Thermal entanglement in a two-qubit Ising chain subjected to Dzialoshinski-Moriya interaction

B. G. Divyamani[1 2 *] Sudha[1 3 †]

[1]*Department of Physics, Kuvempu University, Shankaraghatta, Shimoga-577 451, India.*
[2]*Tunga Mahavidyalaya, Tirthahalli, Shimoga-577 432,India*
[3]*Inspire Institute Inc., Alexandria, Virginia, 22303, USA.*

Abstract. Thermal entanglement of a two-qubit Ising chain subjected to an external magnetic field and Dzialoshinski-Moriya (DM) interaction is examined. The effect of magnetic field, strength of DM interaction and temperature are analyzed by adopting negativity of partial transpose as the measure of entanglement. It is shown that when the DM interaction along the Ising axis is considerable, thermal entanglement can be sustained for higher temperature. The usefulness of longitudinal DM interaction over the one that is perpendicular to the Ising axis, in the manipulation and control of entanglement at a feasible temperature, is illustrated.

Keywords: Thermal Entanglement, Ising Chain, Dzialoshinski-Moriya interaction

1 Introduction

The study of thermal entanglement in quantum spin chains of solid state systems is known to provide a bridge between quantum information processing and condensed matter physics [1]–[12]. The usefulness of entangled spin chains in thermal equilibrium to the future realization of quantum computers [13] has necessitated this potentially rich area of study. The study of thermal entanglement in Ising spin chains subjected to external transverse magnetic field has been carried out quite extensively [9, 10, 11, 12].

Dzialoshinski-Moriya (DM) interaction [14, 15], an anisotropic and antisymmetric exchange interaction, arising due to spin orbit coupling is seen to enhance the thermal and ground state entanglement of Heisenberg spin chains [7, 8]. The anisotropy and antisymmetry of the interaction is evident through its form $\overrightarrow{D}\,.[\overrightarrow{S_1}\times\overrightarrow{S_2}]$ [14, 15]. Thermal entanglement of a two-qutrit Ising system subjected to a magnetic field and DM interaction, both along the direction of Ising axis, is studied by C. Akyüz et. al. [12]. Here we examine the combined effect of external magnetic field and DM interaction, both in the longitudinal as well as transverse directions, on the thermal entanglement of a two-qubit Ising chain [16]. We adopt negativity of partial transpose $N(\rho)$ [17] as the measure of entanglement for its unambiguous determination.

2 Thermal entanglement of an Ising chain with longitudinal magnetic field and DM interaction

The two-qubit Ising chain subjected to magnetic field B and DM interaction [14] both along the direction of the Ising axis is modelled by the Hamiltonian

$$\hat{H}=2J(\hat{\sigma}_{1z}\cdot\hat{\sigma}_{2z})+B(\hat{\sigma}_{1z}+\hat{\sigma}_{2z})+d(\hat{\sigma}_{1x}\cdot\hat{\sigma}_{2y}-\hat{\sigma}_{1y}\cdot\hat{\sigma}_{2x}).$$

Here J, d denote Ising coupling constant and DM interaction parameter respectively. The thermal state $\rho(T)$ of the Ising chain is given by $\rho(T)=\frac{e^{-\frac{H}{kT}}}{Z}$ where $Z=\mathrm{Tr}\left[e^{-\frac{H}{kT}}\right]$ is the partition function and k is Boltzmann's constant. We explicitly evaluate the negativity of partial transpose $N(\rho)$ of the state $\rho(T)$ and present a graphical analysis of its variation with respect to the parameters T, B and d.

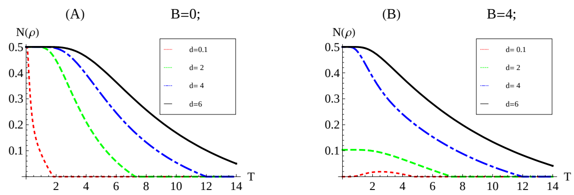

Figure 1: Two dimensional plots showing the effect of magnetic field on $N(\rho)$. ($J=1$.)

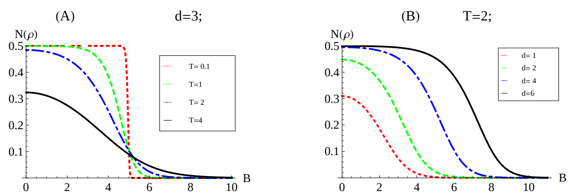

Figure 2: Variation of $N(\rho)$ with magnetic field for fixed d and T ($J=1$.)

It is readily seen that higher DM interaction and smaller longitudinal magnetic field are beneficial in sustaining the thermal entanglement for a larger range of temperature.

*divyaphysics@gmail.com
†arss@rediffmail.com

3 Thermal entanglement in the Ising chain with transverse magnetic field and DM interaction

Instead of the longitudinal magnetic field, if we consider a transverse external magnetic field and the DM interaction perpendicular to the Ising interaction, the Hamiltonian of the system is given by

$$\hat{H} = 2J(\hat{\sigma}_{1x} \cdot \hat{\sigma}_{2x}) + B(\hat{\sigma}_{1z} + \hat{\sigma}_{2z}) + d(\hat{\sigma}_{1x} \cdot \hat{\sigma}_{2y} - \hat{\sigma}_{1y} \cdot \hat{\sigma}_{2x}).$$

Here Ising axis is chosen to be x-axis and the magnetic field, DM interaction along z-axis. If $N^T(\rho)$ denotes the negativity of partial transpose of the corresponding thermal state, its variation with the parameters T, B and d are as shown below.

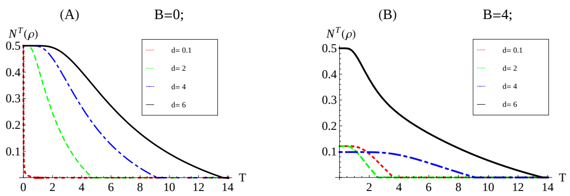

Figure 3: Two dimensional plots showing the effect of transverse magnetic field on $N^T(\rho)$. ($J = 1$.)

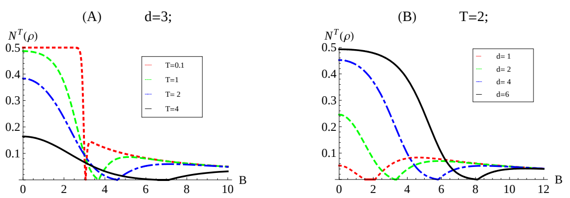

Figure 4: Variation of $N^T(\rho)$ with transverse magnetic field for fixed d and T ($J = 1$.)

An interesting feature in the case of transverse magnetic field, transverse DM interaction is the vanishing and recovery of entanglement at a particular value of magnetic field, that depends both on temperature and strength of DM interaction (See Figs. 4(A) and 4(B)). In particular, at $T = 0$, $N^T(\rho)$ suddenly drops to zero from its maximum value at $B = d$. For $T > 0$, this phenomenon occurs at $B \approx d + T$. But in all these cases the value of $N^T(\rho)$ that is recovered after vanishing is considerably smaller than its value at $B = 0$ (See Figs. 4(A) and 4(B)).

On comparing Figs. 3 and 4 with the corresponding Figs. 1 and 2 one can conclude that "there is a considerable reduction in the range of temperature over which the thermal entanglement is non-zero, at any fixed value of d and transverse magnetic field B in comparison with the same situation with longitudinal magnetic field, DM interaction." The supremacy of an Ising chain with DM interaction along its axis over an Ising chain with transverse DM interaction, in retaining thermal entanglement at not-so-low temperatures, is thus evident.

4 Concluding Remarks

We have evaluated thermal entanglement in the two-qubit Ising model with longitudinal magnetic field, DM interaction and compared the results with that of a transverse Ising model in the presence of transverse DM interaction [16]. We show that a pure DM interaction (without magnetic field) along the Ising axis can give rise to thermal entanglement and its value as well as the range of temperature over which it is non-zero increases with the increase in the DM interaction [16]. The thermal entanglement in the case of transverse DM interaction is seen to last for a smaller temperature range in comparison with that in the case of longitudinal DM interaction. The usefulness of longitudinal DM interaction over the one that is perpendicular to the Ising axis, in the manipulation and control of entanglement at a feasible temperature, is thus illustrated [16]. We conjecture that these results are applicable to the pariwise entanglement in an N-qubit Ising chain in the presence of DM interaction.

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