Separability criteria of $k-$separable $n-$partite quantum states

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Abstract. We present separability criteria of $k-$separable n-partite states in terms of the density matrix elements by combining the theorems formulated by Gao and Hong [T. Gao and Y. Hong, Eur. Phys. J. D **61**, 765 (2011)] on biseparable and fully separable n-partite states. Using our separability criteria one can detect a class of $k-$separable $(k = 2, 3, \ldots, n)$ $n-$qubit states under any partitions.

Keywords: $k-$ separability, multipartite state

1 Introduction

Though the $n-$number of quantum systems can have various kinds of entanglement, the focusing point is the genuine multipartite entanglement because it can be used for various quantum information and quantum computational tasks [1]. Two widely studied multipartite entangled states are Greenberger-Horne-Zeilinger (GHZ) and W states. These two states are inequivalent and maximally entangled ones which are found many applications in quantum information theory such as quantum teleportation, quantum secret sharing, superdense coding, cryptographic conferencing, enhancing the computational power and so on [2–5]. The stronger nonlocality displayed by the multipartite states also lead many theoretical interests in quantum physics [6].

Identifying entanglement in arbitrary multipartite states is nontrivial [1]. Several types of entanglement are observed in multiparticle systems. In addition to the biseparable and genuine entanglement, the multipartite states posses $k-$separable states or partial separable states [1]. The $k-$separability is a major difference between the bipartite and multipartite states. Suppose that a $n-$system under ivestigation is separated into $k-$parts, we say that it is $k-$separable state, say for example if the $4-$qubit state denoted as $ABCD$ is $3-$separable it may exist in any one of the following forms, namely $A|B|CD$, $A|C|BD$, $A|D|BC$, $B|C|AD$, $B|D|AC$ and $C|D|AB$. These are the possible partitions for the $4-$qubit $3-$separable state. However, these $3-$separable state do not tell us which subsystem entangled with the rest. From $k-$producibility one can infer how many subsystems are entangled [7]. The $k-$party entanglement is yet another interesting part of this study. The advantage of determining $k-$party entanglement is that it does not require the complete $N-$number of systems [7]. However, it is necessary to distill genuinely $k-$party entangled states. In the following, we concentrate only on the $k-$separability of $n-$qubit states.

In recent years several separability criteria were proposed to detect the entanglement in multipartite states [8–10]. In addition to the above, several experimentally accessible conditions have also been proposed, see for example Refs. [11–15]. As far as the $k-$separable mixed $n-$partite states are concerned only few conditions are available to detect the entanglement. To name a few we cite the following. Dür and Cirac have developed separability conditions for a class of partial separable states [10]. Seevinck and Uffink have proposed a set of inequalities for partial separability of multiqubit states [11]. Violations of these inequlities provide criteria for the entire hierarchy of $k-$separable entanglement, ranging from the levels $k = 1$ to n. Huber *et.al.* have derived a general framework to detect genuinely multipartite entangled mixed quantum states for arbitrary-dimensional systems [12]. From the later Gabriel *et.al.* have developed an easily computable criterion for detecting $k-$separability in mixed multipartite states [13]. In this direction, recently Gühne and Seevinck have proposed separability criteria for different classses of $3-$qubit and $4-$qubit entanglement, in particular genuine $3-$qubit and $4-$qubit entanglement [14]. These conditions, were derived based on the convex functions, associated with the density matrix elements. Later Gao and Hong have derived separability criteria for biseparable and fully separable $n-$qubit and $n-$qudit states and proved each criterion for any partitions [15]. These conditions are generalization of [14].

In this work we extend the criteria generalized by Gao and Hong to the $k-$separable $n-$partite states [16]. These conditions are able to detect both the GHZ and W classes. Our criteria is able to detect any $k-$separable $n-$qubit states under any partitions. Our results validate the cases, $k = 2$ and $k = n$ as biseparable and fully separable respectively. If a state violates our condition for any value of k $(1 < k \leq n)$ then it is genuinely entangled. The conditions are given in terms of density matrix elements which is a desired one under the circumstances one performs the complete tomography on the density matrix.

2 Definitions

Let us consider a system consists of n subsystems with Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_n$. A system which is in a pure state is fully separable if and only if the state is a product of pure states describing n elementary subsystems. A mixed state which is fully separable if the density matrix of it is a statistical mixture of product

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states, $\rho = \sum_j p_j \rho_j = \sum_j p_j \rho_j^{(1)} \otimes \rho_j^{(2)} \otimes \ldots \otimes \rho_j^{(n)}$, where $p_i > 0$, $\sum_i p_i = 1$ [1].

An $n-$partite pure quantum state $|\psi_{k-\text{sep}}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_k\rangle$ is $k-$separable $(k = 2, 3, \ldots, n)$ if it can be written as a product of k substates $\rho_{k-\text{sep}} = \rho^1 \otimes \rho^2 \otimes \ldots \otimes \rho^k$ [13]. A mixed state $\rho_{k-\text{sep}}$ is called $k-$separable, if and only if it has a decomposition into $k-$separable pure states

$$\rho_{k-\text{sep}} = \sum_i p_i \ \rho^i_{k-\text{sep}}. \tag{1}$$

An $n-$partite state is fully separable if $k = n$ and it is biseparable if $k = 2$. In particular, an n-partite state is genuinely n-partite entangled if and only if it is not k-separable.

3 Criteria for $k-$separability

In the following, we give two inequalities for the k-separable n-qubit states for different classes of entangled states. These two inequalities are obtained by combining the conditions of biseparable and fully separable n-qubit states which were given in Ref. [15]. In order to demonstrate the explicit $k-$ separability we fix certain maximum and minimum values in those expressions [16].

An n-qubit state, $\rho = (\rho_{i,j})_{2^n \times 2^n}$, is k-separable, if and only if the following inequalities hold for the (i)GHZ and (ii)W class

$$\text{(i)} \max_k (2^{k-1} - 1)|\rho_{1,2^n}| \leq \sum_{i=2}^{2^{n-1}} \sqrt{\rho_{i,i}\rho_{2^n-j+1,2^n-j+1}} \tag{2}$$

$$\begin{aligned}\text{(ii)} \sum_{1\leq j<i\leq n} |\rho_{2^{n-i}+1,2^{n-j}+1}| \\ \leq \sum_{1\leq j<i\leq n} \sqrt{\rho_{1,1}\rho_{2^{n-i}+2^{n-j}+1,2^{n-i}+2^{n-j}+1}} \\ + \min_k (\frac{n-k}{2}) \sum_{i=1}^{n} \rho_{2^{n-i}+1,2^{n-i}+1}\end{aligned} \tag{3}$$

respectively. If these two inequalities do not hold for any value of k $(1 < k \leq n)$ for their respective class then ρ is a genuine n-qubit entangled state. In the above k represents the k-separability. In both the cases, the equality holds for pure states.

4 Conclusion

We have derived separability criteria of $n-$partite states from which one can identify certain classes of entanglement. Using our generalized criteria, one can detect any $k-$separable $(k = 2, 3, \ldots, n)$ $n-$qubit states under all possible partitions of it. These inequalities are given in terms of density matrix elements which provide the experimental accessibility as well.

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