Entanglement enhances performance in microscopic quantum fridges

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Introduction

The study of quantum thermal machines has a long history, from the thermodynamic analysis of lasers [1–3], to considerable work on quantum cycles and the second law[4–17]. Recently, models of small self-contained quantum thermal machines [18**?** –21] have attracted growing attention. The key feature of such machines is that they function without any external source of work or control. Only incoherent interaction with thermal baths is required. Interestingly, there exists no fundamental limit on the size of such machines [18], nor on their efficiency [19]. Their main interest resides in their simplicity, which allows us to explore novel ideas in quantum thermodynamics.

The working of such machines is very significant for quantum thermodynamics, furthering our understanding of the notions of work and entropy for quantum systems. In recent work [21], it has been shown that these machines are bound by the same classical Carnot limit as regards efficiency. The notion of work has also been discussed at length since the storage of energy on a quantum scale suffers from practical difficulties arising from decoherence and the uncertainty principle that are not apparent in classical discussions of the same. Clearly for the purpose of quantum computation, or any process that we wish to have control of at the scale of individual qubits, such thermal machines are essential. We can use the quantum refrigerator or heat engine to reset a qubit register to its ground or excited state respectively, or use a quantum work engine to charge a ”quantum battery” as an energy source to drive the unitaries that we run any computation by.

An important question which has not been addressed so far is whether quantum effects play any significant role in such small self-contained thermal machines. Indeed, although these machines are described within the formalism of quantum mechanics, it is not immediately clear to what extent their working is inherently quantum, since one can give an heuristic account of their functioning in classical terms.

In the paper, our aim is to establish the importance of quantum effects in these thermal machines. Our main focus will be on the concept of entanglement, often considered as the defining feature of quantum theory. Hence, if entanglement turns out to play an important role in self-contained quantum thermal machines, this would make it clear that the working of such machines is truly quantum mechanical. Moreover, it would then raise the question of whether entanglement can enhance the performance of such machines. We address these questions focusing our attention on the model of the smallest possible self-contained quantum refrigerator [18, 19], as shown schematically in Fig. 1.

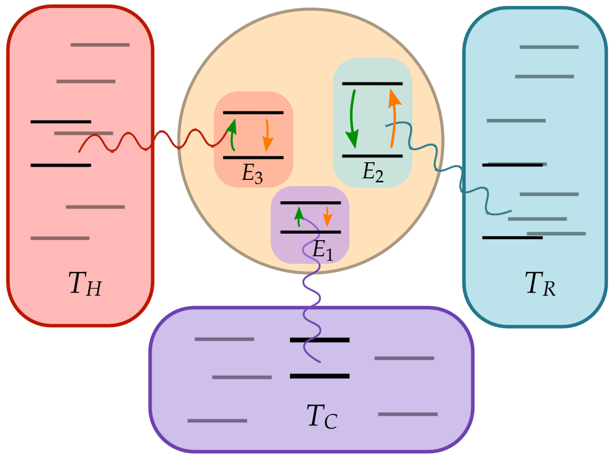

FIG. 1: Schematic diagram of the quantum refrigerator. The fridge contains three qubits (inside the yellow circle), each in weak thermal contact (wiggly lines) with a bath at a different temperature. The qubits interact via the weak interaction Hamiltonian H_{int}, which couples the two degenerate levels $|010\rangle$ and $|101\rangle$, depicted by the arrows. The lower qubit (purple) is the object to be cooled. At equilibrium, it reaches a temperature $T_S < T_C$. The other two qubits (red and blue) are the machine qubits, connected to heat baths at temperatures T_R and T_H.

Summary of Methods and Results

Considering the model of the quantum fridge, the first question addressed was whether there was entanglement present in any regime of the working of the fridge. Since we have a system of 3 qubits, entanglement can be of several kinds, across a bipartition of the system, or genuine tripartite entanglement. Using the class of entanglement witnesses developed in [22, 23], we found that there exist regimes containing entanglement across one, two or all 3 bipartitions of the system, and even regimes demonstrating genuine multipartite entanglement.

In order to determine the effect of entanglement on the working of the fridge, we first questioned its relation to the Carnot point of the fridge, i.e. the set of parameters for which the fridge operates at Carnot efficiency. One of the fundamental results in [21] was that every 3 qubit thermal machine can reach and is bound by the classical Carnot efficiency derived from the relative temperature of the hot and cold thermal baths. Hence we seek to know whether entanglement is present in this high efficiency regime of the fridge.

However we found that in fact the Carnot point itself corresponded to the fridge being in a product state of the 3 qubits of the fridge, and were able to prove that around the Carnot point of any fridge working at finite temperatures, there existed a neighbourhood of states of the fridge that were completely biseparable.

While at first glance this may appear to suggest that entanglement is detrimental to the efficiency of such machines, since we require to depart from Carnot efficiency to find regimes of entanglement, we must take into account that the Carnot point of any thermal machine corresponds to the machine working reversibly, and hence necessarily with *infitesimal* power. However we are interested in whether quantum fridges are physically feasible for cooling, and hence require the effect of entanglement on fridges that achieve the best possible cooling, with respect to power (heat drained per unit time) or even better the temparature to which we can cool our system.

We thus sought the answer to the question :
For a given qubit, is the temparature that a entangled quantum fridge can cool it to lower than the temparature achievable using a completely separable quantum fridge?

Explicitly, this means that we are supplied with a qubit of given energy E_C, immersed in a thermal bath of temparature T_C and coupling with the bath p_C. We are then free to vary the other parameters of the fridge that include the temparatures of the hot and room thermal baths, T_H and T_R respectively, along with the couplings to their respective qubits, p_H and p_R, as well as the energy of the hot qubit E_H and the strength of the interaction between the 3 qubits (g).

Of course there are sensible physical constraints on these parameters, the trivial ones being that $T_R > T_C$, else cooling is achievable by direct transfer between the cold and room qubits, and $T_H > T_R$ for the fridge to cool at all. In order for the energy eigenstates of each qubit to remain the qubit ground and excited state, we require the 3-qubit interaction and thermal couplings to be small with respect to the qubit energies, i.e. $g, p_i \ll E_i$ Checking the separability was done with the same entanglement witnesses referenced earlier.

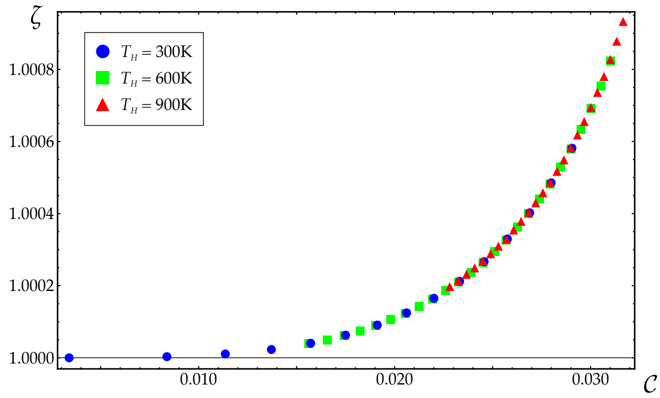

FIG. 2: Cooling advantage is determined by the amount of entanglement. Plot of relative cooling enhancement ζ against amount of entanglement on the bipartition $R|CH$ (measured by the concurrence C), as evaluated from Fig.2a. Since all points lie on a single curve, it follows that ζ is determined solely by the amount of entanglement, and does not depend on the temperatures of the baths T_R and T_H. The fact that the behaviour is monotonic strongly suggests a functional relationship. For convenience the data plotted corresponds to the three horizontal slices of Fig. 2 (a). Taking random sample points from Fig. 2 (a) leads to a similar result.

The result was indeed that there existed situations in which entangled fridges cooled qubits to lower temparatures than fridges constrained to be biseparable, thus conclusively demonstrating the usefulness of entanglement for cooling. Delving further into the properties of such optimal machines, we found that the entanglement present was almost always across the bipartition separating the room qubit from the hot and cold qubits. Considering that heat flows from the hot and cold baths into the room bath, this implies that entanglement for optimal working is between the partition *energy in v/s energy out*, suggesting that quantum coherence enhances energy transport, a phenomenon that has achieved considerable attention in photosynthetic complexes[24].

In fact, when we investigated the relation between the concurrence across the bipartion Room—Cold,Hot and the enhancement in cooling, as measured by comparing the temparatures achievable by entangled v/s separable fridges, we found that the enhancement appeared to be determined entirely by the concurrence.

Conclusion

We have demonstrated that entanglement definitely plays a role in the working of the quantum fridge. While around the Carnot point, that corresponds to the reversible regime of the machine, we find no entanglement, when we require maximal cooling, the optimal machines can be see to be enhanced by the presence of entanglement. The nature of this entanglement is such as to sugges that it is directly involved in enhancing energy tranport between the hot and cold thermal baths to the room bath.

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