HOMEWORK VIII

TOPOLOGY, JAN-APR 2010

(1) Let A be a finitely generated abelian group. Let ϕ be an endomorphism of A. Let $\phi_{\mathbf{Q}}$ be the endomorphism of $A \otimes_{\mathbf{Z}} \mathbf{Q}$ induces by ϕ. Let $\bar{\phi}$ be the endomorphism of A/A_{tor} induced by ϕ. Show that $\bar{\phi}$ and $\phi_{\mathbf{Q}}$ have the same trace (trace is calculated with respect to a basis in the usual way).

(2) Let A be a finitely generated abelian group, B a subgroup, and ϕ an endomorphism of A that maps B into B. Show that the trace of ϕ is the sum of the trace of the restriction to B of ϕ and the trace of the endomorphism of A/B induced by ϕ.

(3) Give an example of a compact polyhedron X and a continous function $f : X \to X$ such that f has Lefscetz number 0 but is not homotopic to any function that has no fixed point.

(4) Let K be a simplicial complex and $f : K \to K$ be a simplicial map such that on each simplex fixed by f, f is the identity. Show that the Lefschetz number of f is the Euler characteristic of the fixed-point set of f.

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