Bell inequalities under one sided relaxation of physical constraints

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Abstract.
Any model simulating quantum correlations must either individually or jointly give up one of the physical constraints (used to frame Bell inequalities) such as no signaling, determinism, measurement independence, etc. Recently, Hall (PRA, **84**, 022102 (2011)) derived different forms of Bell inequalities under the assumption of individual or joint relaxation of those physical properties on both sides. In this work, we have derived a Bell-type inequality under the assumption of joint relaxation of determinism and no signaling on one side. It is shown that Bell-type inequality takes a different form due to increase in measurement settings per party for the previous case. We also relaxed no signaling, determinism and measurement independence simultaneously on one side and framed corresponding Bell-type inequalities. In each case, we have obtained the minimum degree of relaxation of these physical properties for any model violating a standard Bell inequality.

Keywords: Bell Inequality, Determinism, No signaling, Measurement Independence.

In 1964, J.S. Bell introduced an *inequality* [1] thereby showing *no realistic physical theory which is also local in a specified sense can agree with all of the statistical implications of Quantum Mechanics.* Different versions of the theorem (termed as Bell's Thoerem), inspired by the paper [1], are considered as a family and the corresponding inequalities are termed as Bell-type inequalities. Now, various plausible physical postulates are at the background of framing Bell inequalities e.g., no signaling(NS), measurement independence(MI), locality, determinism. Violation of these inequalities by any physical theory give rise to some queries: *are the predictions made by the theory incorrect?* or, *whether at least one of these postulates incompatible with the description of the natural phenomena?* Agreement of quantum mechanical predictions with the experimental data implies that the second query is more relevant. Till now various literatures have dealt with the relaxation of physical constraints for framing Bell inequalities [2, 3, 4, 5, 6, 7]. In [4], Hall considered relaxation of measurement independence. He argued that for violation of Bell-CHSH inequality by any singlet correlations at least 86% percentage of MI must be relaxed. In [5], he introduced a Bell inequality considering joint relaxation of NS and determinism and showed that at least 60% of signaling and 41% of indeterminism must be introduced in the Bell-CHSH model to justify the violation shown by singlet correlations. In [6], a relaxed Bell-type inequality was introduced under the assumption of joint relaxation of NS, determinism and MI. The main objective of all these papers [4, 5, 6] was to simulate singlet correlations assuming both side relaxation of these physical constraints. The question that naturally arises in this context is, whether relaxation of these physical properties on one side can simulate singlet correlations?

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In this work, we have investigated whether relaxation of one sided NS is more useful as a resource for simulation of singlet correlations than that of both sided relaxation of the same. We have succeeded in showing that the minimum degree of relaxation has decreased from 60% to 17%. We also investigate one sided joint relaxation of NS, determinism and MI. Interestingly, for one sided relaxation, maximal violation of Bell inequality cannot be achieved unlike that in the case of both sided relaxation [5, 6]. Moreover, depending on the complementary relation, we obtain two subintervals (2,3) and (2,4) of the total violation (2,4). We now describe briefly our work.

No signaling: In a multi-party system correlations must obey the principle of no signaling: *the choices of observable by one party cannot influence the statistics observed by the remaining parties.* i.e., if $p(a|x, y, \lambda) = p(a|x, y', \lambda)$, $p(b|x, y, \lambda) = p(b|x', y, \lambda)$ hold for all pairs of measurements. The degree of signaling is defined by the maximum shift possible in an underlying marginal probability distribution for one observer, due to the alteration of measurement setting of the other. One may formulate it as follows [5, 6]: $S_{1\to 2} := \sup_{x,x',y,b,\lambda} |p^{(2)}(b|x, y, \lambda) - p^{(2)}(b|x', y, \lambda)|$, $S_{2\to 1} := \sup_{x,y,y',a,\lambda} |p^{(1)}(a|x, y, \lambda) - p^{(1)}(a|x, y', \lambda)|$ where a, b, x, x', y and y' have their usual meanings. The overall degree of signaling, for a given underlying model is defined by, $S := \max\{S_{1\to 2}, S_{2\to 1}\}$.

Determinism: A model is said to be deterministic if the observed statistical correlations are generated by averaging over a set of all possible values of the underlying variable (λ) such that for any fixed value of the variable(λ) all measurement outcomes are fully determined [5]. In such a model all the outcomes being predictable with certainty for any given knowledge of λ, where correlation terms and the marginals are either 0 or 1, i.e., $p(a, b|x, y, \lambda), p(a|x, y, \lambda), p(b|x, y, \lambda) \in \{0, 1\}$. The local degree of indeterminism I_j may be defined [5, 6] as the smallest positive number, such that the corresponding marginal probabilities lie in $[0, I_j] \bigcup [1-I_j, 1]$, i.e., $I_j := \sup_{\{x,y,\lambda\}} \min_z\{p^j(z|x,y,\lambda), 1-p^j(z|x,y,\lambda)\}$. Thus, $I_j = 0$ if and only if the corresponding marginal is deterministic. The overall degrees of indeterminism for the model may be defined as, $I := \max\{I_1, I_2\}$. Hence $0 \leq I \leq 1/2$, with $I = 0$ if the model is fully deterministic. For *one sided indeterministic* model, $I_1 = 0, \quad 0 < I_2 \leq 1/2$. Hence, $I = I_2$.

Duality between Signaling And Determinism: Any deviation in a marginal probability value p, due to signaling, must either keep the value in the same subinterval $[0, I]$ (or, $[1-I, 1]$) $(S \leq I)$, or shift the value across the gap between the subintervals $(S \geq 1-2I)$leading to $I \geq \min\{S, (1-S)/2\}$.

Measurement Independence: It is the property of a model such that the distribution of the underlying variable is independent of the measurement settings chosen by the experimenters. i.e., $p(\lambda|x,y) = p(\lambda|x',y')$ for every joint settings (x,y), (x',y'). It is satisfied by quantum system. Thus, *Measurement dependence(M)* may be interpreted as a measure to quantify the degree of violation of MI by the underlying model. It is defined as [4]: $M := \sup_{x,x',y,y'} \int d\lambda |p(\lambda|x,y) - p(\lambda|x',y')|$. Therefore, for MI, $M = 0$. $M_{max} = 2$ implies complete measurement dependence. The fraction of MI corresponding to a given model is defined by [4], $F := 1 - M/2$. Thus $0 \leq F \leq 1$, and $F = 0$ corresponds $M = 2$. The local degrees of measurement dependence defined by [4], $M_1 := \sup_{x,x',y} \int d\lambda |p(\lambda|x,y) - p(\lambda|x',y)|$; $M_2 := \sup_{x,y,y'} \int d\lambda |p(\lambda|x,y) - p(\lambda|x,y')|$. So, local degrees of MI are, $F_1 := 1 - M_1/2$; $F_2 := 1 - M_2/2$.

Bell Inequality Under Relaxation of Determinism and NS on One Side: To generate models violating Bell inequality, the properties of NS and that of determinism are relaxed to some extent and the extent of relaxation can be quantified with the help of the corresponding relaxed Bell-type inequality [5]. We describe a model now where one sided signaling and indeterminism are introduced.

In a bipartite system (Alice and Bob), it is assumed that determinism and NS are preserved by the correlations shown by Alice's measurement. A signal is sent to Bob by Alice and it is also assumed that the correlations in Bob's part are indeterministic. So for this model $S = S_{1\to 2}$ and $I = I_2$. Then extent of minimum possible relaxation is given by;

Theorem 1: Let x, x' and y, y' denote possible measurement settings for Alice and Bob and label each measurement outcome by 1 or -1. Let $\langle XY \rangle$ be the average product of the measurement outcomes. Then, for any underlying model having values of indeterminism and signaling of at most I and S, Bell inequality takes the form: $\langle XY \rangle + \langle XY' \rangle + \langle X'Y \rangle - \langle X'Y' \rangle \leq B(I,S)$ with upper bound $B(I,S) = 2 + 2I$ for $S < 1 - 2I$(tight upper bound), and $B < 4$ for $S \geq 1 - 2I$.

Implications: Original form of Bell inequality is derivable with $I = S = 0$, i.e., $B(0,0) = 2$. This theorem exerts bounds on the minimum possible degrees of indeterminism and signaling that exist in such a model. For singlet state correlations, $V = 2\sqrt{2} - 2$ [8] where V is the amount of violation. Thus, any singlet state model must assign at least 82% of indeterminism, and/or communicating at least 17% of signaling. Maximal Violation Cannot Be Reached. Due to the duality between indeterminism and signaling, we get two subintervals $\mathfrak{R}_1$ and $\mathfrak{R}_2$ of the interval $\mathfrak{R} = (2,4)$ of violation. For $S < 1-2I$, $\mathfrak{R}_1 = (2,3)$ and for $S \geq 1 - 2I$, $\mathfrak{R}_2 = (2,4)$.

Bell Inequalities Under Simultaneous Relaxation of Determinism, NS and MI on One Side: Here, we assume that joint relaxation is done only on Bob's side, i.e., $I_2 > 0$, $S_{1\to 2} > 0$ and $M_2 > 0$, whereas at the same time Alice maintains $I_1 = 0, S_{2\to 1} = 0$ and $M_1 = 0$. Thus, $I > 0$ and $S > 0$.

Theorem 2: $\langle XY \rangle + \langle XY' \rangle + \langle X'Y \rangle + \langle X'Y' \rangle \leq B(I,S,M)$ where I, S and M are the values of indeterminism, signaling and measurement dependence respectively, for any underlying models. $B(I,S,M) = 4 - (1-I)(2-M)$, for $S < 1 - 2I$ and $M < 2$ (tight upper bound) and $B < 4$, otherwise.

Implications: Any singlet state model must either assign at least 82% of uncertainty and/or predict a change of at least 17% and/or relax MI by 59% for one party in response to a measurement performed on the other party. Here, both $I = 0$ and $S = 0$ and $B(0,0,M) = min\{2 + M, 4\}$. Hence, a local deterministic model exists for simulating a singlet state correlation if and only if $M \geq V = 2\sqrt{2} - 2 \approx 0.82$. So 59% MI is optimal for simulating singlet correlation when measurement dependency is allowed only on one side. Our relaxed Bell inequality gives a general result and [7] can be obtained as a particular case.

In our work, we then considered relaxed Bell inequalities for more than two settings. Thus, we have considered here only asymmetric relaxation of constraints and have framed Bell-type inequalities under such assumptions. We can safely now conclude that the results derived in one sided relaxation scenarios do not tally exactly with that of in the both sided cases. The existing duality between both sided signaling and indeterminism also holds in the one sided case. But the minimal degree of relaxation of the constraints in the one sided cases differ from that of both sided cases. There still remain many other topics of discussion for one sided relaxation of physical constraints.**For details see arXiv:1304.7409**

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