Constructing method of 2-EPP with different quantum error correcting codes

Daichi SASAKI[1] * Tsuyoshi Sasaki USUDA[1] †

[1] *School of Information Science and Technology, Aichi Prefectural University, 1522-3 Ibaragabasama, Nagakute city, Aichi 480-1198, Japan.*

Abstract. For quantum information systems, 'entanglement' is an important resource. However, entangled states are affected by noisy quantum channels if a sender transmits a portion of an entangled state to a receiver creating an entanglement between sender and receiver. To handle noise, entanglement purification protocols (EPPs) were proposed. In a previous study, the superiority of 2-EPPs to 1-EPPs with finite entangled states was shown for a phase-damping channel. We propose here a method of constructing an EPP, which uses two (or more) quantum error correcting codes. Also for phase-damping channels, we assess the performance of the EPP constructed by our method.

Keywords: entanglement, entanglement purification, quantum error correction

1 Introduction

Entanglement purification protocols(EPPs) are important to share entangled states over a noisy quantum channel[1]. EPPs consist of fundamental procedures called "local operations and classical communication(LOCC)". In Ref.[2], a method was given that converted an arbitrary $[n, k]$stabilizer code to a 2-EPP. In the present paper, we consider EPPs constructed from two (or more) quantum error correcting codes and show that our method has higher performance in comparison with those using individual codes.

2 Preliminaries

In this study, we consider the following problem. Alice prepares n Bell states

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \tag{1}$$

and sends half of each to Bob over a noisy quantum channel. Then, each one applies an EPP to these. If they keep their pairs, they share k entangled states.

In Ref.[3], the superiority of 2-EPPs to 1-EPPs with finite entangled states was shown for the phase-damping channel, which is represented as

$$\mathcal{E}_{PD}(\rho) = (1-p)\rho + pZ\rho Z^\dagger, \tag{2}$$

where $0 \le p \le 1$. However, better 2-EPPs are desired for phase-damping channels. For this reason and also for simplicity, we shall assume communications are over a phase-damping channel to examine the performance of our construction method.

3 Method of constructing EPP

3.1 Construction method

In this section, we show a method to construct an EPP that uses two (or more) quantum error correcting codes. We consider two stabilizer codes, $C_1 : [n, k_1]$code and $C_2 : [n, k_2]$code. Let $G^{(1)}$ and $G^{(2)}$ be sets of generators for C_1 and C_2, respectively:

$$\begin{aligned} G^{(1)} &= \{g_1^{(1)}, g_2^{(1)}, \cdots, g_{n-k_1}^{(1)}\}, \\ G^{(2)} &= \{g_1^{(2)}, g_2^{(2)}, \cdots, g_{n-k_2}^{(2)}\}. \end{aligned} \tag{3}$$

Defining

$$\begin{aligned} S^{(1)} &= \langle g_1^{(1)}, g_2^{(1)}, \cdots, g_{n-k_1}^{(1)}\rangle, \\ S^{(2)} &= \langle g_1^{(2)}, g_2^{(2)}, \cdots, g_{n-k_2}^{(2)}\rangle, \end{aligned} \tag{4}$$

as the stabilizers of C_1 and C_2, we introduce the set C obtained from these two stabilizers as

$$C = S^{(1)} \cap S^{(2)}. \tag{5}$$

Let $C' = \{c'_1, c'_2, \cdots, c'_l\}$ be a set of generators of C. Because $C' \subset C \subset S^{(1)}, S^{(2)}$, we have the following generators of C_1 and C_2:

$$\begin{aligned} G^{(1)'} &= \{g_1^{(1)'}, g_2^{(1)'}, \cdots, g_{n-k_1}^{(1)'}\}, \\ G^{(2)'} &= \{g_1^{(2)'}, g_2^{(2)'}, \cdots, g_{n-k_2}^{(2)'}\}, \end{aligned} \tag{6}$$

where

$$g_i^{(1)'} = g_i^{(2)'} = c'_i \ \ (i = 1, \cdots, l). \tag{7}$$

The proposed protocol using $G^{(1)'}$ and $G^{(2)'}$ is performed as follows:

- Alice measures $c'_1, \cdots, c'_l$ on her own quantum states and obtains a measurement outcome $\boldsymbol{a}_{C'} = (a_{c'_1}, \cdots, a_{c'_l})$.
- Bob measures $c'_1, \cdots, c'_l$ on his own quantum states and obtains a measurement outcome $\boldsymbol{b}_{C'} = (b_{c'_1}, \cdots, b_{c'_l})$.
- Alice and Bob send their measurement outcomes to the other and each performs the following two processes according to error syndromes

$$\begin{aligned} \boldsymbol{s}_{C'} &= \boldsymbol{a}_{C'} \oplus \boldsymbol{b}_{C'} \\ &= (a_{c'_1} \oplus b_{c'_1}, \cdots, a_{c'_l} \oplus b_{c'_l}). \end{aligned} \tag{8}$$

*im131007@cis.aichi-pu.ac.jp
†usuda@ist.aichi-pu.ac.jp

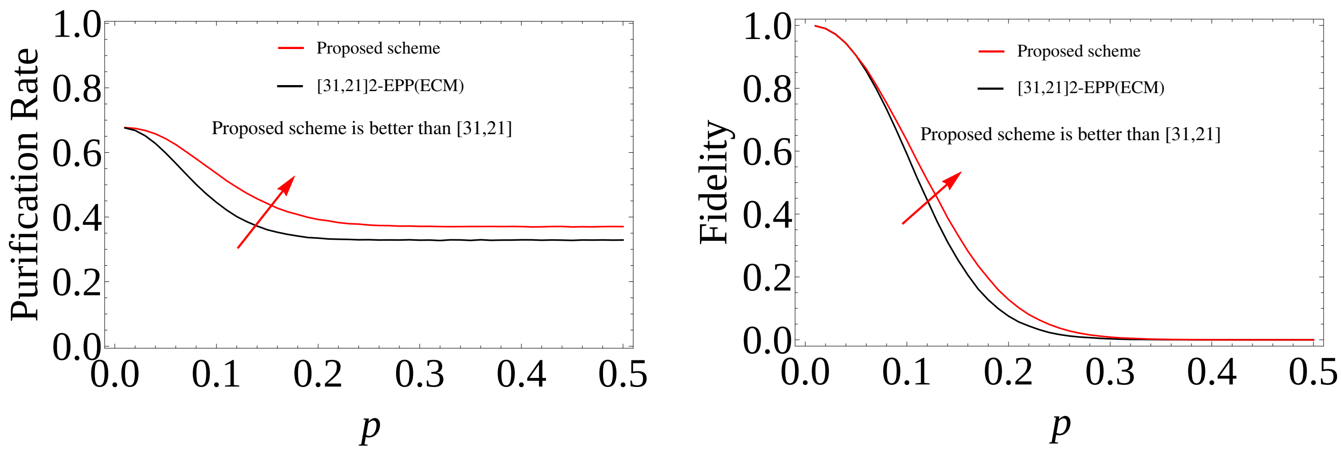

Figure 1: proposed method

- If $s_{C'} \in \mathcal{R}$, Alice and Bob measure the remaining operators $g_{l+1}^{(1)'}, \cdots, g_{k_1}^{(1)'}$, then calculate syndromes and perform further processing depending on all the error syndromes.
- If $s_{C'} \notin \mathcal{R}$, Alice and Bob measure the remaining operators $g_{l+1}^{(2)'}, \cdots, g_{k_2}^{(2)'}$, then calculate syndromes and perform further processing depending on all the error syndromes.

Here, $\mathcal{R} \subset \mathbb{F}_2^l$ is the subset of all syndromes which are obtained by measuring C'. Therefore, a procedure in the protocol is changed according to whether $s_{C'}$ is or is not in $\mathcal{R}$.

3.2 Performance

In this section, we evaluate by simulations the performance of 2-EPPs consisting of different quantum error correcting codes. Here, we use $[31, 21]$code and $[31, 16]$code. Because we are considering phase-damping channels, each generator consists of I or X. The parameters of the protocol are $n = 31$, $k_1 = 21$, $k_2 = 16$, and $l = 10$.

We use the fidelity between k Bell state $|\Phi^+\rangle^{\otimes k}$ and shared states $\rho_{\rm out}$ after purification and a purification rate which is defined as

$$R_P = \frac{k}{n} P_S, \tag{9}$$

where P_S is the success probability of the EPP.

In the following, we consider the 2-EPP from the $[31, 21]$code as the 'standard' protocol and compare it with the other protocols.

We compared the proposed protocol with the standard protocol (Fig.1), from which we find that the proposed protocol is superior to $[31, 21]$2-EPP both in fidelity and in purification rate.

4 Conclusion

In this paper, we proposed a method to construct a 2-EPP which consists of different quantum error correcting codes and by simulations investigated the performance of the 2-EPPs for a phase-damping channel. The proposed protocol showed improved fidelity and purification rate compared with an EPP from a single code when the number of initial shared entanglement is 31.

Although we have shown that the EPP by our method achieves a higher performance, we need to evaluate our method in general quantum channels(e.g. i.i.d. depolarizing channel).

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