FINAL EXAMINATION

TOPOLOGY, JAN-APR 2010

(1) If n is even and $f, g : S^n \to S^n$ are fixed-point-free homeomorphisms then $f \circ g$ has a fixed point.
(2) Prove that, for any abelian group B there is a natural isomorphism
$$\mathrm{Tor}(\mathbf{Q}/\mathbf{Z}, B) \to B_{\mathrm{tor}}.$$
(3) Compute the singular homology groups of $\mathbf{R}P^2 \times \mathbf{R}P^2$ ($\mathbf{R}P^2$ denotes the real projective plane).
(4) Prove that Tor is associative: there is a natural isomorphism
$$\mathrm{Tor}(A, \mathrm{Tor}(B, C)) \tilde{\to} \mathrm{Tor}(\mathrm{Tor}(A, B), C)$$

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