# Tomographic reconstruction of generic three-qubit states on an NMR quantum computer using only two qubit detectors

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**Abstract.** NMR quantum computers are a useful testbed for probing multi-particle quantum entanglement. It has been recently theorized that there exist a set of three qubit generic pure states which can be determined completely from their two party reduced density matrices. On a three-qubit NMR quantum computer, we experimentally generate such a three qubit state as well as the maximally entangled GHZ and W-states. We produce a series of tomographs of the final density matrix including that of the three-qubit generic state and their two qubit subspaces.

**Keywords:** NMR, Tomography, Generic state, GHZ state, W-state, Reduced density matrix

## 1 Introduction

Entanglement is a very important non-local property of quantum systems. $N$-qubit entangled systems can have various types of reducible multi-party entanglement as well as $N$-party irreducible entanglement. For systems composed of more than two qubits, classification of entangled states and quantification of entanglement are not yet fully understood. Entanglement in two qubit systems is well understood. The Bell state is the only type of irreducible entangled state in two qubits. Any Bell state can be converted to any other Bell state under local unitaries.

Three qubit states can be classified as separable, bi-separable and inseparable. Separable states are completely unentangled states and can be written as direct products of individual qubit states while bi-separable states contain entanglement only between two of the qubits, while the third qubit can be written as a tensor product with the other two entangled qubits. Inseparable states are the three-qubit correlated states wherein all the three qubits are entangled with each other. There exist two inequivalent classes of three-qubit entangled states: GHZ class of states and W class of states. These two classes of states are not inter-convertible under local unitary operations. Maximally entangled GHZ and W states are given as:

$$
\begin{aligned}
|\psi_{GHZ}\rangle &= \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\
|\psi_W\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)
\end{aligned}
$$

A non-trivial combination of these two inequivalent classes of entangled states provides a canonical form of three-qubit states

$$
\begin{aligned}
|\psi_G\rangle &= a_1 e^{\iota\phi}|000\rangle + a_2|001\rangle + a_3|010\rangle + a_4|100\rangle \\
&\quad + a_5|111\rangle, \\
\sum_i a_i^2 &= 1
\end{aligned}
$$

$a_i^{'}s$ being positive real [1]. This state is symmetric under permutations of the qubits and we will henceforth refer to it as the generic three qubit state ($|\psi_G\rangle$). The five independent non-zero components (one phase and four moduli) are the minimal number of non-local parameters required to completely specify the state. All the three-qubit pure states are equivalent to states of the form $|\psi_G\rangle$ under local unitary transformations.

It has been recently theorized that in almost all the three-qubit pure states, there does not exist any irreducible three-party correlation [4, 5]. Pure states of three qubits can be uniquely determined by their two-party reduced states with a few exceptions such as the generalized GHZ states:

$$|GHZ(\theta,\phi)\rangle = \cos\theta\,|000\rangle + e^{\iota\phi}\sin\theta\,|111\rangle, \quad \theta \in (0,\pi/2)$$

In such cases the construction of the three-qubit state from its two-party reduced states is not unique as $e^{\iota\phi}$ becomes pairwise inaccessible. In such cases three-qubit correlation is irreducible. In almost all the cases of generic three qubit pure states ($\rho_{ABC} = |\psi_G\rangle\langle\psi_G|$) the three two-party reduced states $\rho_{AB}$, $\rho_{BC}$, $\rho_{AC}$ completely determine the three-party state [2].

## 2 Results and Discussion

In this work we experimentally construct the three-qubit generic pure states on an NMR quantum computer. We propose a general experimental scheme to create various types of such states as well as GHZ and W-states. We produce a series of tomographs [3] of the final density matrix including that of the three-qubit generic state and their two-qubit subspaces. The most general NMR pulse

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sequence to create such states, starting from thermal state as initial state is given in Figure(1). The states thus prepared are of the form, $\psi_G = \cos\frac{\alpha}{2}|000\rangle + \sin\frac{\alpha}{2}\cos\beta\cos\gamma|100\rangle + \sin\frac{\alpha}{2}\cos\beta\sin\gamma|001\rangle + \sin\frac{\alpha}{2}\sin\beta\cos\gamma|010\rangle + \sin\frac{\alpha}{2}\sin\beta\sin\gamma|111\rangle$.

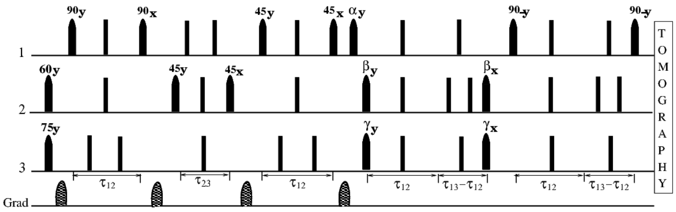


Figure 1: The three qubits are labeled as '1', '2' and '3', 'Grad' refers to the gradient channel. '$\tau_{12}$' corresponds to the '$\frac{1}{2J_{12}}$' evolution and '$\tau_{23}$' is the '$\frac{1}{2J_{23}}$' evolution period. Angles and phases are marked over various shaped pulses. In case of parallel evolutions, '$\tau_{12} - \tau_{13}$' stand for '$\frac{1}{2J_{12}} - \frac{1}{2J_{13}}$' evolution. The filled black rectangles are the $180^0$ pulses refocusing various unwanted evolutions.

Figure (2) contains the tomographically reconstructed three-qubit generic state with $\alpha = 90^0, \beta = 45^0, \gamma = 45^0$, obtained with a fidelity of 0.94.

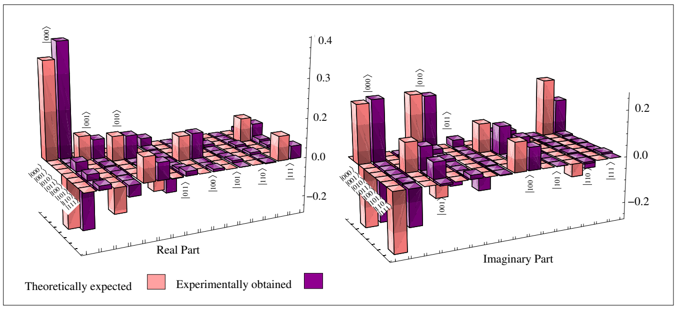


Figure 2: From left: Real and Imaginary parts of the experimentally constructed three qubit generic pure state with $\alpha = 90^0$, $\beta = 45^0$ and $\gamma = 45^0$.

Various three-qubit generic states and GHZ class were prepared by choosing different values of the parameters '$\alpha$, $\beta$, $\gamma$'. W-state prepration needs a slightly different sequence of pulses and was created with a fidelity of 0.97. The states thus prepared were tomographed completely as well as partially using two-qubit detectors. The Table shows the states prepared. The degree of control is reflected in the fidelity achieved in preparing various states.

Table 1: Parameters for experimentally constructed three qubit states and the respective fidelities achieved.

| S.No. | State | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $\alpha$ | $\beta$ | $\gamma$ | Fidelity |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) | GHZ | 0.707 | 0 | 0 | 0 | 0.707 | 90 | 90 | 90 | 0.98 |
| (2) | Generic | 0.707 | 0.354 | 0.354 | 0.354 | 0.354 | 90 | 45 | 45 | 0.94 |
| (3) | Generic | 0.707 | 0.306 | 0.306 | 0.177 | 0.530 | 90 | 60 | 60 | 0.89 |

Three qubit density operator $\rho_{ABC}$ is obtained by complete three qubit quantum state tomography. Also two qubit sub-spaces of the three qubit system are tomographed to reconstruct the two party states : $\rho^e_{AB}$, $\rho^e_{BC}$ and $\rho^e_{AC}$. It is found that the reduced states found experimentally lie close to the respective two party reduced states of $\rho_{ABC}$. Degree of overlap between the experimentally constructed three qubit states $\rho_{ABC}$ and the three qubit state obtained from $\rho^e_{AB}$, $\rho^e_{BC}$ and $\rho^e_{AC}$ was also studied.

## 3 Conclusion

Entanglement is an intrinsic feature of quantum systems and has been exploited as a computational resource. We propose an NMR based scheme to generate a canonical three qubit state and its local variants. It has been shown recently that there is no irreducible three party correlation in a three qubit system and all the information about the complete state is contained in the three 2-party correlations. We have demonstrated this result experimentally in a system of three qubits. It is hoped that such experiments will shed light on how information is stored in multipartite systems.

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