Bell nonlocality for light beams with topological singularities

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Abstract. We consider optical beams with topological singularities which possess Schmidt decomposition and show that such classical beams share many features of two mode entanglement in quantum optics. We demonstrate the coherence properties of such beams through the violations of Bell inequality for continuous variables using the Wigner function. This violation is a consequence of correlations between the (x, p_x) and (y, p_y) spaces which mathematically play the same role as nonlocality in quantum mechanics. The Bell violation for the LG beams is shown to increase with higher orbital angular momenta l of the vortex beam. This increase is reminiscent of enhancement of nonlocality for higher spins in quantum mechanics. The states with large l can be easily produced using spatial light modulators.

Keywords: Bell inequality, optical vortex beams, classical entanglement

The possibility of encoding large amounts of information in light beams with topological singularities has raised the prospects of their applicability in quantum information processing tasks such as computation and cryptography [1]. Vortex beams have interesting coherence properties [2, 3], and such beams with large values of orbital angular momenta have been experimentally realized [4, 6]. It is recognized that Laguerre-Gaussian (LG) beams have a Schmidt decomposition [2, 3, 7], and therefore one would expect many of the ideas developed within the context of quantum mechanics to be applicable to LG beams as well.

In the present work we apply the framework of the Wigner function formulation of the Bell-CHSH inequality [8] for the first time in classical optics to study the continuous variable correlations in light beams with topological singularities. For a two dimensional LG beam which is a physically realizable classical field distribution containing optical vortices with topological singularities, we discuss the framework of obtaining Bell inequalities for continuous variable systems using the Wigner function which can be expressed as an expectation value of a product of displaced parity operators [8]. The Bell inequality in terms of Wigner function (W) for the LG beam is given by

$$\begin{aligned} B &= \pi^2 |W(X1, P_{X1}; Y1, P_{Y1}) + W(X2, P_{X2}; Y1, P_{Y1}) \\ &+ W(X1, P_{X1}; Y2, P_{Y2}) - W(X2, P_{X2}; Y2, P_{Y2})| \\ &< 2 \end{aligned} \tag{1}$$

The two measurement settings on one side are chosen to be either $\{X1, P_{X1}\}$ or $\{X2, P_{X2}\}$, and the corresponding settings on the other side are either $\{Y1, P_{Y1}\}$ or $\{Y2, P_{Y2}\}$. Figure 1 shows the variation of $|B|$ with X and P_Y for three different values of the orbital angular momentum n for the LG beam, for a particular choice of measurement settings $\{X1 = 0, P_{X1} = 0, X2 = X, P_{X2} = 0, Y1 = 0, P_{Y1} = 0, Y2 = 0, P_{Y2} = P_Y\}$. We observe that the Bell-CHSH inequality is violated for LG

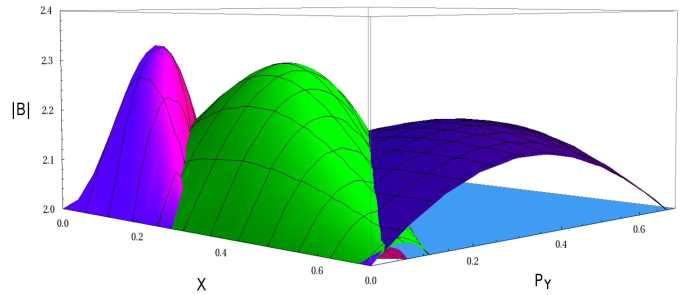

Figure 1: The plot shows the variation of the Bell sum $|B|$ with respect to X and P_Y for three different values of orbital angular momentum. The maximum violation increases with angular momentum.

beams and also show that the violation of the Bell's inequality increases with higher orbital angular momentum of the beam. The above enhancement of violation is analogous to the enhancement of nonlocality in quantum mechanics for larger values of quantum numbers [9].

We further consider the situation where the quadrature phase components of two correlated and spatially separated light fields are measured. The correlations between the quadrature amplitudes $\hat{X}_\theta$ and $\hat{Y}_\phi$ written as

$$\begin{aligned} \hat{X}_\theta &= \cos[\theta]\hat{X} + \sin[\theta]\hat{P_X}, \\ \hat{Y}_\phi &= \cos[\phi]\hat{Y} + \sin[\phi]\hat{P_Y} \end{aligned} \tag{2}$$

are captured by the correlation coefficient [10], $C_{\theta,\phi}$ defined as

$$C_{\theta,\phi} = \frac{\langle \hat{X}_\theta \hat{Y}_\phi \rangle}{\sqrt{\langle \hat{X}_\theta^2 \rangle \langle \hat{Y}_\phi^2 \rangle}}, \tag{3}$$

where $\langle \hat{X}_\theta \rangle = 0 = \langle \hat{Y}_\phi \rangle$. If $|C_{\theta,\phi}| = 1$ for some values of θ and ϕ, the correlation is perfect. The expression of the maximum correlation function is given by

$$C^{max}_{\theta,\phi} = \frac{\langle X P_Y \rangle}{\sqrt{\langle X^2 \rangle \langle P_Y^2 \rangle}} = -\frac{\langle P_X Y \rangle}{\sqrt{\langle P_X^2 \rangle \langle Y^2 \rangle}}. \tag{4}$$

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In Fig.2, we provide a plot of the maximum correlation function for several values of n, m. We observe that the strength of the correlations increases with increase of angular momentum, asymptotically reaching the limit of perfect correlations as angular momentum becomes very large. This feature thus further corroborates our earlier results of increase in Bell violations for larger orbital angular momentum of LG beams.

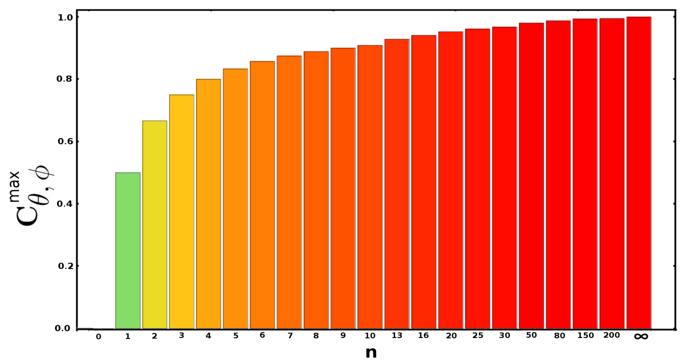

Figure 2: The plot shows the values of the maximum correlation function $C^{\max}_{\theta,\phi}$ for various values of n, where $m = 0$. Similar results are obtained by choosing $n = 0$ and varying m. Note that $C_{\theta,\phi} = 0$ for $n = m = 0$.

To summarize, for classical vortex beams, the magnitude of violation of the Bell inequality is shown to increase with the value of orbital angular momentum of the beam. This feature is further supported by the corresponding increase of the quadrature correlation function. Our predicted values of the correlation function as function of the beam parameters should be not difficult to realize experimentally. Production of such vortex beams have been achieved not only in the optical domain [6], but recently has also been implemented for electron beams [4]. Hence, the feasibility of direct measurement of the two-point correlation function through shear Sagnac interferometry is a potentially promising avenue [11] for experimental verification of our predicted Bell violation and its enhancement for vortex beams with higher angular momentum. The amount of violation of a Bell inequality [12, 13] involving discrete variables, has recently been suggested as a measure to quantify the magnitude of correlation between degrees of freedom of a classical beam [14]. Quantum nonlocality is reinterpreted in classical theory where a violation corresponding to a particular light beam possessing such correlations [15] signifies the impossibility of constructing such a beam using other beams with uncoupled degrees of freedom.

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