TOPOLOGY I

ASSIGNMENT DUE ON 26 SEPTEMBER 2012

(1) For a collection $\{G_\alpha\}$ of abelian groups show that
$$(\bigoplus_\alpha G_\alpha)\hat{} = \prod_\alpha \hat{G}_\alpha.$$
Here the right hand side is viewed as a topological group with the product topology.

(2) Show that $(\mathbf{Q}/\mathbf{Z})_p\hat{}$ is torsion-free.

(3) For each $n \in \mathbf{Z}$, define $\chi_n \in (\mathbf{Q}/\mathbf{Z})_p\hat{}$ by
$$\chi_n(1/p^i) = n/p^i + \mathbf{Z}.$$
Equip $\mathbf{Z}$ with the topology coming from the p-adic norm. Show that the function $n \mapsto \chi_n$ is continuous with dense image.

(4) Show that $\hat{\mathbf{Q}}$ is torsion-free.

(5) Can you find an algebraic condition on G which ensures that $\hat{G}$ is torsion-free and applies to $\mathbf{Q}/\mathbf{Z}$ and $(\mathbf{Q}/\mathbf{Z})_p$?

(6) Show that $\mathbf{R}$ is not homeomorphic to $\mathbf{R}^2$.

(7) Munkres, §24, problems 8, 10, 11.