Gate teleportation using Series entanglement distributions on arbitrary remote states

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Abstract

We consider Series entanglement distributions between remote parties for implementing non-local controlled-Unitary gates. In the familiar network [1], each of the control parties shares one entangled state with the target party whereas in series network, the entanglement distribution forms a series, such that the control parties share entanglement with the adjacent control parties and only one of them shares entanglement with the target party. In general the series network allows controlled-Hermitian gate implementation through local operations and classical communications (LOCC) on arbitrary state using $(n-1)$ ebits and $(n^2+n-2)/2$ cbits. We then explicate the protocol for teleportation of controlled-controlled-Unitary gate and subsequently generalize for 'n'-qubit gate.

Keywords:

Gate teleportation, Bell state, controlled-Unitary gate, controlled-Hermitian gate, Toffoli gate.

Introduction

The counter-intuitive and one of the most striking features of the quantum world is entanglement, which has found practical use in the field of various quantum communication protocols and cryptography. Gate teleportation is another important protocol implementing multi-partite quantum gates between qubits, which are spatially distributed. The protocol deals with the application of a unitary gate, that can not be decomposed into individual local operations i.e., $|\psi\rangle_{\mathcal{AB}} \longrightarrow \mathcal{U}|\psi\rangle_{\mathcal{AB}}$, where $\mathcal{U} \neq \mathcal{U}_{\mathcal{A}} \otimes \mathcal{U}_{\mathcal{B}}$. This can be achieved using entangled channels shared by the remote parties. In the context of distributed quantum computing, it is necessary to produce controlled operations between several remote parties.

In this present work, we consider 'series entanglement network' where the control parties share entanglement with the adjacent control parties and only one of them shares entanglement with the target party. As this network is linear, the target party has to maintain only one entangled channel. We start with a three party scenario, where Alice and Bob simultaneously teleport controlled-Hermitian gate to Charlie in series entangled network, which is then generalized for arbitrary multi-partite state. Next section deals with the teleportation of controlled-controlled-Unitary gate and the generalization to n-controlled Unitary gate teleportation.

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Simultaneous Teleportation of Controlled-Hermitian Gate

Here, we illustrate the scheme for the simultaneous teleportation of controlled-Hermitian (as well as unitary) gates from two parties to one, where the entangled network is in series connection. Suppose Alice, Bob and Charlie possess qubits 1,4 and 7 respectively of the arbitrary state,

$$\begin{aligned}|\psi\rangle_{147} = (d_0|000\rangle + d_1|001\rangle + d_2|010\rangle + d_3|011\rangle \\ +d_4|100\rangle + d_5|101\rangle + d_6|110\rangle + d_7|111\rangle)\end{aligned} \tag{1}$$

with $\Sigma_{i=0}^{7}|d_i|^2 = 1$, on which Alice and Bob want to implement simultaneous controlled-Hermitian gate on Charlie's qubit. Alice and Bob share a Bell state between their respective qubits 2 and 3; Bob and Charlie share a Bell state between their respective qubits 5 and 6 :

$$|\Phi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), |\Phi\rangle_{56} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \tag{2}$$

Here, Alice shares entanglement with another control party Bob and Bob shares entanglement with the target party, Charlie. Thus there is no direct entanglement between Alice and Charlie, which makes a series entanglement connection.

The pictorial representation of local unitary operations, measurements and classical communication for simultaneous

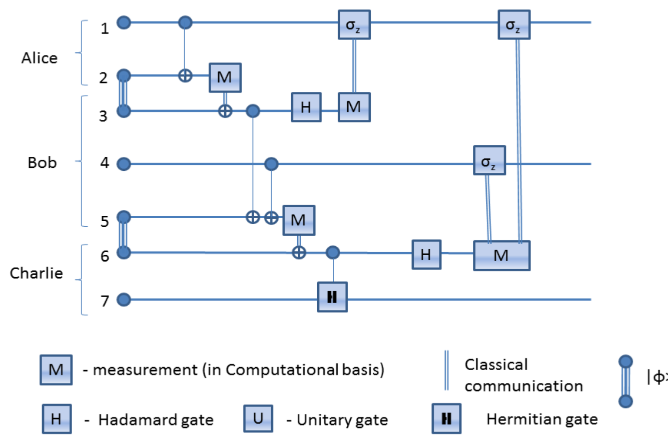

Figure 1: Simultaneous $C^{\mathcal{H}}$ teleportation through series network

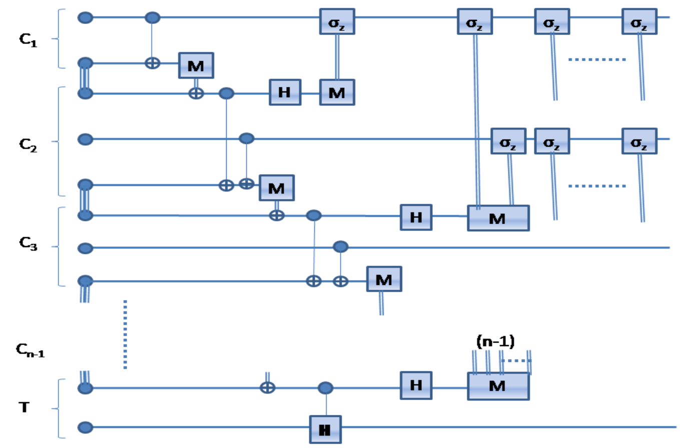

Figure 2: Generalized simultaneous $C^{\mathcal{H}}$ teleportation using the series network

$C^{\mathcal{H}}$ teleportation through series network of Bell states has been depicted in figure1. The simultaneous remote teleportation of controlled-Hermitian gate from two parties to one consumes 2 ebits and total 5 cbits to communicate the measurement outcomes.

n-party generalization : The generalized protocol of simultaneous $C^{\mathcal{H}}$ gate teleportation using the above series network is given in figure 2, where the unknown state given by, $|\chi\rangle = \Sigma_{m=0}^{2^n-1} a_m|i\rangle$, where ' i ' is the binary representation of decimal number ' m '. For n parties the communication cost is $(n-1)$ ebits and $(n^2+n-2)/2$ cbits.

The Unitary as well as Hermitian operators have the additional property of involution (i.e., the operator is same as its inverse), which is responsible for making this protocol deterministic. Most of the important gates like controlled-Pauli gates, controlled-Hadamard gate etc., belong to this category, making this implementation powerful.

Toffoli Gate Teleportation

It has been shown that a more general form of Toffoli gate, i.e., controlled-controlled-Unitary gate can be deterministically teleported using two Bell pairs (2 ebits of entanglement) and 4 cbits [1]. In this present protocol, we implement that same non-local gate with the same communication cost using series entanglement distribution considered in the above section. The pictorial representation of the protocol is given in figure 3.

n-party generalization : The above procedure can be generalized to teleport n-qubit gate, where each $(n-1)$ control qubit belongs to remotely placed control parties and the unitary operator acts on the target qubit, only if, all the control qubits are $|1\rangle$s. The communication cost is $(n-1)$ ebits and $2(n-1)$ cbits which is same as for the network given in Ref.[1]. The details of the protocol have been demonstrated in Ref. [2].

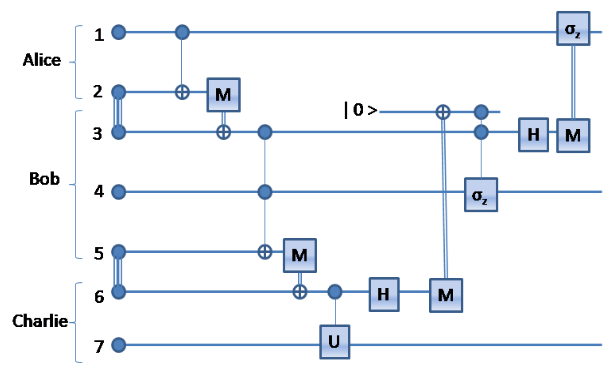

Figure 3: Controlled-controlled-Unitary gate teleportation through series network.

Discussion

In the proposed entanglement network of series configuration, classical communication cost for implementing simultaneous gate teleportation is more as compared to the Eisert et. al. distribution, where as it is same for implementing Toffoli gate. However, this will be advantageous when the entanglement sharing is difficult between each controlling side with the target party. In this scenario the network is less complex compared to the familiar network[1] due to its linearity. Optimal protocol may be a subject of further investigation for the simultaneous controlled-Unitary gate teleportation in series entangled channels.

References

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