Weak measurement can resurrect lost quantum correlation

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Abstract. The projective measurement destroys the quantum correlation of a composite quantum system and make it classical. On the other hand the weak measurement behaves gently to the quantum system and do not force the system state to decohere completely, thus revealing the super quantum discord, a measure of the quantum correlation based on the weak measurement. The SQD is a monotonically decreasing function of the measurement strength, which varies from mutual information to the normal quantum discord. This sheds some light on the long standing problem of classical-quantum boundary and tells that the classical-quantum boundary is a dynamical one and it depends on the observer who is interrogating the quantum system. Remarkably, we prove that the super quantum discord in the post measured state is equal to the lost quantum correlation which is defined as the difference between the super quantum discord and the normal quantum discord in the original state. Thus, the weak measurement has the ability to resurrect the lost quantumness of any composite quantum state. This suggests a conservation law for the extra quantum correlation in any composite state.

Keywords: Weak measurement, Super quantum discord, Extra quantum correlation

The quantum measurement has played many important roles since the inception of the quantum theory. The way the quantum measurement acts on quantum systems is fundamentally and radically different from the measurement, that we are familiar with, in classical mechanics. Typically the kind of quantum measurement discussed in quantum mechanics text books is either projective measurement or positive operator valued measurement (POVM). However, Aharonov, Albert and Vaidman introduced the notion of the weak measurement [1], which has found several fundamental and technological applications in the recent years. Later on, another formulation of the weak measurement was given by Oreshkov and Brun in [2].

It is widely believed that the quantum correlation is one of the prime reasons behind any efficient quantum information processing and computing task compared to their classical counterpart. The quantum discord, introduced by Ollivier and Zurek [3], is one of the well accepted and very successful notion of quantum correlation in the literature of quantum information science. It is nothing but the difference between the total and the classical correlation [4]. Consider a bipartite system in a state ρ_{AB} on which we perform a projective measurement on the subsystem B. The amount of inaccessible information or the quantum correlation is captured by the normal discord $D(\rho_{AB})$. The state of the composite system after the projective measurement is given by $\tilde{\rho}_{AB} = \sum_i p_i \rho_{A|i} \otimes |i\rangle\langle i|$. The effect of projective measurement is to remove the quantum correlation between two parties and make it classical. Specifically, if we calculate the normal quantum discord in the post measured state, then it is zero.

Weak measurements are proved to be gentle measurements which do not alter the state of the quantum system to a large extent. To see this, let us consider the action of the weak operators given by $P(\pm x) = a(\pm x)\Pi_0 + a(\mp x)\Pi_1$, where $a(\pm x) = \sqrt{\frac{(1\mp \tanh x)}{2}}$, on the subsystem B. We can show that after a weak measurement, the joint system AB is still in an entangled state. Consider a bipartite density matrix ρ_{AB}. The state of the composite system after the action of weak operators, denoted by ρ_{AP}, is given by

$$\rho_{AP} = \sum_{i=0,1} (I \otimes \Pi_i)\rho_{AB}(I \otimes \Pi_i) + \mathrm{sech}x\big[(I \otimes \Pi_0)\rho_{AB}(I \otimes \Pi_1) + \mathrm{h.c.}\big]. \quad (1)$$

We can readily say that the state of the composite system, after the weak measurement, is not a quantum-classical (qc) state, unless $x \to \infty$. Therefore, the weak measurements can be utilized to interrogate a composite quantum system in a flexible way which can simultaneously maintain the quantum correlation between the parts of the composite system. Recently, we have proved that the weak measurement can reveal more quantum correlation [5] in a composite state as it captures the otherwise destroyed quantum correlation. We have introduced the notion of super quantum discord (SQD) to felicitate the above. The SQD can be understood as the quantum correlation seen by an observer who performs weak measurement on one of the subsystems of the composite system. Mathematically, SQD is defined as [5]

$$D_w(A:B) := \min_{\{\Pi_i^B\}} S_w(A|\{P^B(x)\}) - S(A|B), \quad (2)$$

where $S_w(A|\{P^B(x)\}) = p(x)S(\rho_{A|P^B(x)}) + p(-x)S(\rho_{A|P^B(-x)})$ is the weak conditional entropy and $S(A|B) = S(\rho_{AB}) - S(\rho_B)$. One of the remarkable features of the SQD is that it is found to exceed the quantum entanglement for the pure maximally entangled bipartite state. In general, it is proved that given a bipartite state ρ_{AB}, the super quantum discord (SQD) revealed by the weak measurement is always greater than or equal to the normal quantum discord with

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the strong measurement, i.e., $D_w(A:B) \geq D(A:B)$. The proof of this result can be found in the reference [5]. Moreover, the SQD is a monotonically decreasing function of the measurement strength and ranges from mutual information to the normal quantum discord for the given quantum state of the composite system, i.e., $\forall$ $(x,y) \in [0,\infty]$ such that $x \geq y$, then $D_w(x) \leq D_w(y)$. The proof of this result can be found in the reference [5]. The SQD satisfies $I(\rho_{AB}) \geq D_w(\rho_{AB}) \geq D_s(\rho_{AB})$, where $I(\rho_{AB})$ is the mutual information and $D_s(\rho_{AB})$ is the normal quantum discord. Thus, the weak measurements reveal more the quantum correlation. Therefore, in a sense, by performing the projective measurement we have destroyed the extra quantum correlation. The extra quantum correlation captured by the weak measurement is defined as the difference between the SQD and the normal quantum discord in the bipartite state, i.e., $\Delta(\rho_{AB}) = D_w(\rho_{AB}) - D_s(\rho_{AB})$. In the strong measurement limit, the extra quantum correlation becomes zero as $\lim_{x\to\infty} D_w(\rho_{AB}) = D_s(\rho_{AB})$. The extra quantum correlation revealed by the SQD is possible only with weak measurements as they disturb the system slightly and thereby, maintaining the quantumness of the original state. This is in contrast with the strong measurement process, which destroys the extra quantum correlation irrevocably and converts it to the classical correlation. Therefore, the SQD shows that the quantum correlation is not only an observer dependent quantity but also depends on the strength of the quantum measurement. Our result shows that if we perform weak measurement to interrogate a composite quantum system, then there will be more quantum correlation between the subsystems which can be consumed for other practical purposes. Thus, the notion of the SQD can be a very robust resource for quantum information processing and computation tasks. It has been shown that the normal quantum discord is a limiting case of the SQD, which is obtained if we take strong measurement limit [5] of the SQD.

We know that the usual strong measurements destroy the quantum correlations between parts of a composite quantum system and make them classical. But as is discussed above the weak measurements do not destroy the quantum correlations completely, so it is natural to ask that what is the amount of the quantum correlation that is lost due to the projective measurement? The amount of lost quantumness due to projective measurement is equal to the extra quantum correlation captured by the SQD for a fixed measurement strength. Next, we ask can there be a procedure to resurrect this lost quantumness? The answer is yes. We have proved that the super quantum discord in the post measured state is equal to the difference between the super quantum discord and the normal quantum discord in the original state. Thus, the weak measurement has the ability to resurrect the lost quantumness of any composite quantum state [6]. We have provided the numerical examples of the Werner state and the pure entangled states to append our result. These examples show that the extra quantumness revealed by the SQD is indeed exactly equal to the SQD in the post measurement state.

The weak measurements have found several applications in recent years ranging from answering the foundational questions of quantum mechanics to the technological advantages. The notion of the super quantum discord is yet another tool based on weak measurements, which can capture the otherwise destroyed extra quantumness of a bipartite quantum state. The SQD may provide deep insights to the problem of quantum classical boundary. It says that the boundary between quantum and classical correlations is dynamical and it depends on the observer as well as the measurement strength. This also strengthens the arguement that the quantum mutual information can behave as if it is exclusively quantum [7, 8]. Since the quantum correlation is a very precious resource for many efficient quantum information and processing tasks, it will be useful if one can consume quantum correlations efficiently. The SQD can be used to interrogate the composite quantum system and at the same time maintaining quantum correlations in the composite state. Moreover, we have proved a surprising result which shows that even if the extra quantum correlation captured by the SQD is destroyed by the projective measurements, we can resurrect this lost quantumness using weak measurements. Specifically, the amount of the lost quantumness in a composite quantum system due to the normal strong measurement is equal to the SQD in the post-measured state. Our result shows that the extra quantum correlation in a state is a property of the system and the measurement strength, and it may be hinting us towards a conservation law for the quantum correlation for a given state. This can be stated as *the amount of extra quantum correlation which is destroyed by the projective measurement in the original state is equal to the amount of extra quantum correlation captured by the weak measurement in the post-measured state.* We believe that our results will provide new ways of thinking about quantum correlation and ways to harness it efficiently.

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