

Protection of an unknown quantum state against decoherence via weak measurement and quantum measurement reversal

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As quantum bit is a basic unit of quantum computers, so its preservation against decoherence is major task in quantum information theory. We study the behavior of a qubit, when it interacts with the environmental noise mainly with amplitude damping. We calculate the trace distance between two density operators in the maximum decoherence limit. The weak measurement and measurement reversal effects after and before decoherence respectively, are also studied and we found that trace distance in this case is decreased as we increase the weak measurement as well as reverse measurement strength.

Key words- Decoherence, quantum bit, trace distance

1. Introduction

The weirdness of quantum mechanics is because of the principle of superposition and it is the heart of the quantum information theory [1] [2]. But it seems from many years that decoherence is fast enemy of superposition and because of decoherence superposition between quantum states destroyed and collapse of wavefunction occurs. The destruction of interference and the disappearance of microphysical world is because of the decoherence phenomenon occurred in nature [3]. A single quantum system, that is a qubit, subjected to decoherence, affects the quantum communication processes. Since a qubit used in quantum teleportation, quantum superdense coding [2], is a building block of quantum computers and major source of quantum information. So its protection against decoherence is quite essential. The historical background of decoherence is well defined in ref. [4]. Decoherence arises due to the interaction of quantum system with the environmental noise. There are many channels which are responsible for decoherence, the depolarisation, dephasing, amplitude damping, thermal emission affecting both the quantum superposition of single quantum state and multipartite entanglement [2]. The illustration of decoherence for single qubit, bipartite entangled systems has recently gives fantastic theoretical and experimental results due to the introduction of these decoherence channels [5]. There are a number of ways to control decoherence which is produced due to various type of the noises present in nature see ref. [6][7][8][9][10][11][12][13].

In this letter, we report that weak measurement and measurement reversal operations are successfully reduce the decoherence as shown by decreased trace distance in case single qubit quantum system. The protection of single qubit against amplitude damping via weak measurement reversal operation is originally described in ref. [10]. The structure of this paper is arranged as follows:

Section 2 will tells us the theory in which we study the dynamics of a single qubit when it interacts with the environment and subjected to amplitude damping decoherence. We gave a theoretical view that how decoherence effects the quantum system by examining trace distance between two density operators. The effect of weak measurement and measurement reversal operations on a single qubit before and after decoherence is discussed in section 3. Finally the conclusion is given in section 4.

2. Consider a single qubit quantum system S initially in the state.

$$|\Psi\rangle_S = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

Where $|\alpha|^2 + |\beta|^2 = 1$ and the environment E is initially in the state $|0\rangle_E$.

The above is described by following quantum map

$$\begin{aligned} |0\rangle_S|0\rangle_E &\rightarrow |0\rangle_S|0\rangle_E \\ |1\rangle_S|0\rangle_E &\rightarrow \sqrt{1-D}|1\rangle_S|0\rangle_E + \sqrt{D}|0\rangle_S|1\rangle_E \end{aligned} \quad (2)$$

Where D is the magnitude of decoherence, sometimes called probability of losing a photon [2][12].

After interaction with environment the qubit subjected to amplitude damping and the state after decoherence is given by,

$$\rho_d = \begin{pmatrix} \beta^2 D^2 + \alpha^2 & 0 & \alpha\beta\sqrt{1-D} \\ \alpha\beta\sqrt{1-D} & 0 & \beta^2 \\ 0 & D(1-D)\beta^2 & 0 \end{pmatrix}$$

The trace distance [14] to be calculated, using the

$$\text{formula, } \delta(\rho, \sigma) = \sqrt{1 - \langle \Psi | \sigma | \Psi \rangle}$$

$$\text{is given by } \delta(\rho_S, \rho_d) = \sqrt{1 - (2\alpha^2 + \beta^2\sqrt{1-D})} \quad (3)$$

Where ρ_S is density matrix for initial quantum state.

3. Effect of weak measurement and measurement reversal operation

Now suppose before decoherence we apply weak measurement.

Since the postulate of quantum mechanics states that, measurements are irreversibly collapse the initial state to one of the eigen states of the measurement operator [2]. So the ordinary projective measurement in the qubit states would collapse the in to $|0\rangle$ or $|1\rangle$ with the probability equal to $|\alpha|^2$ or $|\beta|^2$. Projective measurements do not have mathematical inverse hence irreversible. But in the presence of non-projective measurement we have a different story [15]. it is possible to reverse the measurement induced state collapse. Weak measurements are important when the coupling between system and the apparatus is not strong and measurements are done without affecting the quantum state of the system. The projective and non-projective measurements are defined as:

$$M_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix} M_2 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \text{ respectively [15].}$$

Where p is the weak measurement strength. We are interested here only in non-projective measurement operation. In this paper we apply weak measurement and reverse measurement before and after the decoherence respectively, and calculated trace distance which comes out to be,

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$$\sqrt{1 - \frac{\alpha^4(1-p_r) + \alpha^2\beta_D + \alpha^2\beta_D(1-p_r) + \alpha^2\beta^2D(1-D)}{T}} \quad (4)$$

Where ρ_f is the final qubit state, p_r is the reverse measurement strength. $|\beta_D|^2 = |\beta|^2(1-P)D$ is defined as the probability that the system qubit would experience decoherence and T is the transmittance.[12].

At maximum reverse measurement, $\beta_D \rightarrow 0$

$$\delta(\rho_s, \rho_f) = \sqrt{1 - \frac{\alpha^4(1-p_r) + \alpha^2\beta^2D(1-D)}{T}} \quad (5)$$

Now here we take the ideal condition that decoherence is fully suppressed by weak measurement and measurement reversal operation if the magnitude of decoherence is known we can take $p = p_r$ and from $p_r = p + D(1-p)$ [12] we get $D=0$,

$$\delta(\rho_s, \rho_f) = \sqrt{1 - \frac{\alpha^4(1-p_r)}{T}} \quad (6)$$

At this point we are able to say that at different values of p_r we get the trace distance is constant (fig.1) Suppose decoherence is not equal to zero but have some magnitude then we apply the reversing measurement [12] So that from eq. (5) we calculate trace distance for various value of p , hence for P_r with some fix values of D . taking the case of $|\alpha| > |\beta|$ we get the trace distance is decreased and for maximum weak measurement with probabilistic measurement reversal p_r , we get minimum trace distance (figure 1).

Figure 1

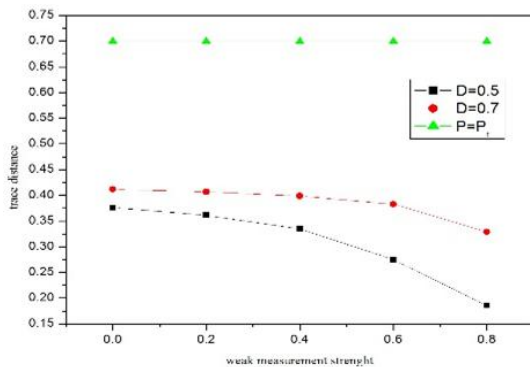


fig. 1 trace distance in the process of decoherence suppression

4. Conclusion

We have shown theoretically that the amplitude damping decoherence in quantum channels can be effectively controlled by introducing a weak measurement and a measurement reversal before and after decoherence channel, respectively. Examining trace distance we conclude that the trace distance comes out to be constant in the ideal condition of minimum decoherence or zero

decoherence. Hence if in any measurement trace distance remains constant then we assume that channel is decoherence free. We also found an interesting result from this study that if decoherence have some known magnitude and if we increase weak measurement strength as well as reverse measurement strength trace distance decreased. This study shows that weak measurements and reverse measurements acts as very powerful weapon to kill decoherence making the quantum states decoherence free.

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