Quantum back-action in a two-component Bose-Einstein condensate

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Abstract. We investigate a non-destructive homodyne measurement of atomic condensates using off-resonant optical laser beam that interacts with atomic Bose-Einstein condensate (BEC). The interaction between laser beam and the atomic BEC creates entangled states of atom and light that are destroyed in a homodyne detection of light. The destruction of the entangled states causes back-action on the atomic states. We derive expression for the measurement operator on the atomic states due to the photon measurement. We characterize the back-action by calculating quantities such as the Q-function and observe the variation of the measured state with the strength of the light-BEC interaction..

Keywords: quantum mesurement, back-action

Information about a property of a system is acquired by performing measurement repeatedly on identically prepared systems or performing measurement repeatedly on a single system. For a quantum system like BEC, projective measurement or dispersive measurement that uses optical techniques are used to acquire information about the condensate. In projective measurement such as absorption imaging [1, 2], optical probe beam that is nearly resonant with atomic transition is used to image the cloud. Because of high optical density of the BEC in atom traps that results in absorption of the probe beam, the BEC are released from the atom trap before projective measurement could be applied. As such, BEC is destroyed in the measurement process therefore requiring identical samples of the condensates to be prepared.

On the contrary, dispersive measurement like the phase contrast imaging [3, 4] uses off resonant beam to measure the properties of small and dense atomic condensates *in situ*. The atoms coherently scatter photons in the probe beam as described in Fig 1. Because dispersive measurement does not destroy the BEC, it is applied repeatedly on the same sample and has been used to image BEC in a number of experiments [5, 6, 7]. Such technique is a useful resource for information read-out in several applications proposing the use of ultra-cold atoms and atomic condensates in metrology [8, 9], quantum information and processing [10], and quantum computation.

The scattered photons in the probe beam carry away information about the atomic condensates. The information is accessed by performing homodyne detection on light beam. Here we analyse dispersive imaging of a two component atomic condensates using off resonant laser pulses. The laser pulses are strongly detuned from atomic resonance such that atomic population is conserved. A light beam of well-known polarisation couples the ground states of the atomic condensates to the excited states as shown in Fig. 1. For sufficiently large detuned light, the ground states follow the light fields adiabatically and results in the entanglement of the light and atomic states. Also, the interaction between atom and light causes the light states to undergo a phase rotation. However, not all incident light beam pass through the condensate. The

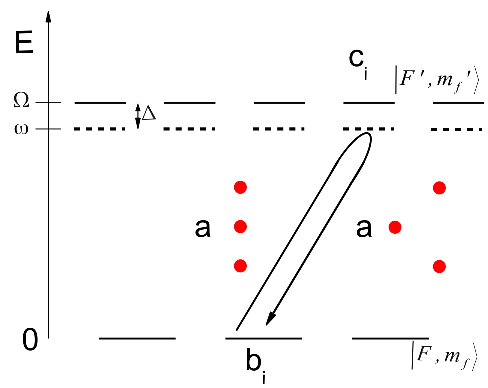

Figure 1: The energy level diagram showing the atomic non-resonant transition during phase contrast imaging. The incident light of frequency ω is detuned from atomic resonance by Δ. The atom in the ground state $|F, m_f\rangle$ absorbs a photon (red ball) from the beam, and makes transition to the quasi-excited state (the dashed lines). It emits the same photon via stimulated emission and transitions back to the same ground state. The emitted photon lags behind the unabsorbed photons thereby introducing a phase shift in the wave-front of light.

beam not deflected and the beam deflected by atomic condensates are interfered at the detector in a homodyne measurement of the photons. Labelling the detectors placed at the output of beam splitter as c and d, the population of photons that arrived at the detector are also labelled as n_c and n_d respectively. The measurement operator $\hat{\Omega}_{n_c,n_d}$ of getting a certain population of photons in the detectors c and d is derived by projecting the state $|n_c, n_d\rangle$ unto the photon subspace

$$\begin{aligned}\hat{\Omega}_{n_c,n_d} = \frac{e^{-\frac{|\gamma_p|^2+|\gamma_u|^2}{2}}}{\sqrt{n_c!}\sqrt{n_d!}} \left[\frac{i}{\sqrt{2}}\left(-i\gamma_p e^{i\hat{\phi}} + \gamma_u e^{i\theta_u}\right)\right]^{n_c} \\ \times \left[\frac{1}{\sqrt{2}}\left(i\gamma_p e^{i\hat{\phi}} + \gamma_u e^{i\theta_u}\right)\right]^{n_d},\end{aligned} \tag{1}$$

where $\hat{\phi} = (2GS_z + g\hat{N})\tau$. The operator $\hat{\Omega}_{n_c,n_d}$ operates in the atomic condensates' space and is used to calculate the signal obtained in homodyne measurement. The result is plotted in Fig. 2 as a function of initial amplitudes of atomic states for different coupling $G\tau$ strength. The

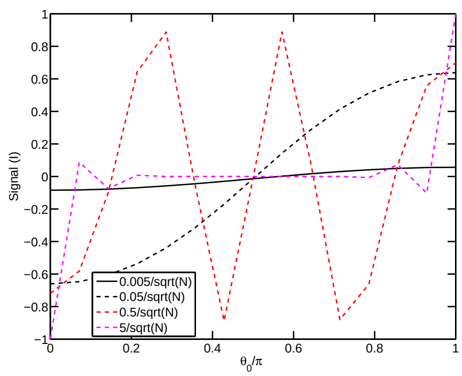

Figure 2: The inferred signal I from the measurement of photon number as a function of θ_0 for $N = 200$, $|\gamma_u| = \sqrt{135}$, and $|\gamma_p| = 2$.

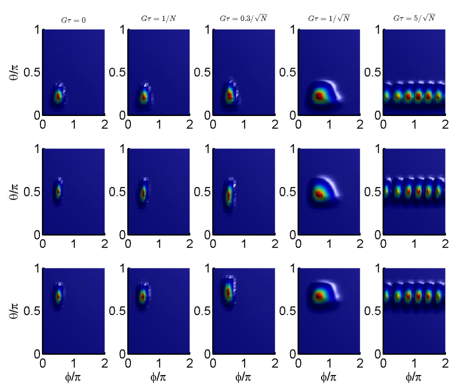

Figure 3: The Husimi Q-distribution plots for different initial conditions at different values of $G\tau$. Each column is plotted at the value of $G\tau$ specified at the column head. The first row is plotted for $\theta_0 = 40°$, $\phi_0 = 90°$, the second row is plotted for $\theta_0 = 90°$, $\phi = 90°$, and the third row is plotted for $\theta_0 = 120°$, $\phi_0 = 90°$. For these plots $|\gamma_u| = \sqrt{80}$, $|\gamma_p| = 2$, $N = 300$, $g\tau = 0.001/\sqrt{N}$, and the phase-plate angle $\theta_u = \pi/3$.

signal diminishes with increased correlation between the atom and light. For a total population of N atoms in the atomic condensates, the signal from the homodyne measurement is lost when the coupling strength is of the order $1/\sqrt{N}$ [see Fig. 2]. We calculated the sensitivity of homodyne measurement to the signal and showed that it depends on the coupling between atom and light, the population of atoms in the atomic condensate and the initial noise of the probe beam.

Besides reading off the information contained in the atomic condensates, the measurement induces back-action on the atomic condensates. To quantify the back-action induced by measurement, we calculate numerically the Husimi Q-distribution by summing over all possible photon number n_c, n_d in the output, $Q(\theta,\phi) = \frac{N+1}{4\pi} \sum_{n_c,n_d} |\langle\langle \alpha,\beta|\psi_m\rangle\rangle|^2$. The Q-distribution is a measure of overlap between the atomic coherent state $|\alpha,\beta\rangle\rangle$ and the state after measurement $|\psi_m\rangle$, and thus describes how much the state after measurement resembles atomic coherent state. The results are presented in Fig. 3. In the regime of small coupling between atom and light, the Q-distribution is Gaussian, with a width that scales roughly as $1/\sqrt{N}$. However, in the large coupling limit the Q-distribution is no longer Gaussian. Instead the width grows and there emerge several satellite peaks in the phase plane.

In conclusion, we have investigated the non-destructive homodyne detection of atomic condensates using off-resonant laser beam. We derived an expression for the operator that describes the homodyne measurement and showed that the relevant time scales for the measurement is that for which the coupling strength $G\tau$ is much less than $1/\sqrt{N}$ ($G\tau \ll 1/\sqrt{N}$). Also we demonstrated that the noise in the measured signal depends on the initial noise of the probe and the atomic condensates. The probe noise could be minimised by allowing considerable number of photons to interact with the condensates, since the noise of the probe is inversely proportional to the average photon number in the probe beam. Finally we quantified the back-action of the measurement using the Husimi Q-distribution that gives a measure of how close the state after measurement is to the atomic coherent state. We found that time scale relevant for non-destructive measurement is roughly of the order of $1/N$.

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