

Experimental Mathematics with Python and Sage

Amritanshu Prasad

Chennaipy

27 February 2016

Binomial Coefficients

$\binom{n}{k} = {}^nC_k =$ number of distinct ways to choose k out of n objects

Binomial Coefficients

$\binom{n}{k} = {}^nC_k =$ number of distinct ways to choose k out of n objects

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

```
sage: [len([k for k in range(n+1) if binomial(n, k)%2 ==1])
for n in range(20)]
[1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8]
```

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

```
sage: [len([k for k in range(n+1) if binomial(n, k)%2 ==1])
for n in range(20)]
[1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8]
```

Write numbers in binary and count number of 1's

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

```
sage: [len([k for k in range(n+1) if binomial(n, k)%2 ==1])
for n in range(20)]
[1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8]
```

Write numbers in binary and count number of 1's

0,	1,	10,	11,	100,	101,	110,	111,	1000,	1001,	1010,	1011,...
0,	1,	1,	2,	1,	2,	2,	4,	1,	2,	2,	3,...

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

```
sage: [len([k for k in range(n+1) if binomial(n, k)%2 ==1])
for n in range(20)]
[1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8]
```

Write numbers in binary and count number of 1's

0	1	10	11	100	101	110	111	1000	1001	1010	1011	...
0	1	1	2	1	2	2	4	1	2	2	3	...

Let $\nu(n)$ denote the number of 1's in binary expansion of n .

How many binomial coefficients are odd?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Question

For how many numbers $0 \leq k \leq n$ is $\binom{n}{k}$ odd?

```
sage: [len([k for k in range(n+1) if binomial(n, k)%2 ==1])
for n in range(20)]
[1, 2, 2, 4, 2, 4, 4, 8, 2, 4, 4, 8, 4, 8, 8, 16, 2, 4, 4, 8]
```

Write numbers in binary and count number of 1's

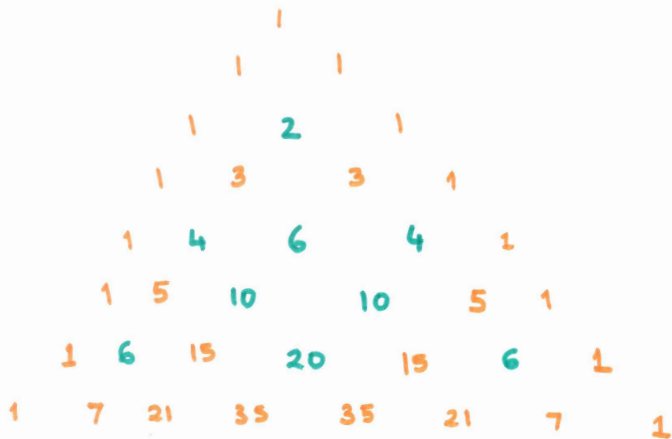
0	1	10	11	100	101	110	111	1000	1001	1010	1011	...
0	1	1	2	1	2	2	4	1	2	2	3	...

Let $\nu(n)$ denote the number of 1's in binary expansion of n .

Conjecture

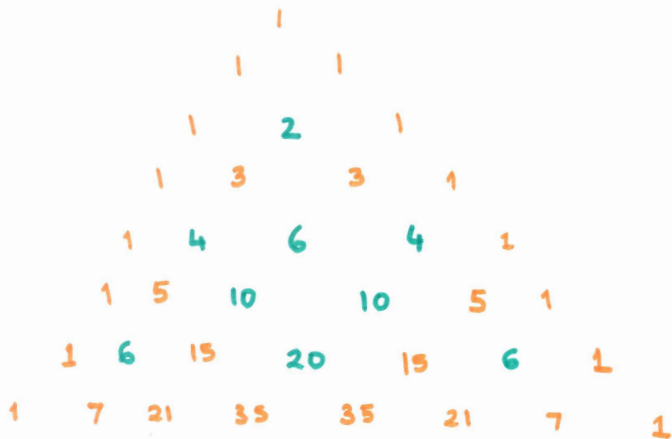
No. of odd binomial coefficients of the form $\binom{n}{k}$ for fixed n is $2^{\nu(n)}$.

Binomial coefficients in Pascal's triangle (Meru Prastaara)

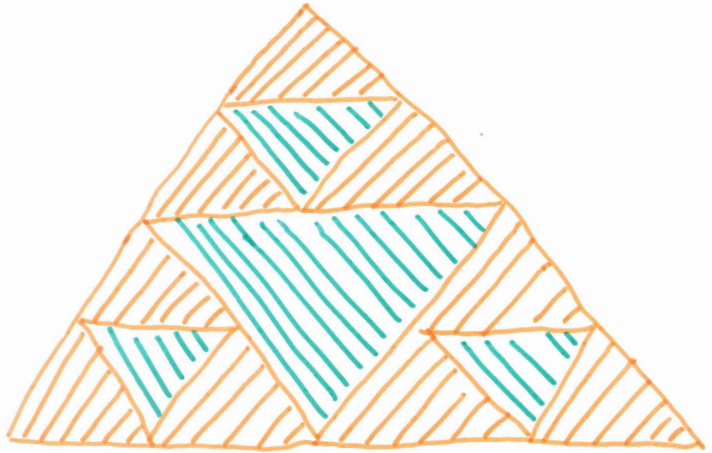


$\binom{n}{k}$ = number of paths from
the apex to the k th position in
the n th row.

Binomial coefficients in Pascal's triangle (Meru Prastaara)



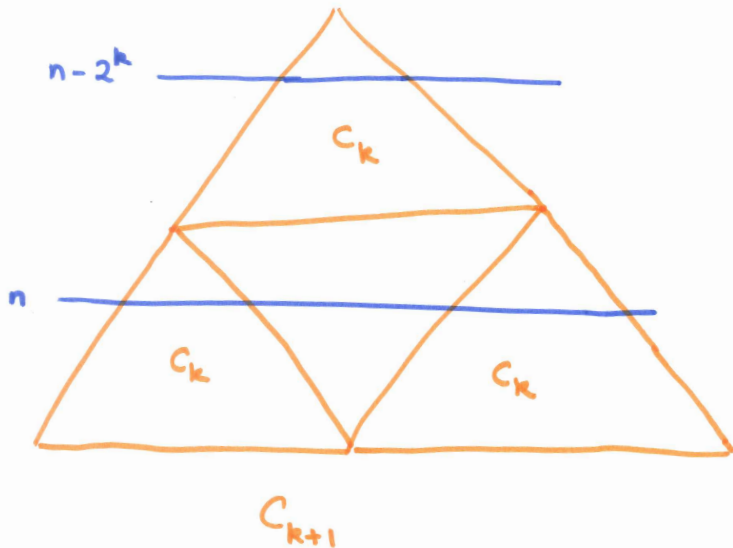
Binomial coefficients in Pascal's triangle (Meru Prastaara)



Binomial coefficients in Pascal's triangle (Meru Prastaara)



Binomial coefficients in Pascal's triangle (Meru Prastaara)



Recurrence Relation

$$\text{If } 2^k \leq n < 2^{k+1},$$

$$a_n = 2a_{n-2^k}.$$

$$a_{42} = 2 \times a_{10} = 2 \times 2 \times a_2 = 2 \times 2 \times 2.$$

$$a_{42} = 2 \times a_{10} = 2 \times 2 \times a_2 = 2 \times 2 \times 2.$$

In terms of binary expansions:

$$42 = 101010$$

$$10 = 1010$$

$$2 = 10$$

$$a_{42} = 2 \times a_{10} = 2 \times 2 \times a_2 = 2 \times 2 \times 2.$$

In terms of binary expansions:

$$\begin{array}{rcl} 42 & = & 101010 \\ 10 & = & \quad 1010 \\ 2 & = & \quad \quad 10 \end{array}$$

Total number of times you double is the number of times 1 appears in the binary expansion.

Integer Partition

$$5 = 5$$

$$= 4 + 1$$

$$= 3 + 2$$

$$= 3 + 1 + 1$$

$$= 2 + 2 + 1$$

$$= 2 + 1 + 1 + 1$$

$$= 1 + 1 + 1 + 1 + 1$$

Integer Partition

$$\begin{aligned}5 &= 5 \\&= 4 + 1 \\&= 3 + 2 \\&= 3 + 1 + 1 \\&= 2 + 2 + 1 \\&= 2 + 1 + 1 + 1 \\&= 1 + 1 + 1 + 1 + 1\end{aligned}$$

$$p(5) = 7$$

```
sage: [number_of_partitions(n) for n in range(19)]  
[1, 1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231,  
 297, 385]
```

Big Open Problem in Mathematics

Find an exact closed formula
for $p(n)$

Hardy and Ramanujan



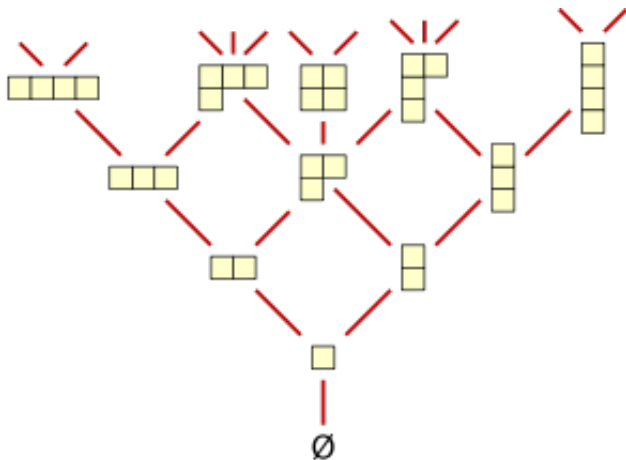
$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp(\pi\sqrt{2n/3}) \text{ as } n \rightarrow \infty.$$

```
sage: number_of_partitions(100000)
2749351056977569651267751632098635268817342931598005475
8203125984302147328114964173055050741660736621590157844
7742962489404930630702004617927644930335101160793424571
9015571894350972531246610845200636955893446424871682878
9832182345009262853831404597021307130674510624419227311
2389997022844086093709355316296978515695698921961084801
58600569421098519
```

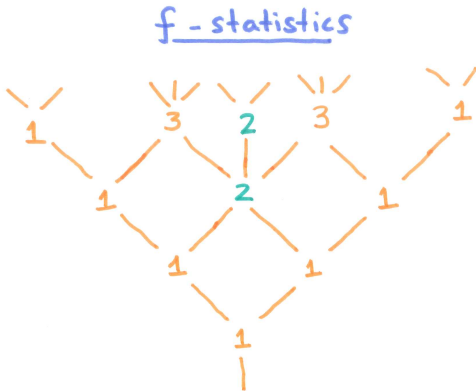
is based on the Hardy-Ramanujan-Rademacher formula:

$$p(n) = \frac{1}{\pi\sqrt{2}} \sum_{k=1}^{\infty} \sqrt{k} A_k(n) \frac{d}{dn} \left(\frac{1}{\sqrt{n - \frac{1}{24}}} \sinh \left[\frac{\pi}{k} \sqrt{\frac{2}{3} \left(n - \frac{1}{24} \right)} \right] \right)$$

Young's lattice



The f -statistic of a partition

$$f_\lambda = \text{Number of paths from the apex to } \lambda \text{ in Young's lattice}$$


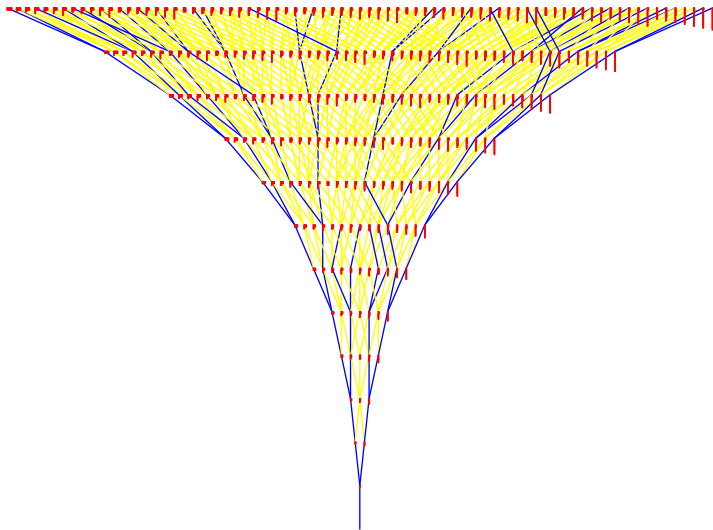
How often in f_λ odd?

```
sage: [len([la for la in Partitions(n) if la.dimension()%2 == 1])
for n in range(19)]
[1, 1, 2, 2, 4, 4, 8, 8, 8, 8, 16, 16, 32, 32, 64, 64, 16, 16, 32]
```

How often in f_λ odd?

```
sage: [len([la for la in Partitions(n) if la.dimension()%2 == 1])  
for n in range(19)]  
[1, 1, 2, 2, 4, 4, 8, 8, 8, 8, 16, 16, 32, 32, 64, 64, 16, 16, 32]
```

Powers of 2 again!



Odd Partitions in Young's lattice form a binary tree

If the binary expansion of n has 1's in the places $k_1, k_2, \dots, k_r$, number of partitions of n for which f_λ is odd is:

$$c_n = s^{k_1+k_2+\dots+k_r}$$

Example

$$n = 42 = 101010 = 2^5 + 2^3 + 2^1$$

$$c_{42} = 2^{5+3+1} = 2^8 = 256.$$

All papers : [Go!](#)

Download:

- License:**

Current browse context:

math.CO

[< prev](#) | [next >](#)

[new](#) | [recent](#) | 1601

Change to browse by:

 $\mathbf{math}$

math_RT

References & Citations

- [NASA ADS](#)

Bookmark [\(what is this?\)](#)



Arvind Ayer, Amritanshu Prasad, Steven Spallone

(Submitted on 8 Jan 2016)

We show that the Hasse diagram of the subposet of Young's lattice consisting of partitions with an odd number of standard Young tableaux is a binary tree. This tree exhibits self-similarities at all scales, and has a simple recursive description.

Comments: 13 pages, 3 figures

Subjects: **Combinatorics (math.CO)**; Representation Theory (math.RT)

MSC classes: 05A15, 05A17, 05E10, 20C30

Cite as: [arXiv:1601.01776 \[math.CO\]](#)

(or [arXiv:1601.01776v1 \[math.CO\]](#) for this version)

Submission history

From: Amritanshu Prasad [\[view email\]](#)

[v1] Fri, 8 Jan 2016 07:01:34 GMT (61kb,D)

Which authors of this paper are endorsers? | [Disable MathJax](#) (What is MathJax?)

Link back to: [arXiv](#), [form interface](#), [contact](#).

partition ^ v Highlight All Match Case 1 of 1 match

Why mathematicians use Sage

- ▶ Easy to program (uses python)
- ▶ Free
- ▶ Developed by research mathematicians - caters to their needs
- ▶ Open source
- ▶ Can also be used to learn and teach calculus, linear algebra, etc.

referenc... Integer p... "msc" (3... Aeropres... Einzug - E... determin... NovoEd's... February... 1 of 1 upl... COURSES X... David Ro... How One... C H Hard... SageM...

www.sagemath.org

Search

RSS - Blog - Trac - Wiki - Questions? Search

Online: SMC, sagem - SageCell or Download, Source Code
v7.9 (2016-01-19) Language

Home Tour Help Library Download Development Links

SageMath is a free open-source mathematics software system licensed under the GPL. It builds on top of many existing open-source packages: NumPy, SciPy, matplotlib, SymPy, Maxima, GAP, FLINT, R and many more. Access their combined power through a common, Python-based language or directly via interfaces or wrappers.

Mission: Creating a viable free open source alternative to Magma, Maple, Mathematica and Matlab.

Do you want to learn how to use SageMath?
Download and read [Sage for Undergraduates](#) by Gregory Bard or
[Calcul mathématique avec Sage](#) (in French).

Use SageMath Online
other: Sagenb, SageMathCell

Download 7.0
Changelog - Source 7.0 - Packages - Git

Help/Documentation
Video - Forums - Tutorial - FAQ - Questions?

Feature Tour
Quickstart - Research - Graphics

Library
Testimonials - Books - Publications - Press Kit

Search
Search...

Random Link: [Quickstart into SageMath](#)

Keywords - 2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19-20-21-22-23-24-25-26-27-28-29-30-31-32-33-34-35-36-37-38-39-40-41-42-43-44-45-46-47-48-49-50-51-52-53-54-55-56-57-58-59-60-61-62-63-64-65-66-67-68-69-70-71-72-73-74-75-76-77-78-79-80-81-82-83-84-85-86-87-88-89-90-91-92-93-94-95-96-97-98-99-100-101-102-103-104-105-106-107-108-109-110-111-112-113-114-115-116-117-118-119-120-121-122-123-124-125-126-127-128-129-130-131-132-133-134-135-136-137-138-139-140-141-142-143-144-145-146-147-148-149-150-151-152-153-154-155-156-157-158-159-160-161-162-163-164-165-166-167-168-169-170-171-172-173-174-175-176-177-178-179-180-181-182-183-184-185-186-187-188-189-190-191-192-193-194-195-196-197-198-199-200-201-202-203-204-205-206-207-208-209-210-211-212-213-214-215-216-217-218-219-220-221-222-223-224-225-226-227-228-229-230-231-232-233-234-235-236-237-238-239-240-241-242-243-244-245-246-247-248-249-250-251-252-253-254-255-256-257-258-259-260-261-262-263-264-265-266-267-268-269-270-271-272-273-274-275-276-277-278-279-280-281-282-283-284-285-286-287-288-289-290-291-292-293-294-295-296-297-298-299-300-301-302-303-304-305-306-307-308-309-310-311-312-313-314-315-316-317-318-319-320-321-322-323-324-325-326-327-328-329-330-331-332-333-334-335-336-337-338-339-340-341-342-343-344-345-346-347-348-349-350-351-352-353-354-355-356-357-358-359-360-361-362-363-364-365-366-367-368-369-370-371-372-373-374-375-376-377-378-379-380-381-382-383-384-385-386-387-388-389-390-391-392-393-394-395-396-397-398-399-400-401-402-403-404-405-406-407-408-409-410-411-412-413-414-415-416-417-418-419-420-421-422-423-424-425-426-427-428-429-430-431-432-433-434-435-436-437-438-439-440-441-442-443-444-445-446-447-448-449-450-451-452-453-454-455-456-457-458-459-460-461-462-463-464-465-466-467-468-469-470-471-472-473-474-475-476-477-478-479-480-481-482-483-484-485-486-487-488-489-490-491-492-493-494-495-496-497-498-499-500-501-502-503-504-505-506-507-508-509-510-511-512-513-514-515-516-517-518-519-520-521-522-523-524-525-526-527-528-529-530-531-532-533-534-535-536-537-538-539-540-541-542-543-544-545-546-547-548-549-550-551-552-553-554-555-556-557-558-559-560-561-562-563-564-565-566-567-568-569-570-571-572-573-574-575-576-577-578-579-580-581-582-583-584-585-586-587-588-589-590-591-592-593-594-595-596-597-598-599-600-601-602-603-604-605-606-607-608-609-610-611-612-613-614-615-616-617-618-619-620-621-622-623-624-625-626-627-628-629-630-631-632-633-634-635-636-637-638-639-640-641-642-643-644-645-646-647-648-649-650-651-652-653-654-655-656-657-658-659-660-661-662-663-664-665-666-667-668-669-670-671-672-673-674-675-676-677-678-679-680-681-682-683-684-685-686-687-688-689-690-691-692-693-694-695-696-697-698-699-700-701-702-703-704-705-706-707-708-709-710-711-712-713-714-715-716-717-718-719-720-721-722-723-724-725-726-727-728-729-730-731-732-733-734-735-736-737-738-739-740-741-742-743-744-745-746-747-748-749-750-751-752-753-754-755-756-757-758-759-760-761-762-763-764-765-766-767-768-769-770-771-772-773-774-775-776-777-778-779-780-781-782-783-784-785-786-787-788-789-790-791-792-793-794-795-796-797-798-799-800-801-802-803-804-805-806-807-808-809-810-811-812-813-814-815-816-817-818-819-820-821-822-823-824-825-826-827-828-829-830-831-832-833-834-835-836-837-838-839-840-841-842-843-844-845-846-847-848-849-850-851-852-853-854-855-856-857-858-859-860-861-862-863-864-865-866-867-868-869-870-871-872-873-874-875-876-877-878-879-880-881-882-883-884-885-886-887-888-889-890-891-892-893-894-895-896-897-898-899-900-901-902-903-904-905-906-907-908-909-910-911-912-913-914-915-916-917-918-919-920-921-922-923-924-925-926-927-928-929-930-931-932-933-934-935-936-937-938-939-940-941-942-943-944-945-946-947-948-949-950-951-952-953-954-955-956-957-958-959-960-961-962-963-964-965-966-967-968-969-970-971-972-973-974-975-976-977-978-979-980-981-982-983-984-985-986-987-988-989-990-991-992-993-994-995-996-997-998-999-1000-1001-1002-1003-1004-1005-1006-1007-1008-1009-1010-1011-1012-1013-1014-1015-1016-1017-1018-1019-1020-1021-1022-1023-1024-1025-1026-1027-1028-1029-1030-1031-1032-1033-1034-1035-1036-1037-1038-1039-1040-1041-1042-1043-1044-1045-1046-1047-1048-1049-1050-1051-1052-1053-1054-1055-1056-1057-1058-1059-1060-1061-1062-1063-1064-1065-1066-1067-1068-1069-1070-1071-1072-1073-1074-1075-1076-1077-1078-1079-1080-1081-1082-1083-1084-1085-1086-1087-1088-1089-1090-1091-1092-1093-1094-1095-1096-1097-1098-1099-1100-1101-1102-1103-1104-1105-1106-1107-1108-1109-1110-1111-1112-1113-1114-1115-1116-1117-1118-1119-1120-1121-1122-1123-1124-1125-1126-1127-1128-1129-1130-1131-1132-1133-1134-1135-1136-1137-1138-1139-1140-1141-1142-1143-1144-1145-1146-1147-1148-1149-1150-1151-1152-1153-1154-1155-1156-1157-1158-1159-1160-1161-1162-1163-1164-1165-1166-1167-1168-1169-1170-1171-1172-1173-1174-1175-1176-1177-1178-1179-1180-1181-1182-1183-1184-1185-1186-1187-1188-1189-1190-1191-1192-1193-1194-1195-1196-1197-1198-1199-1200-1201-1202-1203-1204-1205-1206-1207-1208-1209-1210-1211-1212-1213-1214-1215-1216-1217-1218-1219-1220-1221-1222-1223-1224-1225-1226-1227-1228-1229-1230-1231-1232-1233-1234-1235-1236-1237-1238-1239-1240-1241-1242-1243-1244-1245-1246-1247-1248-1249-1250-1251-1252-1253-1254-1255-1256-1257-1258-1259-1260-1261-1262-1263-1264-1265-1266-1267-1268-1269-1270-1271-1272-1273-1274-1275-1276-1277-1278-1279-1280-1281-1282-1283-1284-1285-1286-1287-1288-1289-1290-1291-1292-1293-1294-1295-1296-1297-1298-1299-1300-1301-1302-1303-1304-1305-1306-1307-1308-1309-1310-1311-1312-1313-1314-1315-1316-1317-1318-1319-1320-1321-1322-1323-1324-1325-1326-1327-1328-1329-1330-1331-1332-1333-1334-1335-1336-1337-1338-1339-1340-1341-1342-1343-1344-1345-1346-1347-1348-1349-1350-1351-1352-1353-1354-1355-1356-1357-1358-1359-1360-1361-1362-1363-1364-1365-1366-1367-1368-1369-1370-1371-1372-1373-1374-1375-1376-1377-1378-1379-1380-1381-1382-1383-1384-1385-1386-1387-1388-1389-1390-1391-1392-1393-1394-1395-1396-1397-1398-1399-1400-1401-1402-1403-1404-1405-1406-1407-1408-1409-1410-1411-1412-1413-1414-1415-1416-1417-1418-1419-1420-1421-1422-1423-1424-1425-1426-1427-1428-1429-1430-1431-1432-1433-1434-1435-1436-1437-1438-1439-1440-1441-1442-1443-1444-1445-1446-1447-1448-1449-1450-1451-1452-1453-1454-1455-1456-1457-1458-1459-1460-1461-1462-1463-1464-1465-1466-1467-1468-1469-1470-1471-1472-1473-1474-1475-1476-1477-1478-1479-1480-1481-1482-1483-1484-1485-1486-1487-1488-1489-1490-1491-1492-1493-1494-1495-1496-1497-1498-1499-1500-1501-1502-1503-1504-1505-1506-1507-1508-1509-1510-1511-1512-1513-1514-1515-1516-1517-1518-1519-1520-1521-1522-1523-1524-1525-1526-1527-1528-1529-1530-1531-1532-1533-1534-1535-1536-1537-1538-1539-1540-1541-1542-1543-1544-1545-1546-1547-1548-1549-1550-1551-1552-1553-1554-1555-1556-1557-1558-1559-1560-1561-1562-1563-1564-1565-1566-1567-1568-1569-1570-1571-1572-1573-1574-1575-1576-1577-1578-1579-1580-1581-1582-1583-1584-1585-1586-1587-1588-1589-1590-1591-1592-1593-1594-1595-1596-1597-1598-1599-1600-1601-1602-1603-1604-1605-1606-1607-1608-1609-1610-1611-1612-1613-1614-1615-1616-1617-1618-1619-1620-1621-1622-1623-1624-1625-1626-1627-1628-1629-1630-1631-1632-1633-1634-1635-1636-1637-1638-1639-1640-1641-1642-1643-1644-1645-1646-1647-1648-1649-1650-1651-1652-1653-1654-1655-1656-1657-1658-1659-1660-1661-1662-1663-1664-1665-1666-1667-1668-1669-1670-1671-1672-1673-1674-1675-1676-1677-1678-1679-1680-1681-1682-1683-1684-1685-1686-1687-1688-1689-1690-1691-1692-1693-1694-1695-1696-1697-1698-1699-1700-1701-1702-1703-1704-1705-1706-1707-1708-1709-1710-1711-1712-1713-1714-1715-1716-1717-1718-1719-1720-1721-1722-1723-1724-1725-1726-1727-1728-1729-1730-1731-1732-1733-1734-1735-1736-1737-1738-1739-1740-1741-1742-1743-1744-1745-1746-1747-1748-1749-1750-1751-1752-1753-1754-1755-1756-1757-1758-1759-1760-1761-1762-1763-1764-1765-1766-1767-1768-1769-1770-1771-1772-1773-1774-1775-1776-1777-1778-1779-1780-1781-1782-1783-1784-1785-1786-1787-1788-1789-1790-1791-1792-1793-1794-1795-1796-1797-1798-1799-1800-1801-1802-1803-1804-1805-1806-1807-1808-1809-1810-1811-1812-1813-1814-1815-1816-1817-1818-1819-1820-1821-1822-1823-1824-1825-1826-1827-1828-1829-1830-1831-1832-1833-1834-1835-1836-1837-1838-1839-1840-1841-1842-1843-1844-1845-1846-1847-1848-1849-1850-1851-1852-1853-1854-1855-1856-1857-1858-1859-1860-1861-1862-1863-1864-1865-1866-1867-1868-1869-1870-1871-1872-1873-1874-1875-1876-1877-1878-1879-1880-1881-1882-1883-1884-1885-1886-1887-1888-1889-1890-1891-1892-1893-1894-1895-1896-1897-1898-1899-1900-1901-1902-1903-1904-1905-1906-1907-1908-1909-1910-1911-1912-1913-1914-1915-1916-1917-1918-1919-1920-1921-1922-1923-1924-1925-1926-1927-1928-1929-1930-1931-1932-1933-1934-1935-1936-1937-1938-1939-1940-1941-1942-1943-1944-1945-1946-1947-1948-1949-1950-1951-1952-1953-1954-1955-1956-1957-1958-1959-1960-1961-1962-1963-1964-1965-1966-1967-1968-1969-1970-1971-1972-1973-1974-1975-1976-1977-1978-1979-1980-1981-1982-1983-1984-1985-1986-1987-1988-1989-1990-1991-1992-1993-1994-1995-1996-1997-1998-1999-2000-2001-2002-2003-2004-2005-2006-2007-2008-2009-2010-2011-2012-2013-2014-2015-2016-2017-2018-2019-2020-2021-2022-2023-2024-2025-2026-2027-2028-2029-2030-2031-2032-2033-2034-2035-2036-2037-2038-2039-2040-2041-2042-2043-2044-2045-2046-2047-2048-2049-2050-2051-2052-2053-2054-2055-2056-2057-2058-2059-2060-2061-2062-2063-2064-2065-2066-2067-2068-2069-2070-2071-2072-2073-2074-2075-2076-2077-2078-2079-2080-2081-2082-2083-2084-2085-2086-2087-2088-2089-2090-2091-2092-2093-2094-2095-2096-2097-2098-2099-2100-2101-2102-2103-2104-2105-2106-2107-2108-2109-2110-2111-2112-2113-2114-2115-2116-2117-2118-2119-2120-2121-2122-2123-2124-2125-2126-2127-2128-2129-2130-2131-2132-2133-2134-2135-2136-2137-2138-2139-2140-2141-2142-2143-2144-2145-2146-2147-2148-2149-2150-2151-2152-2153-2154-2155-2156-2157-2158-2159-2160-2161-2162-2163-2164-2165-2166-2167-2168-2169-2170-2171-2172-2173-2174-2175-2176-2177-2178-2179-2180-2181-2182-2183-2184-2185-2186-2187-2188-2189-2190-2191-2192-2193-2194-2195-2196-2197-2198-2199-2200-2201-2202-2203-2204-2205-2206-2207-2208-2209-2210-2211-2212-2213-2214-2215-2216-2217-2218-2219-2220-2221-2222-2223-2224-2225-2226-2227-2228-2229-2230-2231-2232-2233-2234-2235-2236-2237-2238-2239-2240-2241-2242-2243-2244-2245-2246-2247-2248-2249-2250-2251-2252-2253-2254-2255-2256-2257-2258-2259-2260-2261-2262-2263-2264-2265-2266-2267-2268-2269-2270-2271-2272-2273-2274-2275-2276-2277-2278-2279-2280-2281-2282-2283-2284-2285-2286-2287-2288-2289-2290-2291-2292-2293-2294-2295-2296-2297-2298-2299-2300-2301-2302-2303-2304-2305-2306-2307-2308-2309-2310-2311-2312-2313-2314-2315-2316-2317-2318-2319-2320-2321-2322-2323-2324-2325-2326-2327-2328-2329-2330-2331-2332-2333-2334-2335-2336-2337-2338-2339-2340-2341-2342-2343-2344-2345-2346-2347-2348-2349-2350-2351-2352-2353-2354-2355-2356-2357-2358-2359-2360-2361-2362-2363-2364-2365-2366-2367-2368-2369-2370-2371-2372-2373-2374-2375-2376-2377-2378-2379-2380-2381-2382-2383-2384-2385-2386-2387-2388-2389-2390-2391-2392-2393-2394-2395-2396-2397-2398-2399-2400-2401-2402-2403-2404-2405-2406-2407-2408-2409-2410-2411-2412-2413-2414-2415-2416-2417-2418-2419-2420-2421-2422-2423-2424-2425-2426-2427-2428-2429-2430-2431-2432-2433-2434-2435-2436-2437-2438-2439-2440-2441-2442-2443-2444-2445-2446-2447-2448-2449-2450-2451-2452-2453-2454-2455-2456-2457-2458-2459-2460-2461-2462-2463-2464-2465-2466-2467-2468-2469-2470-2471-2472-2473-2474-2475-2476-2477-2478-2479-2480-2481-2482-2483-2484-2485-2486-2487-2488-2489-2490-2491-2492-2493-2494-2495-2496-2497-2498-2499-2500-2501-2502-2503-2504-2505-2506-2507-2508-2509-2510-2511-2512-2513-2514-2515-2516-2517-2518-2519-2520-2521-2522-2523-2524-2525-2526-2527-2528-2529-2530-2531-2532-2533-2534-2535-2536-2537-2538-2539-2540-2541-2542-2543-2544-2545-2546-2547-2548-2549-2550-2551-2552-2553-2554-2555-2556-2557-2558-2559-2560-2561-2562-2563-2564-2565-2566-2567-2568-2569-2570-2571-2572-2573-2574-2575-2576-2577-2578-2579-2580-2581-2582-2583-2584-2585-2586-2587-2588-2589-2590-2591-2592-2593-2594-2595-2596-2597-2598-259