Broadcasting of correlation and its use in quantum information processing

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Abstract. We apply a state independent local quantum cloner on a $2 \otimes 2$ dimensional system to create more entangled pairs from a given entangled pair. We then analyze the dependence of maximal teleportation fidelity and superdense coding capacity on the broadcasting fidelity for these entangled pairs. Since the amount of quantum correlation plays a vital role in the efficiency of teleportation and superdense coding schemes, so we further study the dependence of these correlations, on broadcasting fidelities. Our motivation is to study how the broadcasting of quantum correlation can be valuably exploited to efficiently perform quantum information processing tasks.

Keywords: Broadcasting, Cloning, Correlation, Discord, Entanglement, Fidelity, Superdense coding, Teleportation

1 Introduction

It often happens in quantum information processing that we are given an entangled pair and asked to create more entangled pairs by using local quantum operations. This need arises from the fact that sometimes there is more priority on increasing number of participants in information processing scheme rather than on the efficiency of it. The two very essential and smart ways of information processing in quantum world are teleportation and super dense coding schemes. Thus the extension of quantum correlations to more pairs, given one, can be of great utility and consequently to broadcast correlation. Fortunately, the method to efficiently disperse entanglement from between a pair to two, has already been successfully addressed earlier by Adhikari et.al. (2006) [4] depending on the method of the use of local quantum cloner. The efficiency of teleportation between two subsystems is portrayed by the measure of maximal fidelity of teleportation [3] and that of dense coding is portrayed by the measure of capacity of super dense coding [5]. Given this knowledge, our motivation is to know that how to effectively utilize the output entangled pairs as a resource for efficiently carrying out the quantum information processing tasks after broadcasting. Since, the extent of quantum correlation between the participating subsystems play a pivotal role in the efficiency of information processing schemes, so we also study the dependence of the quantum correlations, namely entanglement [6] and discord [1] of the input entangled pair and the output entangled pairs on the broadcasting fidelity.

2 Our Approach

Firstly we take the case of a non-maximally entangled state of a system made of two subsystems, namely 1 and 2, and study the maximal teleportation fidelity and super dense coding capacity of that state. We also calculate the entanglement measure and discord measure for analyzing the magnitude of quantum correlation present between the participating subsystems. Next we apply local state independent Buzek-Hillary cloning transformation to generate local cloned copies, namely 3 and 4, from 1 and 2 respectively. By this procedure we intend to broadcast the quantum correlation, namely entanglement, as was addressed by Adhikari et.al. (2006) [4]. This is done to create two entangled pairs from one. Following their methodology next we use the Peres-Horodecki criterion of inseparability [4] to check for which values of the input state parameter they are separable. This provides us the insight to choose states which will have more correlation between the nonlocal copies, say (1 and 4 or 2 and 3) and decrease the same between the local ones (1 and 3 or 2 and 4). Lastly we analytically calculate the broadcasting fidelity and study the dependence of the quantum information processing schemes on it. Further we also study the dependence on entanglement another measure of correlation namely discord [1] between the participating subsystems on broadcasting fidelity. We study the discord measure since at times it gives us a measure of quantum correlation beyond entanglement. We also similarly analyze the results starting with the Werner state to generalize our scheme in case of mixed states as well. Due to limitations of space, the sections below illustrate the main results of our study in the case of the non-maximally entangled state only. The detailed results in both the cases with plots and analytical expressions will be provided in the poster during presentation.

3 Study on Non-maximally entangled state

The Schimdt decomposition [2] form of the state is given by,

$$|\psi\rangle_{12}^{in} = \alpha\,|00\rangle + \beta\,|11\rangle \qquad (1)$$

where α , β are the probability amplitudes and the normalization condition gives $\alpha^2 + \beta^2 = 1$

The range of broadcasting obtained after applying the cloning transformation and Peres-Horodecki theorem [4] is $\frac{1}{\sqrt{2}} - \frac{\sqrt{39}}{16} \leq \alpha^2 \leq \frac{1}{\sqrt{2}} + \frac{\sqrt{39}}{16}$ and the corresponding expression of broadcasting fidelity (FB) is,

$$FB(\alpha^2) = \frac{25}{36} - \frac{4}{9}\alpha^2(1-\alpha^2) \qquad (2)$$

Thus FB varies from $\frac{7}{12}$ to $\frac{125}{192}$.

Dependence of maximal teleportation fidelity on broadcasting fidelity: The dependence of maximal fidelity of

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teleportation [3] of the participating subsystems on broadcasting fidelity [2, 4] before and after applying the cloning transformation is given below,

$$F^{in}_{max}(FB) = \frac{1}{6}\left(4 + \sqrt{25 - 36 * FB}\right) \tag{3}$$

$$F^{c}_{max}(FB) = \frac{1}{54}\left(31 + 4\sqrt{25 - 36 * FB}\right) \tag{4}$$

where F^{in}_{max} and F^{c}_{max} are the maximal teleportation fidelities for the initial and cloned states respectively.

Dependence of superdense coding capacity on broadcasting fidelity: The dependence of superdense coding capacity [3] of the participating subsystems on broadcasting fidelity [2, 4] before and after applying the cloning transformation is given below,

$$\begin{aligned} C_{12}(FB) = &-\frac{u\,ArcCoth\left[\frac{2}{u}\right] - log_e(64)}{log_e(4)} \\ &-\frac{log_e(2-u) + log_e(2+u)}{log_e(4)} \end{aligned} \tag{5}$$

$$\begin{aligned} C_{14}(FB) = &\frac{1}{36log_e(2)}\left\{-12(-2+u)\right\} \\ &* \left\{ArcTanh\left(\sqrt{-\frac{7}{3} + 4FB}\right)\right\} \\ &+\frac{1}{36log_e(2)}\left\{4v * ArcTanh\left(\frac{2}{13}v\right)\right\} \\ &+\frac{1}{36log_e(2)}\left\{-36log_e(3) + 10log_e(5)\right\} \\ &-\frac{1}{36log_e(2)}\left\{6log_e\left(3-u\right)\right\} \\ &-\frac{1}{36log_e(2)}\left\{30log_e\left(3+u\right)\right\} \\ &+\frac{13}{36log_e(2)}\left\{(log_2(13-2v) + log_2(13+2v))\right\} \end{aligned} \tag{6}$$

where C_{12} and C_{14} are the superdense coding capacities for the initial and cloned states respectively and $u = \sqrt{(-21 + 36 * FB)}$ and $v = \sqrt{(180 * FB - 89)}$.

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Dependence of entanglement on broadcasting fidelity: The entanglement between the two participating subsystems is quantified by the method of concurrence [6]. We calculate the difference between the concurrence of the initial and cloned systems and denote it as $\triangle E$. (Refer: Figure 1)

Here FB varies from $\frac{7}{12}$ to $\frac{125}{192}$.

Dependence of discord on broadcasting fidelity: Here the discord between the two participating subsystems has been quantified by the measurement-based approach [1]. We calculate the difference between the discord of the initial and cloned systems and denote it as $\triangle D$. (Refer: Figure 2)

Here FB varies from $\frac{7}{12}$ to $\frac{125}{192}$.

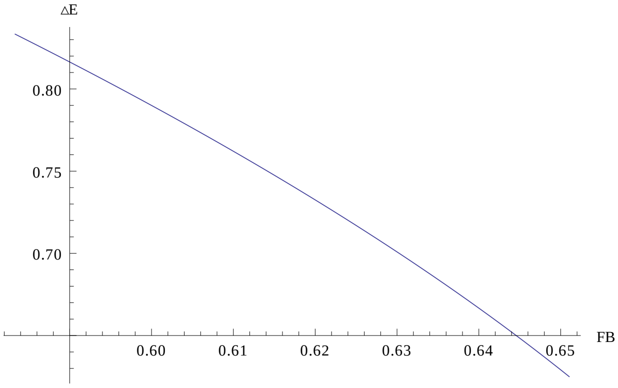

Figure 1: Plot showing the dependence of $\triangle E$ on FB

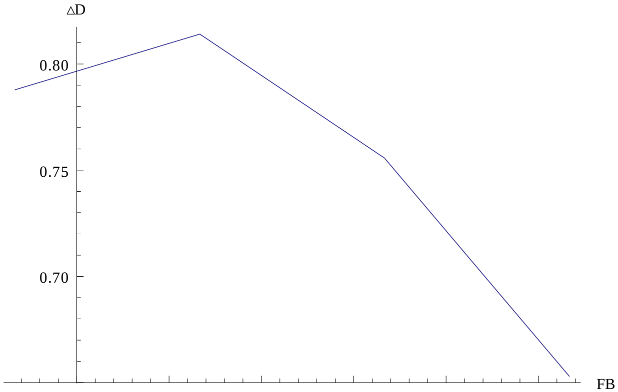

Figure 2: Plot showing the dependence of $\triangle D$ on FB

4 Conclusion

The analytical expressions and plots obtained provides insights and relations to show how the broadcasting of quantum correlation can be valuably exploited to efficiently perform quantum information processing tasks.

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