Aspects of Black Hole Thermodynamics

by

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CERTIFICATE

This is to certify that the Ph.D. thesis titled " Aspects of Black Hole Thermodynamics " submitted by **Arundhati Dasgupta** is a record of bonafide research work done under my supervision. The research work presented in this thesis has not formed the basis for the award to the candidate of any Degree, Diploma, Associateship, Fellowship or other similar titles. It is further certified that the thesis represents independent work by the candidate and collaboration was necessitated by the nature and scope of the problems dealt with.

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ABSTRACT

Black holes are one of the most interesting objects of study in gravitational physics. Classically they trap everything including light. They are characterised by a *event horizon* which encloses a curvature singularity. A study of black hole mechanics shows that the black hole behaves as a thermodynamic system, with the area the event horizon as entropy, and a geometric quantity called surface gravity as the temperature T_H of the black hole. Semiclassical calculations, in which quantum matter fields are studied on a fixed black hole background show that it radiates particles like a blackbody, at temperature T_H. In this thesis, we try to understand black hole thermodynamics mainly from a statistical interpretation using a microscopic theory.

Black hole radiation and subsequent evaporation imply a non-unitary evolution as the final state of evaporation is a mixed thermal state. Here we study how inclusion of back reaction of infalling matter fields on the black hole geometry leads to interactions with the outgoing fields [i]. This interaction is unitary, and shows that back reaction effects are important in order to understand radiation.

However, the complete microscopic description has to come from a quantum theory of gravity, and String theory is one of the candidates for it. In this thesis we study how string theory gives a microscopic description of certain extremal and near extremal black holes which arise in low energy string theory. These black hole solutions correspond to string solitons or D branes wrapped on compact manifolds. The logarithm of the degeneracy of states of the solitons are calculated giving the area law for black hole entropy. In this thesis we study extremal black holes [ii] and show that they occupy a special place in general relativity, infact semiclassical methods show their entropy to be zero. Though the generic extremal black hole in string theory obeys the area law, some special ones do not. To understand the microscopic counting, we propose that the string states correspond to limiting extremal black holes, instead of exactly extremal ones.

Further, we study how the microscopic theory of D branes can be used to study radiation from black holes [iii]. We study fermionic radiation from a four dimensional black hole and show that the radiation rate has a structure which can be reproduced from a 1+1 dimensional Conformal Field Theory. We also give a microscopic calculation in which two open strings collide to give a fermionic closed string mode in

the bulk. This calculation gives the fermion radiation rate from a five dimensional black hole upto coefficients.

There are 2+1 dimensional black holes called BTZ black holes which appear in the near horizon geometry of the above string black holes. We study fermion radiation [iv] from these and show that like the higher dimensional black holes, they have a rate which can be reproduced from a 1+1 dimensional Conformal Field Theory. The 2+1 BTZ black hole is asymptotically anti de Sitter, and it has been conjectured that string theory on anti-de Sitter space is dual to a conformal field theory which lives on the boundary of the AdS space-time (AdS/CFT correspondence). In the light of this, we examine radiation of scalar, fermion and vector particles for all partial waves from a five dimensional black hole [v], by probing the near horizon BTZ geometry. We find that the radiation rate can be *exactly* reproduced from a 1+1 dimensional Conformal field theory, which lies on the boundary of the near horizon geometry.

LIST OF PUBLICATIONS

Publications related to thesis

[i] Fermions in Black Hole Space-time: Hawking Radiation and Back Reaction.
Phys. Rev. **D56** (1997) 4962, with Parthasarathi Majumdar.

[ii] Can Extremal Black Holes Have Non-zero Entropy ?
Mod. Phys. Lett. **A12** (1997) 3067, with Saurya Das, P. Ramadevi

[iii] Black hole fermionic radiance and the Effective String Description.
IMSc-97/06/21.(hep-th/9707124) with Saurya Das, Parthasarathi Majumdar, Tapobrata Sarkar.

[iv] Emission of Fermions from BTZ Black Holes.
Phys. Lett. **B 445** (1999) 279.

[v] Black Hole Emission Rates and the AdS/CFT Correspondence.
IMSc-99/03/07, CGPG-99/7/1 (hep-th/9907116), with Saurya Das.

[vi] Black Hole Thermodynamics,
To be published in proceedings of YATI, 1999, Eds. S. K. Chakrabarty, IJP.

Other publications not related to thesis

[1] High Energy Effects on D-Brane and Black Hole Emission Rates.
Authors: Saurya Das, Arundhati Dasgupta, Tapobrata Sarkar.
Phys. Rev. **D55** (1997) 7693.

[2] Planckian Scattering of D-branes
Authors: Saurya Das, Arundhati Dasgupta, P. Ramadevi and Tapobrata Sarkar.
Phys. Lett. **B428**(1998)51.

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Contents

Chapter 1

Introduction

Black holes are formed when stars with mass around few times the mass of our sun ($M_\odot$) or higher collapse under their own gravitational field. All the mass of the black hole is confined within a radius as small as $2GM/c^2$ [1](~ few km for a star with mass $6M_\theta$) for non-rotating ones. This radius, is called the Schwarzschild radius. Classically nothing, not even light can escape from within this critical radius. At the centre of the black hole is a curvature singularity. Objects which fall inside the black hole inevitably fall to the centre. Since the nature of space-time is undefined at the centre due to the singularity, we cannot determine the physics at that point in the framework of classical physics.

We donot know the complete theory of quantum gravity yet. However, as we shall study in this thesis, our understanding of the quantum black hole has progressed considerably in the last few years. We shall study the semi-classical approach to quantisation in the beginning. In this gravity is treated as classical, whereas all other fields are treated as quantum fields. This approach led to the discovery of the phenomenon of black hole radiation [1]. Semi-classically black holes cease to be objects which can only absorb, but they also radiate particles in a thermal spectrum. This spectrum is very much similar to black body radiation at a temperature related to a geometric quantity called the surface gravity of the black hole. This evidence and other laws of black hole mechanics led to the identification of black hole as a thermodynamic system.

Ordinary thermodynamic systems have a statistical description in terms of microscopic constituents e.g. a gas has molecules as its microscopic constituents. Does the black hole have such a description? We shall precisely address this question in this thesis. String theory is a theory which describes all interactions including

[1] $G \equiv$ Newton's constant, c =speed of light

gravity in a unified frame work, and hopes to give a complete theory of the quantum nature of space-time. We study how this theory helps us give a microscopic description of black hole thermodynamics. Whether it answers more difficult questions of the smoothening of curvature singularity is not very clear at present. There are alternative approaches like canonical quantisation of gravity using Ashtekar variables, which is beyond the scope of this thesis.

In this chapter we shall review aspects of black hole thermodynamics and semi-classical gravity including Hawking radiation. We give a brief introduction to String theory and explain some of the solitonic objects called D-branes which describe black holes. We also describe anti-de Sitter spaces which help us to understand the properties of certain black holes with the help of a recent conjecture relating physical quantities on anti de Sitter spaces to those of a conformal field theory which lives on its boundary.

Black Hole Thermodynamics

A study of classical black hole physics reveals a set of laws of black hole mechanics which are similar to the laws of thermodynamics. Like the irreversibility of entropy in a thermodynamic system, there is an inherent irreversibility in all phenomena associated with black holes. As we know, black holes are solutions of Einstein's equation which relates space-time to matter. The metric which is a second rank tensor under general coordinate transformations, is the field which describes the nature of space-time and hence gravity. The metric of the simplest black hole the Schwarzshild metric:

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{1.1}$$

Where M is the mass of the black hole, and t, r, Ω stand for the time, radius and angular coordinates respectively. There is a curvature singularity at $r = 0$. However the metric singularity at $r = 2GM/c^2$ can be removed by a coordinate transformation. At this critical radius, all non-spacelike curves bend into the black hole, except the light rays given by $r = 2GM/c^2, \theta = constant$, which reach future infinity. All such light rays travel parallel to each other, and constitute a null surface. This surface is called the event horizon of the black hole and lightcones inside the event horizon bend towards $r = 0$ instead of future timelike infinity. A future directed light signal sent from inside the event horizon does not reach future null infinity, but ends up at the singularity. Thus at any space-like slice containing a black hole, there exists future directed non-spacelike curves which do not reach future infinity, and thus in subsequent space-like slices they must remain within a black hole. Thus it is possible for two black holes to unite to form one or new black holes to form at later times. However, black holes once formed cannot disappear or break into two [2]. Bifurcation of black holes is not allowed as any non-spacelike curve inside the black hole can be deformed into another through a sequence of curves. Thus since space-time is connected, in a later slice, the event horizon cannot bifurcate into two. This irreversibility in black hole processes actually leads to the area increasing theorem, which states that in any process, the *area of the event horizon A_{BH} never decreases.*

The generic black holes present in General Relativity posess angular momentum or charge (or both), apart from their masses. It can be shown that in any process the change in the Mass M_{BH}, angular momentum J, charge Q of the black hole are related to the change in area of the event horizon A_{BH} by a simple law. Thus, taken together, the two laws of black hole mechanics are [3]:

$$\delta M_{BH} = \frac{\kappa}{8\pi G}\delta A_{BH} + \Phi_{BH}\delta Q + \Omega_{BH}\delta J \tag{1.2}$$

$$\delta A_{BH} \geq 0. \tag{1.3}$$

Where M_{BH}, J, Q are mass, angular momentum, charge of black hole, A_{BH}, area of horizon, Ω_{BH} and Φ_{BH} are angular momentum and electromagnetic potential at the horizon; κ is surface gravity [2]. This is a geometric quantity, which is defined as the force exerted by an observer standing far away from the black hole to keep a unit mass test particle stationary at the horizon. It is remarkable that this force is constant all over the horizon, and hence is determined in terms of black hole parameters. These laws closely resemble the laws of thermodynamics if we identify the area of the black hole as entropy, and the surface gravity κ as proportional to the temperature of the black hole. Does this imply that the black hole is a thermodynamic system? Thermodynamic entropy has interpretations from information theory, and more importantly from statistical mechanics as the logarithm of the degeneracy of a microscopic states. In [4], it was shown that using the amount of information that is lost when a particle falls into the black hole, the area of the event horizon can be related to entropy. The statistical interpretation is the subject of study of this thesis. Moreover, the fact that the black hole has an intrinsic temperature associ-

[2]. Henceforth we work in natural units h=c=1

ated with it implies that if it is placed in a surrounding at a lower temperature, then it should radiate as a ordinary hot body does. Classically a black hole cannot radiate as the event horizon acts as a one way membrane. On using the semiclassical approximation near a black hole where all fields except gravity are quantum, S. W. Hawking showed that indeed black holes radiate particles at a temperature related to its surface gravity. We review semiclassical gravity and Hawking's derivation of black hole radiation next.

Semiclassical Approximation

In order to estimate the length scale where quantum effects of gravity become important, we determine the Compton Wavelength of a black hole and equate it to its Schwarzschild radius. Hence, the correct relation gives $h/M_{BH}c \sim GM_{BH}/c^2$. Where h is Planck's constant. Using the values of the constants, M_{BH} is determined as $\sim 10^{-5} gm$, and hence the size of the object as $10^{-33} cm$, a very small length indeed. Thus for black holes which are larger in size than $10^{-33} cm$, gravity can be taken as classical. However, for all other interactions, quantum effects are important at much higher length scales. In [1] scalar fields were taken in the background of a collapsing body. In curved space-time, the decomposition of a field into positive and negative frequency components is not unique. For example a positive frequency mode $e^{i\omega t}$ is a mixture of positive and negative modes of $e^{i\omega t'}$, when the time t of one observer is related to the other observer's time $t' = log t$ (such transformations are not allowed in flat space). Hence the concept of particles and anti-particles is not well defined in curved space. However, for asymptotically flat black holes, one can consider modes in the far past, and the modes in the far future and decompose them uniquely. The evolution of these modes can then be traced from the distant past of the black hole to the far future. The support of the massless scalar fields in the past is on the past null infinity surface ($\mathcal{I}^-$), and in the future on the future null infinity surface ($\mathcal{I}^+$). Using a Heisenberg description where the field operator Φ evolves, but the states remains same, it can be shown that the vacuum in $\mathcal{I}^-$ actually has particles in the thermal spectrum in $\mathcal{I}^+$. A detailed derivation of this construction shall be given in Chapter 1. Thus the vacuum defined by $a^- |0\rangle = 0$ where a^- is the annihilation operator of the past, has

$$\langle 0| N_\omega^+ |0\rangle = \frac{\sigma_{\rm abs}}{1 - exp(2\pi\omega/\kappa)}, \tag{1.4}$$

where N_ω^+ is the number operator in the future, and σ_{abs} is the 'Greybody factor' of the black hole. Since the state in the far past did not contain particles, these particles must have been radiated from the black hole. The spectrum has a remarkable similarity to the black body radiation spectrum at a temperature $\kappa/2\pi$. The 'Grey Body factor' or the absorption coefficient of the black hole, is an additional factor which comes due to the geometry of the black hole space-time. Since the black hole geometry is curved, the emitted particles have to travel through a potential barrier to reach an observer far away from the black hole. Thus only a fraction of the particles determined by the transmission coefficient of the potential barrier (also the absorption coefficient) of the black hole is detected by an observer standing far away. For certain black holes like the Schwarzschild black hole, the Greybody factor

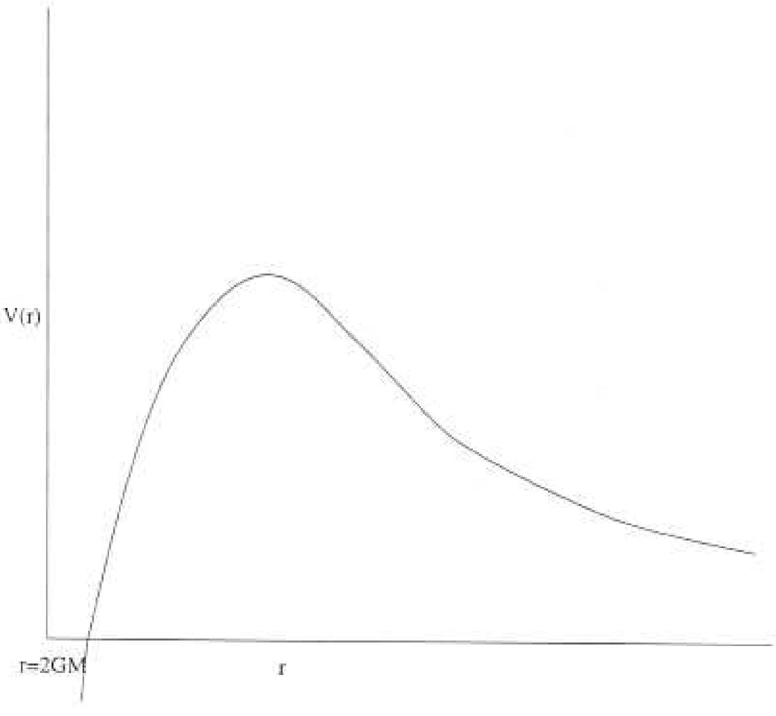

Figure 1.1: *Potential Barrier for a Scalar Field*

is frequency independent and for low frequencies equals the area of the horizon. For these black holes, the spectrum is very similar to black body radiation. For certain other black holes, this factor plays a crucial role as we shall see in Chapter 4, 5 and 6 of this thesis.

Most black holes are not stable against the above radiation. For the Schwarzschild black hole, $T_H = \kappa/2\pi \propto 1/M_{BH}$ This implies that a very small radiation would increase the temperature, and hence the radiation rate. This is a runaway process and the black hole rapidly evaporates away. A complete evaporation of the black hole implies that finally there will be radiation left in flat space-time.

The above lead to the famous 'information loss' paradox. The initial pure state which constitutes the black hole evolves into a thermal state in the future which can only be described by a density matrix. This is a non-unitary evolution, where all information about quantum numbers of particles which fall into the black hole are retained only partially in thermal averaged quantities. Non-unitary evolution is not allowed in quantum mechanics, where pure states can evolve only into pure states. The same situation may arise when a piece of charcoal completely burns away. However, we know the microscopic constituents of coal and there is a unitary microscopic process which explains the burning. Presence of a large number of degrees of freedom for the macroscopic burning coal system necessitates a thermodynamic description. Thus for a black hole also, we expect a resolution of the puzzle from a microscopic description.

The theory of quantum gravity will give the complete picture of Hawking Radiation. Will that theory have a different nature than other quantum theories where unitary evolution is a prime requirement?

On the other hand, we can think of the black hole as made up of microscopic constituents. The degeneracy of those states should give the entropy of the black hole. A interaction of the micro-constituents should give a radiative process which models Hawking radiation. Moreover the origin of thermalisation of these states for black holes have to be determined, as unlike other thermodynamical systems, the black hole is not associated with a heat bath.

In the absence of a quantum description of the black hole, the collapse of a body and its subsequent evaporation was re-examined. The derivation of Hawking radiation ignores the back reaction of the infalling and outgoing Hawking particles on the background geometry which is taken as fixed and classical. In [5] it was shown that as the particles near the horizon travel at the speed of light, their gravitational field is carried in a shockwave. When the shockwave hits the horizon, there is a change in the horizon coordinates. An outgoing Hawking particle sees this shift, and the resultant interaction between infalling and outgoing particles is completely unitary [6, 7]. It thus follows that the complete evolution of the infalling particles into the outgoing radiation may be described as a unitary S-matrix. However the Hawking spectrum can not be determined from the S-matrix description.

In this thesis, we study the back reaction of scalar and fermion matter *fields*

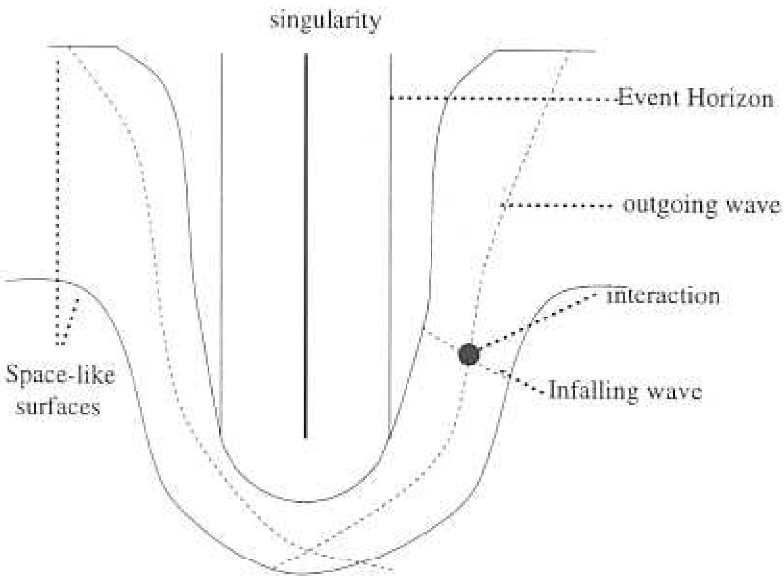

Figure 1.2: *Interaction due to back reaction*

on the black hole geometry [8, 9]. The fields have support on space-like slices very near the horizon. The energy momentum tensor of the matter fields is treated as a small perturbation on the back ground and causes a linearised shift in the black hole metric. As determined in [8], we find that the shift in the metric affects the outgoing particles non-trivially for the fermions as well as the scalars. The wavefunction of the outgoing particles is solved in the shifted metric, and it is shown that it depends on the energy momentum of the infalling particle. On treating the shift as classical, there is an delta function type of interaction between the outgoing fields and the infalling fields. This interaction is ultra local in the angular coordinates as well. Unlike the scalars, the simplest bilinears of the fermions do not see this ultra-local interaction [9]. On promoting the shift to a quantum operator through a correspondence principle, an exchange algebra for the fields is determined, which is not ultralocal as earlier. The fermions and scalars have a behave similarly in this case. Our calculations reveal that back reaction effects are important in the derivation of Hawking radiation and even in the lowest approximation in $\hbar$ lead to an interaction of the matter fields. However, this approach does not help us in determining the entropy of the black hole. The major breakthrough in this respect comes from String Theory. We shall examine the results obtained using string theory in rest of the thesis.

String theory is one of the approaches to quantum gravity. Here the graviton which is the quanta mediating gravitational interactions, arises as a vibrational mode of a string. The 'string' is a one dimensional object which replaces the particle as the fundamental object in all physical interactions. It has a fundamental length associated with it, l_s. As the action for the particle is determined by the proper distance along the trajectory of the particle, the string action is determined by the infinitesimal area swept out as it propagates in space-time. The 2-dimensional surface which arises due to string propagation is called the 'world sheet' of the string. The string world sheet action is thus [10]

$$I = T \int d\sigma d\tau \sqrt{-detg_{\mu\nu} X^{\mu} X^{\nu}} \tag{1.5}$$

where X are the coordinates of the string and $g_{\mu\nu}$, the metric of the space-time on which the string propagates, σ, τ parametrise the space and time directions on the world sheet. This action can be mapped to a 2 dimensional free scalar field action for the X^{μ}'s on the world sheet, with a set of constraints. Infact, the 2dimensional field theory is a conformal field theory, with a given central charge. Thus like in ordinary field theory, the $X's$ can be expanded in normal modes, and the quantisation of these modes gives particle excitations which are representations of the space-time lorentz group. Quantisation of a closed string gives the scalar, an antisymmetric gauge field and the graviton as massless excitations. There are other massive modes, with masses proportional to $1/l_s$. When we confine ourselves to energies much less than the above, field theories in space-time, corresponding to the massless modes are recovered. Thus we see that in string theory, gravity arises naturally in the same way as other fields. In this thesis we shall study the supersymmetric string, where there are fermionic fields for each X^{μ}, and the action is invariant upto a total derivative under 'supersymmetry' transformations of bosons to fermions and vice-versa. Due to consistency requirements, the superstring propagates in 10 dimensional Minkowski space. Low energy theory for superstring yields supergravity in 10 dimensions. [3]

We shall consider the open string in 10 dimensions, here and show how they give rise to the concept of extended solitons called D branes [11]. As the $X's$ are fields on the world sheet, appropriate conditions have to be imposed on the boundary $\sigma = 0$

[3]If the string is purely bosonic, then a tachyon with negative mass is present in the spectrum. This tachyon is not present in the superstring.

and $\sigma = \pi$. Two kinds of boundary conditions can be imposed on the X^μ 's of the open string. One of them is the Newman boundary condition, in which the normal derivative along the boundary is zero. The other boundary condition, which leads to important objects in string theory is the Dirichlet boundary condition. This is given by $X^\mu = constant$ along $\sigma = 0$ or π. In general the open string can have the following boundary conditions:

$$\begin{aligned} \partial_n X^\mu &= 0 \quad \mu = 0....p \\ X^a &= 0, \quad a = p+1...9. \end{aligned} \tag{1.6}$$

In the above, clearly, the open-string end points are fixed on a p+1 dimensional hypersurface which lies along $\mu = 0, 1.., p$ directions. This hypersurface is dynamical. It has a mass which is proportional to $1/g_s$ (g_s is the coupling constant of the string), and hence is infinitely massive in perturbative string theory. Thus these branes, called Dp branes are actually solitons in string theory. They are charged under $p+1$ form field strengths called Ramond-Ramond fields, [11] which arise also in the massless spectrum of super string theory. Thus in perturbative string theory, these solitons are described by open strings stuck on them. Their presence breaks translational invariance along the a directions. The commutator of two supersymmteries gives a translation, and hence only some of the space-time supersymmetries are preserved in the presence of the D brane. States which satisfy a certain bound relating masses to central charges of the SUSY algebra are called BPS states. They preserve $1/2^n$ (where n is an integer) of the supersymmtery transformations. Dp branes precisely satisfy this condition, and this property helps in modelling extremal black holes with the same $M = (1/\sqrt{G})Q$ criterion.

The black holes corresponding to these Dp Branes are obtained when these hypersurfaces are wrapped on compact manifolds like the torus. As the radii of the torus are taken to zero, the branes appear as point like objects in the non-compact directions. In the low energy limit where all the massive particles of the string can be integrated out, the gravitational field due to these objects are determined by solving ordinary supergravity equations. The metric obtained with at least three non-zero charges, e.g. Q_5 D5 branes, Q_1 D1 branes, and momentum along the D1 branes wrapped on $T^4 \times S^1$, are black hole solutions with a stable horizon [12].

In this thesis, we shall study such black holes. We shall also examine briefly black holes which arise in heterotic string theory, and also 11 dimensional supergravity

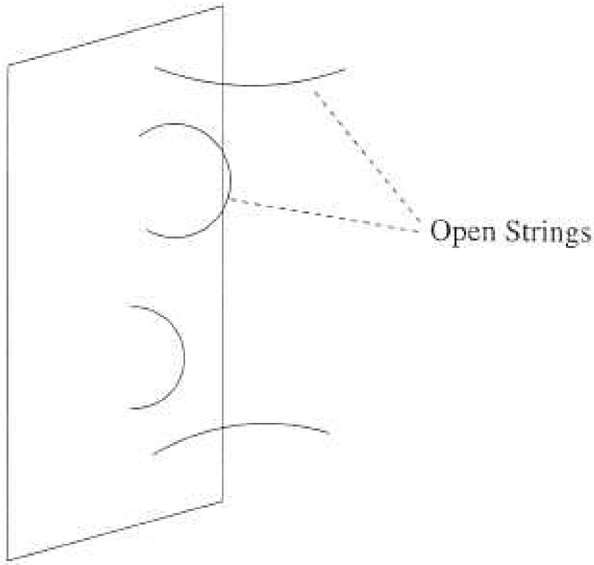

Figure 1.3: *D-branes with open strings stuck on them*

compactified down to lower dimensions. Heterotic string theory arises when only the left (or right) moving modes of the string are supersymmetrised. 11 dimensional supergravity is low energy limit of a theory called M-theory, which gives the strong coupling limit of type IIA theory. Other sueprstring theories are also related to M-theory by various dualities. The spectrum of 11 dimensional supergravity consists of the graviton, the gravitino, and a four form field strength. Hence five-dimensional and 2 dimensional extended branes naturally arise in this theory as they couple to the four form gauge field and its dual in 11 dimensions. As described above, the M-branes like the D-branes can be wrapped on compact dimensions to yield point like excitations in lower dimensions. The black hole solutions have multiple U(1) gauge fields in the lower dimension.

The string black holes for which the entropy was reproduced from a microscopic counting correspond to extremal black holes. In General Relativity extremal black holes occupy a special place as their temperature is zero and their entropy is not proportional to area, some derivations infact show it to be zero. In this thesis we review all aspects of extremal black hole entropy [14]. It can be shown that the entropy of a black hole is proportional to the euler characterestic of the topology of near horizon 2 dimensional $r-\tau$ (Euclidean time) plane. For a generic non-extremal black hole, this topology is that of *disc* whose euler characterestic is 2. However for an extremal black hole this is *annulus*, whose euler characterestic is zero [13]. Hence extremal black holes being topologically different from non-extremal ones have their entropy as zero. We show in this thesis how ordinary black hole processes like Hawking radiation and superradiance cannot take a non-extremal black hole to an extremal one. All this shows that extremal black holes have a special place in general relativity. What about extremal black holes in string theory?

In string theory, the heterotic black holes have a horizon which coincides with the curvature singularity at the center of the black hole space-time. For these black holes semi-classical entropy is zero as area of the horizon is zero [15]. For black holes formed by branes, there are multiple U(1) charges, and generically the near horizon topology is not that of an annulus. For these black holes, entropy is non-zero and equals the area of horizon. However under restrictions that *all* U(1) charges of the black hole are equal, the extremal metric of general relativity is recovered, whose near horizon Euclidean topology of $r-\tau$ plane is that of an annulus. It is expected that the entropy of these black holes is zero.

The above black holes correspond to BPS states in string theory. Since the BPS relation involves relation between the quantum numbers like mass and charge of the state, these quantities donot undergo renormalisation. The counting of string states is done where the string coupling g_s is weak, and the effective gravitational coupling, which is proportional to $g_s Q$, (Q is the charge of the branes)is small. The background space-time is essentially flat in this limit. The degeneracy of states obtained in this limit remains the same as $gQ \gg 1$ and the black hole with the same quantum numbers arises.

It was shown in [15], that the logarithm of the degeneracy of heterotic string states corresponding to the black hole is finite. Due to the presence of high curvatures near the horizon of the black hole, it can be assumed that stringy corrections modify the metric in such a way that the black hole entropy ings finite. Using this consideration, the black hole entropy is shown to be the logarithm of the degeneracy of states upto a numerical factor. This reveals that it is perhaps possible to model black holes by string states.

For black hole in five dimension obtained by compactifying D branes on a five dimensional comapact manifold ($T^4 \times S^1$ or $K^3 \times S^1$), a similar counting of states gives the logarithm of string states to be finite. In this case, since the black hole has a stable and finite horizon, the area of the black hole is finite. Black hole entropy *exactly* equals the degeneracy counting. In this thesis we point out that though the generic D brane black hole obeys the area law, the black hole considered in [16] is obtained when all the U(1) charges are equal and *does not* obey the area law. It's near horizon topology is that of an annulus, and its entropy is zero. Thus there is an apparent contradiction in the results. We point out that this may be resolved by observing that the black hole considered is extremal only in a limiting sense, in which case the area law continues to hold. The nearly extremal black holes are stable under hawking radiation due to vanishingly small hawking temperature, and hence can be modelled by stable states in string theory. However, this resolution of the puzzle is not unique.

To have a complete understanding of the microscopic description of black holes, we have to reproduce Hawking radiation. In this thesis, we shall study radiation from the string black holes and try to model them by a microscopic radiative process. To observe radiation, the black holes have to have non-zero temperature, and we take the 'near' extremal black holes corresponding to the black holes described above. It has been shown that Hawking radiation rate for scalars from these black holes [17, 18] can be reproduced from a 1+1 dimensional CFT. Here we shall study fermion radiation try to see whether a radiative process can be determined to model the radiation [19].

The particular four dimensional black hole we consider in this thesis is a solution of N=4 SUGRA in four dimensions. It is obtained by compactifying on T^7, 1a configuration of three M5 branes intersecting along a line to get a black hole in 4 dimensions [20]. In addition to the three five brane charges, there is momentum along the common line of intersection. This is achieved by boosting the metric in the compactified common intersection line of the five branes. Momentum along compact direction is quantised, and appears as a U(1) charge in the lower dimension. When there is only left moving momentum, the black hole is extremal. Addition of right moving momentum makes the black hole non-extremal, with a non zero Hawking temperature. We study emission rates of minimally coupled fermions, and determine the greybody factor in the above black hole background. The greybody factor has the following form:

$$\sigma_{\text{abs}} = g_{eff}\omega\left(e^{\omega/T_H}+1\right)\rho_B\left(\frac{\omega}{2T_L}\right)\rho_F\left(\frac{\omega}{2T_R}\right) \tag{1.7}$$

Where g_{eff} is a constant depending on the charges, $T_L and T_R$ are quantities proportional to the left and right moving momentum, and the Hawking temperature $T_H = T_L T_R/2(T_L+T_R)$. ρ_B and ρ_F stand for the bose and fermi distributions respectively. For $T_L \gg T_R$, this rate can be reproduced from a 1+1 dimensional

CFT with the central charge which reproduces the entropy of the black hole. The radiative process which gives rise to the above rate is an interaction between CFT excitations with left moving weight 1 and right moving weight 1/2. However, since this minimally coupled fermion is not present in the spectrum of M-theory compactified to 4 dimensions. Therefore we donot attempt to give a microscopic description of the above radiation rate.

We give a plausible microscopic radiative process for fermion emission from the D-brane black hole in 5 dimensions, where the microscopic structure is well understood. This consists of Q_1 D1 branes, and Q_5 D5 branes wrapped around a compact manifold $T^4 \times S^1$. The D1 brane is wrapped on S^1, and in addition, there is momentum along the S^1 direction, which is carried by the open strings stuck on the D branes. The radiative process which models emission from the D brane configuration, [17, 18], is the interaction of a left moving open string colliding with a right moving open string to give a closed string excitation in the bulk. From conservation of spin, one of the open string modes is taken to be fermionic and the other bosonic. The emitted closed string mode is the gravitino with vector polarisation along the T^4 directions. Thus for the non-compact directions, this transforms as a spin 1/2 particle. The amplitude for this process is calculated and it is found that it is proportional to ω, the energy of the emitted gravitino. Since the degeneracy of the massless open string states colliding is very high, we assume that the initial states are thermalised at left and right temperatures T_L and T_R. Thus the emission rate has the form:

$$\Gamma_D \sim \omega \rho_B \left(\frac{\omega}{2T_L}\right) \rho_F \left(\frac{\omega}{2T_R}\right) \frac{d^3k}{(2\pi)^3} \tag{1.8}$$

This upto a coefficient is exactly the emission rate from the 5D black hole as found in [21].

In all the derivations given above, the string theoretic calculations are done when $g_s Q \ll 1$. For near extremal black holes, which are not BPS states, there is no non-renormalisation theorem which prevents the decay rates from receiving corrections as the result is continued to the black hole regime. Thus the matching of decay rates cannot be explained. Moreover, the location of the D-brane degree of freedom or the CFT degree of freedom is obscure. In the black hole space-time, it is not possible to determine where these states lie.

In this thesis we shall study anti-de Sitter spaces and the 2+1 dimensional BTZ black hole [22] formed by certain identifications of AdS_3 space. The near horizon geometries of string black holes are the product of BTZ black hole and a compact manifold. As we shall see, these space-times are closely related to conformal field theories, and provide a better understanding of the black hole micro states.

Anti-de Sitter Space and Conformal Field Theories

Anti-de Sitter space-times are solutions of Einstein gravity with a negative cosmological constant. The curvature for the maximally symmetric metric AdS space-time is constant and equal to inverse of the cosmological constant. It can be thought as a hyperboloid in d+2 dimensions, where 2 of the directions are time like [23]:

$$-u^2 - v^2 + \mathbf{x}^2 = \mathbf{1}, \tag{1.9}$$

where $\mathbf{x}$ denotes d spatial directions. and the metric in the embedding space is:

$$ds^2 = -du^2 - dv^2 + d\mathbf{x}^2 \tag{1.10}$$

The group of isometries of this space is SO(2,d). The space includes closed time like curves which can be removed by going to the covering space of the manifold. The metric can also be mapped to the upper half plane in $d+1$ dimensions. Here, the metric is written in Poincare coordinates as:

$$ds^2 = \frac{1}{x_0^2}\left(dx_0^2 + dx_i dx^i\right) \tag{1.11}$$

where $x_0 \geq 0$ and x_i are such that $i = 1tod - 1$ and include a time like direction. The boundary of the space is given by $x_0 = 0$ and the point at infinity. It can be shown that the boundary is time like, and light rays take finite time to propagate to the boundary. The d dimensional boundary of AdS_{d+1} is Minkowski space, and the global group of isometries SO(2,d) acts as the conformal group on this. From this it is anticipated that physical quantities calculated in the bulk AdS space-time, can have a relation with conformal field theories defined on the boundary. In case of AdS_3, this is closely related to the fact that 2+1 dimensional gravity with or without a cosmological constant can be mapped to a Chern Simons theory [24]. As we know Chern Simons theory is a topological field theory which induces a conformal field theory on the boundary of the manifold. In case of AdS_3, the gauge group of the field theory is $SL(2,R) \times SL(2,R)$. Though the bulk field theory is topological, introduction of a boundary gives rise to boundary degrees of freedom which constitute a Wess-Zumino-Witten Conformal field theory.

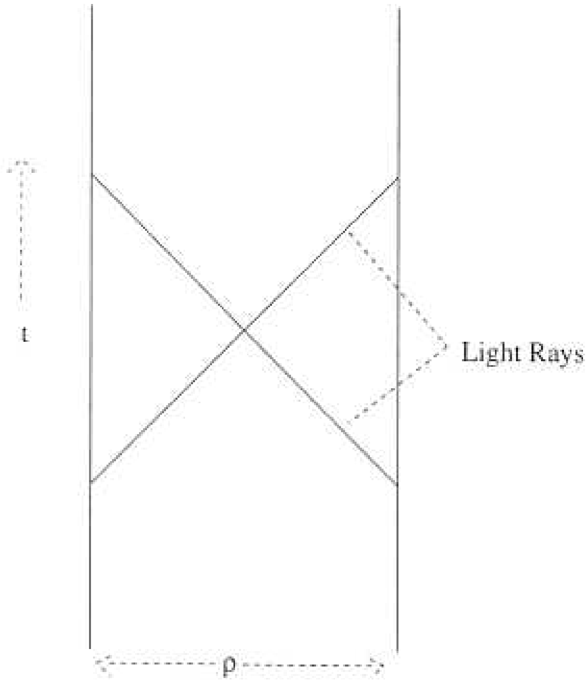

Figure 1.4: *Anti-de Sitter Space-time*

Independently, it was shown by Brown and Henneaux [25], that the asymptotic generator of diffeomorphisms which preserve the AdS metric fall off conditions satisfy a Virasoro algebra, and constitute a Liouville field theory. The close association of asymptotically AdS space with a conformal field theory was used to quantise a 2+1 dimensional black hole, called the Banados-Teitelboim-Zanelli (BTZ) black hole. This black hole solution arises with suitable identifications of AdS_3 space-time. The black hole solution however has no curvature singularity but a constant curvature. It obeys the area law for entropy and the absorption coefficient for various particles can be calculated in a manner similar to higher dimensional black holes.

To quantise the black hole, the horizon was used as a boundary and the conformal field theory induced on it determined. It was shown that for a small cosmological constant and under certain assumptions about what constitutes the physical states, the degeneracy of this conformal field theory gives the entropy of the black hole exactly. Historically this is one of the first microscopic countings of black hole entropy [26].

In this thesis we consider absorption coefficients for fermion particles propagating on the BTZ geometry [27]. The SO(2,2) covariant derivative for the fermions involves the term involving the derivative, the spin connection corresponding to the SO(1,2) symmetry well as the triads which represent the connection along additional SO(2,2) generators. This 'non-minimally' coupled fermion equation is solved exactly to get the fermion wavefunctions. The potential barrier calculation then gives the greybody factor. We initially calculate the GBF for the s-wave fermions only. It is seen that this has a structure very much suggestive of a underlying 1+1 CFT, with a left and right splitting of temperatures, very much similar to that of the D1 D5 system. Also the comparison of the rate with the fermion absorption cross-section of D1 D5 black hole gives the same answer, upto phase-space factors. However, if we want to obtain Hawking emission rates for these fermions, we find that the local temperature for these fermions decreases as ρ^{-1} (radial distance) and is essentially zero at spatial infinity. To obtain the Hawking radiation rate and compare with higher dimensional black holes, we take an observer standing at $\rho = l$, where l is related to the inverse of the cosmological term. To determine a microscopic description of the radiation process, we need to know what the fermions correspond to in the boundary CFT. However, in the frame work of the Chern Simons theory, it is difficult to include particle excitations. To consider particle excitations on AdS_3, a theory with all the particles including gravity, has to be defined on the bulk. The answer to this comes again from String Theory on AdS_3 spaces.

Evidences for the relationship of string theory on AdS spaces and conformal field theories arose in the description of D-branes. The world volume theory of N coincident D3 branes is a super-conformal SU(N) gauge theory. In the low energy limit, the supergravity solutions representing these extended-branes is a metric with a horizon, whose near horizon geometry has the same symmetries as AdS_5 space [28]. The size of AdS space is determined by N. Calculations of the absorption coefficient for the supergravity branes can be reproduced from a 4-dimensional Super-conformal field theory, and as mentioned earlier, the D1 D5 system was shown to be dual to a 1+1 dimensional CFT. The near horizon geometry of the D3 brane solution is $AdS_5 \times S^5$ and that of the D1 D5 system compactified on $T^4 \times S^1$ is $AdS_3 \times S^3$. The boundary of AdS_5 is 4 dimensional, and a $\mathcal{N} = 4$ supersymmetric theory can be defined on it. Similarly, AdS_3 has a 1+1 dimensional CFT defined on the boundary. All these considerations, lead to a conjecture: Superstring theory on AdS spaces are dual to Superconformal theories which lie on the boundary of AdS space-time. In case of the N coincident D brane configurations, the duality relates it to superconformal SU(N)gauge theories. The details of this can be found in [29].

Since AdS spaces are not asymptotically flat, interactions are not cut off at spatial infinity. Thus instead of scattering amplitudes, the relevant physical observables are correlators. Fields in the bulk space-time are dual to operators on the boundary. The bulk field acts as a classical source for the boundary operator. If ψ is the bulk field then, there exists a coupling with the boundary operator O of the form: $\int d^d x \psi_0 O$, where ψ_0 is ψ evaluated at the boundary. The bulk-boundary relation in the low energy limit has the following form [30]:

$$exp(-I(\psi)) = \left\langle exp(\int \psi O) \right\rangle_{CFT} \tag{1.12}$$

The classical supergravity action for the field in AdS space is represented by $I(\psi)$ (given ψ_0). One very important point to note in the above is that classical quantities in the bulk get related to quantum expectation values of operators in the boundary. Since in the supergravity limit $g_s N$ is kept fixed and large, with $g_s \to 0$ and N very large, the 't Hooft limit of Yang Mills theory is realised in the boundary theory. In this limit, only the planar diagrams dominate and physical quantities are calculable.

Using this correspondence, black hole emission rates can be understood very easily. In this thesis we consider emission from a five dimensional black hole of $\mathcal{N} = 8$ supergravity [31]. This black hole can be embedded in Type IIB string theory by modelling it by the D1-D5 system, wrapped on $T^4 \times S^1$. On lifting the 5 dimensional black hole to 6 dimensions, a black string solution is obtained. The near horizon geometry of this configuration is $AdS_3 \times S^3$. We calculate absorption cross-sections for scalars, fermions and vector particles for all partial waves and find that there is a common form for them:

$$\Gamma_H = g_{\rm eff} \exp(-\pi\omega/\kappa) \left|\Gamma(h + i\omega/4\pi T_L)\Gamma(h + s + i\omega/4\pi T_R)\right|^2 \frac{d^4k}{(2\pi)^4} \tag{1.13}$$

The temperatures T_L and T_R satisfy $4\pi/(\kappa) = 1/T_L + 1/T_R$, ω, is the energy of the outgoing particle, h is a number related to the orbital angular momentum of the emitted particle, s is the spin of the particle and $g_{\rm eff}$ depends on the charges of the black hole and kinematic factors. This structure, also noticed earlier from independent calculations for the scalars and fermions, is suggestive, and $h, h+s$, seem to correspond to the left and right weights of conformal operators.

As per the AdS/CFT conjecture, the information about the bulk AdS is contained in the CFT defined on the boundary of the near horizon geometry. The correlators of the CFT are determined using 1.12, for each field separately. A plane wave incident on the CFT from asymptotics then couples to the CFT operator by a coupling of the form: $\int \psi O$. This plane wave is retained as classical, and the operator O is quantum. A quantum mechanical calculation is then done to determine the emission rate of the ψ particles, similar to emission phenomena from atoms in the presence of a classical electromagnetic field. This calculation gives the emission rate as:

$$\Gamma_D \sim \int e^{ip.x} G(x,0) d^2x \tag{1.14}$$

A knowledge of the correlator $G(x,0)$, then gives us the emission rate. We find that this is exactly the same as obtained in the greybody factor calculation. This is a quantum mechanical description of the radiation from a black hole. The CFT degrees of freedom which lie on the boundary of the near horizon geometry constitute the quantum degrees of freedom of the black hole space-time. It should be mentioned that all these emission rates are low energy emission rates from the black hole.

As the above results show, we definitely have a understanding of microscopic states corresponding to certain black holes in string theory. However non-extremal black holes, and the Schwarzschild black hole remain an enigma. Though the Schwarzschild black hole is a possible solution of low energy effective string theory, they cannot be modelled by D-branes. There are some attempts to quantise Schwarzschild black holes in higher dimensions, using Yang Mills theory describing D0 branes, but they are confined to order of magnitude estimates [32]. There is a microscopic counting of Schwarzschild entropy in the framework of canonical quantisation of gravity. In this degrees of freedom also belong to a 1+1 dimensional CFT induced on the horizon [33]. Infact, a recent work by S. Carlip shows that the diffeomorphisms which leave the horizon in any black hole invariant constitute a CFT algebra. The asymptotic density of states of this give the black hole entropy [34]. This semi-classical analyses has to be reproduced from an underlying quantum theory of gravity.

This thesis is organised as follows, in the next chapter, we describe semiclassical effects and study backreaction of fermion and scalar fields in a black hole background. The Schwarzschild black hole is taken for simplicity, and we find the interactions of the fermionic and scalar outgoing and infalling fields, and show that they cannot be ignored in a derivation of Hawking radiation. In the third chapter, we consider extremal black hole entropy and see that semiclassical derivations suggest that it is zero. However for string theoretic black holes only a very special case of extremal black holes has this feature. Since string theoretic counting gives finite entropy for all, we study the special case, and suggest a resolution of the apparent contradiction for those black holes. In the fourth chapter, we try to understand Hawking radiation form a black hole in 4 dimensions, obtained by compactifying M theory on T^7. The radiation rate has a structure reproduceable from a 1+1 CFT. We also give a microscopic description of fermionic radiation from the D1 D5 black hole. In the fifth chapter, we discuss the 2+1 dimensional BTZ black hole, and fermionic radiation associated with it. In the sixth chapter we consider radiation form a five dimensional black hole, whose near horizon geometry is the product of 2+1 dimensional BTZ black hole and a compact manifold. The emission rates for scalars, fermions, and vectors are determined by probing the near horizon geometry. The answers are reproduced from a 1+1 dimensional CFT which lies on the boundary of the near horizon geometry. Lastly in the seventh chapter, we conclude with a discussion of the current status of understanding of black hole thermodynamics.

Chapter 2

Back Reaction Effects in Hawking Radiation

The discovery of black hole radiation is one of the most interesting results in theoretical physics. The horizon, which is a one way membrane classically, ceases to be one in the presence of quantum fields. This phenomenon of 'Hawking Radiation' also leads to the information loss paradox. Evaporation of the black hole due to radiation suggests a non-unitary evolution. We show here that the study of black hole evaporation is incomplete without including the effects of back reaction of the fields on the black hole space-time. In the initial derivation of Hawking's result [1], the gravitational effect of the quantum fields propagating on the black hole space-time were completely ignored. In this chapter, we try to understand why this cannot be done. In the first section we define what is back reaction and show the non-trivial effects infalling particles have on the horizon even classically. We study this for scalar and spin half particles propagating in the background of a massive Schwarzschild black hole. In the next section we study the effect of quantising the fields and how a non-trivial interaction arises between the infalling particles and the outgoing Hawking particles. We also investigate the case where gravitational field due to the matter fields is itself 'quantised' by a correspondence principle. This leads to a non-trivial exchange algebra of the infalling and the outgoing particles. The use of this to give a complete unitary evolution for the black hole space-time, is a work for the future.

2.1 What is back reaction?

We begin this section by analysing the propagation of quantum fields (spin 0 and spin 1/2) with the black hole space-time as a fixed classical background. We observe their behaviour on space-like slices near the horizon. We take a Schwarzschild black hole for simplicity. The extension to charged and rotating black holes is straight forward. The metric for this black hole of mass M, is given in t, r, θ, ϕ coordinates as:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{2.1}$$

The metric is asymptotically flat, as $r \to \infty$, $g_{\mu\nu} \to \eta_{\mu\nu}$, the flat Minkowski metric. The null geodesics of this metric are given by $u = t - r^* = c, v = t + r^* = c$, where $r^* = r + 2GM \log(r/2GM - 1)$, and c is a constant. The coordinates u, v, diverge near the horizon, $r = 2GM$, with $u \to \infty$ and $v \to -\infty$. The asymptotic null infinities are given by $v \to \infty$, $(\mathcal{I}^+)$ in the future, and $u \to -\infty$ $(\mathcal{I}^-)$, in the past. The metric in the above coordinates is singular at $r = 2GM$. The singularity of the metric can be removed by resorting to the u, v coordinates, in which the metric looks like:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dv du + r^2 d\Omega^2 \tag{2.2}$$

We examine matter propagation on this particular geometry in the next subsections.

2.1.1 Scalar Fields in Schwarzschild black hole

Hawkings's [1], initial analyses used asymptotic fields at $\mathcal{I}^+$ and $\mathcal{I}^-$ to derive black hole radiation. Here, we shall study the behaviour the scalar wavefunctions on a Cauchy surface very close to the horizon. The relevant equation of motion of the l^{th} partial wave for the massive scalar in the metric 2.2, is given by [8]:

$$\left[\partial_u \partial_v - e^{(v-u)/4GM} \left(\frac{l(l+1)}{r^2} + m^2 + \frac{2GM}{r^3}\right)\right] r\phi = 0 \tag{2.3}$$

For a collapsing body, only the future event horizon exists, given by $u \to \infty$. From the above, we can ignore terms proportional to e^{v-u} and it follows that, $\partial_u \partial_v \phi = 0$. Hence $\phi = \phi^{in}(v) + \phi^{out}(u)$. Thus near the horizon, the wavefunction splits up into two functions one outgoing $\phi^{out}(u)$, a function of u and the other ingoing $\phi^{in}(v)$, a function of v. Before the collapsing body shrinks behind its Schwarzschild radius, the ingoing modes $\phi^{in}(v)$ travel through the centre of the body to emerge as the outgoing mode $\phi^{out}(u)$. Near the horizon, due to the fact that the local energy of the mode $\omega_l = \omega g_{00}^{-1/2}$, diverges, there is an infinite blue-shift associated with the scalar modes, the geometric optics approximation becomes valid and the waves

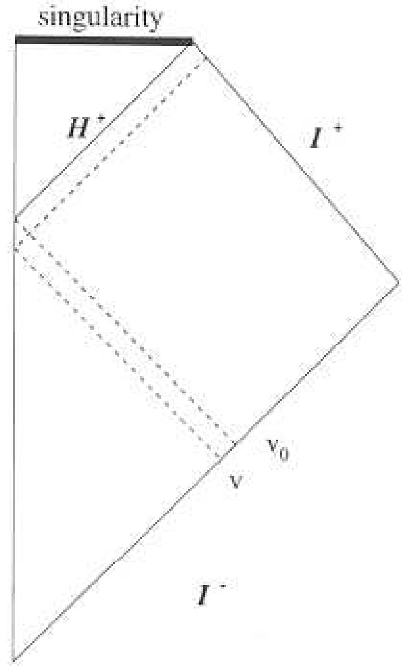

Figure 2.1: *Hawking Construction*

travel along null rays parallel to the horizon. Following Hawking's construction [1], we take a outgoing wave and trace its path back in time. For this purpose, a vector n^a is taken, which satisfies $l.n = -1$, where l^a is tangent to the future horizon. ϵn^a connects a point on the horizon to the constant phase surface of the outgoing mode. This is then parallel transported back in time and it always connects the same constant phase surface. The distance measured along the normal is proportional to the affine parameter along the null geodesics v=constant near the horizon. The affine parameter along these geodesics can be determined by assuming that the Schwarzschild solution is analytically continued into the collapsing body to obtain the past event horizon. Once n^a lies along the past event horizon, which is a $v = \infty$ null surface, $n^a = dx^a/d\lambda$, where λ is the affine parameter. It can be shown that $\lambda = e^{-\kappa u}$. So for small distances along n^a, we assume $dx^a \propto \lambda$, hence $\epsilon \propto \lambda \propto e^{-\kappa u}$. ($\kappa$ is the surface gravity of the horizon and for the Schwarzschild black hole is equal to $1/4GM$.) Thus the coordinate distances measured in u, get translated as $-1/\kappa log\epsilon$. When the null vectors are translated through the centre of the body, they still connect the surface of constant phase. The distance is now measured in $v_0 - v$, so that, $\epsilon \propto v_0 - v$. In order to match the ingoing wavefunction ϕ_i with the outgoing wavefunction translated back, we essentially have a reparametrisation:

$$\phi^{in}(v) = \phi^{out}(u(v)) \tag{2.4}$$

Here $u(v)$ is given as

$$u(v) \equiv v_0 - 4GM \log\left(\frac{v_0 - v}{C}\right) \tag{2.5}$$

Clearly v_0 acquires a special significance in the fact that it gives the critical ingoing time at which any ray released when extended to the future grazes the event horizon [8]. No ingoing wave released after v_0 reaches future null infinity. C is a constant, which we from dimensional analysis fix as $1/\kappa = 4GM$. This reparametrisation is valid only for $v < v_0$. Within Hawking's approximation, $\phi_i(v)$ for $v > v_0$ fall inside the horizon, and are lost forever. However, as we shall, see it is precisely these fields which leads to a non-trivial interaction.

2.1.2 Weyl Fermions in a Schwarzschild black hole

To study these, we take the metric (2.6), in Kruskal coordinates, where $x^+ = e^{v/2GM}, x^- = e^{-u/2GM}$,

$$ds^2 = \frac{32(GM)^3}{r} e^{-r/2GM} dx^+ dx^- - r^2 d\Omega^2 \ . \tag{2.6}$$

A choice of tetrad components $e^{m\ \alpha\dot{\beta}}$ reproducing this metric is given by [35]

$$e^{x^+ \dot{+}+} = -\frac{x^+}{2GM\Delta^{1/2}} \tag{2.7}$$

$$e^{x^- \dot{-}-} = \frac{x^-}{2GM\Delta^{1/2}} \tag{2.8}$$

$$e^{\theta\ \dot{-}+} = -1/r \qquad e^{\theta\ \dot{+}-} = -1/r \tag{2.9}$$

$$e^{\phi\ \dot{-}+} = -\imath/r \sin\theta \qquad e^{\phi\ \dot{+}-} = \imath/r \sin\theta \ . \tag{2.10}$$

Here, $\Delta \equiv (1 - \frac{2GM}{r})$. The dotted-undotted pair of indices indicate a tangent space vector as usual, with dotted (undotted) indices *per se* indicating chiral (antichiral) spinors of the tangent space Lorentz group. The Weyl equation is given by,

$$\imath\nabla^{\alpha\dot{\beta}}\psi_\alpha = 0 = \imath\nabla^{\alpha\dot{\beta}}\bar{\psi}_{\dot{\beta}} \ , \tag{2.11}$$

where

$$\imath\nabla^{\alpha\dot{\beta}}\psi_\alpha \equiv \imath e^{m,\alpha\dot{\beta}}(\,\partial_m\psi_\alpha - \omega_{m\alpha}^{\ \gamma}\,\psi_\gamma)\ etc. \tag{2.12}$$

and $\omega^\beta_{m\alpha}$ are the (chiral) spin connection matrices, given in terms of the tetrad components by the formula

$$\omega^\gamma_{m\alpha} \equiv e^n_{\alpha\dot{\beta}} e^{\gamma\dot{\beta}}_{n;m} \ . \tag{2.13}$$

Their spin connections are given in Appendix A. Near the horizon, $\Delta \to 0$; keeping in mind that length (and time) as measured by asymptotic observers scale by the singular factor Δ in the horizon region, because of infinitely large blue-shifts that the solutions of the dynamical equations undergo, one can rescale $\psi \to \psi/\Delta^{\frac{1}{2}}$. This reduces the pair (2.11) to

$$\partial_v \psi_+ = 0 = \partial_v \bar{\psi}_+ \tag{2.14}$$

$$\partial_u \psi_- = 0 = \partial_u \bar{\psi}_-, \tag{2.15}$$

where u, v are the null coordinates. Thus, the Weyl field decomposes into 'retarded' (outgoing) and 'advanced' (incoming) solutions near the horizon, similar to the scalar field.

$$\psi^{out} = \psi_+(u, \Omega) \tag{2.16}$$

$$\psi^{in} = \psi_-(v, \Omega)\ . \tag{2.17}$$

One can match the advanced and retarded propagation of these solutions at the horizon a la' Hawking [1], leading to the reparametrization

$$\psi^{out}(u(v), \Omega) = \psi^{in}(v, \Omega)\ , \tag{2.18}$$

where, $u(v) = v_0 - 4GM \log(\frac{v_0 - v}{4GM})$. The reparametrization is singular at v_0 which represents the latest reference 'time' at which an incoming wave leaves $\mathcal{I}^-$ to get scattered to $\mathcal{I}^+$ along the event horizon given by $u \to \infty$. For $v > v_0$, all incoming waves are trapped by the black hole.

In Hawking's original asymptotic analyses, this reparametrisation was used to relate modes at $\mathcal{I}^+$ with those on $\mathcal{I}^-$. Following the original treatment [1], the reparametrization above is used to compute the (asymptotic) Bogoliubov coefficients appearing in the Bogoliubov transformations connecting the creation-annihilation operators of fields having support on the two asymptotic null infinities

$$c_\omega = \int d\omega' \left(\alpha_{\omega\omega'} a_{\omega'} + \beta_{\omega\omega'} b^\dagger_{-\omega'} \right), \tag{2.19}$$

where c , $c^\dagger$ (a , $a^\dagger$) are the creation-annihilation operators associated with ψ_{out} (ψ_{in}) on $\mathcal{I}^+$ ($\mathcal{I}^-$). The distinction from the scalar case now manifests as one attempts to calculate the spectral distribution of the outgoing radiation by calculating the expectation value of the number operator $c^\dagger c$ in the vacuum on $\mathcal{I}^-$; For fermions, one must recall that these operators obey an anticommutation algebra instead of a commutation relation. For the scalar case the spectral distribution emerges as a Bose-Einstein distribution and for fermions, it is a Fermi-Dirac distribution. For both, the surface gravity has to be identified with the Hawking temperature [1], [35].

This analysis obviously ignores the change in the black hole geometry induced by infalling and outgoing fermionic matter - the backreaction. In the next section we attempt to incorporate it within the semiclassical approximation: the linearized change in the black hole metric due to infalling fermionic matter is determined, and re-expressed as a shift in the horizon. This is then used to determine an exchange algebra for the fermionic fields.

2.1.3 Classical Backreaction

The derivation of the Hawking radiation as sketched in the above two subsections, show how the scalar and fermion modes behave near the horizon. We find that the gravitational effect of these fields cannot be ignored. In a earlier work by 't Hooft [6, 7], it was shown that the gravitational fields of ordinary test particles lead to a non-trivial effect near the horizon. Consider a massless photon incident on a black hole, along $x^+ = 0$. The particle carries a spherical gravitational shock wave near the horizon. As it falls inside the horizon, a non-trivial shift in the horizon coordinates takes place. The shift, given in Kruskal coordinates is as below:

$$\delta x^+ = p_{\text{in}} f(\Omega, \Omega') \tag{2.20}$$

Where p_{in}, is the infalling particle's momentum, and the shift occurs at $x^- = 0$. $f(\Omega, \Omega')$ is the two dimensional Green's function on the sphere. This shift can also be incorporated as a shift in the metric itself.

Motivated by the above, we try to ascertain the back reaction due to the energy momentum tensor of infalling scalar and fermionic matter, as in [8]. The main observation is that the above shift can be traced to a shift in the metric components. We take an infalling energy momentum tensor near the horizon, whose dominant component is $T_{x^+x^+}(x^+, \Omega)$. We assume the other components are small. This energy momentum tensor is thus supported by fields which are released after $v = v_0$, so that they fall into the horizon. The effect of the infalling fields can be realised by taking an ansatz of the metric of the form:

$$ds^2 = -\frac{32(GM)^3}{r} e^{-r/2GM} dx^+ \left(dx^- + h_{x^+x^+}(x^+, \Omega) dx^+\right) + r^2 d\Omega \tag{2.21}$$

Where $h_{x^+x^+}$ is an infinitesimal, but smooth function. Using this ansatz in Einstein's equation, it is seen that:

$$(\nabla_{\theta,\phi} - 1)\, h_{x^+x^+} = kT_{x^+x^+} \tag{2.22}$$

where k is a constant. As earlier, the shift can also be absorbed by a coordinate transformation: $x^{'-} = x^- + \int h_{x^+x^+} dx^+$. This implies that in this new metric, the affine parameter along the null geodesics near the horizon is $\lambda' = e^{-\kappa u} + \delta v_0$, where $\delta v_0 = \int h_{x^+x^+} dx^+$. Thus for $v > v_0$, we arrive at the following reparametrization:

$$\phi^{out} = \phi^{in}(v(u) + \delta v_0). \tag{2.23}$$

Due to the sign of the shift, an outgoing field which would have reached $\mathcal{I}^+$ now gets trapped behind the horizon. This effect is non-trivial, and crucially shows that the backreaction effects are non-trivial on the outgoing Hawking particles.

We now repeat the above analysis for fermions, using Einstein-Cartan approach for convenience and use the two component notation for the fermions. The dominant effect of backreaction of quantum fermionic matter on the classical black hole geometry can be charecterized by a linearized perturbation of the frame components: $e_m^{\alpha\dot\beta} \to e_m^{\alpha\dot\beta} + h_m^{\alpha\dot\beta}$. The linearized fluctuations $h_m^{\alpha\dot\beta}$ are related to linearized fluctuations of the Schwarzschild metric according to

$$h_{mn} \;=\; e^{\alpha\dot\beta}_{(m} h_{n),\alpha\dot\beta} \;, \tag{2.24}$$

where the $e_m^{\alpha\dot\beta}$ are the Schwarzschild tetrad components given in (2.10). We are specially interested in the effect of infalling fermionic fields on the black hole geometry. Following Hawking's approach to black hole radiation [1], it is sufficient to restrict to waves of very high frequency near the horizon, i.e., adopt the geometrical optics approximation. For fermion fields falling on the horizon, therefore, the largest contribution to the backreaction will come from $T_{x^+,+\dot+}$ and $T_{x^-,-\dot-}$, the nonzero components of the energy momentum tensor in the longitudinal direction, where $T_m^{\alpha\dot\beta}$ may be taken to be (the expectation value of) the fermionic energy momentum tensor near the horizon. The other energy-momentum density components are negligible in the kinematical regime of interest.

As for the scalar case, the shift written in the two component notation gets related to the energy momentum tensor of the fermion.

$$h_{x^+,+\dot+} \;=\; \int d^2\Omega'\; f(\Omega\,,\,\Omega')\; T_{x^+,+\dot+} \;, \tag{2.25}$$

where, $f(\Omega, \Omega')$ is the Green's function of the Laplacian on the two-sphere [5]. With this modification of the geometry near the black hole horizon, consistent propagation of Weyl fermion fields, very close to the horizon, is described by the modified equations (cf. eq.s (2.14, 2.15))

$$x^- \nabla_{x^-} \psi_- \;=\; 0 \;=\; x^+ \nabla_{x^+} \psi_+ \quad , \tag{2.26}$$

where, $\nabla_{x^+} \equiv \partial_{x^+} - h_{x^+x^+} \partial_{x^-}$ etc. Where $h_{x^+x^+} = e^{++}_{x^+}\, h_{x^+,++}$. The formal solution for $\psi^{out} \equiv \psi_-$ may be written as

$$\psi_+ \;=\; \psi_+(x^- \;+\; \int_{x_0^+}^{\infty} dy^+ h_{x^+x^+}(y^+, \Omega)) \, . \tag{2.27}$$

A similar solution exists for $\psi^{in} \equiv \psi_-$ with a shift in the other Kruskal coordinate x^+. The important point to note in (2.27) above is that the integration limit excludes the interval $(0, x_0^+)$; as pointed out in [8], this region is not interesting for our purpose of estimating the effects of backreaction, since for $v < v_0$, all infalling waves reflect back onto $\mathcal{I}^+$. Backreaction effects are important only for particles that get trapped behind the horizon.

Relating the shift δx^- given in (2.27) to the affine parameter λ of the null geodesic generator of the *modified* event horizon (obtained to linear order in the shift of the tetrad components), one can obtain a la' Hawking [1] a matching condition for $v > v_0$ between the incoming and outgoing solutions

$$\psi^{in}(v(u) + \delta v_0) \;=\; \psi^{out}(u) \, , \tag{2.28}$$

where,

$$v(u) \;=\; v_0 - 4GM e^{(v_0 - u)/4GM} \tag{2.29}$$

$$\delta v_0 \;=\; -\int_{v_0}^{\infty} dv \int d\Omega \; f(\Omega, \Omega') e^{(v_0 - v)/4GM} T_{vv} \quad v > v_0 \tag{2.30}$$

Where we have made a transformation of the shift to u, v coordinates, and taken as the lower limit of integration, $v = v_0$. A remark on the similarity of the shift δv_0 found here and that found in [8] for scalar matter is perhaps in order. Recall that near the horizon infalling particles are blue-shifted to enormously high energies, so that one can appeal to the geometrical optics approximation in dealing with these. Now in flat spacetime, it has been shown [36] that in this approximation (which corresponds to the eikonal approximation), fermionic and scalar cross sections become identical for electromagnetic interactions, pointing to an on-shell induced supersymmetry. The similarity between the gravitational effect of fermions and scalars on the horizon seen here may be attributed to such a supersymmetry. Clearly, in the geometrical optics approximation, helicity-flip amplitudes for Weyl fermions vanish. In the next section we study how the above back reaction leads to non-trivial interaction between the outgoing and infalling modes.

2.2 Effects of Backreaction

2.2.1 Semiclassical Approximation

The semiclassical approximation, as always, considers quantum matter in a classical gravitational background, which in this case is a (backreaction-modified) Schwarzschild geometry. In the absence of backreaction, scalar field operators corresponding to *out* and *in* solutions of the covariant Klein-Gordon equation are known to be mutually commuting for $v > v_0$ [8]. It is easy to see that, with the modification discussed above to the geometry, this is no longer the case: in powers of the (c-number) shift δv_0, one can show that

$$\begin{aligned}[\phi^{out}(u,\Omega)\ ,\ \phi^{in}(v',\Omega')] &= 2\pi i\delta v_0\left[1 + \frac{1}{2}\delta v_0\partial_v + \frac{1}{6}(\delta v_0)^2\partial_v^2 + \cdots\right] \\ &\times\ \delta(v-v')\delta^{(2)}(\Omega-\Omega')\ ,\end{aligned} \tag{2.31}$$

where use has been made of the canonical commutation relation for scalar fields and the reparametrisation (2.23).

$$[\phi^{in}(v_1,\Omega_1)\ ,\ \partial_{v_2}\phi^{in}(v_2,\Omega_2)] = 2\pi i\delta(v_{12})\ \delta^{(2)}(\Omega_1-\Omega_2)\ . \tag{2.32}$$

Eq. (2.31) reveals a departure from canonical behaviour in the 'ultralocal' limit $v \to v' \sim v_0\ ,\ \Omega \to \Omega'$ in the form of a power series in the v coordinate shift δv_0 which, we recall, is the central signature of classical backreaction. Thus, it is interesting that even for a c-number coordinate shift of the horizon under gravitational backreaction, albeit for a restricted kinematical range, hints of new features already appear. A similar analysis for fermionic field operators, using the canonical *anti*commuting relations

$$\begin{aligned}\{\ \psi^{in}(v_1,\Omega_1)\ ,\ \psi^{in}(v_2,\Omega_2)\ \} &= 0 \\ \{\ \bar{\psi}^{in}(v_1,\Omega_1)\ ,\ \psi^{in}(v_2,\Omega_2)\ \} &= 2\pi i\delta(v_{12})\ \delta^{(2)}(\Omega_1-\Omega_2)\end{aligned} \tag{2.33}$$

may indeed be performed. It is easy to show that, *to all orders* in the horizon shift δv_0 and all values of the coordinates,

$$\{ \psi^{out}(u(v),\Omega) \, , \, \psi^{in}(v',\Omega') \} \; = \; 0 \, . \tag{2.34}$$

The signature of the horizon shift, does, however, survive in the other anticommutator,

$$\{ \bar{\psi}^{out}(u,\Omega) \, , \, \psi^{in}(v',\Omega') \} \; = \; 2\pi i e^{\delta v_0 \, \partial_v} \delta(v-v') \, \delta^{(2)}(\Omega - \Omega') \, . \tag{2.35}$$

Once again, the rhs of (2.35) survives only in the 'ultralocal' limit $v \to v' \sim v_0$, $\Omega \to \Omega'$. But the physical information about fermions is not completely contained in eq.s (2.34) and (2.35); indeed, as is well known, fermionic fields are themselves unobservable; densities constructed out of fermionic bilinears are the true observable quantities. The simplest bilinear composites constructed out of fermions are the lightcone components of the current J_μ, viz, $J^{out}(u,\Omega)$, $J^{in}(v',\Omega')$[1], defined as

$$J^{in}(v) \; \equiv \; \bar{\psi}^{in}(v)\psi^{in}(v) \; , \; J^{out}(u) \; \equiv \; \bar{\psi}^{out}(u)\psi^{out}(u) \, . \tag{2.36}$$

As sketched in Appendix B , using eq.s (2.34) and (2.35), one now obtains the surprising result that, *to all orders in the classical horizon shift* δv_0, and indeed as given in Appendix A, for all values of the coordinates,

$$[\, J^{out}(u(v),\Omega) \, , \, J^{in}(v',\Omega') \,] \; = \; 0 \, . \tag{2.37}$$

Thus, even in the 'ultralocal' limit of the coordinates, when the horizon shift-dependent terms on the rhs of eq. (2.35) become important, these observables constructed out of the fermion field operators appear not to provide any hint of departure from canonical behaviour. There may be other combinations of the fields (like $\bar{\psi}^{in}\partial_v\psi^{in}$) which do not necessarily commute with J_{out}. Our result in eq. (2.37) above pertains only to the *simplest* fermion bilinear observables in the semiclassical approximation. The observed disparity between bosons and fermions in this very restricted context should not be extrapolated to more general situations. Indeed, as we show in the next section, this disparity is no longer present in the quantum exchange algebra between the same current components.

[1]The other components are negligible in the kinematical situation under consideration. Also, scalar bilinears vanish for chiral fermions.

2.2.2 Quantum Correspondence and Exchange Algebras

With fields as operators, the energy-momentum flux operator[2] $P_v \equiv \int_{v_0}^{\infty} dv\, T_{vv}$ has a nontrivial commutation relation with the incoming field. It generates translations along the v direction. For scalars, this gives:

$$[P_v(v), \phi_i(v)] = 2\pi \imath \delta^2(\Omega - \Omega')\partial_v \phi_i \tag{2.38}$$

The same is true for the fermion field.

$$[P_v(v), \psi^{in}(v')] = 2\pi \imath \delta^2(\Omega - \Omega')\partial_v \psi^{in}(v', \Omega') \tag{2.39}$$

Now, as seen in equation (2.30), the shift δv_0 is related to the energy momentum tensor. If we assume that this classical relation can be promoted as such to a relation between operators by means of some sort of a quantum correspondence principle, it can be used find the commutation relation of δv_0 with the incoming field

$$[\delta v_0(\Omega)\ ,\ \phi^{in}(\Omega')] \;=\; -16\pi \imath f(\Omega, \Omega') e^{(v_0 - v)/4GM} \partial_v \phi^{in} \tag{2.40}$$

$$\left[\delta v_0(\Omega)\ ,\ \psi^{in}(\Omega')\right] \;=\; -16\pi \imath f(\Omega, \Omega') e^{(v_0 - v)/4GM} \partial_v \psi^{in} \qquad v > v_0\,. \tag{2.41}$$

This result can now be used, following Kiem et. al. [8] to determine an exchange algebra between the *in* and *out* field operators. Keeping in mind the canonical (anti)commutation relations obeyed by the fields, this algebra (to lowest order in the backreaction) is given by

$$\begin{aligned} \phi^{out}(u, \Omega)\phi^{in}(v, \Omega') &= \left\{ 1 - 16\pi \imath f(\Omega, \Omega') e^{(u-v)/4GM} \partial_u \partial_v \right\} \\ &\quad \times\ \phi^{in}(v, \Omega')\phi^{out}(u, \Omega)\,. \end{aligned} \tag{2.42}$$

$$\begin{aligned} \psi^{out}(u, \Omega)\psi^{in}(v, \Omega') &= -\left\{ 1 - 16\pi \imath f(\Omega, \Omega') e^{(u-v)/4GM} \partial_u \partial_v \right\} \\ &\quad \times\ \psi^{in}(v, \Omega')\psi^{out}(u, \Omega)\,. \end{aligned} \tag{2.43}$$

In the absence of backreaction $f(\Omega, \Omega') = 0$, we get the standard (anti)commutation relation between the field operators.

Thus to the extent one can trust the procedure of promoting (2.30) to the level of an operator relation, fermionic field operators appear to obey the requirements of Complementarity. The point becomes clearer when one computes, within the same

[2]The Energy Momentum tensor is not well-defined near the horizon, but the momentum flux is [1].

correspondence approach, the exchange algebra of fermionic bilinear densities; this turns out to be similar to the exchange algebra of scalar fields (2.42).

$$J^{out}(u,\Omega)J^{in}(v,\Omega') = \{\ 1\ -\ 16\pi\imath f(\Omega,\Omega')e^{(u-v)/4GM}\partial_u\partial_v\ \}J^{in}(v,\Omega')J^{out}(u,\Omega)\ . \tag{2.44}$$

In other words, observables, mutually commuting in absence of backreaction, now obey a nontrivial exchange algebra (for $v > v_0$). The similarity of the exchange algebras for the fermion bilinears and scalar fields is of course, no surprise, being expected on account of the similarity in the behaviour of fermions and scalars under spacetime translations which lies at the root of the derivation of these algebras. In addition, as already mentioned, since the geometrical optics approximation is essentially being used here, and fermions and scalars behave similarly in this approximation, the only difference that may be expected is from the statistics, i.e., the fact that fermionic field operators obey a canonical anticommutator algebra.

2.3 Conclusions

It should be pointed out that, just like in the scalar field case [8], (2.44) is valid for Ω and Ω' quite distinct, and as such, its domain of validity does not include the 'ultralocal' domain in which a nontrivial result beyond the canonical was obtained in the semiclassical approximation in the scalar case. The absence of a similar noncanonical behaviour for the simplest bilinear observables of spinor fields in the 'ultralocal' semiclassical situation similarly does not imply any disparity with scalar fields outside this domain, or indeed, even within this domain for more complicated fermionic bilinears. Thus, the most general result that one expects for either kind of fields should incorporate both angular domains and may therefore quite possibly turn out to be alike for the two classes of fields. More future work is necessary to resolve the issues to one's satisfaction.

It is important to bear in mind however, that, in the foregoing analysis, as we have taken the effect of the longitudinal components of the energy momentum tensor, transverse gravitational interactions as well as non-gravitational forces have been ignored on the plea that they would be subdominant in the kinematical situation envisaged. Such forces could conceivably change the results somewhat [37], although a complete picture can only be expected after nonperturbative 'quantum gravity' is understood. Furthermore, the relevance of the asymmetry between the semiclassical behaviour of scalar and spinor fields, discerned by us, may emerge in a clearer manner when transverse gravitational backreaction effects are incorporated.

As mentioned earlier, the validity of the exchange algebras derived above hinges on the assumption that the shift δv_0, while a shift in the optimum value of a co-ordinate, can be elevated to the level of an operator on Fock space. Clearly, as an operator, δv_0 is bilocal which might possibly underlie a justifiable suspicion of a violation of microcausality. In the scalar field case, Kiem et. al. introduce a third scalar field ϕ^{hor} to ensure that microcausality is maintained, with ϕ^{in} evolving unitarily to it at the horizon from $\mathcal{I}^-$. It is not clear whether this is a valid procedure without a full quantum theory of gravity to back it up. We therefore adopt a deliberately ambivalent stand on this issue.

Chapter 3

Extremal Black Holes

Charged or rotating black holes are characterised by two horizons. The outer horizon is the event horizon, but the inner horizon is a Cauchy horizon. When the two horizons merge under special restrictions on the charges or angular momentum, these black holes are called extremal black holes. These occupy a special place in black hole physics in the fact that they are stable against Hawking Radiation. When embedded in low energy string theory, they are modelled by states which are invariant under certain supersymmetries. These states are also stable. The extremal black holes were the first ones to be modelled by string theory. In this chapter, we investigate the thermodynamics of these objects and the microscopic interpretation of their entropy using String theory. The classical laws of black hole mechanics lead to the identification of area of the black hole with entropy. The precise identification is:

$$S_{BH} = \frac{A_{BH}}{4G} , \tag{3.1}$$

where S_{BH} stands for the entropy, A_{BH} is the horizon area and G is Newton's constant. We describe in the first section how semi-classical methods show that ordinary extremal black holes do not obey the above area law for entropy. We try to explain why the same does not apply for the extremal black holes which appear in low energy string theory. These black holes are modelled by certain supersymmetric string solitonic states called D-branes. The microscopic counting for these black holes gives as the logarithm of the degeneracy of states precisely relation (3.1).

3.1 Extremal Black Holes in General Relativity

The black holes obtained as solution of Einstein's equation are characterised by their mass (M), and U(1) charge (Q), or angular momentum J. For black holes with charge and angular momentum, certain restrictions on the above parameters give rise to solutions with special properties. These black holes are called extremal black holes. For them,

$$\begin{aligned} M &= (1/\sqrt{G})Q \;;\; \text{(charged black holes)}\; M = (1/\sqrt{G})j\text{(rotating black holes)} \\ M &= \sqrt{\frac{1}{G}(Q^2+j^2)}\text{(rotating and charged black holes)} \qquad (3.2) \end{aligned}$$

These black holes are special as their Hawking temperature is zero, and they are stable against Hawking radiation. The non-extremal counterpart of these, with $M > Q, J$, have non-zero Hawking temperature, as well as non-zero entropy. Initial calculations of entropy for extremal black holes using semiclassical methods all showed that these *do not* obey the area law for entropy. Infact, their entropy is zero. In this section we give evidences for these and also examine physical processes which show that non-extremal black holes cannot transform into extremal ones. All these give the extremal black holes a special place.

3.1.1 Entropy of Extremal Black Holes

Let us consider the four dimensional Reissner-Nordström (RN) metric which is the solution of Einstein's equation coupled with a U(1) Maxwell field;

$$ds^2 = -\left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)dt^2 + \left(1-\frac{2GM}{r}+\frac{GQ^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega^2 \,. \quad (3.3)$$

The following relation holds when the black hole undergoes an infinitesimal change in mass, charge and horizon area :

$$dM = \frac{k}{8\pi G}dA_{BH} + \phi dQ \,, \quad (3.4)$$

where M and Q are the mass and the charge of the black hole respectively. The horizons are at $r_\pm = GM \pm G\sqrt{M^2 - Q^2/G}$. The surface gravity κ is given by $(r_+ - r_-)/2r_+^2$ and $\phi = Q/r_+$ is the electrostatic scalar potential at the outer (event) horizon. The Hawking temperature of the black hole is given by $T_H = \kappa/2\pi$ and M is just the energy E of the black hole. Comparing with the first law of thermodynamics,

$$dE = TdS - PdV \,, \quad (3.5)$$

and replacing PdV by $-\phi dQ$, we see that the entropy S must be identified with $A_{BH}/4G$ [1], thus giving Eq.(3.1) [3].

[1] upto an additive constant , which we set to zero by demanding that $S_{BH} \to 0$ as $M \to 0$

The extremal limit of these black holes is given by $r_+ = r_-$. Clearly, for extremal RN black holes, $T_H = 0$, since $r_+ = r_-$. Thus, we can no longer compare equations (3.5) and (3.6) to obtain (3.1). Moreover, using the relation for temperature

$$T_H^{-1} = \left(\frac{\partial S_{BH}}{\partial M}\right)_Q , \tag{3.7}$$

we see that the right hand side diverges at $T_H = 0$, indicating that the entropy as a function of M has a singularity at the extremal limit $M = Q/\sqrt{G}$. The above arguments hold good for D-dimensional charged black holes as well, for which $T_H = (D-3)(r_+^{D-3} - r_-^{D-3})/4\pi r_+^{D-2}$.

In [4], an alternative approach was used to interpret the area of the black hole as entropy. Simple arguments were given using the definition of entropy and information contained in a thermodynamic system. We analyse in the light of the arguments given in [4], the case of extremal RN black hole. Firstly, the entropy is assumed to be an arbitrary function of area:

$$S_{BH} = f(A) , \tag{3.8}$$

from which, one can write

$$\Delta S_{BH} = \frac{df}{dA}\Delta A , \tag{3.9}$$

where ΔS_{BH} and ΔA correspond to the change in the entropy and area respectively of the black hole when a particle falls into it. The quantity ΔS can be estimated using information theory. Before it enters the event horizon, it is certain that the particle exists. Once it enters the horizon, we loose all information about the particle. It is thus justified to assume that it is equally probable for the particle to exist as it is for it not to exist. Thus the minimum entropy change (ignoring possible internal structures of the particle) is given by

$$(\Delta S)_{min} = \sum_n p_n \ln p_n = \ln 2 , \tag{3.10}$$

where summation over n corresponds to all possible states of the particle. Now, for an infalling particle with mass μ and its center of mass at $r_+ + \delta$, the proper radius b is $\int_{r_+}^{r_+ + \delta} \sqrt{g_{rr}} dr$. Then the minimum change of black hole area accompanied by the absorption of the particle will be [4]

$$\begin{aligned} (\Delta A)_{min} &= 2\mu b , \quad with \\ b &= 2\delta^{1/2}\frac{r_+}{\sqrt{r_+ - r_-}} . \end{aligned} \tag{3.11}$$

where b is obtained using $g_{rr} = (r-r_-)(r-r_+)/r^2$, and non-extremality condition $(r_+ - r_- \gg \delta)$. However, in the extremal limit $(r_+ \to r_-)$, we get:

$$b = \delta + r_+ \ln(r-r_+)|_{r_+}^{r_++\delta} , \tag{3.12}$$

which diverges for any $\delta > 0$. This means that for any finite δ, however small, the corresponding proper radius of the infalling particle is infinite. Thus, the above equation makes sense only for $\delta = 0$. Thus, we take $b = 0$ corresponding to a point particle resulting in $(\Delta A)_{min} = 0$. Thus to satisfy equation (3.9) we require,

$$\left(\frac{\partial f}{\partial A}\right)_{r_+=r_-} \longrightarrow \infty , \tag{3.13}$$

which once again shows that the entropy is not continuous at the extremal limit.

The discontinuous nature of entropy under the transition from non-extremal to extremal black hole asserts that the entropy of extremal black holes cannot be determined as a limit of the non-extremal one. Independent derivations of the entropy for extremal and non-extremal black holes have been given [13] which are in conformity with the above result. It has been shown that the topology of the black hole near the horizon plays a crucial role in determining the entropy. We now briefly review these arguments.

The Euclideanised metric in d dimensions near the horizon is

$$ds^2 = N^2 d\tau^2 + N^{-2} dr^2 + r^2 \, d\Omega_{d-2}^2 . \tag{3.14}$$

For the above metric, the proper angle Θ in the $r - \tau$ plane near the horizon is defined as

$$\Theta \equiv \frac{\text{proper length}}{\text{proper radius}} = \frac{\int_{t_1}^{t_2} \sqrt{g_{\tau\tau}} \, d\tau}{\int_{r_+}^{r} \sqrt{g_{rr}} \, dr} = (NN')|_{r_+}(t_2 - t_1), \tag{3.15}$$

where the prime denotes differentiation with respect to r. It can be shown that N satisfies the following relation:

$$(t_2 - t_1)N^2 = 2\Theta \, (r - r_+) + O[(r-r_+)^2] . \tag{3.16}$$

Also, the two dimensional metric near the horizon can be written in the form

$$ds^2 = d\rho^2 + \rho^2 d\Theta^2 , \tag{3.17}$$

where $\rho \equiv \sqrt{2(r-r_+)/NN'}$. To avoid a conical singularity at the horizon, the period of Θ is identified with 2π, which corresponds to the topology of a *disc* with zero deficit angle in the $r-\tau$ plane. This can always be done for non-extremal black holes, as $(NN')|_{r_+}$ in Eq.(3.15) is non-zero. However, for extremal black holes, the proper radius diverges (see Eq.(3.12)), and hence the proper angle tends to zero. Thus the conical deficit angle becomes 2π and the topology is that of an *annulus* [13]. The topology of the transverse section in either case is S^{d-2}.

Now, we need to see how this topology reflects on the entropy calculation. Treating the black holes as microcanonical ensemble, the action I in the Hamiltonian formulation of gravity is proportional to entropy. The dimensional continuation of Gauss-Bonnet theorem to d dimensions [13, 38] determines

$$I \propto \chi A_{d-2} \tag{3.18}$$

where χ is the euler characteristic of the Euclideanised $r-\tau$ plane and A_{d-2} is the area of the transverse S^{d-2}. The exact expression for the black hole entropy is given by

$$S = \frac{\chi A}{4G} . \tag{3.19}$$

For non-extremal black holes $\chi = 1$ (disc), leading to the area law (3.1), while for extremal black holes $\chi = 0$ (annulus), implying a vanishing entropy.

It has also been argued by Hawking et al [39, 40], that the Euclidean action for extremal black holes is proportional to the inverse Hawking temperature (β) in a canonical ensemble leading to the vanishing entropy. This follows from the relations $S = -\left(\beta\frac{\partial}{\partial\beta} - 1\right) \ln Z$, and $Z = e^{-I}$. [2]

Thus, it is clear that extremal black holes cannot be thought of as limits of non-extremal black holes at least as far as the expression for their entropies are concerned. In the next section, we investigate some physical processes which further support this conclusion.

3.1.2 Physical Processes

For the charged non-extremal black holes, we know that Hawking radiation is dominant in the energy regime $\omega > e\phi$ where e is the charge of the emitted particles. On the other hand, the Penrose process (and its quantum analog - superradiance) is significant when $\omega < e\phi$. We study both the processes, thus spanning all the energy regimes, to confirm that non-extremal black holes cannot transform into extremal ones.

[2] Extremal black hole entropy has been explored by alternative methods as well in [41].

Superradiance and extremality

RN black holes can lose mass and charge by processes like Penrose process and superradiance, which are dominant for low energies of the infalling particles. We examine here, whether a non-extremal black hole can reach extremality through these processes.

The energy of a particle in a curved back ground with a time like killing vector ξ^a is given by $E = p.\xi$, where p_a is the particle's four momentum. For a massive charged particle in $4D$ RN background this energy shall be $m\sqrt{g_{tt}} + eQ/r$, where $\xi_a = \partial_t$ and the electrostatic energy is added. Thus this is given by [42]

$$E = m\sqrt{\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)} + \frac{eQ}{r} . \tag{3.20}$$

Where m and e are the mass and charge of the infalling particle. If this particle has a charge opposite to that of the black hole, then sufficiently close to the horizon, the first term tends to zero, making the energy negative. Hence in this regime,

$$|E| < \frac{|eQ|}{r_+} . \tag{3.21}$$

If two oppositely charged bound particles with total energy E_0 fall near the black hole and separate there, one of the charges can have negative energy by the above argument. The particle with negative energy will fall into the black hole and the other particle escapes. By conservation of energy,

$$E_2' = E_0 + |E_1| \tag{3.22}$$

$$M' = M - |E_1| \tag{3.23}$$

E_2' is the energy of the particle which escapes, E_1 is the energy of the particle which falls into the black hole and M' is the final mass of the black hole. Thus there is a decrease in mass of the black hole, while the escaping particle carries back more energy. Also as an oppositely charged particle is absorbed by the black hole, it's effective charge decreases to become $Q' = Q - e$. Since for a non-extremal black hole ($\sqrt{G}Q/r_+ < 1$). We find from equation(3.21), that

$$\frac{e}{\sqrt{G}} > |E_1| . \tag{3.24}$$

In other words the decrease of mass of the black hole will be less than the decrease of charge due to this process. Hence, the condition of non-extremality $M > Q/\sqrt{G}$ will be maintained as the rate of charge loss will exceed the rate of mass loss.

The quantum analog for this phenomenon is Superradiance. Fields with low energy are shown to be scattered away from the black hole such that the reflection coefficient is greater than one. The charged scalar field equation can be solved in the RN back ground and the following relation for the reflection coefficient $|R|^2$ and transmission coefficient or the absorption coefficient $|T|^2$, can be obtained by explicit calculation as [43],

$$1 - |R|^2 = \frac{1}{k}(\omega - \frac{\epsilon Q}{r_+})\ |T|^2\ . \tag{3.25}$$

Due to the curvature of space-time the reflection and transmission coefficients need not add up to 1. For $\omega < \epsilon Q/r_+$, the reflection coefficient, $|R|^2$ is greater than 1, or the scalar wave takes away energy from the black hole. The condition for superradiance is thus

$$m < \omega < \frac{\epsilon Q}{r_+}\ . \tag{3.26}$$

The rate of charge loss and mass loss for the black hole is

$$\frac{dQ}{dt} = -\epsilon \int_m^{\frac{\epsilon Q}{r_+}} |R|^2 d\omega \tag{3.27}$$

$$\frac{dM}{dt} = -\int_m^{\frac{\epsilon Q}{r_+}} |R|^2 \omega d\omega\ . \tag{3.28}$$

We find that the for the initial value of the integrands, $(\epsilon/\sqrt{G})|R(m)|^2 > m|R(m)|^2$. Thus as equation (3.26) holds for each value of ω,which is bounded from above,

$$|\frac{dQ}{dt}| > \sqrt{G}|\frac{dM}{dt}|\ . \tag{3.29}$$

Hence from the quantum process also it is clear that the $M > Q/\sqrt{G}$ condition will be maintained.

The above result is easily extendible to higher dimensional charged dilatonic black holes. The equation for classical energy of a charged particle in a generic charged black hole back ground is [3]

$$E = m\sqrt{\left(1 - (\frac{r_+}{r})^{D-3}\right)\left(1 - (\frac{r_-}{r})^{D-3}\right)^{1-2a^2/(D-3+a^2)}} + \frac{\epsilon Q}{r^{D-3}}\ . \tag{3.30}$$

Here D is the dimension of space and a stands for a parameter which interpolates between the general relativistic solution $a = 0$ and the dilatonic stringy black hole

[3] for the D-dimensional charged metric, see [44].

$a = 1$. As $r_+^{D-3} = GM + G\sqrt{M^2 - Q^2/G}$, non-extremality will imply $\sqrt{G}Q < r_+^{D-3}$ and equation (3.24), hold for these. The equation for the reflection and transmission coefficients for these black holes has been calculated in [45]. The condition for superradiance equation (3.26), is the same for these black holes.

Apart from the induced process stated above, the black hole loses charge spontaneously by vacuum polarization as shown in [43]. For this the rate of charge loss will also be very high, and the black hole will tend to discharge itself very fast. The $M = Q/\sqrt{G}$ condition, once again, will not be obtained. Thus in the processes considered so far, the extremality condition cannot be attained from a non-extremal state.

Hawking Radiation and Extremality

In this section, we consider mass and charge loss of black holes by Hawking radiation. When the RN black hole radiates, the spectrum of particles is given by the Planck distribution [1] times the greybody factor or the absorption coefficient of the black hole. For our purposes, we use the approximation where the absorption coefficient is proportional to a constant factor so as to give ordinary Planck spectrum for massless particles. Thus the energy loss due to radiation is:

$$dE_\omega = \frac{(\omega - e\phi)^3\, d\omega}{e^{(\omega - e\phi)/T_H} - 1}, \tag{3.31}$$

where dE_ω is the radiation energy in the spectral range ω to $\omega + d\omega$. Integrating over ω from $e\phi$ to ∞, one obtains the rate at which the black hole loses energy, i.e. mass [42]

$$\frac{dM}{dt} = -\sigma T_H^4\, A, \tag{3.32}$$

$A = 4\pi r_+^2$ being the area of the event horizon and σ the Stefan-Boltzmann constant. Thus, for RN black holes with $T_H = (r_+ - r_-)/4\pi r_+^2$, it is given by,

$$\frac{dM}{dt} = -\frac{\sigma}{(4\pi)^3}\frac{(r_+ - r_-)^4}{r_+^6}. \tag{3.33}$$

We integrate (3.33) to get,

$$\int_0^{t_0} dt = -\frac{(4\pi)^3}{\sigma}\int_{M_0,Q_0}^{M',Q'} \frac{r_+^6\, dM}{(r_+ - r_-)^4}. \tag{3.34}$$

Here t_0 is the time taken for the black hole to reach a final state with mass and charge M' and Q' respectively, from their initial values M_0 and Q_0. We are interested in calculating t_0 to reach a final extremal state (i.e. $M' = Q'/\sqrt{G}$). from a non-extremal initial state. For simplicity, let us first assume that the radiated particles are electrically neutral, i.e. $e = 0$ and hence Q is a constant. Then, the time taken for the black hole to become extremal is:

$$
\begin{aligned}
\int_0^{t_0} dt &= -\frac{(4\pi)^3}{\sigma} \lim_{M' \to Q_0/\sqrt{G}} \int_{M_0}^{M'} \frac{r_+^6 \, dM}{(r_+ - r_-)^4}, \\
&= \frac{(4\pi)^3}{\sigma} \lim_{M' \to Q_0/\sqrt{G}} \left[\frac{105}{12} Q_0^3 \sqrt{G} \ln \left(\frac{M + Q_0/\sqrt{G}}{M - Q_0/\sqrt{G}} \right) \right. \qquad (3.35) \\
&+ \left(\frac{328 Q_0^4 - 128 M^4 G^2 - 128 Q_0^2 M^2 G}{12 (M^2 - Q_0^2/G)^{1/2}} \right) \\
&+ \left. \left(\frac{198 Q_0^4 M - 128 M^5 G^2 - 64 Q_0^2 M^3 G}{12 (M^2 - Q_0^2/G)} \right) \right] \Bigg|_{M=M_0}^{M=M'} .
\end{aligned}
$$

Clearly, t_0 diverges. That is, the RN black hole which emits neutral particles, takes an infinite amount of time to reach extremality. Generalizing the proof for Hawking particles carrying charges is not difficult. Then Q is not a constant in Eq.(3.34). However, as before, the integrand on the right hand side diverges as $r_+ \to r_-$ and thus $t_0 \to \infty$. Identical conclusions follow for general relativistic charged black holes in D-dimensions, for which, the rate of mass loss is given by

$$
\frac{dM}{dt} = -\sigma_D \, A_{D-2} \left(\frac{D-3}{4\pi} \right)^D \frac{[r_+^{D-3} - r_-^{D-3}]^D}{r_+^{(D-2)(D-1)}} ,
$$

σ_D being the D-dimensional Stefan-Boltzmann constant and A_{D-2} the area of unit S^{D-2}. Here too the integrand diverges in the extremal limit. In general, $t_0 \to \infty$ whenever $T_H = 0$ for the extremal black hole. We shall see later, that this includes a certain class of stringy black holes.

Thus, we conclude that a extremal black hole state with $T_H = 0$ cannot be reached in a finite time by Hawking radiation from a non-extremal black hole. This is in conformity with the third law of black hole thermodynamics, which asserts that the same cannot be reached in a finite sequence of operations [3]. These also provide pieces of evidence that the area law for the entropy of non-extremal black holes cannot be extended to the case of extremal black holes. In the next section, we will study the entropy of certain extremal stringy black holes which supposedly obey the area law.

3.2 Extremal Black Holes in String Theory

We examine in this section, the black holes which arise in string theory. Some of the extreme black holes in string theory obey the area law, and indeed a microscopic counting gives the same. However, for another set of black holes, though the classical metric appears to fall in the same class as considered in 3.1.1, the string theoretic counting yields the area law. We try to obtain a resolution of this apparent puzzle.

3.2.1 Black Hole Solution

In the low energy limit of string theory, ordinary supergravity is recovered. These supergravity actions have additional fields in their spectrum, which couple to p dimensional extended objects called branes. Since string theory is 10 dimensional, the supergravity solutions, when compactified down to lower dimensions, yield black hole solutions, with multiple U(1) charges. Many of these black hole solutions have two horizons, and extremality for these is defined by the coincidence of these two horizons.

Based on this definition, we can divide extremal stringy black holes into two broad classes:

1. The horizon merges with the curvature singularity. These black holes have zero horizon area and the dilaton field becomes singular at the horizon.
2. The two event horizons coincide as in General relativity. The area for these black holes is non-zero and the dilaton is regular at the horizon.

A few examples of these extremal stringy black holes and their properties are tabulated below.

Type	Example	T_H		Macro Entropy		Micro Entropy	
		NE	E	NE	E	NE	E
1	Het. on T^6	$\neq 0$	$1/4\pi m_0$	$A/4$	0	-	$A_{st}/4$
2	II B on $K^3 \times S^1$	$\neq 0$	0	$A/4$	0	-	$A/4$

where NE = nonextremal, E = extremal and A_{st} is the area of the stretched horizon. Macro≡ semiclassical entropy, Micro≡ Counting of string states. The examples referred to here are taken from Refs. [15, 16].

The first type of extremal black holes [15], obtained by compactifying heterotic string theory on T^6 has $T_H \neq 0$. Hence in accordance with the third law of black hole thermodynamics, the extremal state can be reached in a finite sequence of steps. In particular, one can see from Section 3.1.2, that the time t_0 to reach extremality by Hawking radiation is finite [4]. Similarly, the discussions in section 3.1.1 based on [4, 40] is valid for this example since proper radius is finite. It follows that these extremal black holes can be regarded as limits of non extremal ones and their entropy *obeys* the area law. In order to determine this entropy, we look at the extremal stringy black hole solution of the low energy effective action of heterotic string theory compactified on T^6. The Euclideanised metric near the horizon is [15]:

$$d\bar{s}^2 = \frac{1}{4}\bar{r}^2 d\bar{\tau}^2 + d\bar{r}^2 + \frac{1}{4}\bar{r}^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) , \tag{3.36}$$

and the solution for the dilaton field is $e^{\bar{\phi}} = \bar{r}^2/4$. Here $\bar{r}^2 = 4gr$ and $\bar{\tau} = \tau/m_0$ where g is the string coupling and the parameter m_0 is related to the mass of the black hole. Note that the topology near the horizon ($\bar{r} = 0$) is *disc* $\times S^2$, which is that of a generic non-extremal black hole. Although $S = A/4G$, the entropy vanishes as the horizon area is zero. It has been proposed that stringy corrections near the horizon modifies the metric and the dilaton of this metric. A possible modification of the metric and the dilaton is

$$d\bar{s}^2 = \frac{1}{4}\bar{r}^2 d\bar{\tau}^2 + d\bar{r}^2 + \frac{1}{4}f_1(\bar{r})\left(d\theta^2 + \sin^2\theta d\phi^2\right) \;\; ; \;\; e^{\phi} = f_2(\bar{r}) , \tag{3.37}$$

where $f_1(\bar{r})$ and $f_2(\bar{r})$ are two smoothing functions which are positive constants at $\bar{r} = 0$ and equal $\bar{r}^2/4$ for large $\bar{r}$. Now, the horizon area can be shown to be $\sqrt{m_0 f_1(0)/g}$, which is finite.

The second type of the stringy extremal black holes is obtained from type IIB string theory compactified on $K^3 \times S^1$. The action has the following form in five dimensions:

$$S = \frac{1}{16\pi}\int d^5x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2 - \frac{1}{4}H^2\right) - \frac{1}{4}F\right] \tag{3.38}$$

Where F is a Ramond-Ramond field strength, and H is a three form field strength associated with the NS-NS two form present in the string spectrum. The charges are $Q_H \equiv 1/4\pi^2 \int *e^{-4\phi/3}H, Q_F = 1/16\pi \int *e^{2\phi/3}F$, give the non-zero black hole solution. The horizon area $A \neq 0$ if both NS-NS charge Q_H and R-R charge Q_F,

[4] The grey-body factor for the metric (3.36), for low energy quanta, seems to be zero because of vanishing horizon area. However, as argued in [15], due to the singular nature of the horizon, this metric suffers large stringy corrections resulting in the metric (3.37) which has a non-zero (stretched) horizon area.

are non-zero [16]. The extremal black hole solution from low energy effective theory is the five-dimensional RN metric [44]:

$$ds^2 = -\left(1-\left(\frac{r_e}{r}\right)^2\right)^2 dt^2 + \left(1-\left(\frac{r_e}{r}\right)^2\right)^{-2} dr^2 + r^2 d\Omega_3^2 . \qquad (3.39)$$

where the horizon radius in terms of the charges Q_H and Q_F is $r_e = (8Q_H Q_F^2/\pi^2)^{1/6}$. As discussed in Section 3.1.1, the Euclidean topology of this is *annulus* $\times S^3$ and hence its entropy is zero. The above black hole solution can also be obtained by compactifying a configuration of Q_1 D1-branes, and Q_5 D5-branes, which are charged with respect to the three form Ramond-Ramond gauge field and it's six dimensional dual. The compactification of this configuration on $T^4 \times S^1$ yields a black hole of $N = 8$ supergravity and on $K^3 \times S^1$ yields a black hole of $N = 4$ supergravity in 5 dimensions. The configuration consists of the D5-brane wrapped along $T^4 \times S^1$ and the D1-brane wrapped around S^1. The metric of this in five dimensions is given by

$$ds^2 = -f^{-2/3}dt^2 + f^{1/3}\left(dr'^2 + r'^2 d\Omega^2\right) \qquad (3.40)$$

Where $f = \left(1 + r_1^2/r'^2\right)\left(1 + r_5^2/r'^2\right)\left(1 + r_n^2/r'^2\right)$, $r_1^2 = g_s Q_1/V$ $r_5^2 = g_s Q_5, r_n^2 = r_0^2 \sinh^2\sigma$, and $r_0^2(\sinh 2\sigma/2) = g_s^2 n/(R^2 V)$, where n is momentum added along the S^1 whose radius is R^1 and volume of T^4 is V (small). The entropy of this black hole has a very simple form

$$S = \frac{A}{4G_5} = 2\pi\sqrt{Q_1 Q_5 n} \qquad (3.41)$$

The five dimensional RN metric (3.39) for this configuration is recovered under the restriction $r_1^2 = r_5^2 = r_n^2 = r_e^2$, and with a redefinition of the coordinates: $r'^2 = r_e^2 + r^2$. The standard area law for the extremal RN metric yields the entropy as in (3.41). However, as we know from the arguments given in section(3.1.1), that for this metric, the area law ceases to be valid.

3.2.2 String theoretic counting

The most important observation of the extremal solutions quoted in the earlier section, is that the string charges of these solutions obey the BPS bound. The Bogolmonyi-Prasad-Sommerfield (BPS) bound as it is called, is satisfied by configurations which preserve $1/2n$ where n is an integer, of the supersymmetry transformations in the theory. Since the bound involves the charges and mass of the classical solution, these do not receive any quantum corrections or undergo any renormalisation. Hence, for these black holes, it is reasonable to look at the string theory states which have the same quantum numbers, or global charges. The degeneracy of these states when they undergo gravitational collapse can be identified with that of a black hole solution, with the same charges.

For the black hole considered in [15], the degeneracy counting is obtained by looking at heterotic string states with the same charges as the black hole. The logarithm of the degeneracy of states yields $S = 2\pi m_0/g$, which is same as the entropy of the black hole if $f_1(0) = 4$. This is a remarkable calculations, in which a microscopic interpretation is obtained for black hole entropy.

The degeneracy counting of string states or D-brane states, as given in [16], reproduces the entropy for the above black hole *exactly*. Here for convenience, we give a review of the counting given in [12] for the above black hole obtained by $T^4 \times S^1$ compactification. The D-brane configuration corresponding to the black hole consists of Q_5 D5-branes, and wrapped on $T^4 \times S^1$ and Q_1 D1-branes wrapped on the S^1. The limit of small size of T^4 and larger S^1 radius is taken. The open strings which stretch between the branes connecting the 1 and 5 branes remain massless and contribute most to the entropy. Instead of considering separate $Q_1(5)$ D1(5)-Branes wrapped round the S^1, a single brane of length Q_1Q_5R can be taken, such that the momenta of the massless quanta along the S^1 direction are given by $n/(Q_1Q_5R)$, where n is an integer. The number of bosonic and fermionic species of the massless modes of the theory is obtained as 4 bosonic and 4 fermionic. Hence the central charge c=6(1 for each boson and 1/2 for each fermion). The asymptotic density of states for large N in any CFT is given by $d_N = exp(\sqrt{cN/6})$. Where N is the oscillator level. Here $N = Q_1Q_5n$. Hence, the entropy would be

$$S = 2\pi\sqrt{Q_1Q_5n} \tag{3.42}$$

This is precisely the black hole entropy determined above, if we assume that the area law holds for the metric (3.39). This poses an apparent puzzle. Since semiclassical methods seem to give the extremal black holes a special place due to their topology, but the string theoretic counting does not recognise the above. Moreover, as we shall see in later chapters, non-extremal configurations exist for the string theoretic black holes. The degeneracy counting of those states does not show any discontinuity as the extremal limit is taken [46].

3.2.3 Resolution

As the previous discussion suggests that for the string theoretic extremal black holes the arguments used in section (3.1.1) need not apply. Another relevant suggestion is that there could be string theoretic or other quantum gravity corrections which would prevent the metric near the horizon from being *exactly* extremal, such that the area law continues to be valid. It has been argued that Planck scale effects become important near the horizon [7, 47]. Stringy modifications were also anticipated in [48] on the basis of stability requirements. In view of the above, the modified metric with the correct topology should be of the form,

$$ds^2 = -f(r)\left(1 - \left(\frac{r_0}{r}\right)^2\right) dt^2 + f(r)^{-1}\left(1 - \left(\frac{r_0}{r}\right)^2\right)^{-1} dr^2 + r^2 d\Omega_3^2 , \quad (3.43)$$

where $f(r)$ is a positive definite and bounded function of r in the range $r_0 \leq r < \infty$, such that $f(r_0) \neq 0$, although it can be arbitrarily small. The corresponding Hawking temperature gets modified from zero to $T_H = f(r_0)/2\pi r_0$.

However though the exact nature of the stringy corrections are not ascertained, the metric (3.43) can be understood from an alternative approach. In general the non-extremal metric solution of type IIB action compactified on five dimensional manifold is of the form [49]:

$$ds^2 = -f^{-2/3}\left(1 - \frac{r_0^2}{\bar{r}^2}\right) + f^{1/3}\left[\left(1 - \frac{r_0^2}{\bar{r}^2}\right)^{-1} d\bar{r}^2 + r^2 d\Omega_3^2\right] , \quad (3.44)$$

where,

$$f = \left(1 + \frac{r_0^2 \sinh^2\alpha}{\bar{r}^2}\right)\left(1 + \frac{r_0^2 \sinh^2\gamma}{\bar{r}^2}\right)\left(1 + \frac{r_0^2 \sinh^2\sigma}{\bar{r}^2}\right) \quad (3.45)$$

Here, α, γ and σ are three boost parameters. These parameters along with r_0, radius R of S^1 and volume V of the four dimensional compact manifold determine the number of one branes, five branes, the corresponding anti-branes and the momentum in the S^1 direction. The precise relations are, $Q_1 = (Vr_0^2/2g)\sinh 2\alpha, Q_5 = (r_0^2/2g)\sinh 2\gamma, n = (R^2 V r_0^2/2g^2)\sinh 2\sigma$. This metric differs from the extremal metric in having two horizons. The D-brane configuration corresponding to this has right moving momentum in addition to left moving momentum in the S^1 direction. The five dimensional Reissner-Nordstörm metric is obtained when all these boost parameters are equal, i.e. $\alpha = \gamma = \sigma$, and simultaneously doing the coordinate transformation $r^2 = \bar{r}^2 + r_0^2 \sinh^2 \alpha$:

$$ds^2 = -\left[1 - \frac{(r_0 \sinh^2 \alpha)^2}{r^2}\right]\left[1 - \frac{(r_0 \cosh^2 \alpha)^2}{r^2}\right] dt^2 + \frac{dr^2}{\left[1 - \frac{(r_0 \sinh^2 \alpha)^2}{r^2}\right]\left[1 - \frac{(r_0 \cosh^2 \alpha)^2}{r^2}\right]} r^2 d\Omega_3^2 \; . \tag{3.46}$$

The two horizons are at

$$r_+ = r_0 \cosh \alpha \quad , \qquad r_- = r_0 \sinh \alpha \quad . \tag{3.47}$$

It is evident that the extremal limit ($r_+ \to r_-$) is achieved when the boost parameter becomes indefinitely large, i.e. $\alpha \to \infty$, and $a \to 0$ such that ae^α is held fixed. Clearly, taking this limit does not change the Euclidean topology from $disc \times S^3$ to $annulus \times S^3$ as exact equality of the horizons is not achieved. The metric is of the proposed form (3.43), coinciding with the metric (3.39) only in a limiting way. Therefore the extremal black hole metric considered in the above limiting sense obeys the area law for entropy. The stability of the black hole is ensured by the fact that the Hawking temperature, $T_H = 1/2\pi a \cosh^3 \alpha$, is infinitesimal.

The string theoretic counting for these near extremal states does not show a discontinuity at extremality. This is also not expected for generic extremal black holes in string theory, except for the specific RN extremal black hole. In that case, an additional restriction has to be imposed on the number of charges. For these black holes, the limiting situation correctly yields the area law.

3.3 Conclusions

In this chapter, we have shown that non-extremal and extremal black holes in general relativity are physically quite distinct objects and it is impossible to transform the former to the latter by physical processes. These have different Euclidean topologies and hence they do not share the same entropy formula. Extremal black holes have zero entropy as opposed to non-extremal black holes which obey the area law. On the other hand when the same black hole solutions are embedded in low energy string theory, the degeneracy counting for BPS saturated states follows the area law. We have proposed a form for the string black hole metric which indeed obeys the area law. We have justified this proposal by showing that this metric corresponds to the extremal limit of the black hole solutions in string theory, instead of being exactly extremal. The interesting question remains as to what kind of statistical interpretation can be given to the exactly extremal RN black holes if so whether it shall be a stringy interpretation. There is additional literature, where it is proposed that the area law is valid for extremal black holes when the partition function involves a sum over topologies [41]. However, the last word about this has not been yet said.

Chapter 4

Branes and Hawking Radiation

In this chapter we look at four dimensional black holes embedded in low energy string theory, and try to understand Hawking radiation from them. This black hole is a solution of $\mathcal{N} = 4$ supergravity obtained by compactifying 11 dimensional supergravity on T^7. We probe the black hole with a minimally coupled massless fermion [19]. We show that surprisingly this rate has a structure which is expected from a microscopic 1+1 dimensional CFT. Since the above fermion is not present in the string spectrum, a radiative process modelling the above calculation cannot be reproduced using string theory. The scalar emission from these black holes was initially studied in [50] and emission rates for various spin particles (minimally coupled) in the Neuman-Penrose formalism were considered in [51]. However, as we saw in the earlier chapter, string theory provides a microscopic counting of the Bekenstein-Hawking entropy of certain extremal five dimensional black holes. The near-extremal black holes can be modelled in a similar way using string micro-states. Once the microconstituents are known, an attempt can be made to study a microscopic radiative process which leads to Hawking radiation. It was first shown for scalars in [17, 18], that the D1 brane, D5 brane configuration yields a microscopic decay rate which equals the black hole Hawking radiation rate exactly. Here we attempt to obtain a microscopic calculation for the fermions and obtain a D-brane decay rate, which reproduces the Hawking emission rate of the fermions upto coffecients [19]. In the first section we study the four dimensional black hole configuration, in the second, we derive the Hawking radiation rate for a minimally coupled massless spin half particle and then for a non-minimally coupled fermion. In the last section, we give a string theoretic microscopic derivation of the radiation rate.

4.1 Four dimensional black hole and M theory

11 Dimensional supergravity is considered as the low energy limit of M-theory. The separate string theories, Type I, Type IIA, Type IIB and the heterotic are recovered from this theory by various duality relations. In particular, the strong coupling limit of Type IIA theory yields M theory. 11-dim supergravity contains the graviton, a four form field strength and the gravitino as the complete spectrum. Since a p dimensional membrane couples to a p+2 form field strength, this theory automatically incorporates a two brane and its dual, a five brane. Various supergravity brane solutions exist which satisfy the BPS bound. Here, we look at a configuration of three extremal five branes intersecting over a three brane, which in turn intersect along a string. The solution is then compactified on T^7, to give a black hole in 4 dimensions. Addition of momenta through a boost along the common intersection string gives, a stable solution in 4dimensions. The configuration in 11 dimensional has the following metric [20]:

$$\begin{aligned} ds^2 &= (f_1 f_2 f_3)^{-2/3}\left[f_1 f_2 f_3\left(-dt^2 + dx_1^2 + k\,(dt + dx_1)^2\right)\right. \\ &\left. + f_2 f_3(dx_2^2 + dx_3^2) + f_1 f_3(dx_4^2 + dx_5^2) + f_2 f_1(dx_6^2 + dx_7^2) + dr^2 + r^2 d\Omega_2^2\right] \end{aligned} \tag{4.1}$$

$$\text{where, } f_i = \left(1 + \frac{r_i}{r}\right) \quad [i = 1,2,3], \quad k = \frac{r_n}{r}. \tag{4.2}$$

In the above, r_i are related to the charges of the three five branes, and r_n is related to the momentum along the string. In the above, there is only left moving momentum along the common intersection string, and the solution is extremal. The first brane is along $(x_1, x_4, x_5, x_6, x_7)$, the second along$(x_1, x_2, x_3, x_4, x_7)$ and the third along $(x_1, x_2, x_3, x_4, x_5)$. The radial distance in the rest of three space is denoted by r. The four form field strength is:

$$F_4 = 3\left(*dF_1^{-1} \wedge dy_2 \wedge dy_3 + *dF_2^{-1} \wedge dy_4 \wedge dy_5 + *dF_3^{-1} \wedge dy_6 \wedge dy_7\right), \tag{4.3}$$

where $*$ refers to the three dimensional dual. On compactifying this configuration on$(x_2..x_7)$ T^6, a black string solution wound around a compact x_1 is obtained as:

$$ds^2 = f^{-1}(-dt^2 + dx_1^2 + k\,(dt + dx_1)^2) + f^2(dr^2 + r^2 d\Omega_2^2) \tag{4.4}$$

The four dimensional black hole, obtained by compactifying the above as $ds^2 = ds_4^2 + e^{2\phi}(dx_1^2 + A_\mu dx^\mu)^2$, where $\mu = 8, 9, 10$ and A_μ is a gauge field, gives:

$$
\begin{aligned}
ds_4^2 &= -f^{-1/2}dt^2 + f^{1/2}(dr^2 + r^2 d\Omega^2) , \\
f &= \prod_{i=1}^{4}\left(1 + \frac{r_i}{r}\right).
\end{aligned} \tag{4.5}
$$

where, r_i are related to the four $U(1)$ charges of the black hole, in turn proportional respectively to the numbers of the three five-branes. The entropy of this black hole takes a simple form as seen earlier in [20]

$$
S_{BH} = \frac{A}{4G_4} = 2\pi\sqrt{Q_1Q_2Q_3n}. \tag{4.6}
$$

This as the formula suggests can be reproduced from a conformal field theory with central charge $c = 6Q_1Q_2Q_3$. Or in the long string approximation, $c = 6$. There exist hueristic arguments of the derivation of central charge. We shall again come back to this when we study the BTZ black hole in the next chapter. The near extremal configuration corresponding to the above solution, has small right moving momentum. The black hole for that is given by:

$$
\begin{aligned}
ds_4^2 &= -f^{-1/2}hdt^2 + f^{1/2}(h^{-1}dr^2 + r^2 d\Omega^2) , \\
f &= \prod_{i=1}^{4}\left(1 + \frac{r_i}{r}\right), \quad \text{and} h = 1 + \frac{r_0}{r}.
\end{aligned} \tag{4.7}
$$

The left and right momentum are given by $n_L = r_0 exp(\sigma)$ and $n_R = r_0 exp(-\sigma)$ and $r_n = r_0 \sinh 2\sigma$. The extremal limit is defined by $r_0 \to 0$ and $\sigma \to \infty$, with r_n fixed. The entropy of this black hole is given by:

$$
S = \frac{A}{4G_4} = 2\pi\sqrt{Q_1Q_2Q_3}(\sqrt{n_L} + \sqrt{n_R}) \tag{4.8}
$$

Under the assumption, that the left and the right modes interact very weakly, the above entropy can again be reproduced from a CFT with the left and right oscillator levels being given by n_L and n_R. But having a near extremal black hole, implies a non-zero Hawking temperature, and hence emission of particles. The emission of massless bosons from this black hole gave very interesting results, and the emission rate had a structure which can be reproduced from a 1+1 CFT [50]. In this chapter, we look at fermion emission from these black holes. Firstly we take the simplest case, where a massless neutral fermion propagates on the back ground of this geometry. This calculation reveals that though the above particle does not exist in the supergravity spectrum, it still has a structure that can be reproduced from a 1+1 dimensional CFT.

4.2 Hawking Radiation for fermions

To determine the radiation rate of any particle from a black hole, we essentially need to evaluate the absorption cross-section of the black hole, or the Greybody factor. This is done on the lines of a potential barrier problem. The ratio of the radial flux into the horizon to the incident radial flux at the asymptotics gives the probability of the particle to be absorbed by the blackhole. This is normalised to give the fraction of the plane wave which is absorbed. By definition, it gives the absorption cross-section of the black hole. To determine the fluxes, and the ratio exactly, we have to study the equation of motion in the black hole background. Having obtained the solution for the wavefunction, we subsequently determine the GBF's in the next subsection. The generally covariant Dirac operator in any dimension is given by

$$\nabla\!\!\!\!/ \equiv e^{\mu}_{a}\gamma^{a}\left(\partial_{\mu}+\omega_{\mu}\right) \tag{4.9}$$

Where e^{μ}_{a}, where μ is the world index, a is the tangent space vector index, ω_{μ} is the spin connection associated with the metric. In the first subsection we look at minimally coupled fermion emission from a four dimensional black hole. This fermion simply satisfies the equation $\nabla\!\!\!\!/\psi = 0$.

4.2.1 From 4-dim black hole

A convenient choice for the local tetrad components appropriate to (4.7) is given by

$$\begin{aligned} e^{0}_{t} &= f^{-1/4}h^{1/2}\ , \quad e^{3}_{r} = f^{1/4}h^{-1/2}, \\ e^{1}_{\theta} &= f^{1/4}r\ , \quad e^{2}_{\phi} = f^{1/4}\, r\sin\theta\ , \end{aligned} \tag{4.10}$$

yielding the Weyl equations (for the radial components of the Weyl spinor field, assumed left-handed)

$$\begin{aligned} i\,r\sqrt{f/h}\,\omega\,R_1 + r\sqrt{h}\,\frac{dR_1}{dr} &= -\lambda\,R_2\ , \\ i\,r\sqrt{f/h}\,\omega\,R_2 - r\sqrt{h}\,\frac{dR_2}{dr} &= \lambda\,R_1\ , \end{aligned} \tag{4.11}$$

where λ is a constant, and ω is the frequency associated with the time dependence of the solutions, assumed $\exp(-i\omega t)$.

Near the horizon ($r \to r_0$), we introduce the variable $z \equiv 1 - r_0/r$ and approximate f as

$$f = K\,\left[(1-z)^4\,\sinh^2\sigma + (1-z)^3\right]\ ,$$

where $K \equiv r_1 r_2 r_3 / r_0^3$, and $r_4 = r_0 \sinh^2 \sigma$ defines σ. From eq. (4.11), a second order differential equation is easily deduced for either of the radial functions. For R_1, we take the ansatz

$$R_1 = A \, z^m \, (1-z)^n \, F(z) \, ,$$

and restricting ourselves to the region $r \sim r_0$, we obtain a hypergeometric equation for $F(z)$

$$\begin{aligned} z(1-z)\, \frac{d^2F}{dz^2} &+ \left[\left(2m+\frac{1}{2}\right) - z\left(1+2m+2n+\frac{1}{2}\right)\right] \frac{dF}{dz} \\ &- \left[\left(m+n+\frac{1}{4}\right)^2 + \mu(\omega,\sigma)\right] F = 0 \, . \end{aligned} \tag{4.12}$$

Here, we have already made the choices, $m = -i\left(\frac{a+b}{2}\right)$, $n^2 = \lambda^2$ with $a(resp.\ b) \equiv \omega\sqrt{r_1 r_2 r_3 / r_0}\ e^{\sigma}\ (resp.\ e^{-\sigma})$. In the regime $a \approx b$ corresponding to $\sigma \sim 0$, the function $\mu(\omega,\sigma) = -1/16 + i(a+b)/8$ and hypergeometric function relevant to (4.12) is given by $F(\alpha,\beta,\gamma;z)$ where, $\alpha \approx -i\frac{3}{4}(a+b) + n + \frac{1}{2}$, $\beta \approx -i\frac{1}{4}(a+b) + n$, $\gamma = -i(a+b) + \frac{1}{2}$. The physical significance of this regime may be open to question since it corresponds to $r_4 \ll r_0$, and since all semiclassical considerations pertain to behaviour outside the blackhole horizon. However, we include this for completeness.

In the other regime where $\sigma \geq 1$ corresponding to $a > b$, we have $\mu(\omega,\sigma) = -[1/4 - i(a-b)/2]^2$. The solution for $R_1(z)$ in the near zone can thus be shown to be

$$R_1^{near} = A z^{-i(a+b)/2} \, (1-z)^n \, F(\alpha,\beta,\gamma;z) \, , \tag{4.13}$$

$$\alpha = -ia + n + \frac{1}{2}, \; \beta = -ib + n, \; \gamma = -i\,(a+b) + \frac{1}{2} \, . \tag{4.14}$$

For large σ, $b \approx 0$, and parameters are $\alpha = -ia + n + \frac{1}{2}$, $\beta = n$, $\gamma = -ia + \frac{1}{2}$. This can be derived alternatively by using the Newman-Penrose formalism. In either case, in the limit $z \to 0$, $R_1^{hor}(z) = A z^{-i(a+b)/2}$.

The near zone solutions obtained above are to be matched by extrapolation to the regime $z \to 1$ to the small distance limit of the solution in the far zone ($f \to 1$, $h \to 1$). The latter solution can be shown to be given in terms of the Whittaker function [52] as

$$R_1^{far}(r) = \frac{B}{\sqrt{\omega r}} W_{\frac{1}{2},n}(\omega r) \, . \tag{4.15}$$

Using the small distance limit of the Whittaker function [52], and matching with the near solutions yields the ratio of the constants A and B, in absolute value, to be

$$\left|\frac{B}{A}\right| = \frac{1}{\sqrt{2}}(2\omega r_0)^n \left|\frac{\Gamma(n)}{\Gamma(2n)}\frac{\Gamma(\gamma)\,\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\beta)}\right| . \tag{4.16}$$

Now, with the conserved fermionic flux given by

$$\mathcal{F} = |R_1|^2 - |R_2|^2 , \tag{4.17}$$

the absorption cross section of interest[1] can be expressed as

$$\sigma_{abs} = \frac{2\pi}{\omega^2}\frac{\mathcal{F}_0}{\mathcal{F}_\infty} = \frac{\pi}{\omega^2}\left|\frac{A}{B}\right|^2 . \tag{4.18}$$

where, $\mathcal{F}_0$ $(resp. \mathcal{F}_\infty)$ is the flux entering the black hole horizon (resp. flux arriving from past infinity). Thus, the Hawking radiation rate for fermions is given by

$$\Gamma_H = \frac{\pi}{\omega^2}\left|\frac{A}{B}\right|^2 (e^{2\pi(a+b)}+1)^{-1}\frac{d^3k}{(2\pi)^3} . \tag{4.19}$$

For $a \approx b$, using eq. (4.16) and appropriate values of the parameters α, β *and* γ with $n = -1$ (which corresponds to s-wave solution for the Weyl fermion) we get

$$\Gamma_H = \frac{1}{4}\pi^2 r_0^2(a+b)[\frac{1}{4}+\frac{9}{16}(a+b)^2]\;(e^{\frac{\pi}{2}(a+b)}-1)^{-1}\left(e^{\frac{3}{2}\pi(a+b)}+1\right)^{-1}\frac{d^3k}{(2\pi)^3} \tag{4.20}$$

For $a > b$, we get, for $n = -1$,

$$\Gamma_H = 4\pi^2 r_0^2 \frac{b(\frac{1}{4}+a^2)}{(e^{2\pi a}+1)\,(e^{2\pi b}-1)}\frac{d^3k}{(2\pi)^3} . \tag{4.21}$$

Also, with $n = -1$ one obtains for $a \gg b$

$$\Gamma_H = 2\pi r_0^2\left[\frac{1}{4} + a^2\right]\left(e^{2\pi a} + 1\right)^{-1}\frac{d^3k}{(2\pi)^3}, \tag{4.22}$$

which essentially is the solution found in ref. [53]. This result can be derived using the alternative Newman-Penrose formalism.

4.2.2 Microscopic Description and Branes

Since the above massless minimally coupled fermion does not exist in the spectrum of 11 dimensional supergravity compactified down to 4 dimensions, we cannot look

[1]The contribution of R_2 is negligible.

for a microscopic description of the above. However, under certain considerations of supersymmtery breaking, these particles can arise. It will be interesting to investigate is such a situation. However for the five dimensional black hole, an attempt can be made to understand the emission of fermions through a radiative process in which two open strings collide to give a closed string in the bulk. A dimensional reduction of our calculations might apply for the four dimensional black hole also. (To describe Hawking radiation of scalars from the four dimensional black hole the microscopic formula for the five dimensional black hole obtained in [18] was dimensionally reduced to four dimensions [50].)

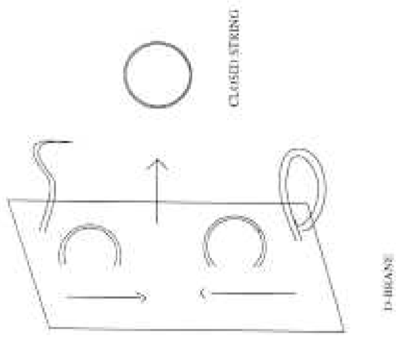

Figure 4.1: *D-brane radiation*

To obtain a microscopic description of Hawking radiation, interaction between the open strings propagating on the D-branes is used. The open strings collide to produce a closed string, which is then emitted from the D-brane. This radiative process is unitary, and the decay rate for scalars, fermions which are present in the closed string spectrum can be reproduced. In ordinary perturbative string theory, scattering amplitudes of open strings interacting to give closed string excitations are easily calculated. In the lowest order in string coupling, the world sheet diagram of two open strings as incoming and outgoing states, can be mapped to a disc diagram with two vertex insertions by a conformal transformation. Vertex operators are local operators denoting the quantum state of the string mode. For the production of a closed string excitation, a vertex operator corresponding to the closed string mode is inserted in the bulk of the disc. In presence of a D-brane, the boundary conditions on the open-string coordinates X^μ, changes to dirichlet in the direction perpendicular to the D-brane directions. On incorporating the appropriate boundary conditions, in the calculations, the D-brane closed string scattering amplitudes are determined as very similar to perturbative string scattering amplitudes.

For the five dimensional black hole, the D-brane configuration consists of D5-

branes wrapped on $T^4 \times S^1$ and D1-branes wrapped on S^1 direction. In the approximation of the volume of $T^4 \to 0$, the entire configuration can be replaced by a D-string along the S^1 direction. So, for our purposes, we study closed string emission amplitudes from a D string. The closed string mode we shall consider is a gravitino mode, with vector polarisation in the compact directions. Gravitinos with polarization vectors transverse to the D-string emerge as chiral spin-1/2 particles akin to those considered in [21]. By spin conservation, one open string vertex insertion is fermionic and the other bosonic. The gravitino vertex operator of the closed string can be decomposed into open string bosonic and fermionic vertex operators. We work in the Green Schwarz formalism, where there is explicit space-time supersymmetry. The bosonic coordinates of the string are X^μ, and the corresponding space-time super partners are θ, where θ are Majorana-Weyl space-time spinors. In the lightcone gauge, the number of fermionic degrees of freedom are reduced to half and are 8 in number, same as bosonic degrees of freedom. In this gauge, the fermionic modes are denoted by S^a, where a stands for a $spin(8)$ spinor index. The vertex operators are:

$$\begin{aligned} V_B(n) &\equiv V_B(\zeta_n, k_n, z_n) \equiv \zeta_n^I B_n^I(k_n, z_n)\ e^{ik_n \cdot X(z_n)} \\ V_F(n) &\equiv V_F(u_n, p_n, z_n) \equiv (u^a F^a(k_n, z_n) + u^{\dot{a}} F^{\dot{a}}(k_n, z_n))\ e^{ip_n \cdot X(z_n)}\ , \end{aligned} \tag{4.23}$$

Where ζ_n stands for the bosonic polarisations, $u^{a,\dot{a}}$ stand for the spinor wavefunctions, k_n is the momentum of the external particle, and z the position of the operator insertion. Here B and F are functions of the field such that vertex operators have the right property under supersymmetry transformations. They are evaluated as [10]:

$$B^+ = p^+ \tag{4.24}$$

$$B^i = \dot{X}^i - R^{ij}k^j \tag{4.25}$$

$$F^a = \left(\frac{p^+}{2}\right)^{1/2} S^a \tag{4.26}$$

$$F^{\dot{a}} = \left(\frac{1}{p^+}\right)^{-1/2} \left[(\gamma.\dot{X}S)^{\dot{a}} + \frac{1}{3} : (\gamma^i S)^{\dot{a}} R^{ij} : k^j\right] \tag{4.27}$$

, where $R^{ij} = \frac{1}{4}\gamma^{ij}_{ab} S^a S^b$. Where :: indicates operator ordering. In the above, p^+ denotes the zero mode of X^+. Due to the presence of the D-string, the X^μ are restricted to the upper half of the complex plane to which the disc can be mapped. However, in those directions where there are Dirichlet boundary conditions, the holomorphic fields get related to the anti-holomorphic fields by the following relations:

$$\tilde{X}_\mu(\bar{z}) = M_\mu^\nu X_\nu(\bar{z}) \tag{4.28}$$

Where M is a matrix which ensures the boundary conditions. On using supersymmetry, the conditions on the S^a can be fixed by using matrices denoted by M_{ab} and $M_{\dot{a}\dot{b}}$. For the four point amplitude, the requisite matrix element may be written, in the light cone Green Schwarz formalism [54]

$$\mathcal{A}(p_1, p_2, p_3, p_4) \;=\; V_{CKG}^{-1} \int \prod_{i=1}^{4} dz_i \;< V_F(1)\; V_B(2)\; V_B(3)\; V_F(4) \;>\, , \tag{4.29}$$

Where, the following identifications are made for the computation of the D-string emission rate:

$$\begin{aligned} 2p_1 &= 2k_1, 2p_2 = 2k_2, 2p_3 = 2k_3, 2p.M = 2k_4 && (4.30)\\ \xi_1 &\equiv \zeta_1 u^a \equiv v_2^a \epsilon.M = \zeta_4 \times v_4 && (4.31)\end{aligned}$$

Where xi_1, u^a is the polarisation of the open strings and ϵ is the gravitino polarisation. Now in case of D-strings, only momentum parallel to the D-string is conserved. The D-brane in the perturbative string approximation is infinitely massive and is like a rigid wall. Thus momentum transverse to the D-string direction need not be conserved. This gives only one invariant which is independent and that is taken to be $t = 4k_1.k_2$. Having done this, we now evaluate the amplitude for the process. The points of insertion of the vertices are fixed as $(x, -x, i, -i)$ since by SL(2,R) invariance, three of them can be fixed, leaving only one independent parameter. The open string modes lie on the real axis. On evaluating the integral, we find:

$$\mathcal{A}_F \sim \frac{\Gamma(-2t)}{\left(\Gamma(1-t)\right)^2}(2t)\left(\bar{u}_1\xi_2 \cdot \gamma\epsilon \cdot k_2 + \ldots\right)\, , \tag{4.32}$$

Here we have only written one representative polarisation term to illustrate the momentum dependence.

Independently, another approach can be used to get the above result. Ward identities corresponding to the surviving spacetime supersymmetries for the D-string [54], which may be expressed schematically as

$$V_{CKG}^{-1} \int \prod_{i=1}^{4} dz_i \;< [\eta Q \;,\; V_F(1) V_B(2) V_B(3) V_B(4)] > \;=\; 0\, . \tag{4.33}$$

This enables us to express the desired amplitude purely in terms of the bosonic amplitude calculated for instance in ref. [55]. Here we merely estimate the energy dependence of the amplitude to check consistency with the semiclassical results. The amplitude for two open string bosons on the D-string fusing into a graviton with polarization ϵ transverse to the D-string is given by [55]

$$\mathcal{A}_B \sim \frac{\Gamma(-2t)}{(\Gamma(1-t))^2} t^2 (\zeta_1 \cdot \epsilon \cdot \zeta_2) . \tag{4.34}$$

Using (4.33) one obtains, once again for vector polarization components of the gravitino being transverse to the D-string, the amplitude (suitably covariantized) the same result as in 4.32. In contrast to standard computations, as is commonly accepted [17], a decaying gas of D-strings does not afford the usual facility of *preparing* an initial state. The standard procedure of *averaging* over initial states for 'unpolarized' initial states must therefore be replaced by a *summation* over all possible initial momentum distributions. If we assume that the gas in question is an ensemble in thermal equilibrium, it follows that the colliding open string Bose and Fermi modes will also 'thermalize' according to their natural statistics, with distributions $\rho_B(\omega, T_B)$ and $\rho_F(\omega, T_F)$ respectively, where, the temperatures T_B, T_F may not be equal. The total decay rate for the D-string with appropriate phase space factors is then given by

$$\Gamma_D \approx \omega\, \rho_F(T_F)\, \rho_B(T_B)\, \frac{d^3k}{(2\pi)^3} . \tag{4.35}$$

4.3 Conclusions

In this chapter, we have seen how fermions behave in the background of a four dimensional black hole. We have taken a minimally coupled fermion which has an interesting emission rate structure. This for $T_L \gg T_R$ has the same structure as that expected from a CFT with the right central charge and which reproduces the entropy of the black hole. For $T_L \approx T_R$, this however is not true, and it is an interesting task to find why this happens. The first step would be to identify the particular fermion mode which approximates the minimally coupled fermion in case supersymmtery is broken. We shall study the non-minimally coupled fermion in this geometry by using a near horizon geometry approximation in subsequent chapters. Our microscopic calculation on the other hand provides an order of magnitude estimate of the fermionic emission amplitude from a five dimensional black hole with the D-string emission rate calculation. Indeed as shown in a paper [21] later, the correct power of ω and the form of the decay rate is reproduced from our calculation. The decay rate for s-wave fermions as determined in [21] is:

A comparison with (4.35), shows that though the exact factors cannot be reproduced, the similarity is striking. The left and right temperatures which appear in the result are also same as in the semiclassical calculation. The above calculations help in strengthening the D-brane black hole correspondence. However, one major problem with this approach is that the D-brane calculations are done in the perturbative regime of string theory, where the black hole does not exist. In the large coupling limit, where g_sQ, g being the string coupling and Q being the charges, is very large, the D-branes are expected to collapse to form black holes. For near-extremal black holes, there is no non-renormalisation process to protect the charges from receiving quantum corrections. Hence the correspondence and exact matching of entropy and radiation rates remains mystery. In the last chapter, we shall try and address some of these questions.

Chapter 5

The BTZ Black Hole

In this chapter, we shall study the 2+1 dimensional BTZ black hole and Hawking emission rates from them. This black hole is a solution of 2+1 gravity with a negative cosmological constant. It was first obtained in a paper by Banados, Teitelboim and Zanelli [22], hence the name BTZ black hole. This black hole was the first for which a microscopic counting of degeneracy of states gave the Bekenstein-Hawking entropy and is associated with a conformal field theory. The near horizon metric of some four and five dimensional black holes are a product of the BTZ black hole and compact manifold. Thus the BTZ geometry can be used as a tool to investigate higher dimensional black holes. In this chapter, we study the BTZ black hole, with the above motivation in focus. We examine radiation from the black holes and whether that has any microscopic interpretation. In the first section we shall study the black hole solution, and find it's Bekenstein-Hawking entropy. We study how 2+1 gravity is associated with a CFT, which lives on any timelike surface and how the black hole entropy can be determined using this. In the next section we study fermions in the BTZ background, and calculate a 'absorption coefficient' of the black hole. Since the BTZ black hole is not asymptotically flat, we define the greybody factor to be just the ratio of particles entering the black hole to the flux of particles passing through a time like surface at $\rho = l$. The rate measured by this observer has a structure which can be reproduced from a 1+1 CFT. In the third section we compare this rate with that from higher dimensional black holes.

5.1 2+1 Gravity and the BTZ Black Hole

Einstein's gravity in 2+1 dimensions is essentially topological, and does not admit black hole solutions. It can be cast into a Chern Simons Theory under suitable redefinitions. We shall study gravity with a negative cosmological constant, which admits a black hole solution. The action is:

$$S = \frac{1}{2\pi}\int d^3x\,(R + 2l^{-2}) + B \tag{5.1}$$

Where R is the scalar curvature and $\Lambda = -(1/l^2)$ is the cosmological constant and B is a boundary term. We have put $G = 1/8$. The solution of this action is a space-time with constant negative curvature i.e. $R_{\mu\nu\lambda\sigma} = -1/l^2(g_{\mu\lambda}g_{\nu\sigma} - g_{\nu\lambda}g_{\mu\sigma})$. The maximally symmteric metric of that is given in t, ρ, ϕ coordinates as:

$$ds^2 = -\left(1 + \frac{\rho^2}{l^2}\right)dt^2 + \left(1 + \frac{\rho^2}{l^2}\right)dr^2 + \rho^2 d\phi^2 \tag{5.2}$$

This is called the anti de Sitter space-time. The global group of isometries of this metric is SO(2,2). This group has additional generators apart from the usual SO(1,2) generators. Asymptotically the metric is of the form:

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{l^2}{\rho^2}dr^2 + \rho^2 d\phi^2 \tag{5.3}$$

The asymptotic fall off conditions are invariant under an infinite dimensional group, which includes the SO(2,2) as a subgroup. The generators of this group satisfy the Virasoro algebra, with a classical central charge. This central charge was determined in [25] as $3l/2G$. The BTZ metric is derived by discrete identifications of anti-de Sitter space time. It can be independently determined as a solution of the action 5.1 along with the boundary term. The boundary term is added to subtract the surface term which comes in the variation of the action. The BTZ metric in coordinates ρ, t and ϕ is $(0 < \rho < \infty, 0 < \phi < 2\pi)$:

$$ds^2 = -\frac{\Delta^2}{l^2\rho^2}dt^2 + \frac{l^2\rho^2}{\Delta^2}dr^2 + \rho^2\left(d\phi - \frac{\rho_+\rho_-}{l\rho^2}dt\right)^2, \tag{5.4}$$

$$\Delta^2 = \left(\rho^2 - \rho_+^2\right)\left(\rho^2 - \rho_-^2\right). \tag{5.5}$$

Clearly, the metric represents a rotating black hole, with two horizons at ρ_+ and ρ_-. The angular momentum of the black hole is $J = 2\rho_+\rho_-/l$, and its mass is $M = (\rho_+^2 + \rho_-^2)/l^2$. These can also be recognised as the conserved quantities associated with the killing directions $L_0 - K_0$ and $L_0 + K_0$ which represent the zeroth mode of the virasoro generators L_n. It is easy to see that the metric is an identifictaion of anti-de Sitter space by going to coordinates defined by:

$$w^{\pm} = \frac{\sqrt{\rho^2 - N}}{\rho}e^{2\pi T_{\pm}(\phi l \pm t)}, \quad y = \frac{\sqrt{N}}{\rho}e^{\pi T_+(\phi l + t) + \pi T_-(\phi l - t)} \tag{5.6}$$

Where $N = \rho_+^2 - \rho_-^2, T_\pm = 2\pi\frac{r_+ \pm r_-}{l^2}$. The metric gets mapped to the Anti de Sitter metric in Poincare coordinates:

$$ds^2 = \frac{l^2}{y^2}\left(dy^2 + dw^+ dw^-\right) \tag{5.7}$$

with the following identifications of the coordinates due to the fact that for $\phi + 2\pi$ is the same point in the BTZ manifold:

$$w^\pm \sim w^\pm e^{4\pi^2 T_\pm} \quad y \sim y^{2\pi^2(T_+ + T_-)} \tag{5.8}$$

These identifications are not present in AdS space-time. In this thesis, we shall take the metric in more convenient coordinates, with the radial coordinate ρ defined in terms of hyperbolic coordinate μ. The redefinition is:

$$\rho^2 = \rho_+^2 \cosh^2\mu - \rho_-^2 \sinh^2\mu. \tag{5.9}$$

The spatial infinity corresponds to $\tanh\mu \to 1$. In this coordinate, the metric takes a form:

$$ds^2 = -\sinh^2\mu\left(\rho_+\frac{dt}{l} - \rho_- d\phi\right)^2 + l^2 d\mu^2 + \cosh^2\mu\left(-\rho_-\frac{dt}{l} + \rho_+ d\phi\right)^2 \tag{5.10}$$

A convenient set of linear combinations of t and ϕ gives us $x^+ = \rho_+ t/l - \rho_-\phi$ and $x^- = -\rho_- t/l + \rho_+\phi$, and the killing directions of the metric are ∂_{x^+} and ∂_{x^-}.

A very interesting observation is that the action 5.1 can be cast as a $SL(2,R) \times SL(2,R)$ Chern Simons action which is:

$$I_{CS} = \frac{k}{2\pi}\int \mathrm{Tr}\left\{A^+ \wedge dA^+ + \frac{2}{3}A^+ \wedge A^+ \wedge A^+ - A^- \wedge dA^- - \frac{2}{3}A^- \wedge A^- \wedge A^-\right\} \tag{5.11}$$

With $A_a^\pm = \omega_a \pm e_a/l$ and $k = l/4G$ in a particular representation of the SL(2,R) generators. The Chern Simons theory is topological, but it induces a WZW conformal field theory on any $\rho = constant$ boundary.The central charge of the induced CFT is related to the Chern Simon's coupling constant k by the formula: $c = 3k/(k+2)$. S. Carlip used this in [26], showed that the at the horizon, the CFT induced is of central charge c=3 +3 for large k. Defining the physical states of the black hole as those which are invariant under the action of $L_0 - K_0$ and $L_0 + K_0$, he found that the log of the degeneracy of such states is $2\pi r_+/4G$, the entropy of the black hole. Independently, A. Strominger used the fact the asymptotic Conformal symmtery and the central charge, to determine the density of states as [56]:

$$S = \pi l^2\left[\sqrt{\frac{M + J/l}{2G}} + \sqrt{\frac{M - J/l}{2G}}\right] \tag{5.12}$$

This is precisely the Bekenstein-Hawking entropy of the black hole if we replace r_+ in terms of M and J. Both these calculations are counting of microscopic degrees of freedom. Though the relation between the above calculations is not exactly clear, it says that the BTZ is described by a CFT at any boundary. In this thesis we then determine Hawking radiation from these black holes and see whether there exists a microscopic interpretation of the Hawking radiation rate.

5.2 Fermion Emission

The scalar emission from these black holes was considered in [57]. It was observed that the greybody factor for this has a non-trivial structure which can be reproduced from a 1+1 dimensional CFT. In this thesis, we study fermion emission from these black holes.

5.2.1 Equation of motion of the fermion in BTZ background

The space with constant negative curvature, is called anti-de Sitter space. This space is invariant under SO(2,2) group, which is larger than the usual Poincare group of flat space time. The appropriate covariant derivative for spin- half fields is:

$$D = \gamma^{\nu}\left(\partial_{\nu} + \omega_{\nu} + g\ e_{\nu a}\gamma^{a}\right) \tag{5.13}$$

Where, ω_ν is the spin connection, $e_{\nu a}$ is the triad and $g = 1/2l$ is related to the cosmological constant. The BTZ metric is derived by appropriate identifications of anti-de Sitter space time, and its asymptotic properties are the same as anti-de Sitter space [22]. Hence the above covariant derivative is relevant for our purposes. The triads are chosen in an appropriate local Lorentz frame in the tangent plane for the metric 5.10.

$$\begin{aligned} e^0_{x^+} &= \sinh\mu \quad e^2_{x^-} = \cosh\mu \\ e^1_{\mu} &= l. \end{aligned} \tag{5.14}$$

The non zero spin connections for this are:

$$\begin{aligned} \omega_{x^+} &= -\frac{1}{2l}\cosh\mu\ \sigma^{01} \\ \omega_{x^-} &= \frac{1}{2l}\sinh\mu\ \sigma^{21}, \end{aligned} \tag{5.15}$$

here $\sigma^{ab} = 1/2[\gamma^a, \gamma^b]$. Using the above, we substitute them in (5.13) and obtain the fermion equation on the BTZ space-time as

$$\gamma^1 \frac{1}{l}\left(\partial_\mu + \frac{\sinh\mu}{2\cosh\mu} + \frac{\cosh\mu}{2\sinh\mu}\right)\psi + \gamma^0 \frac{\partial_{x^+}\psi}{\sinh\mu} + \gamma^2 \frac{\partial_{x^-}\psi}{\cosh\mu} + \frac{1}{2l}\psi = 0 \tag{5.16}$$

We take the representation of gamma matrices to be: $\gamma^0 = \imath\sigma^2, \gamma^1 = \sigma^1, \gamma^3 = \sigma^3$. The killing isometry requires that $\partial_{x^\pm}\psi = -\imath k^\pm\psi$, where $k^\pm$ are constants depending on the energy and azimuthal eigenvalues ω and m respectively. In fact they can be determined, and $k^+ = (\omega - m\Omega)/(2\pi l\, T_H)$ and $k^- = (\rho_-\omega - \rho_+ m/l)/(2\pi l\rho_+ T_H)$. The Hawking temperature is $T_H = (\rho_+^2 - \rho_-^2)/2\pi l^2\rho_+$ and $\Omega = J/2\rho_+^2$. We take the following form for the wavefunction: The radial equations for the two components are determined as

$$\begin{aligned}
\left(d_\mu - \frac{\imath l k^+}{\sinh\mu}\right)\psi_2 &= -\left(\frac{1}{2} - \frac{\imath l k^-}{\cosh\mu}\right)\psi_1 && (5.17)\\
\left(d_\mu + \frac{\imath l k^+}{\sinh\mu}\right)\psi_1 &= -\left(\frac{1}{2} + \frac{\imath l k^-}{\cosh\mu}\right)\psi_2 && (5.18)
\end{aligned}$$

Interestingly, to separate the wave functions, we have to go to a different basis of wave functions. Let us call them ψ_1' and ψ_2', defined as,

$$\begin{aligned}
\psi_1 + \psi_2 &= (1 - \tanh^2\mu)^{-1/4}\sqrt{1 + \tanh\mu}\,(\psi_1' + \psi_2') && (5.19)\\
\psi_1 - \psi_2 &= (1 - \tanh^2\mu)^{-1/4}\sqrt{1 - \tanh\mu}\,(\psi_1' - \psi_2') && (5.20)
\end{aligned}$$

We obtain the equations in coordinates $y = \tanh\mu$ for the ψ' below as:

$$\begin{aligned}
(1-y^2)d_y\psi_2' - \imath l\left(\frac{k^+}{y} + k^- y\right)\psi_2' &= - \quad \{1 - \imath l(k^+ + k^-)\}\,\psi_1' && (5.21)\\
(1-y^2)d_y\psi_1' + \imath l\left(\frac{k^+}{y} + k^- y\right)\psi_1' &= - \quad \{1 + \imath l(k^+ + k^-)\}\,\psi_2' && (5.22)
\end{aligned}$$

The equations are now very easily separable. The second order differential equation obtained from the above two equations can be cast in a simple form in the variable y^2 which we denote as z for convenience.

$$z(1-z)d^2\psi_1' + \frac{1}{2}(1 - 3z)\,d\psi_1' + \frac{1}{4}\left(\frac{-\imath l k^+ + l^2 k^{+2}}{z} + \imath l k^- - l^2 k^{-2} - \frac{1}{1-z}\right)\psi_1' = 0 \tag{5.23}$$

The solution to this equation is determined to be $\psi_1' = z^m(1-z)^n F(\alpha, \beta; \gamma; z)$, where F is a hypergeometric function. For the ingoing function, the constants are as follows: $m = 1/2 + ılk^+/2, n = -1/2$, and the hypergeometric parameters are: $\alpha = ıl(k^+ + k^-)/2 + 1/2, \beta = ıl(k^+ - k^-)/2$ and $\gamma = ılk^+ + 3/2$. The ingoing function is so chosen that at the horizon, the wave function has the dependence $\psi \sim exp(ı(\omega/4\pi T_H) \log z)$. The solution for the other component of the wave function can be determined easily now. It is: $\psi_2 = z^{ılk^+/2}(1-z)^{-1/2}(-(\gamma-1)/\alpha) F(\alpha-1, \beta; \gamma-1; z)$, where α, β, γ are constants as defined above. The flux for this function as shown in the next section is negative, indicating a flow into the black hole.

Thus we see that the fermion equation of motion in the BTZ background is exactly solvable. It is interesting to note that $n = 0$ corresponds to a minimally coupled fermion, and in that case $\gamma = \alpha + \beta$. The hypergeometric solution is not well behaved, and does not converge as $z \to 1$. There can be other kinds of couplings to the BTZ metric, and they will be interesting to investigate. The BTZ black hole is locally anti- de Sitter space, with global identifications. It will be interesting to see whether the solutions obtained here can be related to those obtained for AdS_3 in [58], modulo the global identifications.

5.2.2 Grey Body Factor

The black hole grey body factor is also the absorption coefficient of the black hole. The geometry of the black hole provides a kind of potential barrier for the fields propagating on it. Only a fraction of the incoming flux at infinity is absorbed by the body, and rest is reflected back. In order to determine the total Hawking radiation rate of the observer, sitting far away from the black hole, we need to calculate this absorption rate. Indeed, as in ordinary quantum mechanics, the black hole absorption rate, which we denote by σ_{abs} is related to the ratio of the ingoing flux at horizon and incoming flux at infinity. The fermion flux into the horizon will be determined by the current which enters the horizon. Usually, to probe the black hole geometry, an incoming plane wave is taken at infinity. However, here, for our purposes, we take the incoming flux in the region $\rho \sim l \gg \rho_+$ as the incident flux on the black hole. This would correspond to an BTZ observer, sitting at finite ρ, detecting radiation. Though, the physics of this picture is not very clear , there are a number of reasons for choosing this. In curved space-time, an observer measures a thermal spectrum depending upon his local temperature, which is $T_H/\sqrt{g_{00}}$. In asymptotically flat space time, $\sqrt{g_{00}} \to 1$ as $\rho \to \infty$. However, this is not the case in asymptotically anti-de Sitter space-time where $\sqrt{g_{00}} \sim \rho$ at spatial infinity.

For small mass BTZ black holes, i.e. $\rho_+ \ll l$, it is easily seen that, $\sqrt{g_{00}} \to 1$ when $\rho \sim l$. This motivates the choice of the observer. Moreover, to compare our final answer with higher dimensional black hole rates, going infinitely away from the horizon would imply a modification of the near horizon BTZ geometry, and we are not interested in probing that region. As $\rho \sim l$, the black hole metric is same as asymptotically anti-de Sitter space. Solutions determined in this metric is also the same as that obtained in the vacuum solution of the black hole [59]. The metric is $(\rho \gg \rho_+)$:

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{l^2}{\rho^2}d\rho^2 + \rho^2 d\phi^2 \tag{5.24}$$

To, determine the wave functions we then solve the radial equations:

$$\left(\rho\, \partial_\rho \pm \frac{\imath\omega l^2}{\rho}\right)\psi^f{}_{1(2)} = -\left(\frac{1}{2} \pm \frac{\imath m}{\rho}\right)\psi^f{}_{2(1)} \tag{5.25}$$

To separate this set, we go to a frame in which, $\psi'^f{}_1 = \psi^f{}_1 + \psi^f{}_2$ and $\psi'^f{}_2 = \psi^f{}_1 - \psi^f{}_2$. The equation can be exactly solved in this frame. The solutions are determined, in terms of Bessel functions,

$$\begin{aligned}
\psi'^f{}_1 &= \sqrt{x}\,(A_1 J_0(\Lambda x) + \imath A_2 N_0(\Lambda x)) && (5.26)\\
\psi'^f{}_2 &= \frac{\imath\sqrt{x}}{E}\,(A_1 J_1(\Lambda x) + \imath A_2 N_1(\Lambda x)) && (5.27)
\end{aligned}$$

Where J_n and N_n are bessel functions of the first and second kind. A_1 and A_2 are arbitrary constants of integration. Also $x = 1/\rho, \Lambda = l\sqrt{\omega^2 l^2 - m^2}, E = l(\omega l + m)/\Lambda$. Note that the same solutions will survive when $\rho \to \infty$ in anti-de Sitter space. The interesting aspect about anti-de Sitter space is that $\rho \to \infty$ is a time like surface. Hence, it is necessary to specify boundary conditions, which are either Dirichlet or Nueman on the surface. These boundary conditions, also called reflective boundary conditions [60] can be realised in the set of functions defined in (5.27). On choosing $A_2 = 0$, this condition can be ensured for the above wavefunctions. It is easy to check that in that case $\sqrt{\rho}\psi = 0$ for $\rho \to \infty$. However, here we are not interested in making the wall totally impervious. Instead, we are forced to take A_2 have non-zero values if we want the wavefunction to match the wavefunction determined in (5.23) continued to $z \to 1$. This shows that, our choice of a net inflow of flux into the black hole ensures that we donot have reflecting boundary conditions at infinity. For asymptotic anti- de Sitter space, this might be related to the transparent boundary conditions defined in [60]. Before we can determine the greybody factor using this far solution, we need to match this with the ingoing wavefunction at the horizon since we want the wavefunction to be continuous in space-time. To do that, we continue the solution of (5.23) to $z \to 1$ [61]. Now, with the scalings and redefinitions, the radial wavefunction is:

$$\psi_1 \rightarrow \frac{E_1\sqrt{N(\rho_+ + \rho_-)}}{(2\rho)^{3/2}}\{-(1 - 2\Psi(\alpha)\beta - 2\Psi(\beta+1)\beta) + 4\beta\left(\log\left(\sqrt{N}/\rho\right) + C\right)\} \tag{5.28}$$

$$\psi_2 \rightarrow \frac{\sqrt{\rho_+ - \rho_-}}{\sqrt{2\rho}}E_1 \tag{5.29}$$

Where,

$$E_1 = \left(\frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\right), \tag{5.30}$$

Ψ are the digamma function (subsequently, we write $2\beta(\Psi(\alpha)+\Psi(\beta+1)) \equiv \Psi(\alpha,\beta)$). Also $N = \rho_+^2 - \rho_-^2$, and $\Psi(1) = -C$ (euler's constant). The factors $\sqrt{(\rho_+ \pm \rho_-)/N}$ enter as this wavefunction given in terms of t, ρ, ϕ coordinates is lorentz rotated from the wavefunction obtained in x^+, z, x^- coordinates. Note in the above, we have taken, $\sqrt{N}/\rho \ll 1$ or in other words, $\sqrt{N} \ll l$. The wavefunctions obtained in (5.27) , can be cast in a similar form when $\Lambda/\rho \ll 1$. (For $m = 0$, this indicates that $\omega l \ll 1$)

$$\psi^f{}_1 \approx \frac{1}{\rho^{3/2}}\left(A_1 + \frac{2iA_2}{\pi}\left(\log(\frac{\Lambda}{\rho}) + C\right)\right) \tag{5.31}$$

$$\psi^f{}_2 \approx \frac{2A_2}{\pi E\Lambda\rho^{1/2}}. \tag{5.32}$$

The asymptotic constants are determined from the above equations,

$$A_1 = -\frac{\sqrt{N(\rho_+ + \rho_-)}}{2\sqrt{2}}E_1(1 - \Psi(\alpha,\beta)) \quad A_2 = \frac{\pi E\Lambda}{2\sqrt{2}}\sqrt{\rho_+ - \rho_-}E_1 \tag{5.33}$$

The fermionic flux is given by:

$$\mathcal{F} = \sqrt{-g}J^\rho = \rho\,\bar{\psi}e_1^\rho\gamma^1\psi \tag{5.34}$$

$$\tag{5.35}$$

The incoming flux at $\rho = l$ is determined as (The flux obtained from the above constants is multiplied by l^2/N^2 for normalisations)

$$\mathcal{F}^f = -\frac{l}{8N}|E_1|^2(2 - 2Re\Psi(\alpha,\beta)) \tag{5.36}$$

The absorption coefficient is defined as the ratio of total number of particles entering the horizon with the incoming flux at infinity [62]. The total number of particles entering the horizon is:

$$P = -\int \sqrt{-g}J^{\rho}\rho_{+}d\phi \tag{5.37}$$

This is equal to: $A_H l/4N|(\gamma-1)/\alpha|^2$, where A_H is the area of the horizon. The absorption coefficient is then determined as (for $m=0$):

$$\sigma_{\text{abs}} = \frac{A_H}{1-Re\Psi(\alpha,\beta)}\left(\left|\frac{\gamma-1}{\alpha}\right|^2\right)\left|\frac{\Gamma(\gamma)\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\right|^{-2} \tag{5.38}$$

Here, $\text{Re}\Psi(\alpha,\beta) = -(\omega/4T_L)\tanh(\omega/4T_R) - O(\beta^2)$ for m=0. Using the expressions for the hypergeometric parameters as given in the earlier section, the greybody factor can be written in a interesting form:

$$\sigma_{\text{abs}} = \frac{\omega A_H}{4T_L(1+\omega/4T_L\tanh(\omega/4\pi T_R))}\frac{\exp(\omega/T_H)+1}{(\exp(\omega/2T_L)-1)(\exp(\omega/2T_R)+1)} \tag{5.39}$$

Where, the quantities T_L and T_R are defined by:

$$\frac{1}{T_L} = \frac{1}{T_H}\left(1-\frac{\rho_-}{\rho_+}\right) \quad \frac{1}{T_R} = \frac{1}{T_H}\left(1+\frac{\rho_-}{\rho_+}\right) \tag{5.40}$$

The extremal limit, defined by taking $\rho_+ \to \rho_-$, corresponds to $T_L \gg T_R$. Clearly, taking the above limit in equation (5.40), the absorption coefficient reduces to $A_H/2$. The Hawking radiation rate will be now a product of thermal distributions, instead of being a single fermionic distribution. Infact, it is:

$$\Gamma_H = \frac{\omega A_H}{4T_L}\frac{d^2k}{(exp(\omega/2T_L)-1)(exp(\omega/2T_R)+1)} \tag{5.41}$$

This is precisely the form expected for emission rates from an underlying conformal theory at finite temperature [53]. The fermion in the bulk couples to operators of the 1+1 dimensional conformal field theory. The system is at finite temperature T_H, which can be split into left and right temperatures such that $1/T_L + 1/T_R = 2/T_H$. The decay rate at finite temperature due to the coulping stated above is calculated to have the form of a product of left and right distributions. The fermions are associated with rightmoving temperature, indicating that fermions considered here couple to chiral conformal operator. Using the results of [53] it can be predicted that the conformal fermion couples to operator in the conformal theory of the form O^+O^-, where O^+ is rightmoving, and has conformal weight $1/2$ and O^- is leftmoving with weight 1. It will be interesting to determine the nature of the coupling as that fixes the coefficients exactly.

5.3 Comparison with Higher dimensional black holes

One of the reasons behind the renewed interest in three dimensional black holes, is the fact that near horizon geometries of certain higher dimensional stringy black holes are BTZ times a compact manifold. Near horizon geometry has been extensively used to learn about entropy of the higher dimensional black holes [12, 64, 63]. Here we briefly review this mapping [12, 64] and discuss the implications. The solution due to RR charged one branes and five branes wrapped on $T^4 \times S^1$, and Kaluza Klein momenta along S^1 in 10 dimensions, has a near horizon geometry $BTZ \times T^4 \times S^3$. The radius of the S^3 direction is $l = r_1 r_5$, where r_1 and r_5 are related to the one brane and five brane charges respectively. The time, transverse radial distance ρ and S^1 direction (ϕ) constitute the BTZ black hole coordinates. The ordinary kaluza-klein reduction of the 10-D solution on $T^4 \times S^1$ yields a 5-D black hole, which preserves $N = 8$ supergravity. The entropy of the 5-D black hole is equal to the entropy of the near horizon BTZ black hole and scalar decay rate equals the decay rate for scalar emission from BTZ black holes. Here we make a comparison for fermion decay rates. In [21], it has been shown that the SUGRA fermions of $\mathcal{N} = 8$ supergravity, have a Hawking decay rate for the five dimensional black hole as

$$\Gamma^5_H = A^5_H \frac{\omega}{4T_L} \frac{d^4k}{(exp(\omega/2T_L) - 1)(exp(\omega/2T_R) + 1)} \tag{5.42}$$

Clearly, our decay rate is identical to this decay rate. The temperatures of the left and right distributions are exactly the same as given in (5.40) and A^5_H is the area of the horizon of the five dimensional black hole. A interesting point to note is that the rates can be matched upto exact coefficients if we choose to factor out the phase space factors of S^3 and $\phi = x_5/l$ (x_5 is the S^1 direction, with radius R) from the decay rates, as $A^5_H/A^3_H = \pi l^3/R$. However, an observer in five dimensional space detecting particles at infinity sees all the three dimensions of S^3 as uncompactified. So, it is not clear what the above result implies. However, it can be said that our result confirms the observation about scalar decay rates. The range of frequency for both the calculations, $\omega r_1 \ll 1$, is also same.

The exact matching observed above provides a basis to predict rates for non-minimally coupled fermions which propagate on the background of the four dimensional $\mathcal{N} = 4$ SUGRA black hole obtained by compactifying M-theory (11-D supergravity) on $T^6 \times S^1$ described in the previous Chapter. To identify the required fermion one requires to take the equation of motion in the 11 D Supergravity solution, and take the near horizon geometry limit as described above. All fermions which will couple in the same way as in equation (5.13) in the BTZ part, can then be predicted to have the rate obtained in this paper. The metric in (4.2) is due to 3 M 5-branes wrapped on $T^4 \times S^1$, i.e. directions $x_4..x_{11}$ and a boost in the x_{11}, (S^1) direction. The near horizon limit results in the metric splitting up into a BTZ$\times S^2 \times T^6$, where S^2 is the two sphere of the noncompact t,r,θ, ϕ dimensions of the four dimensional black hole. The radius of the two sphere is $R = l/2 = (r_1 r_2 r_3)^{1/3}$, where r_i are related to the charges of the black hole. As in the five dimensional case here $\phi = x_{11}/R_{11}, \rho^2 = 2R_{11}^2(r_0 + r_0 \sinh^2 \sigma')$ (R_{11} is the radius of x_{11}), and time form the BTZ coordinates. To find the relevant fermions which will have the rate as found in this paper, we start from the 11-D gravitino ψ_M. Clearly gravitino with vector polarisation along x_{11} or the other x_1, x_2, x_3 directions will not satisfy our requirements. We take a representative ψ_5, as in the near horizon limit, all the torus directions are similar, apart from constant scalings. In this limit, since g_{ii} =const, $i = 4..9$, we can split the 11-D equation of motion as:

$$\left(\not{D}_3 + \frac{2}{l} \not{D}_2 \right) \psi = 0 \tag{5.43}$$

Where $\not{D}_3$ and $\not{D}_2$ are the dirac operators in the BTZ and the two sphere metrics respectively. To get simultanouos eigenstates of both the operators, we multiply by the two dimensional chirality matrix $\Gamma_2 = \imath \Gamma_a \Gamma_b$. Thus for $\Gamma_2 \not{D}_2 \psi = \lambda \psi$, the equation of the fermion in the near horizon limit is:

$$\left(\not{D}_{3_\mu}{}' + \frac{2\lambda}{l} \right) \psi = 0 \tag{5.44}$$

For $\lambda = 1/4$, we have the required fermion ($\not{D}' \equiv \Gamma_2 \not{D}$). It is not very difficult to solve the eigenvalue equation stated above. The arguement given here is hueristic, and we have not been careful about the supersymmetry preserved by the background metric. It is to be checked whether the fermion taken above falls in the $N = 4$ multiplet, as the four dimensional black hole preserves $N = 4$ super symmetry. However, it is an interesting calculation, and is under further investigation at present.

5.4 Discussions

In this paper we have calculated emission rate of fermions from BTZ geometry, using techniques of asymptotically flat space-time calculations, like the greybody factor. However, since the physical situation we are interested in is when BTZ occurs as the near horizon geometry of higher dimensional black hole, this is justified. We show that indeed the BTZ calculation reproduces the rate of the non-minimally coupled fermions in the background of a five dimensional black hole whose near horizon geometry is $BTZ \times S^3$. The fact that the rate observed by a BTZ observer at $\rho \sim l$ looks identical to that of an asymptotic observer in a five dimensional black hole is interesting. The physical implications of this are still not clear, but the answer might lie in the location of the degrees of freedom of the underlying conformal field theory. There are several ways to approach the problem. It is known that 2+1 gravity can be cast in the form of Chern Simon theory, which induces a conformal field theory on the boundary. However, on inclusion of matter fields the theory is no longer topological, and the same conclusions cannot be drawn about the entropy. Hence, it is not clear how to study Hawking emission in the above frame work. Recently, matter fields have been treated as a classical perturbation in the Chern Simons action, and the decay rate obtained for scalars[65]. The scalar action is taken as:

$$I_s = \int d^3x \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \tag{5.45}$$

Which gives rise to a boundary term in the t, ρ, ϕ coordinates as:

$$I_B = \int d^2x \sqrt{-g} g^{\rho\rho} \phi^\dagger \partial_\rho \phi + \phi \partial_\rho \phi^\dagger \tag{5.46}$$

The boundary given by $\rho \to \infty$ has the coordinates: $u = t/l - \phi$, $v = t/l + \phi$. The solution at the boundary is of the form:

$$\phi = (1 - ie^{-2\rho})\phi_+(u, v) + (1 + ie^{-2\rho})\phi_-(u, v) \tag{5.47}$$

The background metric can be written in terms of the CS gauge fields. At the boundary, $A_v^+ = A_u^- = 0$. Furthermore, the gauge choice used is: $A_\rho^+ = b^{-1}\partial_\rho b$, $A_\rho^- = b\partial_\rho b^{-1}$, where, $b = exp(\rho T_3)$, T_3 being a generator of $SL(2, R)$. The other field A_ϕ is determined from Gauss law as:

$$A_\phi^+ = \begin{pmatrix} a^{+3}(u) & e^{-\rho} a^+(u) \\ e^\rho \bar{a}^+(u) & -a^{+3}(u) \end{pmatrix}$$

$$A_{\phi}^{-} = \begin{pmatrix} a^{-3}(v) & \epsilon^{-\rho}a^{-}(v) \\ \epsilon^{\rho}\bar{a}^{-}(v) & -a^{-3}(v) \end{pmatrix}$$

Where the functions $a^{+}(u), a^{-}(v)$ are arbitrary fields at the boundary. Under infinitesimal diffeomorphisms of the boundary, the $a^{+}(u)$ and $a^{-}(v)$ transform as $(1,0)$, $(0,1)$ operators, in the boundary CFT. The metric in terms of the $a(u)$ and $a(v)$ goes as:

$$ds^2 = l^2 d\rho^2 - l^2 e^{2\rho} a^{-}(u)\bar{a}^{+}(v) + ... \tag{5.48}$$

In terms of this , the boundary action 5.46 gives:

$$I_B \approx \int dudv \, (a^{+}(v)\bar{a}^{-}(u)) \left[\phi_{+}\phi_{-}^{\dagger} - \phi_{-}\phi_{+}^{\dagger}\right] \tag{5.49}$$

The main result of [65], is the above result, where at the boundary, the scalar field clearly couples to an operator with conformal weights $(1,1)$. A quantum mechanical calculation with the above interaction exactly reproduces the decay rate for the scalars.

For the fermions, a similar procedure can be adopted. However, since the fermion action vanishes onshell, a separate boundary term has to be added. This can be of the form:

$$I_B = C \int dudv\bar{\psi}\psi \tag{5.50}$$

This should give the operator to which the fermion couples as weight $(1, 1/2)$. It is not obvious from the boundary term. This is under investigation at present. The constant C is undetermined, and we can fit it to the semiclassical result to obtain exact matching. The agreement with the black hole decay rate is remarkable for the scalars, and calls for further investigation for the fermions.

Apart from this, the BTZ black hole is asymptotically anti de Sitter, and has a conformal field theory living on it's boundary [25, 56]. With the AdS/CFT correspondence, it is known now, that string theory on orbifolds of AdS_3 times a compact manifold M is dual to a super conformal field theory whose target space is symmetric product of M [12]. In this matter fields are automatically included, and we shall study the decay rates, using this approach in the next chapter.

Chapter 6

The AdS/CFT Correspondence

In the fourth chapter, we studied emission from string black holes. We found that the decay rates have a structure which can be reproduced from a 1+1 CFT. In the next chapter, we studied the BTZ black hole which is also associated with a similar CFT. In this chapter, we use the BTZ black hole and the CFT associated with it to understand the higher dimensional string black holes [31]. We concentrate on studying emission rates of particles from a five dimensional black hole and give a microscopic derivation of the rates. The black hole solution is obtained by compactifying Type II B string theory on $T^4 \times S^1$. On retaining the S^1 as a compact direction with a large radius, it gives a black string solution wrapped around the S^1. The near horizon geometry of this configuration is $BTZ \times S^3$. The emission rates of neutral particles obtained in the black string background are the same as that from the 5-dimensional black hole [12, 66], and the near horizon BTZ geometry has a crucial role in determining the greybody factors [27, 57, 67]. We study the matter fields obtained as perturbations of the given 6-dimensional supergravity background and obtain the equations of motion of particles in the near horizon geometry, by considering a $AdS_3 \times S^3$ compactification of the six dimensional supergravity. Since BTZ space is locally AdS_3, to study the equation of motion of particles it suffices to study $AdS_3 \times S^3$ compactification. The complete wavefunction is determined by taking the wave functions in the near horizon geometry and matching it suitably with the wavefunctions in asymptotically flat spacetime at a distance $r \sim l$ from the horizon, where l is the AdS_3 radius. We look at arbitrary partial waves for the scalar, fermion and vector particles, and determine the greybody factors for each.

In all our microscopic decay rate calculations, we replace the entire near horizon geometry of black string solution by an effective 1+1-dimensional CFT which lies at a finite distance from the horizon, i.e. at $r \sim l \sim \sqrt{r_1 r_5}$. Here l is a measure of the size of the near horizon geometry, and r_1, r_5 are related to the charges of the black hole. A quantum mechanical calculation of the emission rate is done where a plane wave excites the operators of the CFT. The correlators of the CFT operators are determined by the AdS/CFT correspondence according to the prescription as described in Chapter 1 1.12 [24, 29, 68, 69, 70, 71]. The partial wave components of the *plane wave* incident from spatial infinity couple to the CFT operators and excite the CFT. The microscopic calculation using the correlators with their proper normalisation constants reproduces the emission rates *exactly*.

6.1 Five Dimensional Black Holes and Their Near Horizon Geometry

The black hole solutions of string theory that we will consider arise from the low energy effective action of Type IIB string theory in 10-dimensions, by compactifying on $T^4 \times S^1$. The full 10-dimensional metric is given by [72, 73]

$$\begin{aligned} ds^2 &= f_1^{-1/2}\, f_5^{-1/2}\, [-dt^2 + (dx^5)^2 \\ &+ \frac{r_0^2}{r^2}\, (\cosh\sigma dt + \sinh\sigma dx^5)^2 + f_1\, dx_i dx^i] \\ &+ f_1^{1/2}\, f_5^{1/2} \left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2\right] , \end{aligned} \tag{6.1}$$

where x^5 is along S^1 and x^i , $i = 6, .., 9$ are the coordinates on the T^4. The functions f_1 and f_5 are given by:

$$f_1 = 1 + \frac{r_1^2}{r^2} \quad , \quad f_5 = 1 + \frac{r_5^2}{r^2} \ .$$

The resultant black hole metric in 5-dimensions after Kaluza-Klein reduction has six parameters, r_1, r_5, r_0, σ, V [volume of the T^4] and R [length of the S^1]. In the case of the black hole obtained by wrapping Q_5 D-5 branes, Q_1 D-1 branes with momenta n along the 1-D brane the three charges of the black hole viz. Q_1, Q_5, n can be re-expressed as:

$$r_1^2 = \frac{gQ_1}{V} , \quad r_5^2 = gQ_5 , \quad \frac{r_0^2 \sinh 2\sigma}{2} = \frac{g^2 n}{R^2 V} .$$

The black hole in five dimensions obtained by wrapping the D branes, is the same as described in Chapter3. The black hole horizon is at r_0. The non-zero field strength in this background is given by:

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho} \frac{r\, r_1^2}{(r^2 + r_1^2)^2} (f_1 f_5)^{1/4}; \quad H_{abc} = \epsilon_{abc} \frac{r_5^2}{r^3} (f_1 f_5)^{-3/4} \tag{6.2}$$

Where μ, ν.. run over t, x_5, r coordinates and a, b, c denote the angular directions. In the metric (6.1) we take the near horizon limit $r \to r_0$ and in the so-called dilute gas approximation $r_1, r_5 \gg r_0, r_n$. We find that the harmonic functions $f = 1 + r_{1(5)}^2/r^2 \approx r_{1(5)}^2/r^2$. The metric can be split up into three parts,

$$ds^2 = ds^2_{BTZ} + ds^2_{S^3} + ds^2_{T^4} \tag{6.3}$$

where

$$\begin{aligned} ds^2_{BTZ} &= -\frac{\Delta^2}{l^2\rho^2}dt^2 + \frac{l^2\rho^2}{\Delta^2}d\rho^2 + \rho^2\left(d\phi - \frac{\rho_+\rho_-}{l\rho^2}dt\right)^2 \\ \Delta^2 &= (\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2) \ . \end{aligned} \tag{6.4}$$

Clearly this is same as the metric in the earlier chapter, 5.4 and thus is the metric of $(2+1)$-dimensional BTZ black hole, which is a solution of Einstein's equation in 3-dimensions with a negative cosmological constant $\Lambda = -1/l^2$ 5.1. We have made a coordinate change $r^2 = \rho^2 - \rho_-^2$ to get the above metric. The coordinate ϕ, the parameter l and the horizons of the BTZ black hole $\rho_\pm$ are related to the 5-dimensional black hole variables and parameters by the following relations:

$$\phi = x^5/l \ , \ \rho_+ = r_0 \cosh\sigma \ , \rho_- = r_0 \sinh\sigma \ , \ l^2 = r_1 r_5 \ . \tag{6.5}$$

The part $ds^2_{T^4}$ is just the metric on the 4-torus and $ds^2_{S^3}$ is the metric on the three sphere with a constant radius l. The BTZ metric includes time, the periodic x^5 direction and the radial direction of the 5-dimensional black hole.

The above decomposition forms the basis of the approach we are considering, in which all thermodynamic properties of the black hole will be attributed to the 'non-trivial' BTZ part. The relevant near horizon part of the metric thus preserves $SL(2,R) \times SL(2,R)$ symmetry which is absent in the full five dimensional geometry. In fact as shown in earlier cases, the equation of motion of the particles in the five dimensional black hole background show this symmetry near the horizon. The inclusion of the extra direction x^5 does not affect this property, since the extra dimension is a killing direction and does not change the symmetries of the equations of motion.

By compactifying the 10 D metric on T^4, the black string solution in 6D is probed. The black string is wrapped on S^1, and thus has a finite size. Black strings are one dimensional objects with a horizon aound them. This prticular black string is a solution of $\mathcal{N} = 8$ supergravity in 6-dimensions. In $D = 6$, $\mathcal{N} = 8$

supergravity theory, the spectrum consists of 40 fermions apart from 5 anti-self dual, anti-symmetric gauge fields, 16 vector fields, 25 scalar fields and 8 Rarita-Schwinger fields. Out of these, as seen in (6.2), only the anti-symmetric gauge field strength is non-vanishing apart from the metric background. There is a $SO(5,5)$ global symmtery which gets broken due to the black hole background to $SO(4)\times SO(5)$. We look at certain particles in the spectrum, namely the minimally coupled scalars and non-minimally coupled fermions and gauge fields in 6D. The scalars correspond to gravitons along the T^4 direction. The scalars satisfy ordinary Klein Gordon equation in 6D, and on compactification on $AdS_3\times S^3$, are expanded as $\phi = \sum \Phi(t,r,\phi)Y^{(L0)}$, where $Y^{(L0)}$ are spherical harmonics on S^3. The equation of motion for scalar fields for the partial wave L on S^3 satisfies the massive Klein-Gordon equation

$$[\nabla^2 - M^2]\Phi = 0 \tag{6.6}$$

in the AdS_3 spacetime with the mass μ given in terms of L as [74, 75]

$$M^2 = \frac{1}{l^2}L(L+2)$$

In the notation of [76] the fermion equation of motion is

$$\begin{aligned}\frac{i}{2}\Gamma^M D_M \chi^{a\alpha} \quad &- \quad \frac{1}{16}P^a_{\dot{a}\ M}(\gamma^{\dot{a}})^{\alpha}_{\beta}\Gamma^M\Gamma^N\psi^{\beta}_{-N} - \frac{1}{24}F^{\dot{a}}_{MNP}\Gamma^{MNP}\gamma^{\beta\alpha}_{\dot{a}}\chi^a_{\beta} \\ &+ \quad F^{\alpha\dot{a}}_{MN}\left[(\gamma^a)_{\dot{\alpha}\dot{\beta}}\Gamma^{MN}\Gamma^P\psi^{\dot{\beta}}_{+P} - \frac{1}{4}\Gamma^{MN}\chi^{\dot{a}\beta}_{-}(\gamma_a)^{\alpha}_{\beta}(\gamma_{\dot{a}})^{\dot{\alpha}}_{\dot{\beta}}\right] = 0 \end{aligned} \tag{6.7}$$

where $M, N..$ represent 6 dimensional world index, a and $\dot{a}$, $SO(5)\times SO(5)$ vector index , α, β $SO(5)\times SO(5)$ spinor index. The + and − sign denote the chirality of the fermions. In the above P are related to the kinetic term of the scalars, F^a_{MNP} is related to the three form field strength, $F^{\alpha\dot{\alpha}}_{MN}$ are related to the field strengths of the one form gauge fields. Now we study the compactification of this theory to $AdS_3\times S^3$. ¿From (6.2) it can be seen that that the non-zero background fields, the three form field strength is given in the near horizon limit by; $H^a_{\mu\nu\rho} = (1/l)\epsilon_{\mu\nu\rho}\delta^{a5}$ and $H^a_{bcd} = (1/l)\epsilon_{bcd}\delta^{a5}$ where $\mu, \nu, ...$ etc indicate the three AdS directions and $b, c, d...$ are the S^3 directions. This gives the required equation $R_{\mu\nu\rho\lambda} = -1/l^2(g_{\mu\rho}g_{\nu\lambda} - g_{\nu\rho}g_{\mu\lambda})$ for the AdS_3 directions and $R_{bcde} = 1/l^2(g_{bd}g_{ce} - g_{cd}g_{be})$ for the S^3. Next, we factorise the fermion field χ in terms of an undetermined function of the BTZ coordinates, times the harmonic functions on the three-sphere. We also work in the representation where $(\gamma^5)\chi = \chi$. In this linearised approximation, the resultant expression is:

$$\Gamma^M D_M\chi - \frac{1}{12}H_{MNP}\Gamma^{MNP}\chi = 0 \tag{6.8}$$

The expansion in harmonics of S^3 is of the form $\chi = \sum \chi^{(p,\pm 1/2)} Y^{(p,\pm 1/2)}$, and obey $\nabla\!\!\!\!/ Y^{(p,\pm 1/2)} = \pm i(p+1) Y^{(p,\pm 1/2)}$, where p is a half integer, labeling the spin representation. Plugging in this expansion in the equation of motion (6.7), and using the decomposition of 6-dimensional Γ^M matrices into 3-dimensional ones as given in the appendix, the two-component equation takes the form

$$\gamma^\mu D_\mu \chi' + \frac{1}{l}\left(\mp(p+1) - 1\right)\chi' = 0 \tag{6.9}$$

Where $\chi^{(p,\pm 1/2)} = \chi'$, Which can be written as:

$$\gamma^\mu \left(\partial_\mu + \omega_\mu + \frac{1}{l}\left[L + 1/2\right]\right) \chi' \;=\; 0 \quad . \tag{6.10}$$

where $p = L + 1/2$, and we have chosen one of the eigenvalues of the spherical harmonic (choosing the other sign gives $L + 1/2 + 2$ for the mass term). The spin connections correspond to BTZ spacetime. Note that, here L stands for the orbital angular momentum, and in the previous chapter, the calculations were done for $L = 0$. From the above it also follows that the lowest mass term in the BTZ space time is non zero and equals 1/2. This is our basic set of equations for the determination of the fermionic greybody factor. It is interesting that on plugging the three dimensional spin connections and using the relations (6.5), it can be shown that this equation is the same as that of the fermionic fluctuations in the background of the 5-dimensional black hole in the near horizon limit [21]. We confine ourselves to particles without any Kaluza-Klein momentum along the compact direction x^5. In other words, the particles belong to the s-wave sector with respect to the BTZ black hole. Inclusion of the azimuthal quantum number along x^5 will imply charged fermion emission in five dimensions.

Similar decomposition can be made for the vector equations of $D = 6, \mathcal{N} = 8$ supergravity into $AdS_3 \times S^3$. The exercise has been done in [77]. Note that this vector couples to the threeform, and hence its linearised equation of motion reduces to:

$$\nabla^M F^a_{MN} - \frac{1}{6}\epsilon_N^{\ PQRST}(\gamma_a)^\alpha_\beta F^\beta_{PQ} H^a_{RST} = 0 \tag{6.11}$$

The gauge fields when expanded in the spherical harmonics $A_\mu = \sum A_\mu^{(L,\pm 1)} Y^{(L,\pm 1)}$ reduces to:

$$\nabla^\nu \partial_{[\nu} A_{\lambda]} - \frac{1}{l}\epsilon_\lambda^{\ \nu\rho}\partial_{[\nu} A_{\rho]} = \frac{1}{l^2} L(L+2) A_\lambda \tag{6.12}$$

wher we have dropped the indices $(L, \pm 1)$. These set of equations correspond to a massive gauge field in the BTZ background, and we solve for this to get the required greybody factor.

6.2 Greybody Factors

In this section, we solve the scalar, fermion and vector equations of motion of the previous sections to find the absorption cross-sections of the black hole for these particles. Since we study particles of various spins, a Newman-Penrose formalism would have been ideal for the study of particle propagation on the BTZ background. However, this has not been developed in three dimensions, and we separately consider the various equations of motion and find their solutions in the near horizon and in the asymptotic regions.

6.2.1 Scalar Greybody Factor

The scalar greybody factor for arbitrary partial waves for the five dimensional black hole was found earlier in [78]. Here, we exploit the near-horizon (BTZ) geometry of the black holes to solve the scalar wave equations. As stated before, the massless scalar wave equation for an arbitrary partial wave L in the $5D$ background can be reduced to the massive Klein-Gordon equation in the BTZ background. This equation was solved for the massless case in [57].

From (6.6) and (6.4), we we get the massive s-wave scalar equation in BTZ background:

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\Delta^2}{l^2\rho}\frac{d\Phi}{d\rho}\right)+\frac{\omega^2 l^2\rho^2}{\Delta^2}\Phi - M^2\Phi = 0 \ . \tag{6.13}$$

Defining

$$z = \frac{\rho^2-\rho_+^2}{\rho^2-\rho_-^2}$$

and assuming $\Phi(x^\mu) \sim e^{i\omega t}\Phi(\rho)$, the equation takes the form

$$z(1-z)\frac{d^2\Phi}{dz^2}+(1-z)\frac{d\Phi}{dz}+\left[\frac{A}{z}-B-\frac{M^2}{4(1-z)}\right]\Phi = 0 \tag{6.14}$$

where

$$A = (\omega/4\pi T_H)^2,\ B = (\rho_-^2/\rho_+^2)A$$

and

$$T_H = \frac{\rho_+^2-\rho_-^2}{2\pi l^2\rho_+}$$

is the Hawking temperature of the BTZ black hole. Plugging in the ansatz

$$\Phi(z) = z^m(1-z)^n F[z] \tag{6.15}$$

we get

$$\begin{aligned} z(1-z)\frac{d^2F}{dz^2} &+ [(2m+1)-(2m+2n+1)z]\frac{dF}{dz} \\ &+ \left[\frac{m^2+A}{z} + \frac{n(n-1)-M^2/4}{1-z} - (m+n)^2 - B\right] F = 0 \end{aligned} \tag{6.16}$$

Setting the coefficients of the $1/z$ and the $1/(1-z)$ terms to zero, as required by the continuity with the solution very close to the horizon [79], the above equation reduces to the familiar hypergeometric equation

$$z(1-z)\frac{d^2F}{dz^2} + [(2m+1)-(2m+2n+1)z]\frac{dF}{dz} - [(m+n)^2+B]F = 0\ . \tag{6.17}$$

Thus, the final solution is:

$$\Phi(z) = z^m(1-z)^n F[\alpha, \beta; \gamma; z] \tag{6.18}$$

where

$$\begin{aligned} m &= -i\sqrt{A}\ ,\quad n = -\frac{L}{2} \\ \alpha &= -i(\sqrt{A}-\sqrt{B})+n\ ,\quad \beta = -i(\sqrt{A}+\sqrt{B})+n \\ \gamma &= 1-2i\sqrt{A} \end{aligned} \tag{6.19}$$

and we have substituted $M^2 = L(L+2)/l^2$. The flux of particles into the black hole can be calculated form the formula

$$\mathcal{F}_0 = \frac{2\pi}{i}\left[\frac{\Delta^2}{\rho}\Phi^*\frac{d\Phi}{d\rho} - c.c.\right] \tag{6.20}$$

which yields

$$\mathcal{F}_0 = 4\pi\omega l^2\rho_+ \tag{6.21}$$

Now to find the incoming flux at infinity, we have to solve the wave equation at very large distances from the black hole, where space time is almost flat. The corresponding wave equation is solved in the six dimensional black string background, with the metric given in Eq.(6.1), with $r \to \infty$. The solution is expanded as $\sum \Phi(r)Y^{(L0)}$, where the $Y^{(L0)}$ are the spherical harmonics on S^3. Using

$\nabla^2 Y^{(L0)} = -L(L+2)Y^{(L0)}$, where ∇^2 is the Laplacian on the S^3, the radial equation of motion follows:

$$\frac{1}{r^3}\frac{d}{dr}\left(r^3\frac{d\Phi}{dr}\right) + \left[\omega^2 - \frac{L(L+2)}{r^2}\right]\Phi = 0 \tag{6.22}$$

having the ingoing Bessel solution:

$$\Phi = \frac{1}{r}\left(AJ_{L+1}(\omega r) + BN_{L+1}(\omega r)\right) \tag{6.23}$$

The asymptotic expansions of the Bessel functions yields the following flux at infinity:

$$\mathcal{F}_\infty = 2\left[|A|^2 + |B|^2 + i(A^*B - B^*A)\right] \tag{6.24}$$

Since the far solution should smoothly go over to the near horizon (BTZ) solution, we investigate the nature of the solutions near the region $r \sim l$, till which region we assume that the AdS_3 geometry is a good approximation to the black hole spacetime. From Eq. (6.5) and the dilute gas approximation, near $r \approx l \gg r_0 \sinh\sigma$, we get $\rho \sim r$ and hence the angular parts of the wavefunctions are the same. Thus, we simply compare the radial wavefunctions. The intermediate region is obtained by setting $z \to 1$ and $r\omega << 1$ in the hypergeometric and the Bessel solutions respectively to obtain the matching condition [61, 80] :

$$A = N^{-L/2}(L+1)(L!)^2\left(\frac{2}{\omega}\right)^{L+1}\frac{\Gamma(\gamma)}{\Gamma(\gamma-\alpha)\Gamma(\gamma-\beta)}\ . \tag{6.25}$$

where $N = \rho_+^2 - \rho_-^2 = r_0^2$. The other constant B is much smaller by factor of $(N\omega^2)^L$, and hence is neglected in the subsequent calculations.

A interesting point to note is that if we solve for the scalar wavefunction in the asymptotic AdS space, the solutions are obtained as $\Phi = J_{L+1}(\omega l^2/\rho) + N_{L+1}(\omega l^2/\rho)$, and thus at $\rho = r = l$, this has the exact polynomial behaviour as the flat space wave functions as the arguments of the bessel functions both reduce to ωl. Although we do not use the scalar wave functions in asymptotically AdS_3 space to determine the greybody factor, it would be interesting to check whether the above observation has a deeper significance, since the location $r = l$ has no apparent physical significance.

The greybody factor is then evaluated using standard methods of calculation of absorption crossections by taking the ratio of the fluxes. Thus from (6.21), (6.24) and (6.25), the greybody factor is:

$$\begin{aligned}\sigma_{\text{abs}} &= \frac{4\pi}{\omega^3}(L+1)^2\frac{\mathcal{F}_0}{\mathcal{F}_\infty} \\ &= \frac{2\pi}{(L!)^4}\left(\frac{\omega}{2}\right)^{2L}\frac{N^{L+1}}{\omega}\sinh\frac{\omega}{2T_H} \\ &\times \left|\Gamma(1+L/2+i\omega/4\pi T_-)\Gamma(1+L/2+i\omega/4\pi T_+)\right|^2\end{aligned} \tag{6.26}$$

where

$$\frac{1}{T_{-,+}} \equiv \frac{1}{T_H}\left(1-\frac{\rho_-}{\rho_+}\right)$$

and we have included the plane wave normalisation factor $\frac{4\pi}{\omega^3}(L+1)^2$. We have also used the identity $|\Gamma(1-ix)|^2 = \pi x/\sinh \pi x$. The above expression for the greybody factor reduces to the area of the black hole for $L=0$, $T_- \gg T_+$, and $\omega \to 0$ [81].

6.2.2 Fermion Greybody Factor

We shall solve the equation of motion of the fermions equation (6.10) on the BTZ background, in a suitable set of coordinates. We define $\rho^2 = \rho_+^2\cosh^2\mu - \rho_-^2\sinh^2\mu$ and $x^\pm = \pm\rho_\pm t/l \mp \rho_\mp\phi$ and assume the following form of the wavefunctions:

$$\chi'_{1,2} = \frac{e^{i(k^+x^+ + k^-x^-)}}{\sqrt{\cosh\mu\sinh\mu}}\,\psi_{1,2},$$

where (1, 2) refer to the two components of the spinor. The spin connections for the BTZ-metric are:

$$\omega_{x^+} = -\frac{1}{2l}\cosh\mu\sigma^{01} \quad , \quad \omega_{x^-} = \frac{1}{2l}\sinh\mu\sigma^{21}$$

The equation of motion for ψ takes the following form:

$$\gamma^1\partial_\mu\psi + \gamma^0\frac{ilk^+}{\sinh\mu}\psi + \gamma^2\frac{ilk^-}{\cosh\mu}\psi + \left(L+\frac{1}{2}\right)\psi = 0\,. \tag{6.27}$$

Here we work in the representation, $\gamma^1 = \sigma^1, \gamma^0 = \imath\sigma^2, \gamma^2 = \sigma^3$. Then we define a new set of wavefunctions $\psi'_{1,2}$ as

$$\psi_1 + \psi_2 = \left(1-\tanh^2\mu\right)^{-1/4}\sqrt{1+\tanh\mu}\;(\psi'_1+\psi'_2) \tag{6.28}$$

$$\psi_1 - \psi_2 = \left(1-\tanh^2\mu\right)^{-1/4}\sqrt{1-\tanh\mu}\;(\psi'_1-\psi'_2) \tag{6.29}$$

whence the Dirac equation assumes the form:

$$(1-y^2)d_y\psi'_2 - i\left(\frac{k^+}{y}+k^-y\right)\psi'_2 = -[L+1+i(k^+ + k^-)]\psi'_1 \tag{6.30}$$

$$(1-y^2)d_y\psi'_1 + i\left(\frac{k^+}{y}+k^-y\right)\psi'_1 = -[L+1-i(k^+ + k^-)]\psi'_2 \tag{6.31}$$

where we have defined $y = \tanh\mu$. Next, we choose the following ansatz

$$\psi'_{1,2} = B_{1,2} z^{m_{1,2}} (1-z)^{n_{1,2}} F_{1,2}(z) \quad , \tag{6.32}$$

where $B_{1,2}$ are arbitrary constants $z = y^2$ and $F(z)$ are yet undetermined solutions. Substituting in the Dirac equations, separating the equations for ψ'_1 and ψ'_2 and demanding continuity of this solution with the solution obtained very close to the horizon, we finally obtain the following hypergeometric differential equations for $F_1(z)$ and $F_2(z)$:

$$\begin{aligned} z(1-z)\frac{d^2F_i}{dz^2} &+ [(2m_i+1/2)-(2m_i+2n_i+3/2)z]\frac{dF_i}{dz} \\ &- [m_i(m_i+1/2)+n_i(n_i+1/2)+2m_in_i - \frac{ilk_- - l^2k_-^2}{4}]F_i = 0 \end{aligned} \tag{6.33}$$

(6.34)

The constants m_i, n_i, the hypergeometric parameters $\alpha_i, \beta_i, \gamma_i$ and the integration constants B_i are tabulated below

$$\begin{aligned} m_1 &= \frac{1+ilk^+}{2} = m_2 + 1/2 \\ n_1 &= -\frac{1}{2}(L+1) = n_2 \\ \alpha_1 &= m_1 + n_1 + \frac{1}{2} + \frac{ilk_-}{2} = \alpha_2 + 1 \\ \beta_1 &= m_1 + n_1 - \frac{ilk^-}{2} = \beta_2 \\ \gamma_1 &= 2m_1 + \frac{1}{2} = \gamma_2 + 1 \\ B_2 &= -\left[\frac{\gamma-1}{\alpha-(2n+1)}\right] B_1 \end{aligned} \tag{6.35}$$

In our subsequent calculations, we shall normalise $B_1 = 1$. This solution is an exact solution for the BTZ space time and it approximates the fermionic wave function near the horizon.

The flux into the black hole can be calculated using the $\rho \to \rho_+, z \to 0$ limit of this solution. The flux of particles entering the horizon is

$$\mathcal{F}_0 = \sqrt{-g}J^\rho|_{\rho+} = \sqrt{-g}\bar{\psi}e_1^\rho\gamma^1\psi \quad . \tag{6.36}$$

Substituting the above solutions it is clear that ψ_2' dominates the flux, and the latter turns out to be

$$\mathcal{F}_0 = N\left|\frac{\gamma - 1}{\alpha - (2n+1)}\right|^2 . \tag{6.37}$$

Now, to find the incoming flux at infinity, we solve the radial Dirac equation in the 6-dimensional metric (6.1) away from the horizon, i.e. taking $r_n^2/r^2, r_0^2/r^2 \to 0$. Then the metric assumes the following form:

$$ds^2 = -\frac{1}{\sqrt{f_1 f_5}}dt^2 + \frac{1}{\sqrt{f_1 f_5}}dx_5^2 + \sqrt{f_1 f_5}\left(dr^2 + r^2 d\Omega^2\right) \tag{6.38}$$

The spin connections for this metric are:

$$w_t^{01} = -w_{x_5}^{51} = \frac{1}{4\sqrt{f_1 f_5}}\left[\frac{r_1^2}{r^3 f_1} + \frac{r_5^2}{r^3 f_5}\right], \quad w_b^{i1} = \frac{1}{2} - \frac{r}{4}\left[\frac{r_1^2}{r^3 f_1} + \frac{r_5^2}{r^3 f_5}\right]$$

where b stands for the S^3 world indices and i the S^3 tangent space index. Now, in six dimensions the wavefunction is a four component chiral spinor. We start with the appropriate equation of motion in 6D as given in (6.8). Including all the terms, the equation of motion is:

$$\left[(f_1 f_5)^{1/2}\Gamma^0\partial_0 + \Gamma^1\left(\partial_r + \frac{3}{2r} + \frac{1}{8}d_r(\ln(f_1 f_5))\right) + (f_1 f_5)^{1/2}\Gamma^5\partial_{x_5} + \Gamma^b D_b\right]\chi + g(r)\chi = 0 \tag{6.39}$$

Where $D_b = d_b + w_b$, where b denotes the S^3 directions, and $w_b^{ij}\sigma_{ij}$ is the spin connection with the i, j indices running over tangent space S^3 indices only. The function

$$g(r) = \frac{1}{12}\Gamma^{MNP}H_{MNP} = -\frac{1}{4}\left[d\ln(f_1 f_5)\right]\sqrt{\frac{f_1}{f_5}}$$

Using the decomposition of Γ matrices into SO(2,1) and SO(3) parts we can separate out the equation of the components of the 6D chiral wavefunction into two sets of two component wave functions [75]. Again we expand in terms of the spherical harmonics on S^3 as: $\chi = \sum \chi'(x_\mu)Y$, where χ' are two component wave functions. Further, $\chi' = e^{i(\omega t - m\phi)}\,(f_1 f_5)^{-1/8} r^{-3/2}\psi''(r)$ is defined. Then from (6.39), we get:

$$\left[(f_1 f_5)^{1/2}\gamma^0\partial_t + \gamma^1\partial_r + (f_1 f_5)^{1/2}\gamma^2\partial_{x_5}\right]\psi'' = \left[\frac{(L+3/2)}{r} - g(r)\right]\psi'' \tag{6.40}$$

Separating the components gives us the equations:

$$\left(d_r - (f_1 f_5)^{1/2}\imath\omega\right)\psi_1'' = \left(-\frac{L+3/2}{r} + g(r) - (f_1 f_5)^{1/2}\imath m\right)\psi_2'' \tag{6.41}$$

$$\left(d_r + (f_1 f_5)^{1/2}\imath\omega\right)\psi_2'' = \left(-\frac{L+3/2}{r} + g(r) + (f_1 f_5)^{1/2}\imath m\right)\psi_1'' \tag{6.42}$$

Defining, $\psi_1''+\psi_2'' = (f_1f_5)^{-1/4}\psi^+$ and $\psi_1''-\psi_2'' = (f_1f_5)^{1/4}\psi^-$ and with the additional approximation $g(r) = -1/4d_r \ln(f_1f_5)$, the equations reduce to:

$$\left(d_r + \frac{L+3/2}{r}\right)\psi^+ = i(f_1f_5)\omega\psi^- \tag{6.43}$$

$$\left(d_r - \frac{L+3/2}{r}\right)\psi^- = i\omega\psi^+ \tag{6.44}$$

where we have put $m = 0$. The second order differential equation has the following form for ψ_-

$$\left[d_r^2 - \frac{(L+3/2)(L+1/2)}{r^2} + \omega^2(f_1f_5)\right]\psi^- = 0 \tag{6.45}$$

We solve this equation in two regions: $r \sim l$ and $r \geq l$.

Intermediate Region

In the first region $r \sim l$, which we call the intermediate region, we take $\omega^2 f_1 f_5 \approx \omega^2(r_1^2+r_5^2)/r^2 + \omega^2 l^4/r^4$ for low energy emissions. The differential equation in terms of $x = 1/r$ has the form:

$$\left[d_x^2 + \frac{2}{x}d_x - \frac{(L+3/2)(L+1/2) - (r_1^2+r_5^2)\omega^2}{x^2} + \omega^2 l^4\right]\psi^- = 0 \tag{6.46}$$

The solution for the above differential equation is the Bessel function $x^{-1/2}Z_\nu(\omega l^2 x)$ where, $\nu = \sqrt{(L+1)^2 - (r_1^2+r_5^2)\omega^2} \approx L+1$ for low energy emissions $\omega l \ll 1$. Hence explicitly the solutions are:

$$\psi^- = \sqrt{r}\left[a_1 J_{L+1}\left(\omega l^2/r\right) + a_2 N_{L+1}\left(\omega l^2/r\right)\right] . \tag{6.47}$$

And the coupled differential equation for ψ^+ yields:

$$\psi^+ = \frac{il^2}{r^{3/2}}\left[a_1 J_L\left(\omega l^2/r\right) + a_2 N_L\left(\omega l^2/r\right)\right]$$

For $r < l$, the function $f \approx l^4/r^4$ and in the limit we are considering, i.e. $\omega l \ll 1, r \sim l$, we can do a small argument expansion of the bessel function. Hence

$$\psi_1'' + \psi_2'' \approx \frac{il}{\sqrt{r}}\left[a_1\frac{1}{L!}\left(\frac{\omega l^2}{2r}\right)^L + a_2(L-1)!\left(\frac{2r}{\omega l^2}\right)^L\right] \tag{6.48}$$

$$\psi_1'' - \psi_2'' \approx \frac{l}{\sqrt{r}}\left[a_1\frac{1}{(L+1)}!\left(\frac{\omega l^2}{2r}\right)^{L+1} + a_2 L!\left(\frac{2r}{\omega l^2}\right)^{L+1}\right] . \tag{6.49}$$

Which gives the leading order behavior of

$$\chi_{1(2)}' \sim a_2 L!\left(\frac{\omega l^2}{2}\right) r^{L-1/2} \tag{6.50}$$

For $r > l$, $f_1 f_5 \approx 1$ and hence the above wavefunctions go to:

$$\psi_1'' + \psi_2'' \approx \frac{1}{r^{3/2}}\left[a_1 \frac{1}{L!}\left(\frac{\omega l^2}{2r}\right)^L + a_2(L-1)!\left(\frac{2r}{\omega l^2}\right)^L\right] \tag{6.51}$$

$$\psi_1'' - \psi_2'' \approx \sqrt{r}\left[a_1 \frac{1}{(L+1)!}\left(\frac{\omega l^2}{2r}\right)^{L+1} + a_2 L!\left(\frac{2r}{\omega l^2}\right)^{L+1}\right]. \tag{6.52}$$

Which gives the wavefunction in the leading powers of r as:

$$\chi'_{1(2)} = a_2 L!\left(\frac{\omega l^2}{2}\right)^{L+1} r^L \tag{6.53}$$

Far region:

For $r > r_1, r_5$, we approximate $f_1 f_5 \approx 1$, and the second order differential equation for ψ^+ is:

$$d_r^2 \psi'^+ + \left[\omega^2 + \frac{(L+2)^2 - 1/4}{r^2}\right]\psi'^+ = 0 \tag{6.54}$$

This has the solution:

$$\psi'^+ = \sqrt{\omega r}\,(a'_1 J_{L+2}(\omega r) + a'_2 N_{L+2}(\omega r)) \tag{6.55}$$

Now, we can use this solution in the coupled equation (6.43) and get

$$\psi^- = \imath\sqrt{\omega r}\left[a'_1 J_{L+1} + a'_2 N_{L+1}\right].$$

We now see, how the wave functions behave and obtain matching conditions for their smooth joining. Using the expansion for Bessel functions we obtain the leading order behavior of the wavefunctions as: $r \sim l$:

$$\chi'_{1(2)} = a'_1 \frac{\sqrt{\omega}}{(L+1)!}\left(\frac{\omega}{2}\right)^{L+1} r^L \tag{6.56}$$

and For $r \to \infty$ the asymptotic expansion of the bessel functions become important and the wavefunctions go as;

$$\chi'_{1(2)} = a'_1 \frac{1}{\sqrt{2\pi r^3}} e^{-\imath\omega r} \tag{6.57}$$

The flux at infinity entering the black hole spacetime is calculated from the asymptotic expansions of the Bessel functions, which is given by

$$\mathcal{F}_\infty = \frac{|a'_1|^2}{2\pi} \tag{6.58}$$

Matching:

To compare with the near horizon wave function solved in the x^+, r, x^- coordinates, we have to use the properties of the spinor under such transformations from t, r, ϕ coordinates. This gives a rotation on the two component wavefunction by a matrix :

$$[\cosh\left(\frac{\xi}{2}\right) + i\sinh\left(\frac{\xi}{2}\right)\sigma_2]\chi, \quad \cosh\xi/2 = \sqrt{\rho_+ + \rho_-}/N^{1/4} + \sqrt{\rho_+ - \rho_-}/N^{1/4} .$$

The near horizon solution, when extrapolated to $z \to 1$ (keeping the leading term in the expansion) is

$$\chi'_{1(2)} \to \sqrt{\rho_+ - \rho_-}L!2^{1/2}N^{-L/2}G\ \rho^{L-1/2} , \tag{6.59}$$

where

$$G = \frac{\Gamma(3/2 + i\omega/2\pi T_H)}{\Gamma([L+3]/2 + i\omega/4\pi T_+)\Gamma([L+2]/2 - i\omega/4\pi T_-)} .$$

Thus comparing with the intermediate solutions and then with the far solution using equations (6.50,6.53,6.56) we get:

$$a'_1 = 2^{L+3/2}(L+1)L!^2\omega^{-L-3/2}N^{-(L/2)}G \ . \tag{6.60}$$

Substituting in F_∞ we finally get

$$\begin{aligned}
\sigma_{\text{abs}} &= \frac{\pi(L+1)(L+2)}{\omega^3}\frac{F_0}{F_\infty} \\
&= \frac{\pi(L+2)N^{L+1}}{2(L+1)(L!)^4(\rho_+ - \rho_-)}\left(\frac{\omega}{2}\right)^{2L} \\
&\times \cosh(\omega/2T_H)\,|\Gamma(L/2+1/2+i\omega/4\pi T_+)\Gamma(L/2+1+i\omega/4\pi T_-)|^2
\end{aligned} \tag{6.61}$$

where, we have used the fact that $|\Gamma(1/2+ix)|^2 = \pi/\cosh\pi x$ and we have multiplied by the appropriate plane wave normalisation [21]. The wavefunction corresponding to the S^3 spinor $Y^{p,-1/2}$ gives rise to a greybody factor with $T_+ \to T_-$ and vice-versa. Hence the total greybody factor is a sum of two terms, one due to each set of two component fermions.

6.2.3 Vector Greybody Factor

The vector equation of motion is given in (6.12). The higher partial wave in five dimensions gives a mass term for the gauge field in three dimensions. In addition, there is another set of equations as as explained in the Appendix B [77] :

$$\epsilon_\lambda^{\ \nu\rho}\partial_\nu A_\rho = -\frac{L}{l}A_\lambda \tag{6.62}$$

This is derived from the representation theory of one forms on SL(2,R) manifolds. Since the BTZ space is locally anti-de Sitter, whose covering group is SL(2,R)× SL(2,R), the equation of motion gets supplemented by the above. On substituting the above equation in 6.12, the vector equation of motion reduces to (for convenience, we set the AdS radius $l = 1$ in the rest of this section) :

$$\nabla^{\nu}\partial_{[\nu}A_{\lambda]} = L^2 A_{\lambda} \tag{6.63}$$

It is to be observed that (6.63), can now be derived from (6.62) by operating with ∇ on both sides. There is also the consistency condition :

$$A^{\nu}_{;\nu} \;=\; 0 \;. \tag{6.64}$$

We would like to solve the above equations of motion in the background of the BTZ black hole. In the coordinate system (μ, x^+, x^-) that we had adopted previously, the + and − components of (6.12) can be written as:

$$\begin{aligned} \partial^2 A_+ \;&+\; (\tanh\mu - \coth\mu)\partial_\mu A_+ + 2\coth\mu\partial_+ A_\mu \\ -2\tanh\mu\partial_{[-}A_{\mu]} \;&=\; L(L+2)A_+ \end{aligned} \tag{6.65}$$

$$\begin{aligned} \partial^2 A_- \;&-\; (\tanh\mu - \coth\mu)\partial_\mu A_- + 2\tanh\mu\partial_- A_\mu \\ -2\coth\mu\partial_{[+}A_{\mu]} \;&=\; L(L+2)A_- \end{aligned} \tag{6.66}$$

where $\partial^2 \equiv g^{\alpha\beta}\partial_\alpha\partial_\beta = \partial^\mu\partial_\mu + \partial^+\partial_+ + \partial^-\partial_-$, we have taken $\epsilon^{+\mu-} = 1$ and have used the gauge condition (6.64). Defining

$$A_{1,2} = A_+ \pm A_- \tag{6.67}$$

it is clear that the equation for A_2 gets decoupled by adding (6.66) and (6.65). To decouple the equation for A_1, we use the equations (6.62) to substitute for the A_μ terms in (6.65) and (6.66). As a result, we get the following equations for the A_1 and A_2 (These set of equations can also be derived directly from (6.63):

$$\partial^2 A_i + (\tanh\mu + \coth\mu)\partial_\mu A_i = (L^2 - 2\epsilon_i L)A_1 \quad , \tag{6.68}$$

where $i = 1, 2$, $\epsilon_1 = -1$, $\epsilon_2 = 1$. Next, we substitute the solution

$$A_i = e^{ik_+x^+ + k_-x^-}A_i(\mu) \quad ,$$

which is consistent with the isometries of the metric. Substituting in (6.68), and defining $z \equiv \tanh^2\mu$ we get:

$$z(1-z)\frac{d^2A_i}{dz^2} + (1-z)\frac{dA_i}{dz} + \left[\frac{k_+^2}{4z} - \frac{k_-^2}{4} - \frac{L^2 - 2\epsilon_i L}{4(1-z)}\right] A_i \;= 0 \;. \tag{6.69}$$

Next, we substitute the ansatz

$$A_i = e_i z^{m_i}(1-z)^{n_i} F_i(z)$$

in the above equation to obtain (e_i s are constants)

$$z(1-z)\frac{d^2 F_i}{dz^2} + [(1+2m) - z(1+2m+2n)]\frac{dF_i}{dz}$$
$$+ \left[\frac{m^2 + k_+^2/4}{z} + \frac{n(n-1) - (L^2 - 2\epsilon_i L)/4}{1-z}\right] F \tag{6.70}$$
$$- \left[(m+n)^2 + \frac{k_-^2}{4}\right] F = 0 \tag{6.71}$$

Continuity with the corresponding wave equations very close to the horizon ($z \to 0$) gives harmonic solutions in $\log z$ of the form $A_i = e_i^{in} e^{ik_1 \log z} + e_i^{out} e^{-ik_2 \log z}$. To obtain a ingoing solution, we put $e_i^{out} = 0$. To ensure that (6.71) smoothly joins with this, we determine m and n and find that the coefficients of $1/z$ and $1/(1-z)$ terms vanish. The residual part of (6.71) is simply the hypergeometric differential equation. Thus the functions $F_i(z)$ are the hypergeometric functions $F[a_i, b_i; c_i; z]$ and the complete solution for the gauge potentials can be written as

$$A_i = e_i z^{m_i}(1-z)^{n_i} F[a_i, b_i; c_i; z] \quad . \tag{6.72}$$

We can express the various parameters in terms of $k_\pm$ and L :

$$\begin{aligned}
m_i &= -i\frac{k_+}{2} \\
n_1 &= \frac{L}{2}+1 \quad , \quad n_2 = \frac{L}{2} \\
a_1 &= -\frac{i}{2}(k_+ - k_-) + \frac{L}{2} + 1 \quad , \quad b_1 = -\frac{i}{2}(k_+ + k_-) + \frac{L}{2} + 1 \\
a_2 &= a = -\frac{i}{2}(k_+ - k_-) + \frac{L}{2} \quad , \quad b_2 = b = -\frac{i}{2}(k_+ + k_-) + \frac{L}{2} \\
c_i = c &= 1 + 2m_i
\end{aligned} \tag{6.73}$$

$A_\pm$ can now be determined from the definitions (6.67) and the solution for A_μ can be constructed from the μ-component of (6.62):

$$A_\mu = \frac{1}{L\cosh\mu \sinh\mu}\partial_{[+}A_{-]} \tag{6.74}$$

The important point to note is that the two components A_i satisfy equations which are scalar equations in the BTZ background. The spin dependence of the solutions is not obvious. The constants e_1 and e_2 are not independent by virtue of the auxiliary equations (6.62) and the consistency conditions. To determine the ratio, we use the equation with $\mu = +$ in (6.62).

$$- \tanh \mu \left(\partial_\mu A_- - \partial_- A_\mu\right) = L A_+ \tag{6.75}$$

On substituting A_μ from (6.62), and going the z coordinates, the equation reduces to in terms of A_1 and A_2 as,

$$\left[2zd_z - \frac{2k_- k_2}{L} + \frac{L}{1-z}\right] A_1 = \left[2zd_z - \frac{2k_- k_2}{L} - \frac{L}{1-z}\right] A_2 \tag{6.76}$$

On substituting the solutions for A_i, the above simplifies to:

$$\begin{aligned} e_1 &\left[\frac{2abz}{c} F(a+1, b+1; c+1; z) + \left(a + b - \frac{2k_- k_2}{L}\right) F(a, b; c; z)\right] \\ = e_2(1-z) &\left[\frac{2z(a+1)(b+1)}{c} F(a+2, b+2; c+1; z) + \left(a + b + 2 - \frac{2k_- k_1}{L}\right.\right. \\ &\left.\left. - \frac{2(L+1)}{(1-z)}\right) F(a+1, b+1; c, z)\right] \end{aligned} \tag{6.77}$$

Using a series of recursion relations, we get some simplified expressions as explained in the Appendix B. The final expression is written below:

$$\begin{aligned} e_1 &\left[2bF(a, b+1; c; z) + \left(a - b - \frac{2k_- k_2}{L}\right) F(a, b; c; z)\right] \\ &= \frac{e_2}{a} \left[2(b-L)a + (a-b)L + 2k_- k_1\right] F(a, b+1; c; z) \qquad (6.78) \\ &\quad + \frac{e_2}{a} (a - b + 2k_- k_1/L)(a - L) F(a, b, c; z) \end{aligned}$$

(6.79)

From the above, the ratio of constants are now easily determined to be:

$$\frac{e_2}{e_1} = -\frac{b*}{a} \tag{6.80}$$

where $k_{1,2} \equiv [k_+ \pm k_-]/2$. Plugging in this ratio of constants into the solutions and using appropriate recursion relations, the wavefunctions can be written as

$$A_+ = \frac{e_2}{2b^*}(1-z)^{L/2} z^{ilk^+/2} \left[-LF(a, b+1; c; z) + (L - ik^+)F(a, b; c; z)\right] \tag{6.81}$$

$$A_- = -\frac{e_2}{2b^*}(1-z)^{L/2} z^{ilk^+/2} \left[LF(a, b+1; c; z) + ik^- F(a, b; c; z)\right] \tag{6.82}$$

In the above, the solution is actually the real part of the wave function determined above. The flux of the vector field at the horizon of the black hole is calculated using the energy momentum tensor for the massive vector field. Since our wave function is Re A_i, the energy momentum tensor which involves products of the fields will have the square terms proportional to $e^{2i\omega t}$ and $e^{-2i\omega t}$. Under time averaging, these terms go to zero, and hence the steady rate of particle influx is given by cross terms :

$$T_{\nu\lambda} = -\frac{1}{4}(|F|^2 + 2m^2|A|^2)g_{\nu\lambda} + F_{\nu\sigma}F^{*\ \sigma}_{\lambda} + m^2 A_\nu A^*_\lambda \tag{6.83}$$

Where m stands for the mass. For our purposes $m^2 = L(L+2)$. To determine the flux, we incorporate the red-shift factor and integrate over the horizon area to get:

$$\mathcal{F}_0 = \frac{l^2 N^2 L}{2\rho_+} k_+^2 \left|\frac{e_2}{b}\right|^2 \Omega \tag{6.84}$$

where $N \equiv \rho_+^2 - \rho_-^2$, $k^+ = \omega/(2\pi l T_H)$, and we have restored the radius of anti-desitter space. Also $\Omega = 8\pi^2$ denotes the factors which come from the angular integral. The Note that the flux vanishes for $L = 0$, since the latter is a not a dynamical mode [77].

Before determining the waveform at infinity, we solve (6.63) in the asymptotic AdS_3 metric in the coordinates (t, ρ, ϕ)as an interesting exercise, as it sheds light on the boundary behavior of the wavefunction in the BTZ geometry. The wavefunctions, $A_i = e^{i\omega t} B_i$ are solved, with the help of (6.62) as:

$$B_i(x) = \sqrt{\rho}\left[c_i J_{\nu_i}(\frac{\omega l^2}{\rho}) + d_i N_{\nu_i}(\frac{\omega l^2}{\rho})\right] \tag{6.85}$$

where J_ν and N_ν are Bessel functions of the first and second kind respectively, $\nu_1 = L - 1$, $\nu_2 = L + 1$ and c_i, d_i are arbitrary constants. Further, consistency with the equations (6.62) requires that $c_1 = -c_2 \equiv c$ and $d_1 = -d_2 \equiv d$.

To determine the wavefunction at asymptotic infinity which joins with the BTZ wavefunction, we need to look at the vector equation of motion in six dimensions. As given in Eq. (6.11), the vector equation of motion involves all the other A_a components which are scalar in the t, r, ϕ plane as well as the H_{MNP} three form field strength in six dimensions. Since we are interested in that part of GBF which is due to the three dimensional vectors, we take $N = \mu$ in eqn 6.11 and take the limit $r \to \infty$. The equation of motion reduces to:

$$\nabla^\nu F_{\mu\nu} + \nabla^a F_{a\mu} = 0 \tag{6.86}$$

Where we have kept terms of $O(1/r^2)$. In six dimensions $H_{MNP} = \epsilon_{ijkl} d_l f_5$ where ϵ_{ijkl} is the flat space epsilon tensor along the four non-compact directions x_i, which gives the second term in equation 6.11 to be order $(1/r^3)$ form (6.2) and hence can be ignored. In the gauge $\nabla^M A_M = 0$, we assume that $\nabla^\nu A_\nu = \nabla^a A_a = 0$. The main observation is that the A_a's decouple in this gauge. For the wavefunctions $A_\mu = e^{i\omega t} e^{im\phi} A'_\mu(r)/r^{3/2}$, the equation of motion for the $m = 0$ case is of the form:

$$\partial_r^2 A'_{t,\phi} + \left[\omega^2 - \frac{(L+1)^2 - 1/4}{r^2}\right] A'_{t,\phi} = 0 \tag{6.87}$$

The solutions are:

$$\begin{aligned}
A_t &= \frac{1}{r}\left[a_1 J_{L+1}(\omega r) + a_2 N_{L+1}(\omega r)\right] &\quad (6.88)\\
A_\phi &= \frac{1}{r}\left[a'_1 J_{L+1}(\omega r) + a'_2 N_{L+1}(\omega r)\right] &\quad (6.89)\\
A_r &= \frac{1}{r^3}(-\imath\omega)\int_r^\infty r'^3 A_t(r') dr' &\quad (6.90)
\end{aligned}$$

It is interesting to note that the wavefunctions determined here do not share the exact polynomial nature of the wavefunction obtained in (6.85) at $r = \rho = l$, as in the case of scalars. The reason behind this is that due to the loss of $SL(2,R) \times SL(2,R)$ symmetry, the equations (6.62) are no longer valid for the asymptotic metric. Thus the wavefunctions match with each other only in leading order in ωr. Let us find the relation between the coefficients of the solutions (6.72) and (6.90) for which, we compare the two solutions in the region $z \to 1$, and $r\omega << 1$. Using standard results for the behaviour of hypergeometric functions as $z \to 1$, we find the leading behaviour of the wave functions as [61]:

$$A_+, A_- \to \frac{e_2}{2b^*} \frac{(N)^{-L/2} L\Gamma(L)\Gamma(c)}{\Gamma(a)\Gamma(b+1)} \rho^L \tag{6.91}$$

We match the solutions with the far region wavefunctions using the relation: $A_t l = \rho_+ A_+ - \rho_- A_-$ which gives:

$$a_1 l = a'_1 = \frac{e_2}{2b*} (\rho_+ - \rho_-) N^{-L/2} \left(\frac{\omega}{2}\right)^{-(L+1)} \Gamma(L+2)\Gamma(L+1) E_1 \tag{6.92}$$

Where $E_1 = \Gamma(c)/(\Gamma(a)\Gamma(b+1))$. The other constants are negligible and hence ignored. The solutions go as $A'_t \sim \sqrt{1/2\pi\omega} e^{-i\omega r}$ at large distances. The flux, determined from equation (6.83) is:

$$\mathcal{F}_\infty = -\frac{\omega}{2\pi} l^2 |a_1|^2 \, \Omega. \tag{6.93}$$

Taking the ratio of the near horizon and asymptotic fluxes (6.84) and (6.93) and using the above relations for the ratio of the constants, we finally get the probability of absorption of the L^{th} partial wave as

$$\mathcal{P}_L = \frac{\mathcal{F}_0}{\mathcal{F}_\infty} = \frac{\pi L k^{+2} N^L \omega^{2L+1}}{l^2 \rho_+ 2^{2L} (\rho_+ - \rho_-)^2 (\Gamma(L+2)\Gamma(L+1))^2 |E_1|^2} \tag{6.94}$$

This is the general result for the partial wave L. It is clear that the evaluation of the gamma-functions will give rise to the familiar form of the greybody factor with thermal distribution functions corresponding to two incoming particles and one outgoing particle. The latter always is always associated with a Bose distribution function, as can be seen from the relation $|\Gamma(c_1)|^2 = |\Gamma(1 + \omega/2\pi T_H)|^2 = (\omega/2T_H)/\sinh(\omega/2T_H)$. However, the nature of the 'ingoing' distribution functions depend on the value of L that one considers. In particular, on substituting the values of a and b from (6.73) in $\mathcal{P}$, we find that the the gamma-functions in the numerator correspond to fermi distributions for odd-L and bose distributions for even-L. Thus, depending on the partial wave, the vector particle can be thought of arising out of the interactions of two bosons or two fermions.

The Greybody factor or the absorption coefficient of the black hole is determined by multiplying by the plane wave factor as:

$$\begin{aligned} \sigma_{\rm abs} &= \frac{2LN^{L+2}}{(L!)^4 2^{2L}} \frac{\omega^{2L}}{l^2 \rho_+ T_H (\rho_+ - \rho_-)^2} \\ &\times \frac{\sinh \omega/T_H}{\omega} |\Gamma(L/2 + i\omega/4\pi T_+)\Gamma(L/2 + 1 + i\omega/4\pi T_-)|^2 \end{aligned} \tag{6.95}$$

If we include rest of the components of the six dimensional vector, i.e. A_a, then the total GBF will involve a sum of the individual greybody factors. The greybody factors due to A_a are same as that of the scalars. Since those terms do not contain the spin dependence, we ignore them.

The decay rates are obtained from the above greybody factors by multplying with the appropriate Planck or Fermi-Dirac distributions. It has been known for long that the these decay rates can be reproduced form a microscopic calculation using CFT operators. Earlier, the dimensions of the CFT operators were guessed from the structure of the decay rates [53, 82]. However, using the AdS/CFT correspondence, the dimension as well as the exact correlators with correct normalisations can be determined using prescription given in 1.12, [24, 68, 69, 70]. In the next section, we rely on the correspondence to determine the correlators.

6.3 CFT Description

We note that the near horizon approximation of the black holes can be used at most till $r \sim l$. Though the near horizon metric will receive corrections as r approaches l, we ignore them in this region. The correlators are determined in Poincare coordinates for convenience.

The Poincare coordinates are related to the BTZ coordinates by the relations 5.8: The metric in Poincare coordinates is:

$$ds^2 = \frac{l^2}{x_0^2}\left(dx_0^2 + dw^+ dw-\right) \tag{6.96}$$

The Klein-Gordon equation on this background can be written in the following form:

$$\left[\partial_{x_0}^2 - \frac{1}{x_0}\partial_{x_0} + 4\partial_+\partial_- - \frac{L(L+2)}{x_0^2}\right]\phi = 0 \tag{6.97}$$

Substituting:

$$\phi = \int d^2 w \phi_k(x_0) e^{i\vec{k}.\vec{w}} \tag{6.98}$$

The solutions which are ingoing or regular at the black hole horizon are:

$$\phi_k(x_0) = a x_0 K_{L+1}(k x_0)\ .$$

Where, $k = 4k_+k_-$ and a is an arbitrary constant of integration. To determine the correlator corresponding to the above scalar field and look at the bahavior of the wavefunction at $r = l$, which implies $x_0 \sim r_0/l \approx 0$ in the dilute gas approximation. The boundary of the AdS field is taken at $x_0 = \epsilon$ where ϵ is infinitesimally small and set set $\phi_k(\epsilon) = 1$. The action is:

$$I = \frac{1}{2}\int d^2 w dx_0 \frac{1}{2x_0^3}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right] \tag{6.99}$$

On partially integrating, the boundary term from this action at $x_0 = \epsilon$ is:

$$I_B = \int d^2 w \frac{1}{2\epsilon}\lim_{x_0\to\epsilon}\phi d_{x_0}\phi \tag{6.100}$$

On using (6.98) in the above, and using the solutions for $\phi_k(x_0)$, the action (fourier component) consists only of the boundary term at $x_0 = \epsilon$. The fourier component thus is:

$$\lim_{x_0\to\epsilon}\epsilon^{-1}\delta(k+k')\frac{K_{L+1}(kx_0)}{K_{L+1}(k\epsilon)}d_{x_0}\frac{x_0 K_{L+1}(kx_0)}{\epsilon K_{L+1}(k\epsilon)},$$

Using the expansion for

$$\begin{aligned} K_n(kx_0) &= \frac{1}{2}\sum_{k=0}^{k=\infty}(-1)^k\frac{(n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k-n} + \\ & (-1)^{n+1}\sum_{k=0}^{k=\infty}\frac{1}{k!(n+k)!}\left(\frac{z}{2}\right)^{n+2k}[\ln z/2 \\ & -\frac{1}{2}\Psi(k+1)-\frac{1}{2}\Psi(k)\Big], \end{aligned} \quad (6.101)$$

the expression reduces to:

$$\frac{2(L+1)}{(L+1)!L!}\left(\frac{k}{2}\right)^{2L+2}\epsilon^{2L+1}\ln\frac{k\epsilon}{2}$$

The leading non-analytic term has a $\ln(k\epsilon)$ dependence. We keep the coefficient of the term with ϵ dependence as $\epsilon^{2L+1}\ln\epsilon$ and fourier transform to position space to get the correlator

$$G^s(w,w') = 2(L+1)^2\frac{1}{|\vec{w}-\vec{w'}|^{2L+2}} \quad (6.102)$$

For the fermionic correlators, we do a calculation similar to that done in [70, 83]. The boundary is taken at ϵ. The action is taken as:

$$I = \int d^2w dx_0 \frac{1}{2x_0^3}\bar{\psi}(\nabla\!\!\!/ + L + 1/2)\psi + C\int d^2w\bar{\psi}\psi \quad (6.103)$$

Where C is a constant, which gets fixed when we try to obtain exact matching. The solution of the equation of motion in the representation of γ matrices where γ_0 is diagonal, the two components of ψ are:

$$\psi_1 = \int d^2we^{i\vec{k}.\vec{w}}a_1K_L(kx_0) \quad \psi_2 = \int d^2we^{i\vec{k}.\vec{w}}\frac{i\gamma.\vec{k}}{k}K_{L+1}(kx_0) \quad (6.104)$$

On specifying one of the components at $x_0 = \epsilon$, the other component also gets related to it. Using the above solutions, and substituting in the boundary term of the action, one fourier component is read as:

$$\lim_{x_0\to\epsilon}\delta(k+k')\frac{\vec{k}.\gamma_{\alpha\beta}}{k}\frac{K_L(kx_0)}{K_{L+1}(k\epsilon)}$$

Taking the expansion for K_n as given in equation (6.101), and keeping the coefficient of the $\epsilon^{2L+1}\ln\epsilon$ term, the greens function in the fourier transformed space is read off as:

$$G^f(w,w')_{\alpha\beta} = (L+1)\frac{(\vec{w}-\vec{w'}).\gamma_{\alpha\beta}}{|\vec{w}-\vec{w'}|^{2(L+2)}} \quad (6.105)$$

Where $\alpha\beta$ stand for spinor indices. The correlator has also been determined in [83].

To find out the correlators for the CFT operators corresponding to the vectors, it is useful to employ the methods of [70, 71]. We solve the vector equations in AdS_3 space, in Poincare coordinates. The equation of motion for $A_0 = \int d^2we^{i\vec{k}.\vec{w}}A_0$ and $A_i = \int d^2we^{i\vec{k}.\vec{w}}A_i$ have the forms:

$$d^2_{x_0}A_0 - \frac{1}{x_0}d_{x_0}A_0 - \left(k^2 + \frac{L^2-1}{x_0^2}\right)A_0 = 0 \tag{6.106}$$

$$d^2_{x_0}A_\pm + \frac{1}{x_0}d_{x_0}A_\pm - \left(k^2 + \frac{L^2}{x_0^2}\right) = \frac{2}{x_0}ik_\pm A_0 \tag{6.107}$$

The equation for A_0 is easily solved as $A_0 = a_0x_0Z_L(kx_0)$, where $k^2 = 4k_+k_-$. For $k^2 > 0$, $Z_m(kx_0) = K_m(kx_0)$ (modified bessel function of second kind) and for $k^2 < 0$, this is $Z_m(kx_0) = J_m(kx_0)$. However, since we confine ourselves to Euclidean metric, we choose the former solution. The other two components are easily separated using equation (6.62). The solutions for the two components are:

$$A_\pm = a_\pm x^0 K_{L\pm1}(kx_0) \tag{6.108}$$

The use of (6.62) also leads to a relation between the constants $a_i's$, which implies, that only one can be fixed independently by boundary condition. The other arbitrary constants are related to it, and hence are determined. Thus only one component of A_i can be fixed at the boundary, and hence the classical source is actually chiral. The ratio of constants are as follows:

$$a_\pm = a_0\frac{k_\pm}{k} \tag{6.109}$$

This obviously implies that $a_+/a_- = k_+/k_-$. Also, the function A_- falls slower than A_+, and hence we specify A_- at the boundary. The other components then get related to it. The expression for the action is:

$$I = \int d^2wdx_0\frac{1}{2x_0^3}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}L(L+2)A_\mu A^\mu\right] \tag{6.110}$$

The boundary term which comes due to one partial integration is:

$$I_B = \int d^2w\frac{x_0}{2}\left[A_+F_{0-} + A_-F_{0+}\right] \tag{6.111}$$

Using the solutions obtained above, one fourier component of the action evaluated at a distance ϵ is :

$$I = \delta(k+k')\epsilon\frac{K_{L+1}(k\epsilon)}{K_{L+1}(k\epsilon)}\frac{k_-K_{L-1}(k\epsilon)}{k_+K_{L+1}(k\epsilon)}\delta(k+k') \tag{6.112}$$

On using the expansion of (6.101) we find:

$$\begin{aligned}\frac{K_{L-1}}{K_{L+1}} &= \frac{(L-2)!\left(\frac{k\epsilon}{2}\right)^{1-L}+..+\frac{(-1)^L}{(L-1)!}\left(\frac{k\epsilon}{2}\right)^{L-1}\ln(\frac{k\epsilon}{2})}{L!\left(\frac{k\epsilon}{2}\right)^{-1-L}+..+\frac{(-1)^{L+2}}{(L+1)!}\left(\frac{k\epsilon}{2}\right)^{L+1}\ln(\frac{k\epsilon}{2})} \\ &= \frac{(-1)^L\ln(k\epsilon/2)}{L!}\left[\frac{1}{(L-1)!}\left(\frac{k\epsilon}{2}\right)^{2L}-\frac{(L-2)!}{L!(L+1)!}\left(\frac{k\epsilon}{2}\right)^{2L+2}\right]\end{aligned} \qquad (6.113)$$

Thus retaining only the leading power of ϵ in the above and using that in (6.112), we get:

$$I=\epsilon^{1+2L}\ln\epsilon\frac{k_-^2k^{2L-2}}{2^{2L-2}L!(L-1)!}\delta(k+k') \qquad (6.114)$$

The fourier transform of this yields in Poincare coordinates, the correlation function as:

$$\left\langle O^vO'^v\right\rangle=2(L+1)\frac{(w^+-w'^+)^2}{|\vec{w'}-\vec{w}|^{2(L+1)+2}} \qquad (6.115)$$

We now calculate the emission rates from the microscopic theory. The quantum mechanical calculation involves modelling the entire black hole spacetime by a CFT at the boundary of the near horizon geometry $r\sim l$. This is in keeping with the AdS/CFT correspondence, as the information about the near horizon $BTZ\times S^3$ is supposed to be encoded in the boundary of the BTZ space. A plane wave is taken to be incident on the black hole , which couples with the operators of the CFT in the region $r\sim l$. The emission rate due to this excitation is calculated using the results for ordinary stimulated emission. The incident wave is regarded as classical, while the CFT operators are treated as quantum. The plane wave has to be expanded in spherical waves, to get out the partial wave components. The plane wave is expanded in terms of spherical functions as:

$$e^{ikx}=\sum_{L\geq 0}\sqrt{2\pi^2}(L+1)\frac{e^{-i\omega r}}{(r\omega)^{3/2}}e^{i\psi}Z_{l,0}(cos\theta) \qquad (6.116)$$

The spherical wave, near $r=l$ goes as $r^n\phi_0(\phi,t)$ (where n is an integer depending on L). It couples to the CFT operator as $\int d^2x\phi_0\mathcal{O}$.

To determine the dimension of ϕ_0 under conformal transformations, we look at the behaviour of the wave function at $r\sim l$. The part of the wave function which goes as r^n comes from contribution of $J_\nu(\omega r)$ which is analytic in the region we are considering. For the scalars, the wavefunction near $r\sim l$ goes as $r^L\phi_0$. As on the boundary, the theory is invariant under conformal transformations, ϕ_0 should have a definite behavior under transformations which can be scalings like $ds^2 = f^2(r)(ds'^2)$. Here the coordinates scale like $f(r)$ and the wavefunction scales like $\phi = f^L r^L \phi_0'$. Since ϕ is a scalar, ϕ_0 has to scale as f^{-L}. Which gives it a dimension of $-L$. Accordingly, the coupling $\int d^2x \phi_0 \mathcal{O}$ implies the dimension $\Delta_S = L + 2$ for the operator O. This is consistent with the correlator determined earlier.

For the fermions, the two components of the wave function do not fall off in an identical manner, and the eigenstates of γ^1 (which is the chirality matrix for two dimensions) ,$\chi_1 + \chi_2 = \chi_+$ falls slower than $\chi_1 - \chi_2 = \chi_-$ as given in equation (6.50). So for our purposes, we take $\psi_+ \approx 0$. For

$$\chi_- = (1/r)^{1/2-L} \psi_0(t,\phi)$$

and we get the fall off power of χ_- as $\lambda = L - 1/2$. Since the fermion is a scalar under transformations $r' = f(r)r$, ψ_0 has the dimension of $L - 1/2$ under this transformation, which is like a conformal transformation in the boundary metric. Hence by conformal invariance of the term $\int d^2x \phi_0 \mathcal{O}$, the dimension of O is $\Delta_F = 2 + \lambda = L + 3/2$. The operator ψ_0 is a spin 1/2 object under the group $SO(2,2)$, and hence, O is also spin-1/2, but of opposite chirality. Hence, the left and right conformal weights are determined as $h_- + h_+ = L + 3/2$ and $h_- - h_+ = 1/2$, thus

$$h_- = L + 1, h_+ = L + 1/2 \ .$$

This is the same as that appears from the correlator calculation given above.

As for the vector field, it is immediately observed, that the two separable components at the boundary, $A_{1,2} = A_t \pm A_\phi/l$, correspond to left moving and right moving sources in the boundary. The fall off in powers of r is different for the two components and the case we are considering, and as seen from earlier section, A_1 falls slower than A_2, and hence $A_2 \approx 0$. The fall off in A_1 is as follows:

$$A_1 = r^L A_0(t,\phi) \ .$$

Since under the transformation $r \to f(r)r$, A_1 transforms as a covariant vector, the dimension of A_0, $\lambda = L - 1$. Thus A_0 is a source for the CFT operator O_v with weight $\Delta_v = 2 + L - 1 = L + 1$. The left and right weights can now be determined as

$$h_- = L/2 + 1, \qquad h_+ = L/2 \ .$$

All the weights determined above are same as those predicted using group theoretic methods in [77].

We are now ready to compute the emission rate due to the plane wave-CFT coupling $\int \phi_0 O$. If due to this interaction term, the state in the CFT undergoes a transition, $|i\rangle \to |f\rangle$, then, the transition probability for this process is:

$$w_{fi} = |\phi_0|^2 |\langle f|\, O\, |i\rangle\,|^2 \delta(\epsilon_f - \epsilon_i - \omega)$$

Where ϵ_f, ϵ_i are the energies at of the initial and final states of the CFT. The above can be written as an integral over the two coordinates of the boundary, and in case the final state is not a unique state, we sum over the final states which gives:

$$T = \sum_f \int d^2x e^{i\omega t} \langle i|e^{i\epsilon_i t} O^\dagger e^{-i\epsilon_f t}|f\rangle \langle f|O|i\rangle\, |\phi_0|^2 \,.$$

If the initial state is the Poincare vacuum, then the transition probability is:

$$T = \int d^2x e^{i\omega t} \langle 0|O^\dagger(t) O(0)|0\rangle\, |\phi_0|^2 \tag{6.117}$$

Essentially we need $G(w, w')|\phi_0|^2$ to complete the calculations. For ϕ_0 we use the form of the plane wave solutions at $r \sim l$ as determined in section III. However, as these have been determined in BTZ coordinates, we use the conformal dimension of these when we use Poincare coordinates. In effect, $\phi_0^P = (2\pi T_+ w^+)^{h_+ - 1}(2\pi T_- w^-)^{h_- - 1}(Nl^2)^{h_+ + h_-}\phi_0^{BTZ}$. An additional power of $(Nl^2)^{h_+ + h_-} = (4\pi^2 T_- T_+ l^4)^{h_+ + h_-}$ enters, since in BTZ coordinates, we assume that the wave function scales like $r^{h_+ + h_-}$ at the boundary and in the poincare coordinates $x_0^{-(h_+ + h_-)}$. ($x_0 = \sqrt{N}/r$ at the boundary, and we use l to make the scalings in both the coordinates dimensionless) Using the fact that $t = (1/4\pi T_+)\ln w^+ + (1/4\pi T_-)\ln w^-$, we get the integral in the transmission coefficient to be:

$$I = \int dw^+ dw^- \frac{(w^+)^{i\omega/4\pi T_+ + h_+ - 1}(w^-)^{i\omega/4\pi T_- + h_- - 1}}{(w^+ - 1)^{2h_+}(w^- - 1)^{2h_-}}$$

where we take the initial

$$w'^{\pm} = e^{2\pi T_\pm (t \pm \phi)}$$

at the origin of the BTZ coordinates. The range of $w^\pm$ is from 0 to ∞. Changing from $w^\pm \to -w^\pm$, and using $B(x, y) = \int_0^\infty dt\; t^{x-1}/(1+t)^{x+y}$, the integral can be done:

$$I = \frac{1}{\Gamma(2h_+)\Gamma(2h_-)} e^{-\omega/2T_H} |\Gamma(h_+ + i\omega/4\pi T_+)|^2 |\Gamma(h_- + i\omega/4\pi T_-)|^2$$

The emission rate is evaluated as:

$$(2\pi l^2 T_+)^{2h_+-1}(2\pi l^2 T_-)^{2h_--1}|\phi_0|^2 CI$$

Where C is a normalisation constant, which includes the plane wave normalisation. Plugging in Inputting the correct normalisation for each of the correlators, and using the appropriate ϕ_0, the emission rates are exactly same as the semiclassical calculations. For the scalars, $\phi_0 = 1/(L+1)!(\omega/2)^{L+1}$. Using this, as well as $h_- = h_+ = L/2+1$, the relation that $4l^4\pi^2 T_+ T_- = N$,and multiplying by the appropriate factor to get the plane wave normalisation, we get the emission rate as:

$$\begin{aligned}
\Gamma^S_{\text{cft}} &= 2\pi N \frac{(N\omega^2)^L}{2^{2L}(L!)^4}\frac{exp(-\omega/2T_H)}{\omega} \\
&\times \left|\Gamma\left(L/2+1+i\omega/4\pi T_+\right)\Gamma\left(L/2+1+i\omega/4\pi T_-\right)\right|^2 \qquad (6.118)
\end{aligned}$$

A comparison with equation (6.26), shows that the semiclassical calculation has been reproduced exactly. There is an alternative derivation for the s-wave emission in [65].

For the fermions, the wave is chosen to be of a given chirality, and hence in the expression for the emission rate, $\psi_0 = \omega^{L+3/2}2^{-(L+1)}/(L+1)!$ with $h_- = L/2+1, h_+ = L/2+1/2$, the emission rate is determined as:

$$\begin{aligned}
\Gamma^F_{\text{cft}} &= \frac{\pi(L+1)^2(L+2)(2\pi l^2 T_-)^{L+1}(2\pi l^2 T_+)^L}{4(L+1)!^2\Gamma(L+1)\Gamma(L+2)}\left(\frac{\omega}{2}\right)^{2L} \\
&\times e^{-\omega/2T_H}\left|\Gamma\left(L/2+1+i\omega/4\pi T_-\right)\left(L/2+1/2+i\omega/4\pi T_+\right)\right|^2 \quad (6.119)
\end{aligned}$$

Using the expressions for the temperatures, it can be seen that the above expression exactly matches that obtained in equation (6.61) after multiplying by the Fermi-Dirac distribution, $1/(\exp(\omega/T_H)+1)$. The special case of s-waves for $T_- \gg T_+$ was obtained in [84]. The GBF for the other set of two component wave functions in six dimension can be obtained by the same procedure above, but now with h_+ and h_- interchanged.

For the vector coupling, we retain the component of the wave function which falls slower as a function of r at $r \sim l$. This couples to the operators on the boundary. Hence for the vector $\phi_0 = A_t + A_\phi = 1/(L+1)!(\omega/2)^{L+1}$. This along with $h_- = L/2+1, h_+ = L/2$, yields the emission rate as:

$$\begin{aligned}
\Gamma^V_{\rm cft} &= 2\pi \left(\frac{\omega}{2}\right)^{2L} \frac{(L+1)^3}{\Gamma^2(L+2)} \frac{(2\pi l^2 T_+)^{L+1}(2\pi l^2 T_-)^{L-1}}{\Gamma(L)\Gamma(L+2)} \\
&\times \frac{e^{-\omega/T_H}}{\omega} |\Gamma(L/2 + i\omega/4\pi T_+)\Gamma(L/2+1+i\omega/4\pi T_-)|^2 \\
&= 2\pi \left(\frac{N\omega^2}{4}\right)^L \frac{NL}{(L!)^4(\rho_+ - \rho_-)^2} \\
&\times \frac{e^{-\omega/2T_H}}{\omega} |\Gamma(L/2 + i\omega/4\pi T_+)\Gamma(L/2+1+i\omega/4\pi T_-)|^2 \quad (6.120)
\end{aligned}$$

This is same as eqn (6.95) multiplied by the Planck distribution with temperature T_H. Thus we see for each of the cases stated above the matching is exactly obtained. It is interesting to note how the various factors conspire among themselves to yield this exact matching, using the AdS/CFT correspondence.

6.4 Discussions

In this chapter, we studied the emission rate for particles for arbitrary partial waves by probing the near horizon geometry of a 5-dimensional near extremal black hole. We determined the greybody factors of scalars, spinors and vector particles by solving their respective equation of motion in the BTZ back ground and matching them with wavefunctions obtained at large distances from the black hole. For fermions, the matching was non-trivial, and we solved the equation of motion in an intermediate region; $r \sim l$. The answers obtained for the scalars and spinors reproduced the results obtained previously for the five dimensional black hole. Our calculation for non-minimally coupled vector particles is the first calculation for emission rates for the given configuration.

Next, we used the conformal field theory at the boundary to obtain the quantum mechanical spontaneous emission rates. This is in the spirit of the AdS/CFT correspondence, in which all information regarding the bulk degrees of freedom are entirely encoded in the degrees of freedom at the boundary. Indeed, we used the various 2-point functions which have been calculated from the AdS/CFT correspondence to find the decay rates, and the latter perfectly matches with the semi classical Hawking radiation rates, *for all partial waves*. The asymptotic plane waves that excite the CFT near $r \sim l$ carry non-trivial kinematical information and influence the spontaneous emission rate. Thus our calculation shows how the AdS/CFT correspondence can be successfully used to predict the emission rates from black holes.

The exact matching suggests that the thermodynamical properties of these black holes are 'holographically' encoded in the boundary CFT.

It is to be noted that the CFT is at a finite distance from the horizon, and the role of the horizon degrees of freedom are not very clear, unlike the CFT determined in [26]. Also the thermodynamics of non-extremal black holes like the Schwarzschild black hole remains unaddressed, as the near horizon $BTZ \times S^3$ geometry emerges only for near extremal black holes.

Chapter 7

Conclusions

More than twenty years of research in black hole thermodynamics has revealed many interesting features of the nature of quantum gravity. A complete picture incorporating all the facts learned is now a challenge for the future. This thesis has tried to understand some of the important results in the last few years mainly using String Theory. The calculation of black hole entropy using the canonical quantisation of gravity, is one important result which is beyond the scope of this thesis. In this concluding chapter, we try to see whether there are any features common to all of the above approaches. These features might help us in determining a final and unified theory of the black hole as a quantum system.

In the introductory chapter of this thesis, we reviewed the laws of black hole thermodynamics, and saw how the concept of entropy arises from black hole mechanics. Elementary considerations of black hole processes show that the area of event horizon can never decrease. Further, the change in angular momentum, charge, mass of a black hole are related to the area of the black hole and the surface gravity (a geometric quantity) in a manner very similar to ordinary laws of thermodynamics if we relate area to entropy and surface gravity to temperature. Moreover, the use of quantum field theory in the background of a black hole reveals that black holes radiate particles in a thermal spectrum, with temperature proportional to the surface gravity of the black hole. Thus the black hole does function as a hot body at a finite temperature, except that the temperature of stellar black holes is expected to be very small ($10^{-8}K$). Primordial black holes, which are of much smallar mass and have a higher temperature can be formed in the early universe and are yet to be detected. However, even without experimental evidence the puzzle of understanding black hole entropy and radiation using a microscopic theory, remains one of the outstanding theoretical problems.

One of the main problems which arises with the result of black hole radiation is the loss of unitarity associated with it. The nature of Hawking evaporation is against the laws of quantum theory. The loss of information due to the complete evaporation of the black hole may not happen if there exists a Planck size remnant. However, the amount of information that has to be retained in this remnant is enormous, and this theory did not provide much answers.

Inclusion of backreaction of the infalling particles on the black hole metric however changes the situation somewhat as described in Chapter 2 of the thesis. The resultant interaction of infalling and outgoing particles as described in Chapter 2, is completely unitary. A S-matrix can be written for the evolution of such individual particles in the far past to the far future. It is expected that a averaging of this leads to Hawking radiation. But this has not been derived, nor the black hole entropy recovered from the S-matrix theory. In this thesis, we have approached the effect of backreaction in the frame work of field theory. We show that even in the classical regime, the ingoing and outgoing scalar fields have a delta function type of interaction, absent in the simplest fermion bilinears. The promotion of the shift in the metric to a quantum stature, gives a very interesting exchange algebra for the ingoing and outgoing fields showing that back reaction effects need to be understood in the complete evolution of the black hole. This feature has to be investigated to its conclusion. The validity of promotion of the shift to a quantum operator has to be checked in the full framework of quantum gravity. It is to be determined how the nature of the final state of the evaporating black hole shall change on inclusion of backreaction. Though with the advent of string theoretic approach to the understanding of black hole entropy the focus has shifted, it is not clear how backreaction effects can be included in the string picture also. It is indeed an important problem to investigate. Till now string theory has helped us to understand the semi-classical nature of Hawking radiation. Inclusion of backreaction does modify the picture, and the complete theory of black holes should include these.

The string theoretic counting of black hole entropy openes a new dimension to black hole thermodynamics by giving a quantum description of entropy and radiation. The rest of the thesis follows this approach and we come across many interesting results. But as we realise in the end, the main limitation of the approach is a special preference for extremal and near extremal black holes. Motivated by this, we have investigated extremal black hole entropy in Chapter 3. As we show in that chapter, extremal black holes are also special in the framework of general relativity. In the case of charged or rotating black holes, there are usually two horizons, which coincide for the extremal black hole. These black holes have zero temperature and some semi-classical methods show that their entropy is not proportional to area, but is infact zero. In case of black holes which occur as solutions of low energy string theory, and are multiply charged, there are solutions with two horizons (non-extremal). The coincidence of the horizons gives the extremal black hole. Not all of these extremal black holes have zero entropy except for one particular case where under certain restriction of charges, the extremal black hole of General Relativity is recovered.

The microscopic counting of string states which are used to model the black holes gives a finite result for black hole entropy for all the extremal black holes. Hence there is an apparent contradiction for these special black holes. We address this issue and observe that the string theoretic counting models the special extremal black holes only in a limiting sense. The black holes which are not exactly extremal (but very close to extremality), are stable against Hawking evaporation due to vanishingly small temperatures, and satisfy the area law for entropy. However, the question which naturally arises is that which string states then model the exactly extremal ones? If there exists such a string state then it must be unique. This is indeed a problem for the future.

In the fourth chapter, we investigate the microscopic description of Hawking radiation using string theory. In all the derivations of Hawking radiation rate using the string black holes, the common feature is a radiation rate of a product of two thermal distributions, one at temperature T_L and the other at temperature T_R, such that $2/T_H = 1/T_L + 1/T_R$. This suggests an underlying quantum theory, where the excitations constitue a 1+1 dimensional gas. From simple considerations, it can be shown that in the case of very weak interaction between the left and right moving modes, the characterestic left and right temperatures are T_L and T_R. In Chapter 2 we study radiation rate for fermions as the scalars have been investigated previously. The radiation rate for $T_L \gg T_R$ has the expected structure of a product of two thermal distributions, one Fermi-Dirac and the other Bose-Einstein distribution with the characterestic temperatures. Infact, the radiation rate can be reproduced from a CFT where a right moving excitation of weight 1/2 interacts with a left moving excitation of weight 1 to give the result.

To model Hawking radiation rate using string theory, we have to embed the black hole in 11 dimensional M-theory. Since the fermion considered above does not lie in the spectrum of this theory, we have not been able to proceed here. However for another black hole in five dimensions, the microscopic structure is clearer. Here, the entire configuration of D branes is replaced by a single D string. The decay rate due to the interaction of a right moving fermionic open string mode with a left moving bosonic open string mode gives a closed string fermion in the bulk. The rate is calculated. However, the initial open string states are taken to be thermalised. This assumption is justified in the limit where the degeneracy of the open strings is very large. Since their is no heat bath in contact with the D branes, the ensemble is microcanonical. This is approximated for low energies with a canonical ensemble. The temperatures are reproduced as T_L and T_R. Our calculation gives the result upto coefficients, but the origin of thermalisation remains obscure.

Another aspect of the string theoretic description is that the microscopic picture and the black hole exist in two different coupling constant regimes. To see the D-branes, and use the perturbative string techniques, the effective coupling, $g_s Q$ has to be very small. However, a black hole solution exists when $g_s Q \gg 1$. Thus there is an explicit assumption of non-renormalisation of the theory. However, though the extremal black holes correspond to BPS states, the near extremal black holes do not (The corrections are probably small). Also, since the quantum description is in the D brane regime, where space-time is essentially flat, the nature of the quantum degrees of freedom when the black hole exists remains obscure.

In the fifth Chapter, we investigate a 2+1 dimensional black hole which is associated with a CFT. The remarkable feature which we highlight in Chapter 6 also, is the fact that this black hole appears in the near horizon limit of the string black holes. The clue about the location of the quantum degrees of freedom of the higher dimensional black hole lies in the near horizon geometry. With this motivation in mind, we study fermion emissions from the 2+1 dimensional asymptotically anti- de Sitter BTZ black hole. As the black hole is not flat asymptotically, the absorption cross-section or the grey body factor of the black hole which comes with the fermi factor cannot be calculated so easily. However, since we keep the fact that the BTZ black hole approximates the near horizon geometry in mind, we take an observer around $\rho = l$, where l is the radius of the anti de Sitter black hole. The rate detected for s-wave fermions, by such an observer is exactly same as that from a five dimensional black hole. Both the above rates are reproduceable from a CFT, with the excitations which collide to give radiation in the bulk having weights (1,1/2). The importance of the $\rho = l$ observer is not clear, as there is no special physical significance associated with her. Perhaps, the nature of the potential of the black hole is such that at $\rho = l$ the metric flattens out, simulating an asymptotically flat observer.

A more complete picture is obtained in the sixth chapter, where we take a five dimensional black hole of $\mathcal{N} = 8$ supergravity. We take scalar, fermion and vector particles which lie in the $\mathcal{N} = 8$ spectrum and study their behavior in 6 dimensions, by lifting the black hole to six dimensions and obtaining a black string solution. The equation of motion become much simplified in the near horizon geometry which is $BTZ \times S^3$. This helps greatly in solving for the wave functions. The near horizon wavefunctions then join with the asymptotic flat wavefunctions to give the complete wavefunction for the black hole. Using this, the decay rate is determined.

For a microscopic calculation, we then show that the entire near horizon geometry can be replaced by a 1+1 dimensional CFT, which lies on the boundary of this geometry. A plane wave incident from infinity couples to the operators of the CFT. To determine the correlators of the CFT operators, we appeal to the AdS/CFT correspondence. This recent conjecture relates the bulk quantities of AdS space, to those on the boundary, with a set of prescriptions. To determine the correlators of the boundary operators, we calculate the bulk field action. The bulk field action has the form: $I = \psi(x)G(x,x')\psi(x')$, where ψ is the bulk field, and $G(x,x')$ is the correlator in the boundary. x, x' represent the coordinates in the 1+1 dimensional boundary. The remarkable fact is that the correlators come out as those at finite temperature T_H with the required T_L, T_R splitting. Our microscopic calculation gives the decay rate exactly. Earlier for the D brane decay rates, the thermalisation of the CFT was imposed using arguements of high degeneracy of open string states. In this approach, the thermal correlators emerge naturally in the given framework. One point to be noted is that our work is not a test of AdS/CFT correspondence, as it uses that to define the correlators of the boundary operators. The correspondence suggests that string theory on AdS_3 is dual to a $SU(2|1,1) \times SU(2|1,1)$ Conformal Field theory. An independent derivation of the decay rates from that theory would be a real test. However, due to the identifications which result in the BTZ black hole, the boundary theory is expected to be only a sector of the above theory. All these are for future investigations.

In conclusion the work presented in this thesis, definitely points to an underlying CFT for the string black holes. The non extremal black holes, specially the Schwarzschild black holes however remain outside the scope of this work. It is expected that non-perturbative string theory will tell us how to study gravitational collapse, and address more realistic black holes. Work focussing on non-BPS [85] states in string theory will also help us in studying these. We hope to have many answers in the nera future.

The alternative approach to quantise gravity is canonical quantisation using Ashtekar variables. This theory is free of the requirements of the presence of supersymmetry and higher dimensions. The recent calculation of Schwarzschild black hole entropy using this approach eventually counts CFT degrees of freedom [33, 86]. The CFT is induced on the horizon by a Chern Simons' theory. Thus a feature underlying all descriptions of black holes is the presence of a 1+1 dimensional CFT.

This fact appears as a universal phenomena associated with black holes in a recent work by S. Carlip [34]. The diffoemorphisms which leave the black hole horizon invariant are shown to constitute a Virasoro algebra. The central charge of this algebra is determined classically. If we assume that quantum mechanics does not add corrections to the central charge, then the asymptotic density of states found using Cardy's formula gives the black hole entropy correctly. This is a universal result, irrespective of the underlying quantum theory of gravity. It seems thus that the presence of a CFT in all approches to quantise black holes is not a surprise. However, one major crucial difference remains in the fact that the string theory CFT degrees of freedom do not correspond to horizon degrees of freedom. This question definitely has to be addressed clearly. We hope to work on all these issues in the future.

Appendix A: Exchange algebra for fermions

The linear order perturbation to the tetrads is shown to satisfy the following equation:

$$(\nabla_{\Omega}^{2} - 1)h_{x^+,+\dot{+}} = T_{x^+,+\dot{+}} \quad . \tag{A.1}$$

The tetrad is perturbed to linear order $e_{m}^{\alpha\dot{\beta}} \to e_{m}^{\alpha\dot{\beta}} + h_{m}^{\alpha\dot{\beta}}$. Hence, to linear order in h, the two form,

$$\begin{aligned} R_{mn}^{\alpha\dot{\beta},\gamma\dot{\delta}} &= \partial_m \, \delta\omega_{n}^{\alpha\dot{\beta},\gamma\dot{\delta}} - \partial_n \, \delta\omega_{m}^{\alpha\dot{\beta},\gamma\dot{\delta}} + \delta\omega_{m}^{\alpha\dot{\beta},\, \alpha'\dot{\beta}'}\omega_{n,\alpha'\dot{\beta}'}^{\gamma\dot{\delta}} + \omega_{m}^{\alpha\dot{\beta},\, \alpha'\dot{\beta}'}\delta\omega_{n,\alpha'\dot{\beta}'}^{\gamma\dot{\delta}} \\ &\quad - \delta\omega_{m,\alpha'\dot{\beta}'}^{\gamma\dot{\delta}} \, \omega_{n}^{\alpha\dot{\beta},\alpha'\dot{\beta}'} - \omega_{m,\alpha'\dot{\beta}'}^{\gamma\dot{\delta}} \, \delta\omega_{n}^{\alpha\dot{\beta},\alpha'\dot{\beta}'} \, . \end{aligned} \tag{A.2}$$

Where $\delta\omega$ stands for the linearised spin connection. The change in the tetrad, is only along the longitudinal direction as near the horizon the energy momentum tensor of the matter field is dominant in these directions only.

$$\delta e_{x^+,+\dot{+}} = h_{x^+,+\dot{+}}(x^+, x^-, \Omega), \quad \delta e_{x^-,-\dot{-}} = h_{x^-,-\dot{-}}(x^+, x^-, \Omega) \tag{A.3}$$

The changes in the spin connection very near the horizon (neglecting terms proportional to x^+ and higher powers),are calculated and the self dual parts are:

$$(\delta\omega_{x^+})_{\pm}^{\pm} = \mp\frac{h_{x^+,+\dot{+}}}{\Delta^{1/2}} \tag{A.4}$$

$$(\delta\omega_{x^+})_{+}^{-} = -\frac{1}{r}\left(\partial_\theta + \frac{i}{\sin\theta}\partial_\phi\right) \, h_{x^+,+\dot{+}} \tag{A.5}$$

$$(\delta\omega_{x^+})_{-}^{+} = \frac{i}{r sin\theta}\partial_\phi \, h_{x^+,+\dot{+}} \; . \tag{A.6}$$

Similar equations are obtained for $\delta\omega_{x^-}$. The spin connections of the original metric were,

$$
\begin{aligned}
(\omega_{x^+})_{\pm}^{\pm} &= \pm\frac{1}{2x^+}\frac{4(GM)^2}{r^2} & \text{(A.7)}\\
(\omega_{x^-})_{\pm}^{\pm} &= \pm\frac{1}{2x^-}\frac{4(GM)^2}{r^2} & \text{(A.8)}\\
{\omega_{\theta\pm}}^{\mp} &= \mp\,\Delta^{1/2} & \text{(A.9)}\\
{\omega_{\phi\pm}}^{\pm} &= \mp\,\imath\cos\theta & \text{(A.10)}\\
{\omega_{\phi\pm}}^{\mp} &= \imath\sin\theta\Delta^{1/2}\,. & \text{(A.11)}
\end{aligned}
$$

Using the above the perturbed curvature can be calculated from equation A.2. The contraction with the vierbein gives $R_{m,\gamma\dot{\delta}} = e^{n,\alpha\dot{\beta}} R_{mn,\,\alpha\dot{\beta}\gamma\dot{\delta}}$. From Einstein-Cartan equation:

$$
R_m^{\alpha\dot{\beta}} - \frac{1}{2}e_m^{\alpha\dot{\beta}}R = T_m^{\alpha\dot{\beta}} \quad , \qquad \text{(A.12)}
$$

equation (A.1) follows.

Commutator to all orders in δv_0

Through all orders in the classical horizon shift δv_0, the commutator of the currents is calculated as

$$
\begin{aligned}
\left[J^{out}(u,\Omega)\,,\;J^{in}(v',\Omega')\right] &= \left[\,\bar{\psi}^{out}\psi^{out}(u,\Omega)\,,\;\bar{\psi}^{in}\psi^{in}(v',\Omega')\right]\\
&= \bar{\psi}^{out}\left\{\psi^{out}\,,\;\bar{\psi}^{in}\right\}\psi^{in} - \left\{\bar{\psi}^{out}\,,\;\bar{\psi}^{in}\right\}\psi^{out}\psi^{in}\\
&+\;\bar{\psi}^{out}\bar{\psi}^{in}\left\{\psi^{out}\,,\;\psi^{in}\right\} - \bar{\psi}^{in}\left\{\bar{\psi}^{out}\,,\;\psi^{in}\right\}\psi^{out} \qquad \text{(A.13)}
\end{aligned}
$$

Using the anticommutators given in equations (2.33), (2.34) and (2.35), and the relation of the incoming and outgoing fields,

$$
\psi^{out}(u) = \psi^{in}(v+\delta v_0)\,, \qquad \text{(A.14)}
$$

the above commutator is simplified to:

$$
\begin{aligned}
&\left[J^{out}(u,\Omega)\,,\;J^{in}(v',\Omega')\right] = 2\pi\imath\; e^{\delta v_0\,\partial_v}\delta(v-v')\delta^{(2)}(\Omega'-\Omega)\\
&\left(e^{\delta v_0\,\partial_v}\bar{\psi}^{in}(v)\psi^{in}(v') - \bar{\psi}^{in}(v')e^{\delta v_0\,\partial_v}\psi^{in}(v)\right)\\
=\;& 2\pi\imath\delta^{(2)}(\Omega-\Omega')\left\{e^{\delta v_0\,\partial_v}\left(\bar{\psi}^{in}(v)\,\delta(v-v')\right)\psi^{in}(v') - \bar{\psi}^{in}(v')e^{\delta v_0\,\partial_v}\left(\delta(v-v')\,\psi^{in}(v)\right)\right\}\\
=\;& 0\;. \qquad \text{(A.15)}
\end{aligned}
$$

Appendix B.
$AdS_3 \times S^3$ Compactification

Here we study the S^3 compactifications. For that, we need to know the d'Alembertian on the Wigner functions. The eigenvalues can be determined from group theory. For S^3, where the symmetry group is $SO(4)$, this is determined to be:

$$\nabla_y^2 Y_{(s)}^{(l_1,l_2)} = -\left[l_1(l_1+2)+l_2^2 - s(s+1)\right] Y_{(s)}^{l_1 l_2} \tag{B.16}$$

Where the SO(4) representation is given by (l_1, l_2) and the SO(3) representation by s. We consider the cases of interest:

$$\begin{aligned} \nabla_a^2 Y^{(l0)} &= l(l+2)Y^{(l0)} & \text{(B.17)}\\ \nabla_y^2 Y_a^{(l,\pm1)} &= \left[2-(l+1)^2\right] Y_a^{(l,\pm1)} \quad , \nabla^a Y_a^{(l,\pm1)} = 0, \\ \epsilon_a^{bc}\partial_b Y_c^{(l,\pm1)} &= \pm(l+1)Y_a^{(l,\pm1)}. & \text{(B.18)}\end{aligned}$$

A $SL(2,R)$ group has similar representations as $SU(2)$. Since $SO(4) \equiv SU(2) \times SU(2)$, transverse one forms on the $AdS_3(SL(2,R) \times SL(2,R))$ manifold also satisfy equation B.18.

The spinor harmonics obey:

$$\nabla_y^2 Y^{(p,\pm1/2)} = \left[\frac{3}{2}-(p+1)^2\right] Y^{(p,\pm1/2)}, \quad \not{\nabla} Y^{(l,\pm1/2)} = \pm i(p+1)Y^{(l,\pm1/2)} \tag{B.19}$$

The gamma matrix decomposition for the $SO(1,5) \to SO(1,2) \times SO(3)$ is of the form:

$$\Gamma^\mu = \gamma^\mu \times 1 \times \sigma_1, \Gamma^a = 1 \times \gamma^a \times \sigma^2. \tag{B.20}$$

. Where Γ are SO(1,5) matrices.

Using various recursion relations of HG functions, in equation (6.77), we try to derive equation (6.78) here. The LHS of (6.77) is:

$$\epsilon_1 \left[\frac{2abcz}{c} F(a+1,b+1;c+1;z) + \left(a+b-\frac{2k_-k_2}{L}\right) F(a,b;c;z)\right] \tag{B.21}$$

In this we use:

$$azF(a+1,b+1;c+1;z) = cF(a,b+1;c;z) - cF(a,b;c;z) \tag{B.22}$$

This gives LHS of equation 6.78. The RHS of (6.77)is:

$$\begin{aligned} e_2(1-z)\Bigg[\frac{2z(a+1)(b+1)}{c}F(a+2,b+2;c+1;z) + \Bigg(a+b+2-\frac{2k_-k_1}{L} \\ - \frac{2(L+1)}{(1-z)}\Bigg)F(a+1,b+1;c,z)\Bigg] \end{aligned} \tag{B.23}$$

In this we use the following recursion relations:

$$\begin{aligned} (a+1)(b+1)z(1-z)F(a+2,b+2;c+1;z) \\ = -c(c-a-1)F(a,b+1,c) \\ -c(c-(b+1)z-(a+1)z)F(a+1,b+1,c) \end{aligned} \tag{B.24}$$

$$\begin{aligned} a(1-z)F(a+1,b+1,c;z) = (c-a-b-1)F(a,b+1;c;z) \\ +(c-b-1)F(a,b;c;z) \end{aligned} \tag{B.25}$$

to get the RHS of equation (6.78)

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