

$$G_1 \cup G_2 < G$$

G : conjugacy class of G

$$\text{Then } |C \cap G_1| = |C \cap G_2|$$

$$k_1(i, j) = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{o/w} \end{cases}$$

$$k_2(i, j) = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{o/w} \end{cases}$$

$$k_1 * k_1 = k_1$$

$$k_1 * k_2 = k_2 = k_2 * k_1$$

$$k_2 * k_2 = (n-1)k_1 + (n-2)k_2$$

$$\text{Q } \varepsilon_1 = \varepsilon = \alpha_1 k_1 + \alpha_2 k_2$$

$$\varepsilon_2 = k_1 - \varepsilon$$

$$(\alpha_1 k_1 + \alpha_2 k_2)^2 = \alpha_1 k_1 + \alpha_2 k_2$$

$$\alpha_1^2 k_1 + 2\alpha_1 \alpha_2 k_2 + \alpha_2^2 [(n-1)k_1 + (n-2)k_2]$$

$$\underbrace{[\alpha_1^2 + (n-1)\alpha_2^2]}_{\alpha_1} k_1 + \underbrace{[2\alpha_1 \alpha_2 + \alpha_2^2 (n-2)]}_{\alpha_2} k_2$$

$$2\alpha_1 + \alpha_2(n-2) = 1$$

$$\alpha_1 = \frac{1 - \alpha_2(n-2)}{2}$$

$$\alpha_1 = \alpha_2 = \frac{1}{n} \quad n\alpha_2 = 1$$

$$n(n-1)\alpha_2^2 = \alpha_2 \cdot 1$$

$$E_1 = \frac{1}{n}k_1 + \frac{1}{n}k_2$$

$$E_2 = \frac{n-1}{n}k_1 - \frac{1}{n}k_2$$

k_1 does nothing to a fn.

$$k_2 f(i) = \sum_{j \neq i} f(j)$$

$$\begin{aligned} E_1 f(i) &= \frac{1}{2} \left[f(i) + \sum_{j \neq i} f(j) \right] \\ &= \frac{1}{n} \sum_j f(j) \end{aligned} \quad \left. \vphantom{\begin{aligned} E_1 f(i) \\ = \frac{1}{n} \sum_j f(j) \end{aligned}} \right\} \begin{array}{l} \text{const} \\ \text{fus. as} \\ \text{image.} \end{array}$$

$$\begin{aligned} E_2 f(i) &= \frac{1}{n} \left[(n-1)f(i) - \sum_{j \neq i} f(j) \right] \\ &= \frac{1}{n} \left[n f(i) - \sum_j f(j) \right] \\ &\Rightarrow f \mapsto f - av(f) \end{aligned} \quad \left. \vphantom{\begin{aligned} E_2 f(i) \\ = \frac{1}{n} [n f(i) - \sum_j f(j)] \end{aligned}} \right\} \begin{array}{l} \text{sum} \\ \text{zeros.} \\ \text{as image.} \end{array}$$

$$K \left[\binom{m}{2} \right]$$

$$k_i(S, T) = \begin{cases} 1 & \text{if } |S \cap T| = i \\ 0 & \text{o/w} \end{cases}$$

$$k_2 = \text{id.}$$

$$k_1,$$

$$k_0.$$

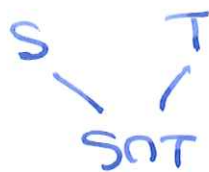
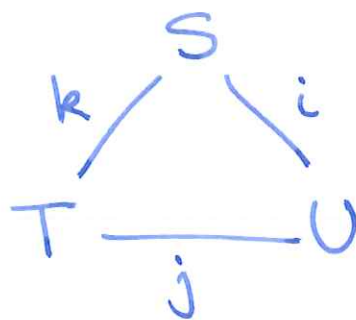
$$k_i * k_j(S, T) = \# \{ U \mid \begin{array}{l} |S \cap U| = i \\ |T \cap U| = j \end{array} \}$$

$$|S \cap T| = k$$

Combinatorial problem:

Given i, j, k , take $S, T \ni |S \cap T| = k$.

Find no. of $U \ni |S \cap U| = i, |T \cap U| = j$.



$$\sum_{l=0}^k \binom{|S|}{i-l} \binom{|T|}{j-l} \binom{k}{l}$$

$$l = |S \cap T \cap U|$$

$$\sum_{l=0}^k \binom{k}{l} \binom{s-k}{i-l} \binom{t-k}{j-l} \binom{n-(s+t-k)}{u-(j+i-l)}$$

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SNTNU UNS-
 UNT

$u - (j+i-l)$
 $s+t-k$

In our example, $s = t = 2 = u$

Now ~~do it~~ compute the e_1, e_2, e_3
 for $\text{End}_{S_n} K\left[\binom{n}{2}\right]$.

$$n = 3, \quad s, t, u = 2$$

$$k = 1, \quad i, j = 2, \quad j = 1$$

$$S = \{1, 2\}, \quad T = \{2, 3\}$$

$$\# \left\{ U \mid \begin{array}{l} |S \cap U| = 2 \\ |T \cap U| = 1 \end{array} \right\} \quad U = S = \{1, 2\}$$

$$\sum_{l=0}^1 \binom{1}{l} \binom{2-1}{2-l} \binom{2-1}{1-l} \binom{2-3+0}{2-l}$$