Quantum Algorithm to Solve a Maze: Converting a Maze Problem into Search Problem

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Abstract. We propose a quantum methodology to approach a Maze problem, which is otherwise a NP complete problem. The paper aims to convert the maze problem to a quantum search algorithm and apply iterative Grover search mechanism to move towards the solution. Our solution deals with two dimensional perfect mazes with no closed loops. We encode all possible individual paths from the starting point of the maze into a quantum register. A quantum fitness operator applied on the register encodes each individual with its fitness value. We propose an oracle design which marks all the individuals above a certain fitness value and use Grover search algorithm to find one of the marked states. Iterating this method, we approach towards the optimum solution.

Keywords: Maze, NP complete, Grover search, Individual, Fitness operator

1 Introduction

Certain Quantum algorithms(QA) employing quantum superposition and entanglement principles perform computations with fewer resources as compared to most efficient classical algorithms. For instance, Grover's algorithm[1] provides a O($\sqrt{n}$) speedup over the best classical search algorithm O(n), for sample size n . Here we look at the Maze problem(NP complete) with a quantum approach to the solution.

2 Maze Creation

- A maze is a complex system of passages or paths separated by walls or hedges. It is analogous to a binary tree where each line of the maze representation has one input, and two outputs, until we reach the end of the maze in which there is no further output. This method yields a perfect maze with no closed loops.
- This action of one input, and two outputs doubles in size each time, thus making the problem exponentially in time domain to solve as the number of decisions increases. A typical maze problem is a NP complete [2].
- The maze creation algorithm uses a recursive backtracker algorithm [3]. For a 2D maze, each grid in the maze is called a Room and a square m×m maze is constructed, starting with an initial empty stack.

3 Formulation and Approach

- **Superposition:** A individual $|x\rangle$ is a sequence of gates $|N\rangle$,$|E\rangle$,$|S\rangle$,$|W\rangle$ of a size **n**. A Path register $|\Omega\rangle$ is created storing all possible combinations of the gates of size n. $|\Omega\rangle = \frac{1}{2^n}\sum_{x\epsilon\{N,E,S,W\}^n}^{4^n-1}|x\rangle$
- **Fitness:** A fitness operator **F** operates on the Path register $|\Omega\rangle$ thus resulting in the formation of encoded register $|\Psi\rangle$ containing the superposition of all possible individuals encoded with their fitness values. $|\Psi\rangle = \mathbf{F}|\Omega\rangle = \frac{1}{2^n}\sum_{x\epsilon\{N,E,S,W\}^n}^{4^n-1}|x\rangle \otimes |fit_x\rangle$
- **Oracle:** An oracle structure is defined which marks all the individuals in the register $|\Psi\rangle$ that have the fitness value greater than a random fitness value $cutoff = |fit_x\rangle$.
  $[|x\rangle \otimes |fit_x\rangle] \rightarrow (-1)^{f(fit_x)}[|x\rangle \otimes |fit_x\rangle]$, where
  $f(fit_x) = 1$, $if, fit_x > cutoff$,
  and -1 , otherwise.
- **Grover Iteration:** After the oracle marks certain individuals, Grover iteration is employed to find one of those individuals. In the next step, the value of *cutoff* is updated by the fitness of one of the marked individual found by Grover's algorithm. In the subsequent number of iterations of updating the *cutoff* value with a higher fitness, we approach towards the optimum individual.

4 Figurative Analysis

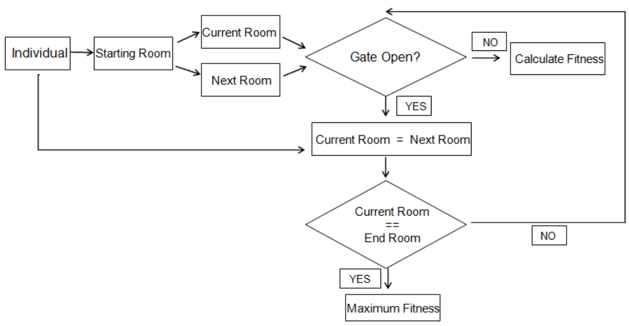

Figure 1: The flowchart depicting functionality of Fitness function

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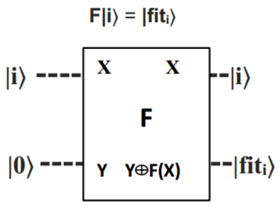

Figure 2: Figure depicting operation of Fitness operator on an individual

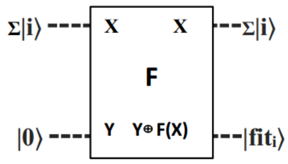

Figure 3: Figure depicting operation of Fitness operator on an Path register

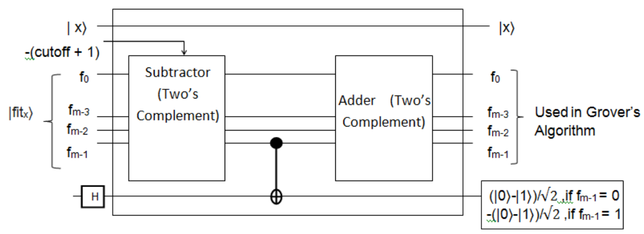

Figure 4: Implementation of the Oracle to mark the fitness state having greater than cutoff value

5 Conclusion

The paper focuses on the conversion of a perfect maze into a quantum search problem and to approach towards the optimum solution in finite number of steps using Grovers Algorithm. The fitness gate is defined to calculate the fitness of each individuals, which is a criteria in selecting the optimum individuals. An Oracle structure has been proposed to mark certain individual states having fitness levels above the cutoff value. Grovers operator iterates subsequently, and with each iteration, the closeness to the optimum solution increases.

In the first iteration, if **m** individuals have fitness levels above the *cutoff*, an O($\sqrt{\frac{n}{m}}$) steps are used in the first iteration, with **n** being the total number of individuals. Next iteration is performed on the previous marked states, and so on, until the end of iteration is reached, or an optimum solution is found out.

Grovers algorithm provides only the quadratic speedup, hence a solution is not guaranteed in the polynomial iteration steps. Subsequent search algorithms with more speedup would boost the approach towards the optimum solution.

References

[1] L. K. Grover. A fast quantum mechanical algorithm for database search. *Proceedings of 28th Annual ACM Symposium on Theory of Computing (STOC)* pages 212-219, May 1996.

[2] L. Hyafil and R. L. Rivest. Constructing Optimal Design Trees is NP-Complete. *Information Processing Letters* , 1976.

[3] H. A. Priestley and M. P. Ward. A Multipurpose Backtracking Algorithm. *J. Symb. Comput.* 18(1994) 1-40.

[4] H. F. Legard, P. E. Mcquaid, A. Singer. From Baker Street to Binary: An Introduction to Computers and Computer Programming. (*New York: McGraw-Hill* , 1983).

[5] M. A. Nielsen and I. L.Chuang. Quantum Computation and Quantum Information *New York: Cambridge University Press* , 2000.

[6] D. Goswami. Laser phase modulation approaches towards ensemble quantum computing *Physical Review Letters* 88(17), 177901/1-177901/4 , 2002.

[7] N. S. Choubey. A-Mazer with Genetic Algorithm *International Journal of Computer Applications* (0975 8887), 2012.