# Remote State Preparation of Arbitrary Two-Qubit State with Unit Success Probability


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**Abstract.** Two schemes for the remote state preparation (RSP) of arbitrary two-qubit state were presented by Zha et al. [Opt. Commun. **284** (2011) 1472] and Wang et al. [Opt. Commun. **284** (2011) 5853]. In both these schemes, RSP could be realized with probability 1/4, in general but in some special cases this probability could be improved to 1/2 or even 1. Here, we present a scheme for RSP of arbitrary two-qubit state with unit success probability based on the protocol of An et al. [Adv. Nat. Sci.: Nanosci. Nanotechnol. **2** (2011) 035009] for RSP of one qubit.



**Keywords:** RSP, sequential measurement, projective measurement, unitary transformation


Quantum information processing is not possible without quantum entanglement [1]. Quantum teleportation (QT) [2] and remote state preparation (RSP) [3, 4, 5, 6] are two important applications of quantum entanglement. QT is used to transmit an unknown state by a sender to a receiver at distant location. On the other hand, RSP is used to prepare a known state by a sender at a remote receiver's end. Recently, many RSP schemes have been proposed for arbitrary two-qubit state [7, 8, 9, 10, 11, 12, 13, 14, 15]. Zha et al. [8] obtained RSP of two-qubits by using four-qubit cluster state. Wang et al. [9] obtained the same using joint RSP using six-qubit cluster state and an extra sender who makes projective measurements. In both, success is 1/4 only in general but can be even 1 for some special cases. An et al. [6] used resource with an extra qubit and sequential measurements to obtain RSP of one qubit with success unity. In this paper we generalize this for RSP of two-qubit state using four-qubit entangled state and two ancillary qubits with perfect success for all cases.

Consider an arbitrary two-qubit state, possessed by Alice, described by

$$|I\rangle = \lambda_0|00\rangle + \lambda_1 e^{i\delta_1}|01\rangle + \lambda_2 e^{i\delta_2}|10\rangle + \lambda_3 e^{i\delta_3}|11\rangle, \quad (1)$$

where $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ are non-negative real coefficients and $0 \leq \delta_1, \delta_1, \delta_1 < 2\pi$ are the phase angles with the normalization condition $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$. Suppose that given arbitrary two-qubit state is known completely to Alice, but not to Bob. Initially, Alice takes two ancillary qubits $|00\rangle_{12}$, and she shares four-qubit state with Bob as quantum channel given by the expression

$$|E\rangle_{3456} = \frac{1}{2}(|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle)_{3456}, \quad (2)$$

where particles (3, 4) are in the possession of Alice and particles (5, 6) belongs to Bob. Now, Alice performs two controlled-NOT (CNOT) gates on the qubits (1, 3) and (2, 4), with 3 and 4 are control qubits and 1 and 2 are targets, respectively. As a result, the four-qubit entangled state $|E\rangle_{3456}$ and the states $|00\rangle_{12}$ become a six-qubit entangled state

$$|E'\rangle_{123456} = \frac{1}{2}(|000000\rangle + |010101\rangle + |101010\rangle + |111111\rangle). \quad (3)$$

At this stage, Alice measures the particles (1, 2) and (3, 4) in different bases. For the particles (1, 2), the measurement basis is defined as

$$\begin{pmatrix} |\xi_0\rangle_{12} \\ |\xi_1\rangle_{12} \\ |\xi_2\rangle_{12} \\ |\xi_3\rangle_{12} \end{pmatrix} = \begin{pmatrix} \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1 & -\lambda_0 & \lambda_3 & -\lambda_2 \\ \lambda_2 & -\lambda_3 & -\lambda_0 & \lambda_1 \\ \lambda_3 & \lambda_2 & -\lambda_1 & -\lambda_0 \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix},$$

while for the particles (3, 4) measurement basis is

$$\begin{pmatrix} |\zeta_0\rangle_{34} \\ |\zeta_1\rangle_{34} \\ |\zeta_2\rangle_{34} \\ |\zeta_3\rangle_{34} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \epsilon_1 & \epsilon_2 & \epsilon_3 \\ 1 & -\epsilon_1 & \epsilon_2 & -\epsilon_3 \\ 1 & -\epsilon_1 & -\epsilon_2 & \epsilon_3 \\ 1 & \epsilon_1 & -\epsilon_2 & -\epsilon_3 \end{pmatrix} \begin{pmatrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{pmatrix},$$

where $\epsilon_j = e^{-i\delta_j}, j = 1, 2, 3$. After the measurements, Alice transmits four classical bits information about her measurement outcomes to the receiver Bob. Bob then reconstructs the original state on his particles 5 and 6 conditioned on the classical information from Alice.

Under these two sets of bases, i.e., if measurements of the particles (1, 2) in the basis $\{|\xi_m\rangle_{12}\}$ and measurements of the particles (3, 4) in the basis $\{|\zeta_m\rangle_{34}\}(m = 0, 1, 2, 3)$ are carried out independently, it can be shown that Bob gets the original state with success 1/4 only. However, with the strategy of sequential measurements [6], where outcome of first measurement decides a unitary transformation to be done by Alice on her two particles (3, 4). The first measurement is to be done on the particles (1, 2) in the basis $\{|\xi_0\rangle_{12}, |\xi_1\rangle_{12}, |\xi_2\rangle_{12}, |\xi_3\rangle_{12}\}$, whose outcome is specified by $m = 0$ (1, 2, 3) if $|\xi_0\rangle_{12}$, $(|\xi_1\rangle_{12}, |\xi_2\rangle_{12}, |\xi_3\rangle_{12})$ is found. Then, depending on the outcome $m$, Alice performs the unitary phase shift operator, $\prod^{(m)}$, on the particles (3, 4), which are given by the expressions $\prod^{(0)} = \hat{I} = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$, $\prod^{(1)} = e^{i\delta_1}|00\rangle\langle 00| + e^{-i\delta_1}|01\rangle\langle 01| + e^{i(\delta_3-\delta_2)}|10\rangle\langle 10| + e^{i(\delta_2-\delta_3)}|11\rangle\langle 11|$, $\prod^{(2)} = e^{i\delta_2}|00\rangle\langle 00| +$


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$e^{i(\delta_3-\delta_1)}|01\rangle\langle01| + e^{-i\delta_2}|10\rangle\langle10| + e^{i(\delta_1-\delta_3)}|11\rangle\langle11|$ and $\prod^{(3)} = e^{i\delta_3}|00\rangle\langle00|+e^{i(\delta_2-\delta_1)}|01\rangle\langle01|+e^{i(\delta_1-\delta_2)}|10\rangle\langle10|+e^{-i\delta_3}|11\rangle\langle11|$. After this, she measures her particles (3, 4) in the basis $\{|\zeta_0\rangle_{34}, |\zeta_1\rangle_{34}, |\zeta_2\rangle_{34}, |\zeta_3\rangle_{34}\}$. Now, Alice transmitts four bits of classical information to Bob for identifying her sixteen possible measurement outcomes in the following way:

'0000', '0001', '0010', '0011',
'0100', '0101', '0110', '0111',
'1000', '1001', '1010', '1011',
'1100', '1101', '1110' or '1111',

if she found
$|\xi_0\rangle_{12}|\chi_{00}\rangle_{34}, |\xi_0\rangle_{12}|\chi_{01}\rangle_{34}, |\xi_0\rangle_{12}|\chi_{02}\rangle_{34}, |\xi_0\rangle_{12}|\chi_{03}\rangle_{34}$,
$|\xi_1\rangle_{12}|\chi_{10}\rangle_{34}, |\xi_1\rangle_{12}|\chi_{11}\rangle_{34}, |\xi_1\rangle_{12}|\chi_{12}\rangle_{34}, |\xi_1\rangle_{12}|\chi_{13}\rangle_{34}$,
$|\xi_2\rangle_{12}|\chi_{20}\rangle_{34}, |\xi_2\rangle_{12}|\chi_{21}\rangle_{34}, |\xi_2\rangle_{12}|\chi_{22}\rangle_{34}, |\xi_2\rangle_{12}|\chi_{23}\rangle_{34}$,
$|\xi_3\rangle_{12}|\chi_{30}\rangle_{34}, |\xi_3\rangle_{12}|\chi_{31}\rangle_{34}, |\xi_3\rangle_{12}|\chi_{32}\rangle_{34}$ or $|\xi_3\rangle_{12}|\chi_{33}\rangle_{34}$,
respectively, where $|\chi_{jk}\rangle_{34} = \prod^{(j)} |\zeta_k\rangle_{34}$ with $j, k = 0, 1, 2, 3$. On the basis of Alice's classical information, Bob performs suitable unitary operation on his particles (5, 6) to prepare the required state (1). Results are summarized in the following Table 1.

Table 1: Alice's first measurement basis (FMB) $\{|\xi_m\rangle_{12}\}$ on particle (1, 2), whose outcome decides the unitary phase shift operator (UPSO) $\prod^{(m)}$ on particle (3, 4) followed by second measurement basis (SMB) $\{|\zeta_m\rangle_{34}\}$, four bits of classical information (CI) sent to Bob and Bob's unitary transformation (UT) on particle (5, 6).

| FMB (1, 2) | UPSO (3, 4) | SMB (3, 4) | CI | UT (5, 6) |
|---|---|---|---|---|
| $\vert\xi_0\rangle$ | $\prod^{(0)}$ | $\vert\zeta_0\rangle$ | 0000 | $\hat{I}\otimes\hat{I}$ |
| | | $\vert\zeta_1\rangle$ | 0001 | $\hat{I}\otimes\hat{\sigma}_z$ |
| | | $\vert\zeta_2\rangle$ | 0010 | $\hat{\sigma}_z\otimes\hat{\sigma}_z$ |
| | | $\vert\zeta_3\rangle$ | 0011 | $\hat{\sigma}_z\otimes\hat{I}$ |
| $\vert\xi_1\rangle$ | $\prod^{(1)}$ | $\vert\zeta_0\rangle$ | 0100 | $\hat{I}\otimes\hat{\sigma}_x\hat{\sigma}_z$ |
| | | $\vert\zeta_1\rangle$ | 0101 | $\hat{I}\otimes\hat{\sigma}_x$ |
| | | $\vert\zeta_2\rangle$ | 0110 | $\hat{\sigma}_z\otimes\hat{\sigma}_x$ |
| | | $\vert\zeta_3\rangle$ | 0111 | $\hat{\sigma}_z\otimes\hat{\sigma}_x\hat{\sigma}_z$ |
| $\vert\xi_2\rangle$ | $\prod^{(2)}$ | $\vert\zeta_0\rangle$ | 1000 | $\hat{\sigma}_x\hat{\sigma}_z\otimes\hat{\sigma}_z$ |
| | | $\vert\zeta_1\rangle$ | 1001 | $\hat{\sigma}_x\hat{\sigma}_z\otimes\hat{I}$ |
| | | $\vert\zeta_2\rangle$ | 1010 | $\hat{\sigma}_x\otimes\hat{I}$ |
| | | $\vert\zeta_3\rangle$ | 1011 | $\hat{\sigma}_x\otimes\hat{\sigma}_z$ |
| $\vert\xi_3\rangle$ | $\prod^{(3)}$ | $\vert\zeta_0\rangle$ | 1111 | $\hat{\sigma}_x\hat{\sigma}_z\otimes\hat{\sigma}_x$ |
| | | $\vert\zeta_1\rangle$ | 1101 | $\hat{\sigma}_x\hat{\sigma}_z\otimes\hat{\sigma}_x\hat{\sigma}_z$ |
| | | $\vert\zeta_2\rangle$ | 1110 | $\hat{\sigma}_x\otimes\hat{\sigma}_x\hat{\sigma}_z$ |
| | | $\vert\zeta_3\rangle$ | 1111 | $\hat{\sigma}_x\otimes\hat{\sigma}_x$ |

In conclusion, we prepare an arbitrary two-qubit state remotely via four-qubit entangled state with unit success probability with the use of two ancillary qubits and the sequential measurement techniques. Since after performing two CNOT gates the combined state of particles 1, 2, 3, 4, 5 and 6 becomes six-qubit entangled state (3), one can say that our present scheme is RSP of an arbitrary two-qubit state via six-qubit entangled state with unit success probability. The advantage of our scheme is that we achieve perfect success for all cases.

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