Solution to problem 5 on homework 2:

Let $x \in A$ be an element of infinite order. For every $t \in T$, there exist characters $\chi \in \hat{A}$ such that $\chi(x) = t$. Therefore, the map $\phi : \hat{A} \to T$ defined by $\phi(\chi) = \chi(x)$ is surjective. It is clearly continuous. Since $\hat{A}$ is compact, ϕ is open.

Now, if K is a closed (hence compact) and open subset of $\hat{A}$, then $\phi(K)$ is a compact open subset of T. Therefore $\phi(K) = T$. If $\hat{A}$ were zero-dimensional Hausdorff, then the set of compact open neighborhoods of 0 would have intersection $\{0\}$. On the other hand, their images under ϕ would have intersection T, which is absurd.