$$n \le \frac{d_1+d_2}{2} - 1$$

$$c_n - c_{n-1} = \sum a_i b_{n-i} - \sum a_i b_{n-1-i}$$

$$= \sum_{i \le \frac{d_1}{2}-1} a_i b_{n-i} - \sum_{i \le \frac{d_1}{2}-1} a_{i-1} b_{n-i}$$

$$+ \sum_{i \ge d_1}$$

$(x, y, z) \mapsto (x+a, y+b, z+c)$ translation

$(x, y, z) \mapsto (a_{11}x + a_{12}y + a_{13}z, a_{21}x + a_{22}y + \dots)$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\langle A\vec{u}, A\vec{v} \rangle = \langle \vec{u}, \vec{v} \rangle \quad (*)$$

<u>Ex</u>: ~~Proving~~ So $\langle Av, Av \rangle = \langle v, v \rangle \ \forall v$

$\Rightarrow (*) \ \forall v$.

$A^T A = I$ - orthogonal matrices $\leftarrow$

$(x, y, z) \mapsto (x, y, -z)$

$$\begin{pmatrix} 1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \qquad A = I,\ \vec{\alpha} = 0$$

Qn

Let $G = \left\{ \underbrace{(x,y,z)}_{} \overset{}{\mapsto} A\vec{v} + \vec{\alpha} \;\middle|\; \begin{matrix} A^T A = I \\ \det(A) = 1 \end{matrix},\ \vec{\alpha} \in \mathbb{R}^3 \right\}$ where $\vec{v} = (x,y,z)$, $(A, \vec{\alpha})$

Show that $\forall\ g \in G\ \exists$ cts. fn. $\gamma : [0,1] \to G$ such that $\gamma(0) = I$ and $\gamma(1) = g$.

$\forall\ \vec{v} \in \mathbb{R}^3$

$\gamma_v(t) : t \mapsto \gamma(t)\vec{v}$

$t \mapsto \gamma_v(t)$ is cts. $\forall\ v \in \mathbb{R}^3 \iff \gamma$ is cts.

$$\theta \mapsto \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

C : point configuration. $\subseteq \mathbb{R}^3$

$$G_C := \{ g \in G \mid g(c) \in C \ \forall c \in C \}$$

$$C = \{(0,0,0)\}$$

$$G_C = SO_3 := \{ (A, \vec{\alpha}) \in G \mid \vec{\alpha} = 0 \}$$

Ex: What is $G_{\{(1,0,0)\}}$?

$$\mathbb{R}^4 \supseteq \mathbb{R}^3 = \{ (x, y, z, w) \mid x + y + z + w = 1 \}$$

$$\vec{v}_1 = (1,0,0,0)$$

$$\vec{v}_2 = (0,1,0,0)$$

$$\vec{v}_3 = (0,0,1,0)$$

$$\vec{v}_4 = (0,0,0,1)$$

Ex: Use this trick to write down the coordinates of four points in space $\mathbb{R}^3$ which form the vertices of a regular tetrahedron.

$C =$ vertices of a regular tetra.

$= \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$

$$G_C \subseteq S_4$$

$$G_C \neq S_4 \qquad |G_C| \leq 12$$

$$\boxed{\begin{array}{ll} \underline{\text{id.}} & (2,3,4),\ (1,2,3),\ (1,2,4),\ (1,3,4) \\ & (4,3,2),\ (3,2,1),\ (4,2,1),\ (4,3,1) \\ & (12)(34),\ (13)(24),\ (14)(23) \end{array}}$$

$$\|$$

$$G_C \cong A_4$$

Ex: Express the elements in the last row as compositions of elements in the first two rows

Ex: Does there exist a configuration C of four pts. in $\mathbb{R}^3 \ni |G_C| = 24$?

Decahedron:

$$\boxed{1, 2, 3, 4, 5}, \boxed{6\ 7}$$

$$S_5 \times S_2$$

$$C_5 \times C_2 \qquad 10$$

$$H < G_c$$

$H = \{ h \in G_c \mid h(6) = 6 \}$ has order 5

$$H' = \{ h \in G_c \mid h(6) = 7 \}$$

$$G_c = H \amalg H'$$

Suppose $x \in H'$

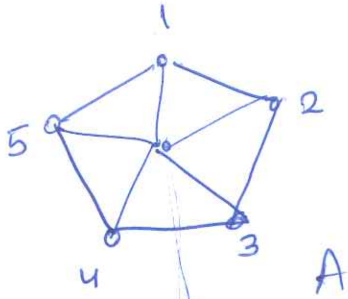
$\{xh \mid h \in H'\} = xH = xH'$

$H' = x^{-1} H$

$$|H'| \leq 5$$

D_{10} D_5 – ambiguous.

dihedral gp. of order ten.

C =  regular pentagon.

$$\mathcal{G} = \{ (x,y) \mapsto \overset{A}{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \mid A^T A = I, \ \cancel{\det A = 1}, \ \alpha_1, \alpha_2 \in \mathbb{R} \}$$

$\mathcal{G}_C \cong$ Dihedral gp. of order 10.

$C_1 = \{ (\varepsilon_1, \varepsilon_2, \varepsilon_3) \mid \varepsilon_i \in \{\pm 1\} \} \subseteq \mathbb{R}^3$ vertices of a cube

<u>Qu</u>: What is G_{C_1}?

<u>Trick</u>: 3 elements of G fix any given vertex. There are 8 possible destinations for that vertex. — so $3 \times 8 = 24$.

<u>Qu</u>: Is $G_{C_1} \cong S_4$?

<u>Octahedral gp</u>

$C_2 = \{ (\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) \}$

Same trick $\Rightarrow |G_{C_2}| = 24$.

<u>Qu</u>: Is $G_{C_1} \cong G_{C_2}$?