Violation Of Entropic Leggett-Garg Inequality in Numclear Spins

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Abstract. We report an experimental study of recently formulated entropic Leggett-Garg inequality (ELGI) by Usha Devi *et al.* (Phys Rev A 87 052103,(2013)). This inequality places a bound on the statistical measurement outcomes of dynamical observables describing a macrorealistic system. Such a bound is not necessarily obeyed by quantum systems, and therefore provides an important way to distinguish quantumness from classical behavior. Here we study ELGI using a two-qubit nuclear magnetic resonance system. The experimental results show a clear violation of ELGI by over four standard deviations. We also study the reason of violation of ELGI.

Keywords: Entorpic LGI, NMR

1 Introduction

Macrorealism a notion imposed on classical objects, is based on two criteria: (i) the object remains in one or the other of many possible states at all times, and (ii) the measurements are *noninvasive*, i.e., they reveal the state of the object without disturbing the object or its future dynamics. Quantum systems are incompatible with these criteria and therefore violate bounds on correlations derived from them. For instance, Leggett-Garg inequality (LGI) sets up macrorealistic bounds on linear combinations of two-time correlations of a dichotomic observable belonging to a single dynamical system [1]. In this sense, LGI is regarded as a temporal analogue of Bell's inequality. Quantum systems do not comply with LGI, and therefore provide an important way to distinguish the quantum behavior from macrorealism. Violations of LGI by quantum systems have been investigated and demonstrated experimentally in various systems [2, 3].

For understanding the quantum behavior it is important to investigate it through different approaches, particularly from an information theoretical point of view.

Recently Usha Devi *et al.* [4] have introduced an entropic formulation of LGI in terms of classical Shannon entropies associated with classical correlations.

Here we report an experimental demonstration of violation of entropic LGI (ELGI) in an ensemble of spin 1/2 nuclei using nuclear magnetic resonance (NMR) techniques. Although NMR experiments are carried out at a high temperature limit, the nuclear spins have long coherence times, and their unitary evolutions can be controlled in a precise way. The large parallel computations carried out in an NMR spin ensemble assists in efficiently extracting the single-event probabilities and joint-probabilities. The simplest ELGI study involves three sets of two-time joint measurements of a dynamic observable belonging to a 'system' qubit.

Further, it has been argued in [4] that the violation of ELGI arises essentially due to the fact that the joint probabilities do not originate from a legitimate grand probability (of which the joint probabilities are the marginals). Here we also describe extracting the three-time joint probability (grand probability) using a three-qubit system, and demonstrate experimentally that it can not reproduce all the marginal probabilities substantiating this feature.

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2 Entropic Leggett-Garg Inequality

Usha Devi *et al.* have formulated the following entropic Leggett-Garg Inequaltiy [4]:

$$\sum_{k=2}^{n} H(Q_k|Q_{k-1}) \geq H(Q_n|Q_1). \tag{1}$$

The above equation says the information gained when we measure a dynamical observable $n-1$ times will always be greater than or equal to when we measure is only once. The above inequality (1) scaled in units of $\log_2(2s+1)$ is termed as the information deficit $\mathcal{D}$. For n-equidistant measurements, it can be written as [4]

$$\mathcal{D}_n(\theta) = \frac{(n-1)H[\theta/(n-1)] - H[\theta]}{\log_2(2s+1)} \geq 0. \tag{2}$$

3 Experimental Results

To calculate the entropies required to obtain information deficit, we need to extract the probabilities of the measurement of an dynamical observable. For this purpose, we utilize an ancilla qubit initialized in the state $|0\rangle\langle 0|$. The CNOT gate encodes the probability of the outcomes in the diagonal elements of ancilla qubit since,

$$\begin{bmatrix} P(0_i) & a \\ a^\dagger & P(1_i) \end{bmatrix}_S \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_A \xrightarrow{\text{CNOT}} \begin{bmatrix} P(0_i) & 0 & 0 & a \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a^\dagger & 0 & 0 & P(1_i) \end{bmatrix}_{SA},$$

where a is the off-diagonal element of the system density matrix. The probabilities $P(0_i)$ and $P(1_i)$ can now be retrieved by tracing over the system qubit and reading the diagonal elements of the ancilla state.

In order to extract these probabilities noninvasively we employ 'ideal negative result measurement' (INRM) procedure [3]. The idea is as follows. The CNOT gate is able to flip the ancilla qubit only if the system qubit is in state $|1\rangle$, and does nothing if the system qubit is in state $|0\rangle$. Therefore after the CNOT gate, if we measure the probability of unflipped ancilla, this corresponds to an 'interaction-free' or 'non-invasive measurement' of $P(q = 0)$. Similarly, we can implement an anti-CNOT gate, which flips the ancilla only if the system qubit is in state $|0\rangle$, and does nothing otherwise, such that the probability of unflipped qubit now gives $P(q = 1)$. Note that in both the cases, the probabilities of states wherein the system interacted with the ancilla, resulting in its flip, are discarded.

The theoretical and experimental values of $\mathcal{D}_3$ for various rotation angles θ are shown in Fig. 1(a). We find a general agreement between the mean experimental $\mathcal{D}_3$ values with that of the quantum theory. The error bars indicate the standard deviations obtained by a series of independent measurements. According to quantum theory, a maximum violation of $\mathcal{D}_3 = -0.134$ should occur at $\theta = \pi/4$, experimentally we obtained $\mathcal{D}_3(\pi/4) = -0.114 \pm 0.027$. Thus we found a clear violation of ELGI over 4 standard deviations.

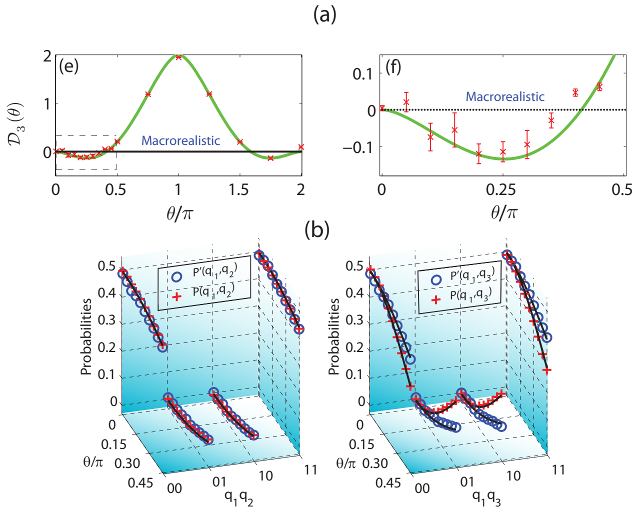

Figure 1: (a) Information deficit $\mathcal{D}_3$ versus θ by INRM procedure . The boxed area in the left plot is magnified in the right plot. The mean experimental $\mathcal{D}_3$ (in bits) values are shown as symbols. The curves indicate theoretical $\mathcal{D}_3$ (in bits). The horizontal line $\mathcal{D}_3 = 0$ indicates the lower bound of the macrorealism territory. (b) Joint probabilities $P(q_1, q_2)$, marginal probabilities $P'(q_1, q_2)$, and joint probabilities $P(q_1, q_3)$, marginal probabilities $P'(q_1, q_3)$. The lines correspond to theoretical values and the symbols are mean experimental values.

4 Reason for Violation

In principle, it is possible to generate the two-time joint probabilities as marginals $P'(q_i, q_j)$ of three-time joint probabilities:

$$P'(q_1, q_2) = \sum_{q_3} P(q_1, q_2, q_3);$$
$$P'(q_2, q_3) = \sum_{q_1} P(q_1, q_2, q_3);$$
$$P'(q_1, q_3) = \sum_{q_2} P(q_1, q_2, q_3). \quad (3)$$

In a macrorealistic world $P'(q_i, q_j) = P(q_i, q_j)$, where $P(q_i, q_j)$ stands for the two-time joint probabilities that are obtained directly from experiments

The experimental results of $P(q_1, q_2)$ and $P'(q_1, q_2)$ are shown in Fig. 1(b). It is evident that the marginals agree quite well with the corresponding joint probabilities. Similarly experimental results of $P(q_1, q_3)$ and $P'(q_1, q_3)$ are also shown in Fig. 1(b). However, here we see significant deviation of marginal probabilities from joint probabilities.

These results show, in contrary to the macrorealistic theory, that the grand probability $P(q_1, q_2, q_3)$ can not reproduce all the two-time joint probabilities as the marginals. Therefore the grand probability is not legitimate in the quantum case, which is the fundamental reason for the violation of ELGI by quantum systems [4].

It is interesting to note that even for those values of θ for which $\mathcal{D}_3$ is positive, the three-time joint probability is illegitimate. Therefore, while the violation of ELGI indicates the quantumness of the system, its satisfaction does not rule out the quantumness.

One distinct feature of the entropic LGI is that, the dichotomic nature of observables assumed in the original formulation of LGI can be relaxed, thus allowing one to study the quantum behavior of higher dimensional systems such as spin $> 1/2$ systems.

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