TOPOLOGY I

ASSIGNMENT DUE ON 22 AUGUST 2012

(1) Let M be a monoid. Let $\underline{M}$ denote the category with one object $*$ and $\mathrm{Mor}(*,*) = M$. Let id denote the identity functor $\underline{M} \to \underline{M}$. What are the natural transformations $\mathrm{id} \to \mathrm{id}$?

(2) In the category of sets, given two objects X and Y, find an object Z such that the functors $\underline{\mathrm{Sets}} \to \underline{\mathrm{Sets}}$ given by $U \mapsto \mathrm{Mor}(U, Z)$ and $U \mapsto \mathrm{Mor}(U, X) \times \mathrm{Mor}(U, Y)$ are naturally isomorphic.

(3) In the category of sets, given two objects X and Y, find an object Z such that the functors $\underline{\mathrm{Sets}} \to \underline{\mathrm{Sets}}$ given by $U \mapsto \mathrm{Mor}(Z, U)$ and $U \mapsto \mathrm{Mor}(X, U) \times \mathrm{Mor}(Y, U)$ are naturally isomorphic.

(4) Show that the functors $\underline{\mathrm{Sets}} \to \underline{\mathrm{Sets}}$ given by

$$Z \mapsto \mathrm{Mor}(X, \mathrm{Mor}(Y, Z)) \quad \text{and} \quad Z \mapsto \mathrm{Mor}(X \times Y, Z)$$

are naturally isomorphic.

(5) [1] If ^{i} is an operator which carries subsets of X into subsets of X, and $\mathcal{T}$ is the family of all subsets of X such that $A^i = A$, under what conditions will $\mathcal{T}$ be a topology on X and ^{i} be the operator which takes every subset of X to its interior?

(6) Munkres, Section 20, problems 3, 5, and 11.

[1] Kelley, *General Topology*, p. 56