Purification and retrieval of entanglement via single local filtering

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Abstract. The effect of filtering operation with respect to purification and concentration of entanglement in quantum states are discussed here. It is shown, through examples, that the local action of the filtering operator on a part of the composite quantum state allows for purification of its remaining part. The retrieval of entanglement in the subsystems of a noise affected state is shown to be due to the filtering action on the non-decohering part of the system. The varying effects of the filtering parameter, on the entanglement transfer between the subsystems, depending on the choice of the initial state is illustrated.

Keywords: Local Filtering, Purification, Entanglement Sudden death

1 Introduction

The concept of quantum entanglement is very useful in several areas like quantum teleportation, quantum cryptography, quantum dense coding and quantum computation [1] but its fragility to environmental interaction is a cause of great concern for all its technological applications [2]. It is well known that when a pure entangled state interacts with the environment, in addition to reduction in its entanglement, it loses its coherence becoming a mixed state [2]. Purification and concentration of entanglement has thus been an issue of great importance. Several protocols including collective measurements, Local Quantum Operation with Classical Communications (LQCC) and local filtering operations are proposed to perform this task [3]–[15]. The effect of *single* local filtering operation with respect to purification and concentration of entanglement in quantum states are discussed in this work.

2 Single local filtering on 3-qubit pure states.

To begin with, we have analyzed the effect of filtering on a single qubit of the 3-qubit pure states belonging two inequivalent SLOCC families. Filtering is a non-trace-preserving map that is seen to be capable of increasing entanglement with some probability and it represents a dichroic environment the extreme examples of which are polarizers [10]. In the computational basis it is given by,

$$F = \sqrt{(1-k)}|0\rangle\langle 0| + \sqrt{k}|1\rangle\langle 1|;\ 0 \le k \le 1 \qquad (1)$$

On subjecting the 1st qubit of 3-qubit W state ($|W\rangle$) and superposition of W, obverse W states ($|W\bar{W}\rangle$), we have shown that redistribution of entanglement takes place between the subsystems $\rho_{12}(=\rho_{13})$ and ρ_{23}. In both the states, we have illustrated the purification of the subsystem ρ_{23} not acted upon by the filtering operator. Inspite of both 3-qubit GHZ and $|W\bar{W}\rangle$ states belonging to the same SLOCC class [16], the subsystems of GHZ state are unaffected by single local filtering whereas the entanglement of the subsystem ρ_{23} of $|W\bar{W}\rangle$ increases at the expense of entanglement in the filtering affected part $\rho_{12}(=\rho_{13})$. Figs. 1 and 2 capture the dependence of purity and concurence of the subsystems on the filtering parameter.

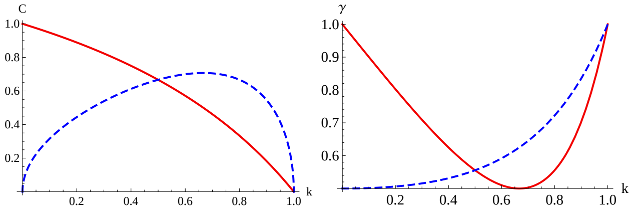

Figure 1: The variation of concurrence(C) and purity(γ) of the subsystems $\rho_{12}(=\rho_{13})$ (dashed line) and ρ_{23} (solid line) of the filtering affected W state with respect to filtering parameter k. The redistribution of entanglement and purity among the subsystems is readily seen.

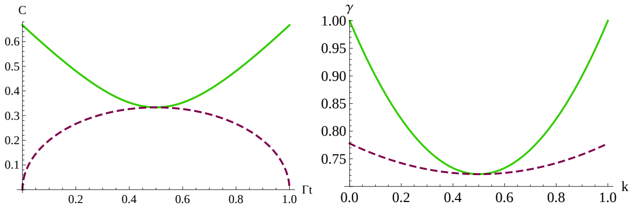

Figure 2: The redistribution of entanglement (C) and purity (γ) between the subsystems $\rho_{12}(=\rho_{13})$ (dashed line), ρ_{23} (solid line) in the filtering affected $|W\bar{W}\rangle$ state as a function of the filtering parameter k;

3 The effect of single local filtering on noise affected states.

In this section, we have demonstrated the probabilistic retrieval of entanglement due to single local filtering on the non-decohering 1st qubit of both $|W\rangle$, $|W\bar{W}\rangle$ undergoing Entanglement Sudden Death (ESD) [17, 18] by the depolarizing noise acting on their remaining two qubits. For all values of the filtering parameter k, delay in the

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onset of ESD and increase in the entanglement of ρ_{23} is seen for $|W\bar{W}\rangle$ but in the case of $|W\rangle$, such a phenomenon happens only in the range $0 \leq k \leq 0.5$. For $0.5 < k < 1$, an early onset of ESD and decrease in entanglement of ρ_{23} which is not directly affected by filtering is observed in $|W\rangle$. This knowledge of the filtering-parameter dependence of the delaying of ESD (or otherwise) in the subsystems of a composite quantum state will help in choosing the right filter for the task under consideration. We have given a graphical illustration of the above mentioned results in Figs. 3 and 4.

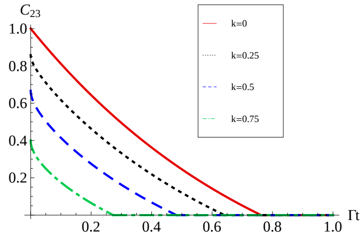

Figure 3: The variation of concurrence of the subsystem ρ_{23} of the noise affected W state after local filtering on its 1st qubit, with the dimensionless time parameter Γt as a function of k. Notice that when $k = 0.5$, F corresponds to identity implying that the state is unaffected by filtering.

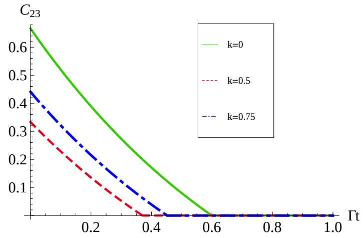

Figure 4: Temporal variation of concurrences of the subsystems ρ_{23} of the noise-affected state $|W\bar{W}\rangle$ after local filtering on its 1st qubit, with respect to k. For all values of k, gain in the entanglement of ρ_{23} and delay in the onset of ESD (compared to that without filtering) is observed.

While Ref. [15] deals with the effect of a local filter on the probabilistic retrieval of entanglement after ESD due to Generalized Amplitude Dampling (GAD) [1, 15], the authors have illustrated this result choosing only a particular filtering parameter. Our choice of the depolarizing noise is in view of the observation that it is more effective in causing ESD in quantum states [19]. In [20], we have given a detailed analytical and graphical illustrations of the results mentioned here.

4 Concluding Remarks

We have shown here that it is possible to increase the subsystem entanglement of pure quantum states by single local filtering on a part of the system. An increase in the purity of one subsystem owing to the decrease in the other is also shown to be caused due to local filtering. The results are seen to be extendible to multiqubit states with $N > 3$. We hope that this work will shed more light on the significance of local filtering action on quantum states. It is an open question to study the filtering action upon higher dimensional states and utilize the enhanced entanglement coherences and purity to the desired applications.

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