Entanglement enhances performance in microscopic quantum fridges

Nicolas Brunner,[1,2] Marcus Huber,[3,4,5] Noah Linden,[3] Sandu Popescu,[2] Ralph Silva,[2] and Paul Skrzypczyk[4]

[1]*Département de Physique Théorique, Université de Genève, 1211 Genève, Switzerland*
[2]*H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom*
[3]*Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom*
[4]*ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain*
[5]*Universitat Autonoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

Understanding the thermodynamics of quantum systems is of fundamental importance, from both theoretical and experimental perspectives. A growing interest has been recently given to small self-contained quantum thermal machines, which require no external source of work or control, but only incoherent interactions with thermal baths. Their simplicity makes such machines ideal test-beds for exploring quantum thermodynamics. So far, however, the importance of quantum effects in these machines has remained elusive. Here we show that entanglement plays a fundamental role in small self-contained quantum refrigerators, as it can enhance cooling and energy transport – except notably near the Carnot limit. Furthermore, the amount of entanglement alone quantifies the enhancement in cooling.

Keywords. — **thermodynamics, entanglement, thermal machines, fridge, Carnot, efficiency**

Introduction. — The study of quantum thermal machines has a long history, from the thermodynamic analysis of lasers [1–3], to considerable work on quantum cycles and the second law[4–17]. Recently, models of small self-contained quantum thermal machines [18–21] have attracted growing attention. The key feature of such machines is that they function without any external source of work or control. Only incoherent interaction with thermal baths is required. Interestingly, there exists no fundamental limit on the size of such machines nor on their efficiency [18]. Their main interest resides in their simplicity, which allows us to explore novel ideas in quantum thermodynamics.

The working of such machines is important to further our understanding of the notions of work and entropy for quantum systems. In recent work [21], it has been shown that these machines are bound by the classical Carnot limit as regards efficiency. The notion of work has also been discussed at length since the storage of energy on a quantum scale suffers from practical difficulties arising from decoherence and the uncertainty principle that are not apparent in classical discussions of the same. Clearly for the purpose of quantum computation, or any process that we wish to have control of at the scale of individual qubits, such thermal machines are essential. We can use the quantum refrigerator or heat engine to reset a qubit register to its ground or excited state respectively, or use a quantum work engine to charge a "quantum battery" as an energy source to drive the unitaries that we run any computation by.

An important question which has not been addressed so far is whether quantum effects play any significant role in such small self-contained thermal machines. Indeed, although these machines are described within the formalism of quantum mechanics, it is not immediately clear to what extent their working is inherently quantum, since one can give an heuristic account of their functioning in classical terms.

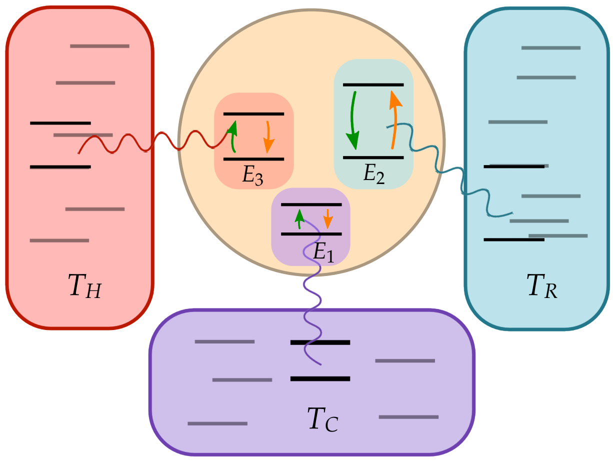

FIG. 1: The fridge contains three qubits (yellow circle), each in weak thermal contact (wiggly lines) with a bath at a different temperature. The qubits interact via a weak interaction Hamiltonian H_{int} which couples the two degenerate levels $|010\rangle$ and $|101\rangle$, depicted by the arrows. The lower qubit (purple) is the object to be cooled. At equilibrium, it reaches a temperature $T_S < T_C$. The other two qubits (red and blue) are the machine qubits, connected to heat baths at temperatures T_R and T_H.

In the paper, our aim is to establish the importance of quantum effects in these thermal machines. Our main focus will be on the concept of entanglement, often considered as the defining feature of quantum theory. Hence, if entanglement turns out to play an important role in such machines, it would be clear that their working is truly quantum mechanical. Moreover, it would then raise the question of whether entanglement can enhance the performance of such machines. We address these questions focusing our attention on the model of the smallest possible self-contained quantum refrigerator [18], as shown schematically in Fig. 1.

Summary of Methods and Results. — Considering the model of the quantum fridge, the first question addressed was whether there was entanglement present in any regime of the working of the fridge. Since we have a system of 3 qubits, entanglement can be of several kinds, across a bipartition of the system, or genuine tripartite entanglement. Using the class of entanglement witnesses developed in [22, 23], we found that there exist regimes containing entanglement across one, two or all 3 bipartitions of the system, and even regimes demonstrating genuine multipartite entanglement.

We then investigated the Carnot point of the fridge, i.e. the set of parameters for which the fridge operates at Carnot efficiency. One of the fundamental results in [21] was that every 3 qubit thermal machine can reach and is bound by the classical Carnot efficiency derived from the temperatures of the hot and cold thermal baths.

However we found that in fact the Carnot point itself corresponded to the fridge being in a product state of the 3 qubits of the fridge, and were able to prove that around the Carnot point of any fridge working at finite temperatures, there existed a neighbourhood of states of the fridge that were completely biseparable.

While at first glance this may appear to suggest that entanglement is detrimental to the efficiency of such machines, we must take into account that the Carnot point of any thermal machine corresponds to the machine working reversibly, and hence necessarily with *infitesimal* power. However we are interested in whether quantum fridges are physically feasible for cooling, and hence require the effect of entanglement on fridges that achieve the best possible cooling, with respect to the temparature to which we can cool our system.We thus sought the answer to the question : *For a given qubit, is the temparature that a entangled quantum fridge can cool it to lower than the temparature achievable using a completely separable quantum fridge?*

The result was indeed that there existed situations in which entangled fridges cooled qubits to lower temparatures than fridges constrained to be biseparable, suggesting the usefulness of entanglement for cooling. Furthermore, we found that the entanglement present was almost always across the bipartition separating the room qubit from the hot and cold qubits. Considering that heat flows from the hot and cold baths into the room bath, this implies that entanglement for optimal cooling is between the partition *energy in v/s energy out*, suggesting that quantum coherence enhances energy transport, a phenomenon that has achieved considerable attention in photosynthetic complexes[24].

Indeed, we found that the enhancement in cooling appeared to be determined entirely by the entanglement across the bipartition Room—Cold,Hot. (Fig 2.)

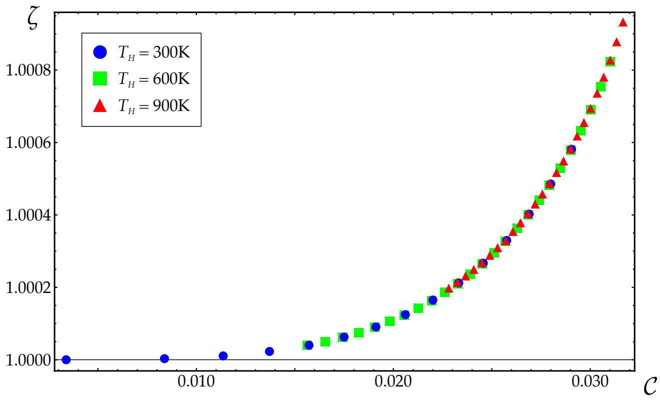

FIG. 2: Plot of relative cooling enhancement ζ against concurrence across the bipartition $R|CH$ for different temperatures of the hot bath. The monotonic behaviour strongly suggests a functional relationship.

[1] N. F. Ramsey, Phys. Rev. **103**, 20 (1956).
[2] H. E. D. Scovil and E. O. Schulz-DuBois, Phys. Rev. Lett. **2**, 262 (1959).
[3] J. E. Geusic, E. O. Schulz-DuBois, and H. E. D. Scovil, Phys. Rev. **156**, 343 (1967).
[4] E. Geva and R. Kosloff, J. Chem. Phys. **104**, 7681 (1996).
[5] J. P. Palao, R. Kosloff, and J. M. Gordon, Phys. Rev. E **64**, 056130 (2001).
[6] C. M. Bender, D. C. Brody, and B. K. Meister, Proc. Roy. Soc. A **458**, 1519 (2002).
[7] B. Lin and J. Chen, Phys. Rev. E **67**, 046105 (2003).
[8] M. O. Scully *et. al*, Science **299**, 862 (2003).
[9] T. E. Humphrey and H. Linke, Physica E **29**, 390 (2005).
[10] D. Segal and A. Nitzan, Phys. Rev. E **73**, 026109 (2006).
[11] M. J. Henrich, M. Michel, and G. Mahler, Europhys. Lett. **76**, 1057 (2006).
[12] M. J. Henrich, G. Mahler, and M. Michel, Phys. Rev. E **75**, 051118 (2007).
[13] E. Boukobza and D. J. Tannor, Phys. Rev. Lett. **98**, 240601 (2007).
[14] H. T. Quan, Y.-X. Liu, C. P. Sun, and F. Nori, Phys. Rev. E **76**, 031105 (2007).
[15] A. E. Allahverdyan, K. Hovhannisyan, and G. Mahler, Phys. Rev. E **81**, 051129 (2010).
[16] M. O. Scully *et. al*, Proc. Nat. Acad. Sci. USA **108**, 15097 (2011).
[17] M. Büttiker and B. Sothmann, Europhys. Lett. **99**, 27001 (2012).
[18] P. Skrzypczyk, N. Brunner, N. Linden, and S. Popescu, J. Phys. A: Math. Theor. **44**, 492002, (2011).
[19] L. A. Correa, J. P. Palao, G. Adesso, D. Alonso, arXiv:1212.4501 (2012).
[20] A. Levy and R. Kosloff, Phys. Rev. Lett. **108**, 070604 (2012).
[21] N. Brunner, N. Linden, S. Popescu and P. Skrzypczyk, Phys. Rev. E **85**, 05111 (2012).
[22] O. Gühne and M. Seevinck, New J. Phys. **12**, 053002 (2010).
[23] M. Huber, F. Mintert, A. Gabriel, B. Hiesmayr, Phys. Rev. Lett. **104**, 210501 (2010).
[24] G. S. Engel *et al.*, Nature **446**, 782786 (2007).