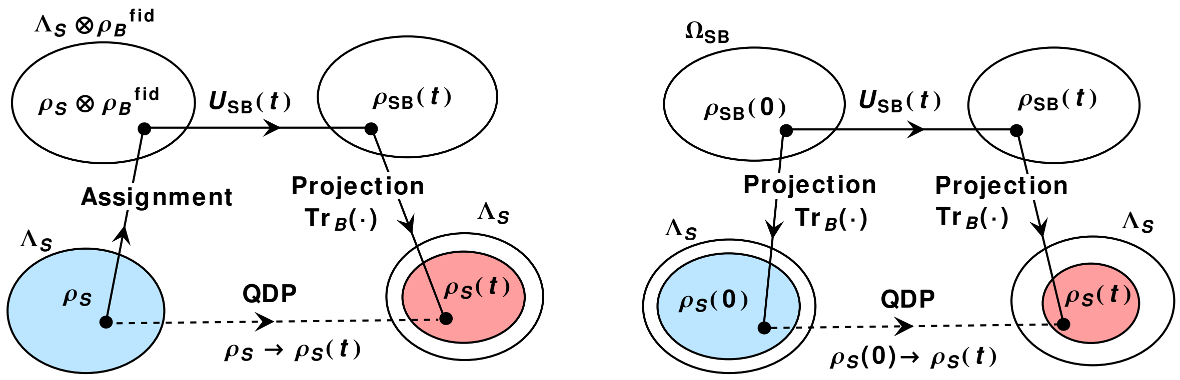


FIG. 1: **The folklore scheme (left) and the SL scheme**. In the folklore scheme, initial system states $\rho_S$ are elevated to product states of the composite, for a *fixed* fiducial bath state $\rho_B^{\rm fid}$, through the assignment map $\rho_S \to \rho_S \otimes \rho_B^{\rm fid}$. These uncorrelated system-bath states are evolved under a joint unitary $U_{SB}(t)$ to $U_{SB}(t)\, \rho_S \otimes \rho_B^{\rm fid}\, U_{SB}(t)^\dagger$ and, finally, the bath degrees of freedom are traced out to obtain the time-evolved states $\rho_S(t) = {\rm Tr_B}\left[{\rm U_{SB}(t)}\rho_{\rm S} \otimes \rho_{\rm B}^{\rm fid}\, {\rm U_{SB}(t)}^\dagger\right]$ of the system. The resulting quantum dynamical process (QDP) $\rho_S \to \rho_S(t)$, parametrized by $\rho_B^{\rm fid}$ and $U_{SB}(t)$, is completely positive by construction. In sharp contrast, there is no such assignment map in the SL scheme. The distinguished bath state $\rho_B^{\rm fid}$ is replaced by a collection $\Omega^{SB}$ of (possibly correlated) system-bath initial states $\rho_{SB}(0)$. The dynamics gets defined through $\rho_{SB}(0) \to \rho_{SB}(t) = U_{SB}(t)\, \rho_{SB}(0)\, U_{SB}(t)^\dagger$ for all $\rho_{SB}(0) \in \Omega^{SB}$. With reduced system states $\rho_S(0)$ and $\rho_S(t)$ defined through the imaging or projection map $\rho_S(0) = {\rm Tr_B}\, \rho_{\rm SB}(0)$ and $\rho_S(t) = {\rm Tr_B}\left[{\rm U_{SB}(t)}\, \rho_{\rm SB}(0)\, {\rm U_{SB}(t)}^\dagger\right]$, this unitary dynamics of the composite induces on the system the QDP $\rho_S(0) \to \rho_S(t)$. Whether the Shabani-Lidar QDP so described is well-defined and completely positive is clearly an issue answered solely by the nature of the collection $\Omega^{SB}$. It is clear the folklore scheme is a particular case of the SL scheme corresponding to $\Omega^{SB} = \{\, \rho_S \otimes \rho_B^{\rm fid} \,|\, \rho_B^{\rm fid} = {\rm fixed}\,\}$. The Hilbert spaces $\mathcal{H}_S$ and $\mathcal{H}_B$ of the system and the bath are of dimensions $d_S$, $d_B$ respectively. The $(d_S^2 - 1)$-dimensional (convex) state space $\Lambda_S$ is a subset of $\mathcal{B}(\mathcal{H}_S)$ whereas $\Omega^{SB} \subset \mathcal{B}(\mathcal{H}_S \otimes \mathcal{H}_B)$, the convex hull of $\Omega^{SB}$ being denoted $\overline{\Omega^{SB}}$. In both schemes, initial system states are identified by the blue region and the final states by the red.

*if the quantum discord vanishes for all $\rho_{SB} \in \Omega_{SB}$, i.e., if and only if the initial system-bath correlations are purely classical.*

The SL theorem has come to be counted among the more important recent results of quantum information theory, and has influenced an enormous number of authors [8]. But the very recent work of Brodutch et al. [9], co-authored also by some of the authors of the original 'breakthrough' paper, contests the claim of SL and asserts that vanishing quantum discord is sufficient but not necessary condition for complete positivity. Our analysis proves that both the SL theorem and the assertion of Brodutch et al. are too strong to be tenable.

We begin with the (almost) obvious. In order that the QDP in equation (1) be *well defined* in the first place, the SL set $\Omega^{SB}$ should satisfy two requirements or properties: **(1)** no system state $\rho_S(0)$ can have two or more pre-images in $\Omega^{SB}$ (see Fig. 1); **(2)** while every system state $\rho_S(0)$ need not have a pre-image *actually enumerated* in $\Omega^{SB}$, the set of states $\rho_S(0)$ having pre-image should be large enough that the QDP in equation (1) can be extended by linearity to all states of the system, i.e., to the full state space $\Lambda_S$.

*Our entire analysis rests on these two (almost obvious) requirements on the part of* $\Omega^{SB}$.

It leads, in contradistinction to the SL claim, to the following conclusion :

> No initial correlations—*even classical ones*—are permissible in the SL scheme. That is, quantum discord is no less destructive as far as CP property of QDP is concerned.

The stated goal of SL was to give a *complete characterization* of possible initial correlations that lead to CP maps. It is possibly in view of the (erroneous) belief that there was a large class of permissible initial correlations out there within the SL framework, and that that class now stands fully characterized by the SL theorem, that a large number of recent papers tend to list complete characterization of CP maps among the principal achievements of quantum discord [8]. Our result implies, with no irreverence whatsoever to quantum discord, that characterization of CP maps may not yet be rightfully paraded as one of the principal achievements of quantum discord.

The SL theorem has influenced an enormous number of authors, and it is inevitable that those results of these authors which make essential use of the sufficiency part of the SL theorem need recalibration in the light of our result.

There are other, potentially much deeper, implications of our finding. Our analysis—strictly within the SL framework—has shown that this framework brings one exactly back to the folklore scheme itself, as if it were a *fixed point.* This is not at all a negative result for two reasons. First, it shows that quantum discord is no 'cheaper' than entanglement as far as complete positivity of QDP is concerned. Second, and more importantly, the fact that the folklore product-scheme survives attack under this powerful, well-defined, and fairly general SL framework demonstrates its, perhaps unsuspected, *robustness.* In view of the fact that this scheme has been at the heart of most applications of quantum theory to real situations, virtually in every area of physical science, and even beyond, its robustness the SL framework has helped to establish is likely to prove to be of far-reaching significance.

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