Adiabatic Effect in Open Quantum Systems: Implications for Quantum Information

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Extended Abstract:

Adiabatic theorem refers to a situation in which the original Hamiltonian of the system is gradually changed to a new Hamiltonian. It played an important role in the development of quantum mechanics [1, 2, 3]. An energy eigenstate, of the original Hamiltonian, becomes approximately an eigenstate for the new Hamiltonian, if the switch-on of the energy difference is sufficiently slow. This implies that the slowness of variation needs to be compared with an inherent slow system frequency, for e.g., the minimum of splitting of energy levels, say ω. The time variation of the Hamiltonian introduces another frequency, χ. For the adiabatic regime to hold, $\chi \ll \omega$, which implies that the Hamiltonian does not change significantly during the system characteristic cycle of motion. A canonical example of the adiabatic phenomena can be seen from the case of a one dimensional harmonic oscillator perturbed by $-e\mathcal{E}Xe^{-\frac{t^2}{\tau^2}}$, where

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τ is the time scale associated with the perturbation and $\mathcal{E}$ is the electric field. Applying time dependent perturbation theory, the probability of transition of the oscillator from the ground state to the first excited state is $\frac{\pi e^2 \mathcal{E} \tau^2}{2m\hbar\omega} e^{-\frac{\omega^2\tau^2}{2}}$. In the adiabatic regime ($\tau \sim \frac{1}{\chi}$), $\omega\tau \gg 1$, the system is seen to remain in the ground state.

The above simple estimate, justifying the adiabatic approximation, has been subject, in a number of works, to rigorous mathematical analysis, related to first order estimates of the spectral gap, of the spectral projection of the ground state separated from the rest of the spectrum [4, 5, 6]. These estimates have been extended to systems without a gap [7]. In [8], rigorous estimates were made for Hamiltonians which at any time t possess two spectral projectors, $P_1(t)$ and $P_2(t)$, and which are spectrally isolated. Considering systems with avoided level crossing, the adiabatic analysis lead to a rigorous derivation of the well known Landau-Zenner formula. In [9] use was made of the adiabatic theorem to introduce the concept of topological states of matter in order to distinguish gapped many body ground states of non-interacting systems and mean field superconductors, respectively, regarding their global geometrical features.

In recent times, adiabatic approximation has been used as a method of quantum computation [10]. The Hamiltonian of interest is $H(t) = (1 - g(t))H_0 + g(t)H_1$. In most applications of the adiabatic theorem to quantum computation one is interested to find out how certain quantities, such as the running time of a computer program, grow (or decrease) with the parameter, n, which describes the size of the system [11]. In [12], the time evolution of a quantum system in the adiabatic limit was shown to have a geometric origin, leading to the concept of the geometric phase, an important tool in holonomic quantum computation.

A general applicability of these ideas requires an extension to the arena of open quantum systems. Open quantum systems are ubiquitous in the sense that any system can be thought of as being surrounded by its environment (reservoir or bath) which influences its dynamics. They provide a natural route for discussing damping and dephasing. One of the first testing grounds for open system ideas was in quantum optics [13]. Its application to other areas gained momentum from the works of Caldeira and Leggett [14], and Zurek [15], among others. Depending upon the system-reservoir ($S - R$) interaction, open systems can be broadly classified into two categories, viz., quantum non-demolition (QND) or dissipative. A particular type of quantum nondemolition (QND) $S - R$ interaction may be achieved when the Hamiltonian H_S of the system commutes with the Hamiltonian H_{SR} describing the system-reservoir interaction, i.e., H_{SR} is a constant of the motion generated by H_S [16, 17, 18, 19, 20]. This results in pure dephasing without dissipation. A dissipative open system would be when H_S and H_{SR} do not commute resulting in dephasing along with damping [21]. Impressive progress has been made on the experimental front in the manipulation of quantum states of matter towards quantum information processing and quantum communication. Myatt *et al.* [22] and Turchette *et al.* [23] performed a series of experiments in which they engineered both the pure dephasing as well as dissipative type of evolutions.

Efforts have been made to develop an understanding of the adiabatic effect in open quantum systems. In [24], estimates were made for systems evolving under a Lindbladian evolution. Some rigorous estimates for adiabatic evolution of Lindbladian open quantum systems, with and without a gap, was made recently in [25]. Since an open system evolution would, in general, be non-unitary, it cannot be described by a Hermitian Hamiltonian. In some recent works, attempts have made to use an effective Hamiltonian approach to this problem [26]. Here we aim at providing simple, physically motivated examples aimed at an understanding of adiabatic effects in the context of open quantum systems. An interesting analysis can be made from the prespective of thermodynamics. Here it is easy to show that for systems, undergoing Lindbladian evolution, where the Lindbladian commutes with the Hamiltonian, the system is adiabatic from the perspective of thermodynamics, but is not informationally isolated from its environment. In fact, this is the above discussed QND regime which is subject to decoherence. We take up a simple model of a two-level system, undergoing a general open system evolution. This is then converted into an equivalent problem of a spin precession around an effective magnetic field, which is described in terms of the open system parameters [27]. From the time scales in this problem, a simple understanding is reached about the adiabatic regime in a simple, but instructive, open system model.

The technical paper will appear soon in the quant-ph archive.

References

[1] M. Born and V. Fock, Z. Phys. **51**, 165 (1928).

[2] A. Dass, *Lectures on Quantum Mechanics* (Hindustan Book Agency, 2003).

[3] A. Messiah, *Quantum Mechanics* (North-Holland Publishing Company, 1962).

[4] T. Kato, J. of Phys. Soc. Japan **5**, 435 (1950).

[5] G. Nenciu, J. Phys. A **13**, L15 (1980).

[6] J. E. Avron, R. Seiler and L. G. Yaffe, Comm. of Math. Phys. **110**, 33 (1987).

[7] J. E. Avron and A. Elgart, Comm. of Math. Phys. **203**, 445 (1999).

[8] A. Joye, *Geometrical and Mathematical Methods of the Adiabatic Theorem in Quantum Theory* (Thesis No 1022 (1992), Ecole Polytechnique Federale De Lausanne).

[9] J. C. Budich and B. Trauzettel, arXiv:1210.6672.

[10] E. Farhi, J. Goldstone, S. Gutmann and M. Sipser, arXiv:quant-ph/0001106: E. Frahi *et. al*, Science **292**, 472 (2001).

[11] S. Jansen, R. Seiler and M-B. Ruskai, arXiv:quant-ph/0603175.

[12] M. V. Berry, Proc. R. Soc. London A **392**, 45 (1984).

[13] W. H. Louisell, *Quantum Statistical Properties of Radiation* (John Wiley and Sons, 1973).

[14] A. O. Caldeira and A. J. Leggett, Physica A **121**, 587 (1983).

[15] W. H. Zurek, Phys. Today **44**, 36 (1991); Prog. Theor. Phys. **87**, 281 (1993).

[16] J. Shao, M-L. Ge and H. Cheng, Phys. Rev. E **53**, 1243 (1996).

[17] D. Mozyrsky and V. Privman, Journal of Stat. Phys. **91**, 787 (1998).

[18] G. Gangopadhyay, M. S. Kumar and S. Dattagupta, J. Phys. A: Math. Gen. **34**, 5485 (2001).

[19] S. Banerjee and R. Ghosh, J. Phys. A.:Math. Theor. **40**, 13735 (2007).

[20] V. B. Braginsky, Yu. I. Vorontsov and K. S. Thorne, Science **209**, 547 (1980).

[21] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, 2002).

[22] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, *et al.*, Nature **403**, 269 (2000).

[23] Q. A. Turchette, C. J. Myatt, B. E. King, C. A. Sackett, *et al.*, Phys. Rev. A **62**, 053807 (2000).

[24] O. Oreskov and J. Calsamiglia, Phys. Rev. Lett. **105**, 050503 (2010).

[25] J. E. Avron, M. Fraas, G. M. Graf, and P. Grech, *arXiv:1106.4661* (quant-ph).

[26] X. X. Yi, D. M. Tong, L. C. Kwek, and C. H. Oh, *arXiv:0606203* (quant-ph).

[27] S. Banerjee, unpublished notes.