

# Entanglement in two site Bose-Hubbard model

K. V. S. Shiv Chaitanya<sup>1 \*</sup>

Sibasish Ghosh<sup>2 †</sup>

V. Srinivasan<sup>3 ‡</sup>

<sup>1</sup> BITS Pilani, Hyderabad Campus, Jawahar Nagar, Shameerpet Mandal, Hyderabad, India.

<sup>2</sup> Optics & Quantum Information Group, The Institute of Mathematical Sciences, C.I.T Campus, Taramani, Chennai, India, 600113.

<sup>3</sup> Department of Theoretical Physics, University of Madras, Guindy, Chennai, India, 600025.

**Abstract.** In this paper, the decoherence and entanglement properties for the two site Bose-Hubbard model in the presence of a non-linear damping is studied. The techniques of thermo field dynamics and Hartree-Fock approximation are applied to solve the corresponding master equation (ME). The expectation values of the approximated field operators appearing in the solution of ME, are computed self-consistently. We solve this master equation for a small time  $t$  so that we get the analytical solution, thereby we compute the decoherence and entanglement properties of the solution of the two-mode bosonic system.

**Keywords:** Entanglement, Decoherence, Thermo field dynamics and Master equation.

## 1 Introduction

In recent years, there is lot of interest generated in the study of entanglement properties of ultra cold atoms [1, 2, 3, 4, 5, 6]. In one such study, the single-site addressability in a two-dimensional optical lattice [7] has been demonstrated which could be a natural resource for applications of quantum information processing with neutral atoms. In all the experimental demonstrations of ultra cold atoms, loss is an important role which gives rise to decoherence and in turn destroying the quantum correlations. The losses due to decoherence can be modelled by a master equation. One such model is examined in the ref [5, 6] for a linear damping using the Bose-Hubbard model. The Bose-Hubbard model [8] is one of the popular models used to study the evolution of cold atoms and the Bose-Einstein condensates in optical lattice. In this paper, we examine the two-site Bose-Hubbard model to study the entanglement and decoherence properties of two-mode states under the action of non-linear damping. We consider the following master equation for density matrix  $\rho$  in a non-linear medium

$$\frac{\partial}{\partial t}\rho = \frac{i}{\hbar}[H, \rho] + \kappa \sum_{k=1}^K ([a_k b_k, \rho a_k^\dagger b_k^\dagger] + [a_k b_k \rho, a_k^\dagger b_k^\dagger]), \quad (1)$$

here  $\kappa$  is a damping coefficient,  $a_k$  and  $b_k$  bosonic annihilation operators referring to atoms in the internal states  $|N_1\rangle$  and  $|N_2\rangle$ , respectively, with one boson in the  $k$ th lattice site and  $K$  is the number of lattice sites and  $H$  is the Hamiltonian for the Bose-Hubbard model which describes the optical lattice. In this paper, we are studying the model in the presence of non-linear damping corresponding to the term associated with  $\kappa$ .

For solving this master equation we use the techniques of thermo field dynamics (TFD) [9, 10] and thereby the Hartree-Fock approximation to convert the two-site Bose-Hubbard model into a two-mode bosonic system (see ref [11] for details). The two-site Bose-Hubbard model is

used to study Josephson tunneling between two Bose-Einstein condensates. The expectation values of the approximated field is computed self-consistently. We solve the corresponding master equation for a small time  $t$  so that we get the analytical solution, thereby we compute the decoherence and entanglement properties of the two-mode bosonic system.

In TFD, any master equation reduces to,

$$\frac{\partial}{\partial t}|\rho(t)\rangle = -i\hat{H}|\rho\rangle, \quad (2)$$

where  $|\rho\rangle$  is a vector in the extended Hilbert space  $\mathcal{H} \otimes \mathcal{H}^*$  and  $-i\hat{H} = i(H - \tilde{H}) + L$ , here  $L$  is Liouville term,  $H$  and  $\tilde{H}$  corresponds to non tildian and tildian Hamiltonians respectively, and  $H^*$  is the added Hilbert space (for further details, see ref [11]).

## 2 Two site Bose-Hubbard Model

To study the decoherence and the entanglement properties of Bose-Hubbard model, for simplicity, we consider a toy model, in which the Bose-Hubbard model is written for the two site interaction only. Then the master equation for the two site Bose-Hubbard model is obtained by taking  $K = 1$  in the equation (1)

$$\begin{aligned} \frac{\partial}{\partial t}\rho &= -i\omega(a^\dagger a \rho - \rho a^\dagger a) - iJ(a^\dagger b \rho - \rho a^\dagger b) \\ &\quad - iJ(b^\dagger a \rho - \rho b^\dagger a) + i\frac{U_a}{2}(a^\dagger a^\dagger a a \rho - \rho a^\dagger a^\dagger a a) \\ &\quad - i\omega(b^\dagger b \rho - \rho b b^\dagger) + i\frac{U_b}{2}(b^\dagger b^\dagger b b \rho - \rho b^\dagger b^\dagger b b) \\ &\quad + i\frac{U_{ab}}{2}(a^\dagger b^\dagger a b \rho - \rho a^\dagger b^\dagger a b) \\ &\quad + \frac{\kappa}{2}(2ab\rho a^\dagger b^\dagger - a^\dagger b^\dagger a b \rho - \rho a^\dagger b^\dagger a b). \end{aligned} \quad (3)$$

The interaction term  $J$  in the Hamiltonian describes the induced hopping between adjacent cells.  $\omega$  is the frequency of the atom in the lattice. The on-site interactions of atoms are described by  $U_a$  and  $U_b$ , and a nearest-neighbour interaction by  $U_{ab}$ . For further details see ref [8].

\*username1@domainname1

†username2@domainname2

‡username3@domainname3

At first we consider the special case to solve this master equation with  $J = U_a = U_b = 0$  and  $U_{ab} = U$ , which corresponds to the Mott insulating phase. We apply the thermo field dynamics techniques to convert the master equation (3) into a Schrödinger equation (2) by applying  $|I\rangle$  from the right to the eq (3), (The state vector  $|I\rangle$  takes a normalized vector to an another normalized vector in the extended Hilbert space  $\mathcal{H} \otimes \mathcal{H}^*$  for detail ref [11]) and using Hartree Fock approximation we get the solution to be

$$|\rho(t)\rangle = (\exp[-i \int dt H_1] \otimes \exp[-i \int dt H_2]) |\rho(0)\rangle, \quad (4)$$

where  $|\rho(0)\rangle$  is an initial state in  $\mathcal{H} \otimes \mathcal{H}^*$ . Here

$$H_1 = \omega(a^\dagger a + b^\dagger b) + \frac{i\kappa\Delta(t)}{2} (ab - a^\dagger b^\dagger) - \frac{U\Delta(t)}{2} (a^\dagger b^\dagger + ab) \quad (5)$$

and similar  $H_2$  is with the tildian terms. By exploiting the  $su(1, 1)$  symmetry of the Hamiltonian and calculating the  $\Delta(t)$  ( is given in eq (32) ref [11]) self consistently one gets the solution to be, for details refer to [11].

### 3 Entanglement

The solution of the master equation (3) in the system Hilbert space  $\mathcal{H}$  is given by :

$$\rho(t) = (\exp[\Gamma_{a+}\mathcal{K}_+] \exp[\ln(\Gamma_{a3}\mathcal{N}) \exp[\Gamma_{a-}\mathcal{K}_-] \rho(0) \times (\exp[\Gamma_{b+}\mathcal{K}_+] \exp[\ln(\Gamma_{b3}\mathcal{N}) \exp[\Gamma_{b-}\mathcal{K}_-]), (6)$$

where  $\rho(0)$  is taken to be the initial state,  $\mathcal{N} = a^\dagger a + b^\dagger b$ ,  $\mathcal{K}_+ = a^\dagger b^\dagger$ ,  $\mathcal{K}_- = ab$  and  $\Gamma_{i\pm} = -\frac{\Delta(0)\zeta t}{2} (1 + \frac{\omega^2 t^2}{4}) e^{\pm i\phi}$ , here  $i$  stands for  $a$  and  $b$ . One can clearly see that this a two-mode squeezed state. It is well known that two-mode squeezing gives rises to entanglement [12]. By taking the initial state  $\rho(0)$  to be the two mode thermal state, the amount of entanglement in  $\rho(t)$  is given in terms of logarithmic negativity

$$E_N(r) = -\frac{1}{2} [\text{Log}(e^{-4r}/n)], \quad (7)$$

where  $r$  is the squeezing parameter and  $n = n_1 = n_2$  is are the symplectic eigenvalues of covariance matrix of two-mode thermal state (for details ref to [11]).

To calculate decoherence effects of  $\rho(t)$  we compute  $\{\rho(t)\}^2 = \rho^2(t)$  and is given by eq (56) in ref [11]. One can see immediately that for the short time it self as the value of damping coefficient increases the system decoheres faster (see figures 1 and 2 in ref [11] for further details).

### 4 Conclusion

We show that the entanglement for two site Bose-Hubbard model for a short time increases when the initial state is a two-mode thermal state. We interpret this behaviour due to the existence of the non-linear medium.

To get the exact picture for the long time behaviour of the entanglement, one has to do the numerical studies. It can be seen from the decoherence plot, (see figures 1 and 2 in ref [11]), that as the value of the damping coefficient increases the damping in the system is faster as expected. We expect that the further numerical studies using this model will give better results and these results may be applied to condensed matter systems.

### References

- [1] I. Bloch, *Quantum coherence and entanglement with ultracold atoms in optical lattices. Nature* **453**, 1016, (2008).
- [2] L. O. Castanos and R. Jauregui *Entanglement properties of an ultracold atom interacting with a cavity quantized electromagnetic field. Phys. Rev. A* **82**, 053815, (2010).
- [3] R. V. Mishmash and L. D. Carr, *Quantum Entangled Dark Solitons Formed by Ultracold Atoms in Optical Lattices. Phys. Rev. Lett.* **103**, 140403 (2009)
- [4] F Benatti, R Floreanini and U Marzolino, *Entanglement and squeezing with identical particles: ultracold atom quantum metrology. J. Phys. B: At. Mol. Opt. Phys.* **44**, 091001, (2011).
- [5] K. Pawłowski and K. Rzazewski, *Background atoms and decoherence in optical lattices. Phys. Rev. A* **81**, 013620, (2010)
- [6] K. Pawłowski, P. Zin, K. Rzazewski, M. Trippenbach, *Revivals in an attractive Bose-Einstein condensate in a double-well potential and their decoherence. Phys. Rev. A* **83**, 033606, (2011)
- [7] P. Wurtz, T. Langen, T. Gericke, A. Koglbauer, and H. Ott *Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice. Phys. Rev. Lett* **103**, 080404, (2009).
- [8] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller *Cold Bosonic Atoms in Optical Lattices. Phys. Rev. Lett.* **81**, 3108, (1998).
- [9] S. Chaturvedi and V. Srinivasan, *Solution of the Master Equation for an Attenuated or Amplified Nonlinear Oscillator with an Arbitrary Initial Condition. J. Mod. Opt.* **38**, 777, (1991).
- [10] S. Chaturvedi and V. Srinivasan, *Class of exactly solvable master equations describing coupled nonlinear oscillators. Phy Rev A* **43**, 4054, (1991).
- [11] K. V. S. Shiv Chaitanya, Sibasish Ghosh and V. Srinivasan *Entanglement in two site Bose-Hubbard model. arXiv : 1302.5238 (quant-ph) and references therein.*
- [12] R. Simon, *Peres-Horodecki Separability Criterion for Continuous Variable Systems. Phy Rev Lett*, **84**, 2726, (2000).