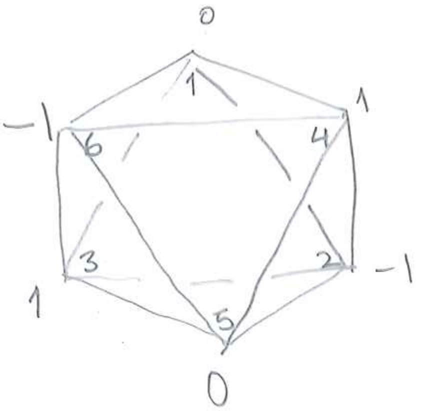
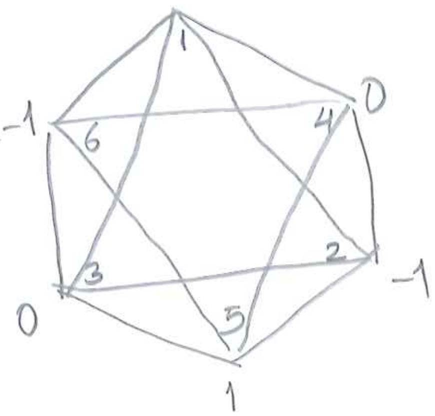
A two-dimensional rep. of S_4: 1

$v_1 =$ 

$v_2 =$ 

$s_1 = (15)(23)(46)$

$$\rho(s_1) = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}\begin{matrix} 1 \\ -2 \end{matrix}$$

$$\rho(s_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\rho(s_3) = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$\rho\otimes\varepsilon(s_1) = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$\rho\otimes\varepsilon(s_2) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\rho\otimes\varepsilon(s_3) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Want an intertwiner:

$$T : V \longrightarrow V$$

$$T\rho(s_i)v = -\rho(s_i)Tv \qquad (*)$$

So if v is a ± 1 eigenvector for $\rho(s_1)$,

Tv should be a ∓ 1 eigenvector.

-1 eigenvector: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = v_1$ $= w_1$

$+1$ eigenvector: $\begin{pmatrix} 1 \\ -2 \end{pmatrix} = v_1 - 2v_2$ $= w_2$

~~With respecto~~ $T(v_1) = c_1(v_1 - 2v_2)$ (1)

$$\cancel{T(v_2) = c_2(}$$

$$T(v_1 - 2v_2) = c_2 v_1 \qquad (2)$$

$$\cancel{T}\frac{(1)-(2)}{2} \; ; \; T(v_2) = \frac{c_1(v_1 - 2v_2) - c_2 v_1}{2}$$

$$= \cancel{\frac{c_1}{2}v_1 - (c_1 + c_2)}$$

$$\frac{c_1 - c_2}{2} v_1 - c_1 v_2$$

$$T = \begin{pmatrix} c_1 & \frac{c_1 - c_2}{2} \\ -2c_1 & -c_1 \end{pmatrix}$$

$$\underline{\underline{c_1 = 1}}\; T = \begin{pmatrix} 1 & \frac{1 - c_2}{2} \\ -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & \frac{1-c_2}{2} \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & \frac{1-c_2}{2} \\ -2 & -1 \end{pmatrix} = 0$$

(put $i = 2$ in $(*)$) $\qquad T = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$

$$\begin{pmatrix} \frac{1-c_2}{2} & 1 \\ -1 & -2 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 1 & \frac{1-c_2}{2} \end{pmatrix} = 0$$

$$\frac{1-c_2}{2} = 2 \qquad 1 - c_2 = 4 \quad c_2 = -3$$

Catalog of irreducible representations

S_2 : $\underset{\text{dim } 1}{\underline{1}}$, $\underset{\text{dim } 1}{\varepsilon}$

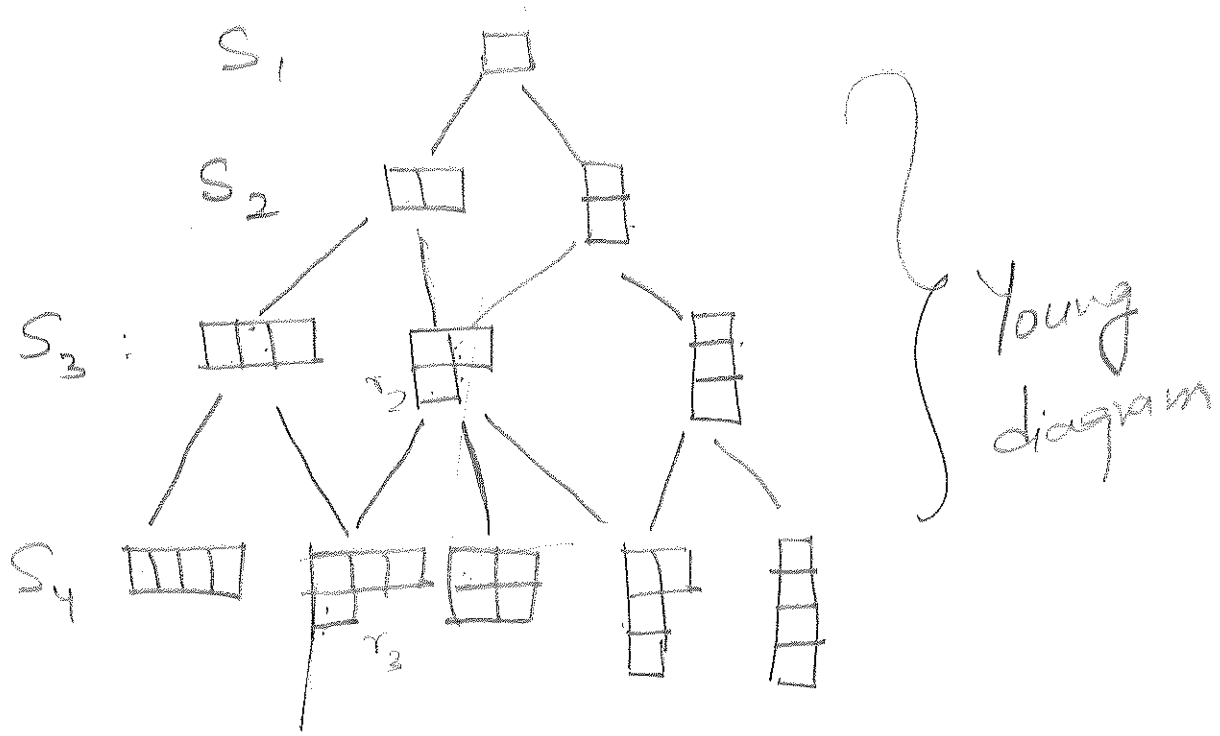
S_3 : $\underline{1}$, τ , ε

1 dim, 2 dim, 1

Alperin & Bell

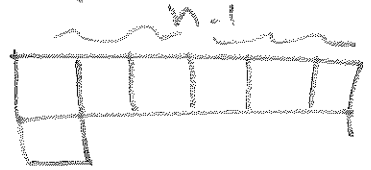
S_4 : $\underline{1}$, τ , ρ , $\tau \otimes \varepsilon$, ε

1 dim, 3 dim, 2 dim, 3 dim, 1 dim.



Young diagram

boxes left & top justified

S_n ~ 

$= r_{n-1}$

V

$\mathbb{Q}^n = \mathbb{Q} \oplus V$