$$p(t) = (t+2)(t-6)(t-16)$$

$$p_{-2}(t) = a_0 + a_1 t + a_2 t^2$$

$$a_0 - 2a_1 + 4a_2 = 1 \Rightarrow a_0 = 2a_1 - 3$$

$$a_0 + 6a_1 + 36a_2 = 0 \Rightarrow 8a_1 + 33 = 0$$

$$a_0 + 16a_1 + 256a_3 = 0$$

$$\cancel{a_1 = -\frac{33}{8}}$$

$$\cancel{a_0 = -\frac{33}{4} - \frac{12}{4} = -\frac{45}{4}.}$$

$$\cancel{-\frac{45}{4} - \frac{16 \times 33}{8} + 256}$$

$$p_{-2}(t) = \frac{2}{3} - \frac{11}{72} t + \frac{1}{144} t^2.$$

$$\cancel{p_6(t) = a_0}$$

$$p_6(t) = \frac{2}{5} + \frac{7}{40} t - \frac{1}{80} t^2$$

$$p_{16}(t) = -\frac{1}{15} - \frac{1}{45} t + \frac{1}{180} t^2$$

$$\mathbb{Q}\left[\binom{10}{2}\right] \quad - \quad \text{dimension} = 45$$

$$\mathbb{Q}[\underline{10}] = \mathbb{Q} \oplus V_{9,1}$$
$$9$$

$$\overset{??}{\parallel}$$

$$\mathbb{Q} \oplus V_1 \oplus V_2$$
$$1 \quad 9 \quad 35$$

$$\mathrm{End}_{S_n}\left(\mathbb{Q}\left[\binom{10}{2}\right]\right) = \mathbb{Q}[t]/(p(t))$$

$$p(t) = (t-\alpha)(t-\beta)(t-\gamma)$$

$$= \frac{\overset{1}{\mathbb{Q}[t]}}{t-\alpha} \oplus \frac{\overset{0}{\mathbb{Q}[t]}}{t-\beta} \oplus \frac{\overset{0}{\mathbb{Q}[t]}}{t-\gamma}.$$

$$= \mathrm{End}_S(\mathbb{Q}) \oplus \mathrm{End}_{S_n} V_1 \oplus \mathrm{End}_{S_n} V_2.$$

$$p_\alpha(t) \equiv 1 \mod (t-\alpha)$$
$$0 \mod (t-\beta)$$
$$0 \mod (t-\gamma)$$

$$p_\alpha(t) = a_0 + a_1 t + \cancel{a_2} t^2$$

$$a_0 + a_1\alpha + \alpha^2 = 1$$
$$a_0 + a_1\beta + \beta^2 = 0$$
$$a_0 + a_1\gamma + \gamma^2 = 0$$

$$V_1 = \sum_{T \in \mathrm{Hom}_{S_n}(\mathbb{Q}[\underline{10}],\ \mathbb{Q}[\binom{10}{2}])} \cancel{\mathrm{Im}(T)}\ T(V_1)$$

$$= \cancel{\mathrm{span}\{ F_0}\ T_0(V_1) + T_1(V_1)$$

$$= \underbrace{(T_0 + T_1)(V_1)}_{=0} + T_1(V_1)$$

$$= T_1(V_1)$$

$$= \left\{ S \mapsto \sum_{i \in S} f(i) \ \middle|\ \sum f(i) = 0 \right\}$$

$$= \left\{ \{i,j\} \mapsto f(i) + f(j) \ \middle|\ \sum f(i) = 0 \right\}$$

$$V_0 \oplus V_1 = \left\{ \{i,j\} \mapsto f(i) + f(j) \right\}.$$

$$V_2 = (V_0 \oplus V_1)^{\perp}.$$

<u>Problem:</u> Find ten nice conditions to define ~~the subs~~ 35 dimensional subspace V_2 of $\mathbb{Q}[\binom{10}{2}]$.

Guess:

$$\dim V_3 = \binom{10}{3} - \binom{10}{2}$$

$$V_3 = \left\{ f \in \mathbb{Q}\left[\binom{10}{3}\right] \;\middle|\; \sum_{s \supset t} g(s) = 0 \quad \forall\, t \in \binom{10}{2} \right\}$$

$$W = \{ \{i,j\} \mapsto f(i) + f(j) \mid f \in \mathbb{Q}[\underline{10}] \}$$

$$= \operatorname{span} \{ f_\ell \mid \ell \in \underline{10} \}$$

$$f_\ell(s) = \begin{cases} 1 & \text{if } \ell \in s \\ 0 & \text{o/w.} \end{cases}$$

$$g \in W^\perp \iff \sum_s f_\ell(s) g(s) = 0 \quad \forall \ell$$

$$\iff \sum_{s \ni \ell} g(s) = 0. \quad \forall \ell.$$

$$V_2 = \{ f \in \mathbb{Q}[\tbinom{10}{2}] \mid \sum_{s \ni \ell} g(s) = 0 \ \forall \ell \in \underline{10} \}$$

$$d_{k\ell} = \dim \operatorname{End}_{S_n}\left(\mathbb{Q}[\tbinom{10}{k}], \mathbb{Q}[\tbinom{10}{\ell}]\right)$$

	1	2	3
1	2	2	2
2	2	3	3
3	2	3	4

	1	2	3
1	2	2	2
2	2	3	3
3	2	3	4

$\dim \operatorname{End}_{S_n}[\tbinom{10}{3}] = 4$, and is commutative (hw)

$$V_0 \oplus V_1 \oplus V_2 \oplus V_3$$