HOMEWORK VI

LOCALLY COMPACT ABELIAN GROUPS

(1) Let $G = \mathbf{Z}_p^{\mathbf{N}}$ (sequences in $\mathbf{Z}_p$), and let S be the subgroup of G consisting of sequences of the form

$$p\psi_1, p\psi_2 - \psi_1, p\psi_3 - \psi_2, \dots,$$

for some sequence $\{\psi_n\}$ in $\mathbf{Z}_p^{\mathbf{N}}$. Show that G/S is isomorphic to $\mathbf{Z}_p$.

(2) Think of the Prüfer group $\mathbf{Z}(p^\infty)$ as $(\mathbf{Q}/\mathbf{Z})_p$. Let $\phi : \mathbf{Z}^{\oplus \mathbf{N}} \to \mathbf{Z}(p^\infty)$ be the homomorphism which takes the nth standard basis vector to p^{-n}. Let K denote the kernel of ϕ. Use the projective resolution

$$0 \longrightarrow K \xrightarrow{i} \mathbf{Z}^{\oplus \mathbf{N}} \xrightarrow{\phi} \mathbf{Z}(p^\infty) \longrightarrow 0$$

to show that $E(\mathbf{Z}(p^\infty), \mathbf{Z}_p) \cong \mathbf{Z}_p$.

(3) If L is a locally compact abelian group which appears in the extension

$$0 \to \mathbf{Z}_p \to L \to \mathbf{Z}(p^\infty) \to 0$$

corresponding to some element α that lies in the subgroup $p^m E(\mathbf{Z}(p^\infty), \mathbf{Z}_p)$ of $E(\mathbf{Z}(p^\infty), \mathbf{Z}_p)$ but not in $p^{m+1} E(\mathbf{Z}(p^\infty), \mathbf{Z}_p)$, show that L is isomorphic to $\mathbf{Q}_p \times \mathbf{Z}/p^m\mathbf{Z}$ when $m > 1$.

(4) An involution on $E(\widehat{\mathbf{Z}/p^n\mathbf{Z}}, \mathbf{Z}/p^n\mathbf{Z})$ is obtained by applying Pontryagin duality to each extension. What is it?

Date: due on Wednesday, October 7, 2009 (before class).