Generation of free-traveling optical trio coherent states

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Abstract. Trio coherent states (TCSs) are three-mode entangled states that can be useful for continuous variables quantum tasks. We propose a scheme to generate such TCSs in free-traveling optical fields, using standard physical resources such as coherent states, beam-splitters, phase-shifters, nonideal threshold photodetectors and realistic weak cross-Kerr nonlinearities, without single-photon sources or homodyne measurements. We study dependences of the fidelity of the generated state with respect to the target TCS. Theoretically, the fidelity can be high enough for whatever nonlinearities τ and photodetector efficiency η, provided that the amplitude $|\alpha|$ of an input coherent state satisfies the condition $|\alpha| \geq 5\eta^{-1/2}\tau^{-1}$.

Keywords: AQIS, Trio coherent state, continuous variables, cross-Kerr nonlinearities

The trio coherent states (TCSs) introduced in Ref. [1] are non-Gaussian three-mode entangled states which promise applications to continuous variables quantum information processing. Their nonclassical properties were investigated [2] and stable TCSs can be produced for the vibrational motion of an ion trapped in a 3D isotropic harmonic potential [3]. However, such vibrational TCSs are confined inside a crystal. In order to perform long-distance quantum communication and distributed quantum computation the states should be shared beforehand among distant parties. Here we propose a scheme to generate optical TCSs that can easily be distributed to intended remote parties.

By definition [1] the TCS is the simultaneous eigenstate of the three operators $\widehat{A}_{123} = \hat{a}_1\hat{a}_2\hat{a}_3$, $\widehat{N}_{21} = \hat{a}_2^\dagger\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_1$ and $\widehat{N}_{32} = \hat{a}_3^\dagger\hat{a}_3 - \hat{a}_2^\dagger\hat{a}_2$, with $\hat{a}_i^\dagger$ ($\hat{a}_i$) the bosonic creation (annihilation) operator of mode $i = 1, 2, 3$ of an optical field. That is, the TCS denoted by $|\Psi_{p,q}(\xi)\rangle_{123}$ with complex amplitude ξ is the solution of the three equations

$$\widehat{A}_{123}\,|\Psi_{p,q}(\xi)\rangle_{123} = \xi^3\,|\Psi_{p,q}(\xi)\rangle_{123}\,, \tag{1}$$

$$\widehat{N}_{21}\,|\Psi_{p,q}(\xi)\rangle_{123} = q\,|\Psi_{p,q}(\xi)\rangle_{123}\,, \tag{2}$$

$$\widehat{N}_{32}\,|\Psi_{p,q}(\xi)\rangle_{123} = p\,|\Psi_{p,q}(\xi)\rangle_{123}\,. \tag{3}$$

In terms of Fock states, it reads [1]

$$|\Psi_{p,q}(\xi)\rangle_{123} = \mathcal{N}_{p,q}(r)\sum_{n=0}^{\infty}\frac{\xi^{3n+2q+p}}{\sqrt{n!(n+q)!(n+q+p)!}}\,|n\rangle_1\,|n+q\rangle_2\,|n+q+p\rangle_3\,, \tag{4}$$

where

$$\mathcal{N}_{p.q}(r) = \left(\sum_{n=0}^{\infty}\frac{r^{2(3n+2q+p)}}{n!(n+q)!(n+q+p)!}\right)^{-1/2} \tag{5}$$

with $r = |\xi|$.

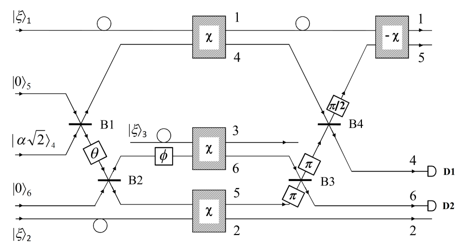

Figure 1: Schematic setup for generation of free-traveling optical TCSs. B1 to B4 are balanced beam-splitters, a square with, say, θ, is a phase-shifter $\hat{P}(\theta)$, a shaded rectangle with $\pm\chi$ is a cross-Kerr medium with nonlinearities $\pm\chi$, while D1 and D2 are nonideal on-off photodetectors. The inputs 1, 2 and 3 are coherent states whose amplitude $|\xi|$ is the same as that of the TCS to be prepared, but the amplitude $|\alpha|\sqrt{2}$ of the input 4 must be large enough (see text), while nothing is injected into the inputs 5 and 6. The circles on paths 1, 2 and 3 denote devices that synchronize the modes' incoming to the cross-Kerr media.

Our scheme shown in Fig. 1 uses coherent states as inputs and employs standard optical elements such as balanced beam-splitters, phase-shifters and nonideal on-off photodetectors. As coherent states are classical and classical states cannot be transformed into nonclassical ones by means of linear optics elements, we employ cross-Kerr nonlinearities for our purpose. Four coherent states with amplitudes ξ, ξ, ξ and $\alpha\sqrt{2}$ are used as input modes 1, 2, 3 and 4, respectively, where ξ is the amplitude of the TCS to be generated and α may be varied to manage the fidelity and success probability. All the beam-splitters are balanced, but the phase shifts are set to be $\theta = \tau q$ and $\phi = \tau p$ with $\tau \ll 1$ being a measure of cross-Kerr nonlinearity. Two photodetectors D1 and D2 are arranged to detect photon numbers of modes 4 and 6, respectively.

We first analyze theoretically the case when the two photodetectors are ideal, i.e., they can resolve the incoming photon number. The calculation shows that the fidelity vanishes whenever either or both photodetectors

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fire. That is, we would succeed only when both the photodetectors register no photons, implying that on-off photodetectors suffice.

We thus proceed to considering on-off photodetectors with efficiency η ($\eta < 1$ implies nonideal on-off photodetectors, while the ideal ones have $\eta = 1$). As a result, the silence of both the on-off photodetectors heralds generation of the target TCS in the output modes 1, 2 and 3, that can travel freely in space.

We have derived explicit analytical expressions for the success probability $P \equiv P(\alpha, \tau, \eta)$ and fidelity $F \equiv F(\alpha, \tau, \eta)$ of the generated state with respect to the desired TCS, which for $\tau \ll 1$ are given, to a good approximation, by

$$P \simeq e^{-3r^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \frac{r^{2(n+m+k)}}{n!m!k!} e^{-\frac{\eta|\alpha|^2\tau^2}{4}[\frac{1}{2}(m-k+p)^2+(n-m+q)^2+(n-k+p+q)^2]} \quad (6)$$

and

$$F = \frac{e^{-3r^2}}{P\mathcal{N}^2_{p,q}(r)}. \quad (7)$$

The above formulae reveal that the probability and thus the fidelity depend collectively on the product $\eta|\alpha|^2\tau^2$ rather than individually on each of η, $|\alpha|$ and τ. Figure 2 shows the dependence of the fidelity and the corresponding probability on the collective parameter $Z = \sqrt{\eta}|\alpha|\tau$ for various values of r. One can recognize that the fidelity is almost perfect when $Z \geq 5$. The inequality $Z = \sqrt{\eta}|\alpha|\tau \geq 5$ is meaningful. It signifies that our scheme still works even for weak cross-Kerr nonlinearity and low photodetector efficiency if the coherent beam injected into mode 4 is intense enough. In theory, the fidelity can be high enough for whatever weak nonlinearities χ and low photodetector efficiency η, provided that the amplitude $|\alpha|$ of an input coherent state is large enough, namely, $|\alpha| \geq 5/(\sqrt{\eta}\tau)$. Also seen from (7), the product PF is independent of η, $|\alpha|$ and τ. So, the price to pay for a high fidelity at a fixed r is a reduced success probability, a fact that cannot be avoided in many postselection-based schemes for quantum state engineering. For a reasonably high fidelity the success probability is small but still much larger than the rate of single photon generation through parametric down conversion which is about one in million. Furthermore, the fact that our generation scheme is nondeterministic causes no problems, because the TCSs we generate will be supplied off-line for a subsequent given quantum task. So we can repeat the whole process until success.

Our scheme does not require strong nonlinearities and high detector efficiency, provided the amplitude of the input mode 4 is large enough. For example, even with an inefficient on-off photodetector with η as low as $\eta \sim 0.8$, a laser pulse with about 10^6 mean photon numbers (i.e., $|\alpha| \sim \mathcal{O}(10^3)$) would compensate the smallness of the cross-Kerr nonlinearity $\tau \sim \mathcal{O}(10^{-3})$. Such weak (but not tiny) cross-Kerr nonlinearities can potentially be engineered in practice within the present technologies using various means such as doped optical fibers, cavity quantum electrodynamics, electromagnetically induced transparency, etc.. (see, e.g., [4]). This means that strong cross-Kerr nonlinearities are not at all a compulsory precondition and weak cross-Kerr nonlinearities could be very useful for quantum information processing and quantum computing [5]. Furthermore, not as in many other schemes/protocols, we require neither single-photon sources nor homodyne/heterodyne measurements.

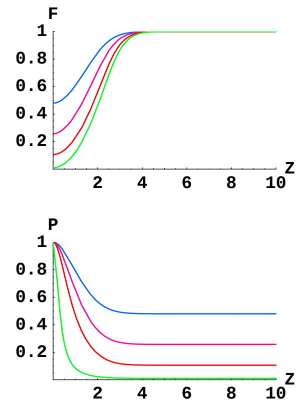

Figure 2: The fidelity F, Eq. (7), of the generated state with respect to the target TCS, Eq. (4), and the corresponding generation probability P, Eq. (6), versus $Z = \sqrt{\eta}|\alpha|\tau$ for $p = q = 0$ and $r = 0.5$, 0.7, 1.0 and 3.0 (upwards for the fidelity and downwards for the probability).

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