MAKE-UP MIDSEMESTER EXAMINATION

LOCALLY COMPACT ABELIAN GROUPS

(1) Let L be a locally compact abelian group which admits a compact open subgroup. Let $\nabla : L \to \hat{L}$ be an isomorphism. Show that there exists a compact open subgroup M such that if $M^{\perp}$ is defined as

$$M^{\perp} = \{\chi \in \hat{L} \ : \ \chi(m) = 0 \text{ for all } m \in M\},$$

then $M \subset M^{\perp}$ and $M^{\perp}/M$ is finite.

(2) Let $\mathbf{A}$ denote the ring of adèles of $\mathbf{Q}$. Describe the automorphism group of $\mathbf{A}^n$ for a positive integer n.

(3) If the identity component of a locally compact abelian group L is compact and open then L is a product of a compact connected abelian group and a discrete abelian group.

(4) Let $G = (\mathbf{Z}/p\mathbf{Z})^2$. Show that there exists a non-split extension in $E(\hat{G}, G)$ whose Pontryagin dual is equivalent to its inverse in the Baer group.

Date: Wednesday, October 14, 2009 (2:00pm–4:00pm).