Degree of Quantum Coherence of the Light Field from an Optical Mach–Zehnder Interferometer with a NOON State

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Abstract. Using the NOON state and a photon-number-resolving detector, the accuracy of phase measurement with a Mach–Zehnder interferometer reached the Heisenberg limit. In an experimental demonstration using the continuous-wave NOON state, the coherence time was much longer than the response time of a photon-number-resolving detector. Therefore, we investigated the time correlation of photon detection events in the light field from the Mach–Zehnder interferometer. As a result, it was found that the photon detection events were anti-bunching. This result suggests that we need a photon-number-resolving detector with a fast response time and a NOON state with a much longer coherence time in order to use a large-photon-number NOON state.

Keywords: degree of quantum coherence, anti-bunching, continuous-wave NOON state

1 Introduction

Reducing measurement uncertainty is the most important issue for any scientist. Usually, the uncertainty of a measurement, δx, decreases as the inverse square root of the total number of measurements, $1/\sqrt{n}$. This relationship is called the standard quantum limit.

However, we can beat the standard quantum limit using quantum entangled states as the measurement probe. In this case, the uncertainty of the measurement decreases as the inverse total number of measurements, $1/n$. This relationship is called the Heisenberg limit [1-3].

In optical metrology, various optical interferometers are widely applied in various measurement systems. Therefore, for phase-shift measurement in optical interferometer, if we can reduce measurement uncertainty to below the standard quantum limit, then we might realize some benefits. In gravity wave detection, for example, studies have been carried out to improve phase measurements beyond the standard quantum limit [4,5]. To achieve phase measurement approaching the Heisenberg limit, the uncertainty of phase estimation, $\delta\varphi$, decreases with the inverse photon number $1/N$ of the quantum entangled state as [6,7]: $\delta\varphi \propto 1/N$.

In a Mach–Zehnder interferometer, to achieve Heisenberg-limited phase estimation, we use the so-called " NOON " state [8], which is the entangled photon state of the photon number between the two arms of a Mach–Zehnder interferometer,

$$\frac{1}{\sqrt{2}}\left(|N,0\rangle_{1,2} + |0,N\rangle_{1,2}\right), \tag{1}$$

where N is the photon number, and the subscripts 1 and 2 represent the two aim modes of the optical interferometer. Ultrashort pulsed lasers have been used in previous experiments for generating NOON states [9-11]. However, the NOON states generated using continuous-wave (CW) lasers are more applicable to particular applications such as gravitational wave detection experiments.

In the NOON state generated using a CW laser source, the coherence time is much longer than the response time of the photon-number-resolving detector that is used for measuring the photon number from the Mach–Zehnder interferometer to estimate the phase shift. That is, multiphoton counting events occur during a NOON state detection. Therefore, we have investigated the time correlation of photon detection events.

2 Assumed experimental condition

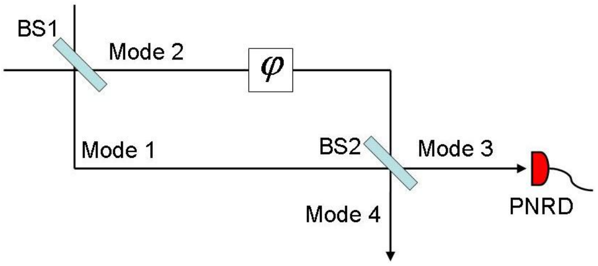

Figure 1: Schematic experimental setup. BS and PNRD mean beam splitter and photon-number-resolving detector, respectively.

Figure 1 shows a schematic of the experimental setup. We would like to measure the phase in the interferometer using the NOON state. After the first beam splitter (BS1), we suppose that the NOON state is generated. The NOON state is changed through the phase shift φ in Mode 2 as follows:

$$\frac{1}{\sqrt{2}}\left(|N,0\rangle_{1,2} + \exp(iN\varphi)|0,N\rangle_{1,2}\right). \tag{2}$$

In the NOON state generated using a CW laser source, the coherence time is much longer than the response time of the photon-number-resolving detector. That is, multiphoton counting events occur during a NOON state detection.

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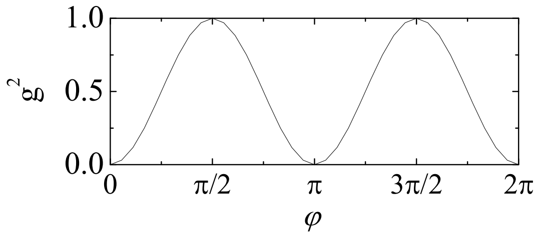

Figure 2: The calculated quantum degree of second-order coherence, g^2, for a NOON state with $N = 2$ as a function of initial phase, φ, in the interferometer.

3 Results and Discussion

To investigate the time correlation of a photon detection event, we calculated the quantum degree of second-order coherence, g^2:

$$g^2 = \frac{\langle \Psi_{\text{out}} | \hat{a}_3^\dagger \hat{a}_3^\dagger \hat{a}_3 \hat{a}_3 | \Psi_{\text{out}} \rangle}{\langle \Psi_{\text{out}} | \hat{a}_3^\dagger \hat{a}_3 | \Psi_{\text{out}} \rangle^2}, \tag{3}$$

where, $|\Psi_{\text{out}}\rangle$ is the output state from the interferometer including Modes 3 and 4; $\hat{a}_3$ and $\hat{a}_3^\dagger$ are annihilation and creation operators, respectively, in Mode 3. Figure 2, for example, shows the calculated result of g^2 for the NOON state with $N = 2$. When $0 < g^2 < 1$, the photon detection events are anti-bunching, so in almost every case, the photon detection events are anti-bunching.

This result is important for carrying out the experiment. This is because the photon-number-resolving ability of the novel photon detector is insufficient for detecting huge photon number states. With current technology, the maximum photon number resolution is few dozens of photons [12-14]. Therefore, we must restrict the photon number when generating the NOON state. In the end, we cannot sufficiently improve the accuracy of phase measurement with the interferometer. However, because the result showed that the photon detection events are anti-bunching, we do not require high photon-number-resolving ability in the photon detector, but we do need fast response time. For the anti-bunching condition, we can intuitively estimate detectable the photon number of τ_c/t_r, where τ_c is the coherence time of the NOON state, and t_r is the response time of the photon-number-resolving detector. With current technology, τ_c is more than 100 ns, and t_r is less than 1 ns [14]. Therefore, we might count hundreds of photons.

4 Summary

To investigate the time correlation of a photon detection event in a Mach–Zehnder interferometer with a continuous-wave NOON state, we calculated the quantum degree of coherence of output quantum states from the interferometer. As a result, we found that the photon detection events were anti-bunching. This result suggests that we can carried out the demonstration of phase super-sensitivity using a large-photon-number NOON state with a much longer coherent time and a photon-number-resolving detector with a fast response time.

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