Adiabatic Effect in Open Quantum Systems: Implications for Quantum Information

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Abstract. The Adiabatic theorem plays a very prominent role in the development of quantum mechanics. There have been many invest pert to this effect in quantum mechanics. A general applicability of these ideas requires an extension to Open Quantum Systems. In This work, we study two recently proposed attempts to understand adiabaticity in Open Quantum Systems, by applying them to a host of open system models. While they agree in general, there are regimes of disagreement. We further develop a simple intuitive approach to this problem by mapping a commonly studied open system model to a model of spin precession.

Keywords: Adiabatic Effect,Open Quantum Systems

Adiabatic theorem refers to a situation in which the original Hamiltonian of the system is gradually changed to a new Hamiltonian. It played an important role in the development of quantum mechanics [1, 2, 3]. An energy eigenstate, of the original Hamiltonian, becomes approximately an eigenstate for the new Hamiltonian, if the switch-on of the energy difference is sufficiently slow. This implies that the slowness of variation needs to be compared with an inherent slow system frequency, for e.g., the minimum of splitting of energy levels, say ω. The time variation of the Hamiltonian introduces another frequency, χ. For the adiabatic regime to hold, $\chi \ll \omega$, which implies that the Hamiltonian does not change significantly during the system characteristic cycle of motion.

The above simple estimate, justifying the adiabatic approximation, has been subject, in a number of works, to rigorous mathematical analysis, related to first order estimates of the spectral gap, of the spectral projection of the ground state separated from the rest of the spectrum [4, 5, 6]. These estimates have been extended to systems without a gap [7]. In [8], rigorous estimates were made for Hamiltonians which at any time t possess two spectral projectors, $P_1(t)$ and $P_2(t)$, and which are spectrally isolated. Considering systems with avoided level crossing, the adiabatic analysis lead to a rigorous derivation of the well known Landau-Zenner formula. In [9] use was made of the adiabatic theorem to introduce the concept of topological states of matter in order to distinguish gapped many body ground states of non-interacting systems and mean field superconductors, respectively, regarding their global geometrical features.

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In recent times, adiabatic approximation has been used as a method of quantum computation [10]. The Hamiltonian of interest is $H(t) = (1 - g(t))H_0 + g(t)H_1$. In most applications of the adiabatic theorem to quantum computation one is interested to find out how certain quantities, such as the running time of a computer program, grow (or decrease) with the parameter, n, which describes the size of the system [11]. In [12], the time evolution of a quantum system in the adiabatic limit was shown to have a geometric origin, leading to the concept of the geometric phase, an important tool in holonomic quantum computation.

Efforts have been made to develop an understanding of the adiabatic effect in open quantum systems. In [13], estimates were made for systems evolving under a Lindbladian evolution. Some rigorous estimates for adiabatic evolution of Lindbladian open quantum systems, with and without a gap, was made recently in [14]. Since an open system evolution would, in general, be non-unitary, it cannot be described by a Hermitian Hamiltonian. In some recent works, attempts have made to use an effective Hamiltonian approach to this problem [15]. In another approach[16], the adaibaticity of the open system was analyzed using the Jordan block diagonalization of the Lindbladian superoperator, generating the open system evolution. Here we aim at providing simple, physically motivated examples aimed at an understanding of adiabatic effects in the context of open quantum systems. An interesting analysis can be made from the prespective of thermodynamics. It is easy to show that for systems, undergoing Lindbladian evolution, where the Lindbladian commutes with the Hamiltonian, the system is adiabatic from the perspective of thermodynamics, but is not informationally isolated from its environment. In fact, this is the above discussed QND regime which is subject to decoherence. We make a comparison of the Jordan block [16] and effective Hamiltonian [15] approaches to adiabaticity in open quantum systems, by applying them on a number of open system models. We then take up a simple model of a two-level system, undergoing a general open system evolution. This is then converted into an equivalent problem of a spin precession around an effective magnetic field, which is described in terms of the open system parameters. From this analysis, a simple understanding is possible about the adiabatic in open quantum systems.

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