HOMEWORK VI

LOCALLY COMPACT ABELIAN GROUPS

(1) Let $\mathbf{F}_q$ be a finite field with q elements, where q is a power of a prime number p. Let $\psi : \mathbf{F}_p \to T$ be any non-trivial homomorphism. Show that $x \mapsto \psi_x$, where $\psi_x(y) = \psi(\mathrm{trace}(xy))$ is an isomorphism $\mathbf{F}_q \to \widehat{\mathbf{F}_q}$. Here, $\mathbf{F}_q$ is thought of as a vector space of $\mathbf{F}_p$ and each element of $\mathbf{F}_q$ as a linear endomorphism of this vector space, so that trace makes sense.

(2) Show that every abelian group is isomorphic to a subgroup of a divisible abelian group.

(3) Show that the extension

$$0 \longrightarrow G \xrightarrow{j} L \xrightarrow{q} A \longrightarrow 0$$

is split if and only if there exists a homomorphism $s : A \to L$ such that $q(s(a)) = a$ for all $a \in A$.

(4) Let A and G be abelian groups. Let $i : G \to D$ be an injective homomorphism into a divisible group D. Let $Q = D/i(G)$ and let $p : D \to Q$ be the quotient map. Construct a canonical homomorphism $\partial : \mathrm{Hom}(A, Q) \to E(A, G)$ for which there is an exact sequence

$$0 \longrightarrow \mathrm{Hom}(A,G) \xrightarrow{i\circ} \mathrm{Hom}(A,D) \xrightarrow{p\circ} \mathrm{Hom}(A,Q) \xrightarrow{\partial} E(A,G) \longrightarrow 0$$

(do it without using any homological algebra–this involves many steps).

(5) Suppose G and A are finite abelian groups whose cardinalities are coprime. Show that $E(A, G) = 0$.

(6) Let G be a compact topological p-torsion group and A a discrete q-torsion group. Show that $\mathcal{E}(A, G) = 0$.

(7) For finite abelian groups A and G, show that $\mathrm{Hom}(A, G)$ and $E(A, G)$ are isomorphic (but it seems that there is no canonical isomorphism).

Date: due on Wednesday, September 16, 2009 (before class).