Macroscopic Schrödinger cat resistant to particle loss and local decoherence

Utkarsh Mishra * R. Prabhu † Aditi Sen(De) ‡ Ujjwal Sen §

[1] *Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India*

Abstract. The concept of a quantum superposition in states of a large number of particles form an important aspect of quantum physics with significance on the fundamental front as well as in useful applications. There is an ongoing effort towards experimental realization of such states in a variety of physical substrates. We have consider a large multiparty quantum state, whose macroscopic part is modelled by a space spanned by two quantum states, of a large number of particles, that drastically differ by their amounts of violation of Bell inequalities. We show that this state is robust against loss of a finite fraction of its particles and simultaneously against local depolarizing channels, modelling a local decoherence mechanism. We compare our results with other potential multiparticle states, including the Greenberger-Horne-Zeilinger state.

Keywords: Macroscopic quantum superposition, particle loss, local decoherence, entanglement.

The recent developments in computation and communication tasks have underlined the necessity to preserve quantum coherence in states shared by a large number of quantum systems [1]. Feynman proposed that complex and large quantum systems can be efficiently simulated only by using a quantum computer [2]. Shors algorithm demonstrated that quantum algorithms can be used to efficiently solve problems that may not be possible with classical ones [3]. To build a viable quantum computer that can compile and implement a quantum algorithm, which outperforms the ones running on classical machines, requires quantum coherence preserved in a system of about 1000 qubits [4]. Coherence in quantum states of a large number of particles is one of the essential ingredients for building a quantum communication network [5]. Such exciting developments on the theoretical front were accompanied by several experimental proposals and realizations, by using, e.g., photons, ion traps, cold atoms, and nuclear magnetic resonance.

A macroscopic entangled state was first introduced by Schrödinger in his seminal 1935 paper [6] through the concept of the Schrödinger cat, which is an entangled state between a microscopic system and a macroscopic one. The microscopic system can be an atom, which can decay spontaneously, with the undecayed state $|\text{up}\rangle$ and the decayed state $|\text{down}\rangle$ making up a two-dimensional complex Hilbert space (qubit). The macroscopic system was also conceived as a qubit made up of the alive and dead states of a cat, respectively denoted as $|\text{alive}\rangle$ and $|\text{dead}\rangle$. The quantum state of the combined micro-macro system was considered to be $\frac{1}{\sqrt{2}}(|\text{up}\rangle|\text{alive}\rangle + |\text{down}\rangle|\text{dead}\rangle)$. While such micro-macro superposition have since been found to be significant in technological pursuits, they are also fundamentally important, e.g., for understanding the quantum measurement problem and the quantum-to-classical transition [7, 8].

It is of vital importance to investigate the effects of environmental noise on quantum superpositions of large systems, for understanding their fundamental as well as technological implications. Usually, the environmental effect that is considered for such a state is decoherence. Here we consider its coherence properties after it has been subjected, *simultaneously* as well as separately, to local decoherence channels, in the form of local depolarizing channels, on all its constituent particles (in the micro as well as the macro sectors) and to loss of a finite fraction of its particles (in the macro part).

The classic example of such micro-macro superposition is the Greenberger-Horne-Zeilinger (GHZ) state [10]. The two macroscopic sectors of GHZ state is different in terms of their average magnetization. Noise effects on the GHZ state have been studied by using a variety of models [9]. However, as is well-known, the GHZ state loses *all* quantum coherence if even a single qubit is lost.

Here we have consider micro-macro superposition state called H_C state [11]. The macroscopic sectors of the H_C state are macroscopically distinguished by their amount of Bell inequality violation. We show that the H_C state is robust, i.e. can preserve quantum coherence in the form of quantum correlations between its micro and macro sectors, against loss of a finite fraction of its particles and against local depolarizations on all its particles, and with the simultaneous action of both these noise effects. In particular, we find that for a finite number of particles in the macroscopic part, the H_C state is more robust to local depolarizing noise than the GHZ state.

To quantify the amount of quantum correlation that a cat state can sustain, after the effect of local noise, in its micro-macro bipartition, we have calculated logarithmic negativity and quantum discord. The quantum discord is calculated numerically.

In the case of particle loss, the entanglement varies as $\log_2\left(2 - \frac{m}{N}\right)$, where m is the number of particles that are traced out from macroscopic sector. If m and N are such that the ratio $\frac{m}{N}$ is a finite constant, then the entanglement between the microscopic and macroscopic parts will be less than unity, but can still be substantially higher than zero. We have also derived the quantum discord in the same situation, and found that the qualitative behav-

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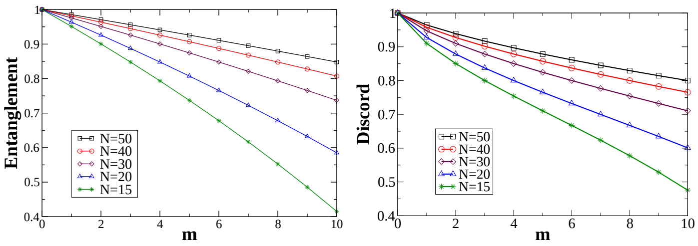
[*] `utkarsh@hri.res.in`

[†] `prabhurama@hri.res.in`

[‡] `aditi@hri.res.in`

[§] `ujjwal@hri.res.in`



(a) (b)

Figure 1: (Figs. (a)). Entanglement after particle loss in the H_C state. The horizontal axis represents the number of particles lost (m) from the macroscopic part of the H_C state while the vertical one represents the entanglement between the micro and the macro parts of the H_C state after particle loss. The entanglement with respect to m is plotted for different initial number of particles, N. The vertical axis is measured in ebits, while the horizontal one is in particles. (Figs. (b)). The same is for quantum discord. The vertical axis is measured here in bits.

ior is the same, except for slight changes in the curvature properties. In particular for $N = 50$, while about 85 % of the entanglement (Figs. 1(a)) is retained after 10 particles are lost, it is about 80 % for quantum discord (see Figs. 1(b)).

We now consider the situation where the H_C state is affected by local decoherence as well as by particle loss. We assume that m particles are lost (from the macro part) and that the remaining $N - m + 1$ particles are all affected by local decoherence as modelled by the depolarizing channel. The entanglement in the micro-macro bipartition is analyzed for the resulting $(N - m + 1)$-party state. The entanglements are plotted in Figs. 2(b) for the case when macroscopic part consists of 10 particles. The entanglement of the locally decohered H_C and GHZ state is compared in the micro-macro bipartition in Figs. 2(a) . Interestingly, we obtain that the H_C state is more resistant to local decoherence than the GHZ state, and for example, for 10 particles in the macroscopic part, the H_C state can preserve entanglement up to 44% of local decohering noise, while the GHZ state remain entangled until 28% of the same noise (see Figs. 2(a)). Quantum discord has a qualitatively similar behavior, and e.g. for $N = 7$, the quantum discord in the micro-macro partition for the GHZ state becomes $< 5 \times 10^{-4}$ for $p \approx 0.44$, while the same happens for the H_C state for $p \approx 0.85$.

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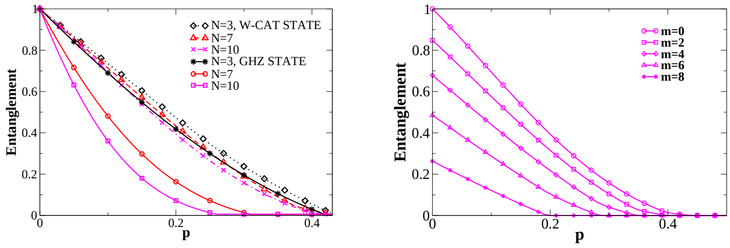

(a) (b)

Figure 2: (Figs. (a)). Entanglement of GHZ and H_C states against local decoherence. While the discontinuous lines are for the GHZ states, the continuous ones are for the H_C. The horizontal axis represents the dimensionless (decohering noise) parameter p, and the vertical axis is the entanglement in the micro-macro bipartition (in ebits). (Figs. (b)). Effect of local decoherence and particle loss on H_C state. Entanglement in the micro-macro bipartition is plotted on the vertical axis against a base of the dimensionless depolarizing parameter p, and the number of lost particles (m). The H_C state under consideration is of 11 qubits, so that $N = 10$.

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