STUDIES
IN
MUON CAPTURE BY COMPLEX NUCLEI

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By

V. N. SRIDHAR, M.Sc.,

MATSCIENCE

THE INSTITUTE OF MATHEMATICAL SCIENCES,

MADRAS - 600 113. (India)

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CONTENTS

P R E F A C E

This thesis embodies the research work done by the author on 'Muon Capture by Complex Nuclei' during the years 1977-1982 at the Institute of Mathematical Sciences, under the guidance of Dr.R.Parthasarathy. It is devoted to a study of the following problems in muon capture (i) Gamma-Neutrino Angular Correlations (ii) recoil nuclear polarization (iii) total capture rates and (iv) an investigation of (V+A) admixture in the muon capture Hamiltonian.

Part I of the thesis provides a brief review of the theory of muon capture process and the density matrix formalism which is employed in most of the ensuing chapters.

In Part II, a systematic and detailed investigation of gamma-neutrino angular correlations in unpolarized and polarized muon capture by ^{28}Si is carried out employing density matrix methods. The existence of another observable in muon capture (apart from average recoil polarization), namely β_2 (the $\gamma-\nu$ angular correlation coefficient) is pointed out which is nearly insensitive to nuclear models. Interesting relations among the $\gamma-\nu$ angular correlation coefficients and other observables in muon capture process have been obtained, which are independent of nuclear models and muon capture coupling constants. A reliable numerical value for $(g_P + g_T)$ is deduced by comparing β_2 with available experimental data. Meson Exchange effects have been taken into account through the time part of the axial vector current and are found to be negligible.

Part III is devoted to the study of average recoil in muon capture by ^{12}C, incorporating the effect of the excited states of ^{12}B (predominantly the 1^- level at 2.62 MeV) on $^{12}B(1^+; g.s.)$ polarization. A numerical value for $(g_P + g_T)$ is obtained by comparing with experiment, which is consistent with our value for $(g_P + g_T)$ obtained in part II for the A = 28 system. Part IV contains a discussion of total capture rates in heavy nuclei within the context of the Salam-Strathdee idea of the vanishing of Cabibbo angle at large magnetic fields. In absence of a clear indication of such large nuclear magnetic fields, our results show the importance of nucleon momentum dependent terms in the analysis of total capture rates.

In part V, we discuss (V+A) admixture in muon capture by hydrogen motivated by the left-right symmetric gauge theory of electro-weak interactions, and obtain a lower limit for the mass of right handed gauge boson.

The results of the thesis are summarised in the Introduction.

Based on this thesis, the following five papers have been published in International Journals:-

1. Gamma-Neutrino Angular Correlations in Muon Capture by ^{28}Si (with R.Parthasarathy)
Phys. Rev. <u>C18</u> (1978) 1796.

2. Gamma-Neutrino Angular Correlations in Muon Capture by ^{28}Si-II (with R.Parthasarathy)
Phys. Rev. <u>C23</u> (1981) 861.

3. Quenching of Cabibbo Angle and Total Muon Capture Rates (with R.Parthasarathy)
Can. J. Phys. <u>56</u> (1978) 1606.

4. A Note on the Induced Pseudo-Scalar Coupling Constant in $\mu^- + {}^{12}C(0^+) \longrightarrow {}^{12}B(1^+; g.s.) + \nu_\mu$ (with R.Parthasarathy)
Phys. Lett. <u>B82</u> (1979) 167.

5. Effect of Meson Exchange Corrections on allowed Muon Capture (with R.Parthasarathy)
Phys. Lett. <u>106B</u> (1981) 363.

and in Conferences:

1. (V+A) Admixture in Muon Capture by Hydrogen (with R.Parthasarathy)
Silver Jubilee Physics Symposium, BARC, Bombay (1981).

Collaboration with my guide Dr.R.Parthasarathy was necessitated by the nature of the problem and it is gratefully acknowledged. Available reprints are attached at the end of the thesis.

I deem it a proud privilege and a great honour to thank Dr.R.Parthasarathy for constant encouragement and inspiring guidance without which this work would not have been completed.

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V.N. Sridhar

(V.N. Sridhar)

INTRODUCTION

The study of muon capture by nuclei is of two fold interest. It can be used to obtain information about the strong interaction induced weak hadronic form factors in general and the induced pseudoscalar form factor (g_P) in particular. Since the strong interaction induced form factors are dependent on the momentum transfer involved, the muon capture process is a better probe than β decay to gain information about the induced form factors. This is due to the fact that the momentum transfer (q) is ~ 100 MeV/c in muon capture whereas $q \sim 0$ in β-decay. Secondly, having obtained reliable information about these form factors, the muon capture process can be used as a probe to study the nuclear structure. The six weak hadronic form factors are governed by the CVC hypothesis of Feynman and Gell-Mann [1] and the PCAC hypothesis of Gell-Mann and Levy [2]. Using CVC and the near equality of neutron and proton masses, assuming fermions to be on the mass shell, g_V, g_M, and g_S are determined to be $g_V = 0.987G$, $g_M = 3.70\ g_V$ and $g_S = 0$, at their static limits. The Goldberger-Treiman [3] relation gives $g_A = -1.25 g_V$ in agreement with the Adler-Weisberger sum rule [4] and a recent analysis of Wilkinson [5] on β-decay. The one pion pole dominance and PCAC gives $g_P \sim 7\ g_A$ for muon capture by proton. In the case of finite nuclei, there is no clear and unambiguous treatment of g_P at present; such a treatment requires the knowledge of π-nuclear coupling which in turn involves the explicit use of nuclear wavefunctions. While CVC implies $g_S = 0$, PCAC gives no information about g_T. In this thesis, g_P and g_T are treated as unknown and are determined by comparing with experiment. Further, these form factors possess an intrinsic q^2 dependence which is expected to be weak, since the momentum transfer involved here is relatively small.

Weinberg [6] has classified the six form factors as first and second class under G-parity transformation ($G = Ce^{i\pi I_2}$) and it is generally believed that the second class form factors (g_S and g_T) do not exist. In fact CVC itself rules out g_S and recent experiments in β-decay and muon capture [7] seem to rule out the existence of g_T. However, in view of the fact that g_P and g_T always occur as a linear combination in muon capture, it is worthwhile to consider this combination as unknown, to be determined by appealing to experiment.

In order to determine the above combination of g_P and g_T in a reliable manner, it is necessary to examine those observables in muon capture which are to a large extent free from nuclear wavefunction uncertainties. The various observables in muon capture process are partial and total capture rates, recoil nuclear polarization and asymmetry in the angular distribution of the recoil nucleus, gamma-neutrino angular correlation coefficients, asymmetry in the angular distribution and the longitudinal polarization of the emitted neutrons, alignment and longitudinal polarization of the recoil nucleus. Extensive studies on partial and total capture rates [8] reveal that they are very sensitive to nuclear models. The asymmetry and longitudinal polarization of emitted neutrons are very sensitive not only to the bound nuclear proton wavefunction but also to the final state interaction of the emitted neutron [9] with the residual nucleus. There is as yet no experimental determination of the asymmetry in the angular distribution of the recoil nucleus. The experimental uncertainties in alignment and longitudinal polarization [10], charged particle multiplicity [11] are rather large.

It has been shown by Devanathan, Parthasarathy and Subramanian [12] that average recoil polarizatiom is almost insensitive to nuclear wavefunction uncertainties but sensitive to g_P and hence is a suitable observable to obtain a reliable value for g_P. This observable has been measured by the Louvain-ETH-Saclay group [13] and its nuclear model insensitivity has been examined recently by Kobayashi et. al. [14], Ciechanowicz [15] and Rosenfelder [16] using different nuclear models. In this thesis we take into account the corrections to the recoil polarization due to the gamma decay of the excited states of the recoil nucleus and also consider possible meson exchange effects as a means of improving the impulse approximation procedure.

We find after examining the gamma-neutrino angular correlation coefficients in muon capture by ^{28}Si that only one of the angular correlation coefficients β_2 (see Chapter III for definition) is nearly free from nuclear wavefunction uncertainties and this observable has been measured rather precisely by the William and Mary group [17]. Thus we compare the values of $(g_P + g_T)$ obtained from these two observables and find that they are consistent although the nuclei involved are different.

This thesis is devoted to the theoretical study of the following problems in muon capture :

(1) Gamma-Neutrino angular correlations in muon capture by ^{28}Si.
(2) Effective average recoil nuclear polarization in muon capture by ^{12}C.
(3) Total capture rates in certain heavy nuclei.
(4) Analysis of $(V + A)$ admixture in muon capture by proton.
(5) Study of Generalised Meson Dominance Model for muon capture Hamiltonian.

Before summarising the main results of our study, we now proceed to review briefly earlier works in the above topics and then point out how our work is either different from or an improvement over them.

The general theory of γ-ν angular correlations in muon capture has been developed in a series of papers by Popov et. al. [18] based on the multipole expansion similar to orbital electron capture and it has been applied to ^{28}Si by Ciechanowicz [19]. By comparing with the experiment of Miller et. al. [17], he obtains $-4.9\ g_A < g_P < 1.2\ g_A$ with a claim that this value indicates a downward renormalization of g_P from the Goldberger-Treiman value, for the $A = 28$ system. While this claim is consistent with the idea of quenching in nuclear matter [20], such a large amount of quenching is quite unlikely in light nuclei such as ^{28}Si. Also, while the treatment of $\gamma - \nu$ angular correlations in Ref. [19] is essentially based on the impulse approximation approach which treats the nucleons in nucleus as free, the reason for the renormalization of g_P is the many body effect [20] (possible scattering of virtual pions by nucleons and introduction of the pion optical potential). Further, the numerical values of the angular correlation coefficients in Ref. [19] for the PCAC estimate of g_P and $g_T = 0$ are not in agreement with experiment [17], as noted by Mukhopadhyay [21]. This problem has been studied by Devanathan and Subramanian [22] using density matrix methods, who applied it to the case of muon capture by ^{16}O for which there are no experimental measurements available at present.

In this thesis, we develop a formalism to study $\gamma - \nu$ angular correlations in muon capture by spin-zero nucleus for both unpolarized and polarized muon capture using density matrix methods. The formalism developed is general and can be applied to a general cascade of the type $|J_i\, M_i\rangle \xrightarrow{\mu^-} |J_f\, M_f\rangle \xrightarrow{\gamma} |J_F\, M_F\rangle$, as long as the initial nucleus is unoriented. Detailed and exact expressions for the correlation coefficients are derived taking into account the nucleon momentum dependent terms and higher order partial waves for the outgoing neutrino. During the course of our study we obtain very interesting relations among the correlation coefficients and other observables in muon capture. They are given below:

$$-\alpha = 1 + \frac{3}{2} P_L$$

$$\beta_1 = 1 - \frac{3}{2} \frac{P_N}{P_\mu}$$

$$\beta_2 = -1 + \frac{3}{2} \frac{P_N}{P_\mu} - \frac{3}{2} P_L$$

$$\beta_1 + \beta_2 = 1 + \alpha$$

where α, β_1 and β_2 are the γ- ν angular correlation coefficients, $\vec{P}_L$, $\vec{P}_N$ are the longitudinal and average polarization of the intermediate nucleus and $\vec{P}_\mu$ is the muon polarization at the instant of capture . These relations are independent of nuclear models and muon capture coupling constants. The first relation has been derived by Devanathan and Subramanian [22] . Identifying the $^{28}Al^*(1^+$, 2202 KeV) level as the isobaric analogue of $^{28}Si(1^+$; 13.67 MeV), we have used the particle-hole wavefunctions of Donnelly and Walker [23] to evaluate the correlation coefficients in the process

$$\mu^- + {}^{28}Si(0^+) \longrightarrow {}^{28}Al^* (1^+ ;\ 2202\ KeV) + \nu_\mu$$
$$\hookrightarrow {}^{28}Al(0^+ ;\ 973\ KeV) + \gamma$$

In Table I, we summarize our values for α, β_1 and β_2 (Parthasarathy and Sridhar [24,25]) and compare them with the values obtained by Cechanowicz [19,21] and experiment [17].

TABLE I

Comparison of the values of α, β_1 and β_2 in FPA and exact calculation with experiment.

Correlation coefficient	Ciechanowicz [19,21]	FPA	Ours [24,25]	Expt. [17]
α	0.4	0.2925	0.4203	0.15 ± 0.25 0.29 ± 0.30
β_1	0.88	0.0809	0.19243	0.02 ± 0.03
β_2	0.53	1.2115	1.2278	1.12 ± 0.1

FPA means Fujii-Primakoff Approximation, wherein we neglect nucleon momentum dependent terms (MDT) and confine only to S wave neutrinos. Then the nuclear matrix elements cancel out in the expressions for α, β_1 and β_2 ; the exact calculation (Column 4) includes the effect of MDT and higher order neutrino partial waves. It is easily seen from the table that our values for α, β_1 and β_2 are in better agreement with experiment and more importantly, it is the correlation coefficient β_2 which is nearly nuclear model insensitive.

The FPA and exact values for β_2 are seen to differ by a very small amount indicating that nuclear structure effects do not play an important part in contrast to α and β_1 which are obviously sensitive to nuclear models. This coefficient β_2 has been measured rather precisely by Miller et. al. [17] as compared with large experimental uncertainties in α and β_1. Thus we have another observable in muon capture, other than the average recoil polarization, which is almost free from nuclear wavefunction uncertainties. By comparing with experiment [17], we find

$$(g_P + g_T) = (13.5 \begin{smallmatrix} +3.5 \\ -5.5 \end{smallmatrix}) \; g_A$$

a value reasonably free from nuclear wavefunction uncertainties. In Chapter III, we compare this value of $(g_P + g_T)$ with other estimates and the conclusion is that it is in good agreement with them.

We have referred to the importance of average recoil nuclear polarization as a reliable observable for determining g_P. The interest in this topic has been recently activated by a remeasurement by Possoz et. al. [13] of the $^{12}B(1^+; g.s.)$ average recoil polarization in muon capture by $^{12}C(0^+)$. They have considered the contribution from the gamma-decay of $^{12}B(1^-\,;\,2.62\ \mathrm{MeV})$ state to $^{12}B(1^+\,;\,g.s.)$ average polarization following a theoretical calculation by Ciechanowicz [15] using the generalised Helm model and conclude that $(g_P + g_T) = (7.1 \pm 2.7) g_A$, implying the validity of nucleon PCAC in nuclei and the absence of g_T. Intrigued by the large correction from the $^{12}B(1^- ; 2.62$ MeV) gamma decay to $^{12}B(1^+ ;$ g.s.) recoil polarization, especially when the capture rate to $^{12}B(1^- ; 2.62$ MeV) level is very small compared to that of $^{12}B(1^+ ,$ g.s.), we have studied this problem in detail. We have calculated the correction due to the $^{12}B(1^-)$ gamma feed by appealing to a theorem of Rose [26], which states that if a nuclear level is polarized, the state to which it decays by gamma-emission (parity conserving transition) will also be polarized, the two being related a simple Racah coefficient. We first calculate the recoil polarization of $^{12}B(1^- ; 2.62$ MeV) and then the correction to the $^{12}B(1^+ ;$ g.s.) polarization due to the gamma feed from $^{12}B(1^- ; 2.62$ MeV) level. Thus $^{12}B(1^+ ;$ g.s.) will be polarized by (i) direct muon capture denoted by $P^{\mu}_{av.}(^{12}B(1^+))$ and (ii) gamma decay of $^{12}B(1^- ; 2.62$ MeV) to $^{12}B(1^+ ;$ g.s.) denoted by $P^{\gamma}_{av.}(^{12}B(1^+))$. The resultant or effective recoil polarization of $^{12}B(1^+ ;$ g.s.) will now be a <u>statistical sum</u>

$$P^{res.}_{av.}(^{12}B(1^+)) = \left[\frac{\lambda(1^+)}{\lambda(1^+) + \lambda(1^-)}\right] P^{\mu}_{av.}(^{12}B(1^+)) + \left[\frac{\lambda(1^-)}{\lambda(1^+) + \lambda(1^-)}\right] P^{\gamma}_{av.}(^{12}B(1^+)),$$

where $\lambda(1^+)$ and $\lambda(1^-)$ are partial capture rates to $^{12}B(1^+)$ and

$^{12}B(1^-)$ level respectively. For the spin sequence involved, we show that

$$P^{\gamma}_{av.}(^{12}B(1^+)) = 0.5\, P^{\mu}_{av.}(^{12}B(1^-))$$

where $P^{\mu}_{av.}(^{12}B(1^-))$ is the average recoil polarization of $^{12}B(1^-\,;\, 2.62\text{ MeV})$ in muon capture by $^{12}C(0^+)$. We have calculated $\lambda(1^+)$, $\lambda(1^-)$, $P^{\mu}_{av.}(^{12}B(1^+))$ and $P^{\mu}_{av.}(^{12}B(1^-))$ using the particle -hole wavefunctions of Gillet and Vinh Mau [27] and Donnelly and Walker [23] . We find a small correction to $P^{\mu}_{av.}(^{12}B(1^+))$ and with DW wavefunctions, the correction to $P^{\mu}_{av.}(^{12}B(1^+)) = 0.5792$ is 0.0247 at $g_P = 7.5\, g_A$. Comparing with the Louvain-ETH-Saclay experiment, we find 28

$$(g_P + g_T) = (13.3 \pm 1.8)\, g_A$$

in good agreement with our determination of $(g_P + g_T)$ from the γ- ν angular correlation coefficient β_2 in ^{28}Si.

This problem has been studied recently by many authors. In particular, the calculation of the Ciechanowicz [15] which is based on the generalised Helm model has been criticised by Kobayashi et. al. [14] and Truttman [29] on the grounds that the use of Helm model for $^{12}B(1^-)$ may not give correct results due to the fact that capture rates calculated by the Helm model are not in good agreement with experiment. Further, the Helm model parameters are taken from inelastic scattering data which are not well known for the $^{12}B(1^-)$ level. This is reflected in the capture rate calculation of Devanathan and Subramanian [30] . Kobayashi et. al. [14] have calculated the resultant average polarization using Cohen-Kurath wave functions and they obtain

$$(g_P + g_T) = (10.3 \pm 2.7)\, g_A$$

which is consistent with our value of $(g_P + g_T) / g_A$. In Table II, we present values for $\lambda(1^-)$, $P^{\mu}_{av.}(1^-)$ and compare them with recent estimates.

Table II

Partial Capture Rate $\lambda(1^-)$ in 10^3 sec^{-1} and average recoil polarization of $^{12}B(1^-)$.

	$\lambda(1^-)$	$P^{\mu}_{av.}(^{12}B(1^-))$
Ciechanowicz [15]	0.23	- 0.25
Ours [28]	0.593	0.6523
Kobayashi [14]	1.40	0.4310
Expt. [13]	0.38 ± 0.1	$0.6\ {}^{+0.1}_{-0.3}$

It is to be noted that our results are in better agreement with experiment, both for partial capture rate and recoil polarization.

We now briefly discuss the effect of meson exchange currents as a means of improving the impulse approximation approach.

The advent of soft pion theorems and current algebra techniques have given a new impetus to the study of two body meson exchange corrections (MEC) to impulse approximation approaches. The earliest evidence for MEC effects was found in the $np \longrightarrow d\gamma$ reaction where a 7% discrepancy in the rate between theory and experiment was resolved by including MEC effects and Δ-isobar, as shown by Riska and Brown [31]. In the context of weak interactions, it has been shown by Kubodera, Delorme and Rho [32] that, assuming one pion exchange (OPE) dominance of the two body exchange current, the <u>space component</u> of the single particle (IA) <u>vector current operator</u> V_μ is enhanced by MEC effects whereas the time part of the single particle (IA) <u>axial vector current</u> operator A_μ is enhanced by MEC effects. There is some evidence for such an enhancement of the time part of A_μ, in the calculation of partial capture rates in muon capture by ^{16}O [33]. We have incorporated [34] MEC effects in the Fujii-Primakoff Hamiltonian in a phenomenological way and studied its effects on β_2 and $P^{res.}_{av.}(1^+)$. We find that Meson exchange corrections affect directly the nucleon momentum dependent term $\int(\hat{\nu}\cdot\vec{p_i})$. This is the reason why the $^{16}O(0^+) \longrightarrow {}^{16}N(0^-)$ partial capture rate which is very sensitive to such relativistic terms, is in turn sensitive to MEC effects. This was first pointed out by Rood [35] in the context of the importance of relativistic terms. Our calculations show (with 50% MEC) that MEC effects are negligible, as is expected for allowed Gamow-Teller transitions which are dominated by the space part of the axial vector current . Consequently our values for $(g_P + g_T)$ remain almost unchanged. A brief discussion on our values of $(g_P + g_T)$ is now in order. Our values are to a large extent free from nuclear wavefunction uncertainties. First of all, in muon capture it is impossible to disentangle g_P and g_T in the Fujii-Primakoff Hamiltonian; they always occur in the combination $(g_P + g_T)$. This is the price paid when we perform the non-relativistic reduction. Secondly, ubiquitous nuclear physics uncertainities do not hinder us as the two observables β_2 and $P_{av.}$ are almost insensitive to nuclear models. Also, these are not plagued by final state interaction effects since the outgoing particle is just the neutrino. Thirdly, the Goldberger-Treiman estimate for $g_P (\sim 7 g_A)$ has been shown on general grounds to be the upper bound for g_P in a nucleus by Castro and Dominguez [36] . Then our results indicate that the upper bound on g_T could be $(5.8 \pm 3) g_A$. At a first glance, this could be interpreted as a prima facie argument for the existence of second class currents. However, as pointed out by Wilkinson [5] , such a conclusion could be true only in a <u>phenomenological</u> sense ; the Lorentz invariant form factor g_T can at best be a qualitative indicator of second class currents (SCC) and one has to pinpoint the relevant meson exchange which generates SCC, similar to the spirit in which the OPE diagram dominates the g_P form factor. On the experimental side, the recent measurement of the ratio $P_{av.} / P_L$ by Truttman [29] and ^{12}B alignment by Roesch et. al. [10] in $A = 12$ system seem to show the absence of g_T. In view of these recent experimental measurements, our value can be interpreted as $g_P = (13.3 \pm 3)\, g_A$, which is consistent with the recent Argonne National Laboratory measurement on β-decay and muon capture in $A = 16$ system by Galiardi et. al. [37].

We proceed now to the discussion of total capture rates. The standard prescription for the evaluation of total capture rates which is the sum of partial capture rates to all the final nuclear levels energetically possible, has been that of Primakoff [8] who used the closure approximation to sum over the final nuclear levels. The problem is then essentially reduced to the ground state (initial nuclear state) expectation value of the relevant muon capture operators. The very convenient $SU(4)$ symmetry relations of Foldy and Walecka [38] reduce the computational burden considerably and there are very many attempts in this direction (see the review of Mukhopadhyay [21]). In an interesting paper, Salam and Strathdee [39] have advanced the viewpoint that the Cabibbo angle θ_c could vanish at high magnetic fields ($\sim 10^{16}$ Gauss). It was pointed out by Suranyi and Hedinger [40] and Lee and Khanna [41] that such large magnetic fields could possibly be present in the interior of odd-proton nuclei. In fact, Hardy and Towner [42] point out that the long standing anomaly in the value of $\sin\theta_c$ in ^{35}Ar as compared with other nuclei, can be removed if one accepts the idea of the vanishing of Cabibbo angle. Following the suggestion of Salam and Strathdee [39] Watson [43] argued that the anomalously large value of the total muon capture rate in ^{93}Nb as compared with that of ^{92}Zr could be due to the vanishing of θ_c. In his analysis, hyperfine effects and nucleon momentum dependent (MDT) terms have not been considered at all. In order to make sure that the presently available large experimental capture rate in ^{93}Nb do in fact support the idea of vanishing of Cabibbo angle, one has to examine carefully other possible corrections and improvements which are unrelated to θ_c. Since an exact nuclear physics calculation for heavy nuclei is not feasible, we [44] have considered other corrections namely, hyperfine effects and nucleon MDT. We have carried out the calculation of total capture rates in ^{93}Nb, ^{235}U, ^{239}Pu, ^{92}Zr and ^{232}Th taking into account (i) hyperfine effects following Bernstein et. al. [45] (ii) nucleon MDT following Rood [35] and (iii) an improved formula of Goulard and Primakoff [46] for the evaluation of momentum independent terms. Our results indicate that while these improvements bring theory into better agreement with experiment, there is still a residual discrepancy in odd-proton nuclei which could be explained by the vanishing of θ_c. However, this cannot be considered as an unambiguous indication of the vanishing of θ_c. Recently, Suzuki [47]* has made an extensive study of total capture rates in many nuclei with improved experimental techniques and his results for ^{93}Nb do indeed show a large capture rate when compared with neighbouring nuclei. Also, Wilcke et. al. [48] show that the large capture rates in Actinide nuclei (such as the ones we are considering) can be explained to some extent on the basis of the resonance model of Kozlowski and Zglinski [49]. In view of these considerations and in absence of a clear indication of the existence of high magnetic fields in nuclei, our calculations show the importance of hyperfine effects and nucleon MDT which should be taken into account before drawing conclusions regarding the vanishing of Cabibbo angle.

We now discuss briefly some elementary particle aspects of muon capture. By now, it is an accepted fact that the $SU(2)_L \times U(1)$ model of Salam [50] and Weinberg [51] is the most successful and renormalizable model which unifies weak and electromagnetic interactions. This model reduces to the standard (V-A) theory at low energies and its prediction of neutral currents via the neutral Z boson has been confirmed by experiments. There have been numerous attempts to enlarge the gauge group and in particular the $SU(2)_L \times SU(2)_R \times U(1)$ theory [52] has received much attention. In such left-right symmetric theories there are left and right handed gauge bosons which mediate the charged and neutral current

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* We are grateful to Professor Measday for providing us the thesis of Suzuki.

(V-A) and (V+A) interactions respectively. The Higgs potential is such that parity is violated spontaneously, that is to say the parameters of the Higgs potential yield M_W^R (mass of the right handed vector boson) $\gg M_W^L$ (mass of the left handed gauge boson) so that the weak interaction is predominantly (V-A) in character at present day available energies. There have been many attempts [53] to set a lower limit on the M_W^R based on neutral current neutrino interactions and the general consensus is that $M_W^R > 300$ GeV. We have analysed [54] hyperfine muon capture rates in hydrogen, writing the muon capture Hamiltonian as a linear combination of (V-A) and (V+A) form. With available data on singlet capture rate, we obtain a lower limit on M_W^R as $M_W^R > 420$ GeV, a value not inconsistent with gauge theoretic estimates. Finally, we give a generalized meson dominance picture for the six hadronic form factors following Igarishi et. al. [55] and deduce interesting limits on g_T, the second class axial current.

We now summarise the salient features of the thesis.

(1) A convenient formalism for the description of γ- ν angular correlations in muon capture is developed employing density matrix methods and applied to the process

$$\mu^- + {}^{28}\mathrm{Si}(0^+) \longrightarrow {}^{28}\mathrm{Al}^*(1^+ ;\ 2202\ \mathrm{KeV}) + \nu_\mu$$
$$\hookrightarrow {}^{28}\mathrm{Al}(0^+ ;\ 973\ \mathrm{KeV}) + \gamma$$

both for polarized and unpolarized muons.

(2) Interesting relations among the correlation coefficients and $P_{av.}$, P_L of the recoil nucleus are derived and are shown to be independent of nuclear models and muon capture coupling constants.

(3) Closed expressions for the correlation coefficients are derived including nucleon MDT and higher order neutrino partial waves, which can be evaluated in any nuclear model.

(4) Numerical values for the γ- ν angular correlation coefficients have been computed using the particle hole wavefunctions for various values of $(g_P + g_T)$.

(5) The recoil polarization of $^{28}Al^{*}(1^{+}$; 2202 KeV) and the partial capture rate are calculated using particle-hole wavefunctions for various values of $(g_P + g_T)$.

(6) The effect of the excited states of ^{12}B, especially the 1^{-} level at 2.62 MeV on the average recoil polarization of $^{12}B(1^{+};g.s.)$ are calculated.

(7) The partial capture to and the recoil nuclear polarization of $^{12}B(1^{-})$ have been calculated and are found to be in excellent agreement with recent experimental measurements.

(8) The effect of MEC corrections on β_2 and $P_{av.}$ are studied in a phenomenological way and are found to be negligible, since the processes studied are allowed transitions.

(9) By comparing the numerical value of β_2 with experiment, a reliable value for $(g_P + g_T)$ which is to a large extent free from nuclear wavefunction uncertainities has been obtained as

$$(g_P + g_T) = (13.5 \; {}^{+\,3.5}_{-\,5.5}) \, g_A$$

in the $A = 28$ system. By comparing the corrected effective recoil nuclear polarization of $^{12}B(1^+)$ with Louvain-ETH-Saclay experiment we find

$$(g_P + g_T) = (13.3 \pm 1.8) \, g_A$$

in the $A = 12$ system, in good agreement with the value obtained in the $A = 28$ system. These values are almost free from nuclear wavefunction uncertainties and are shown to be nearly the same even after taking into account MEC effects. These values are consistent with the recent ANL measurement in the $A = 16$ system which yields $(g_P + g_T) = (10.0 \pm 2.5) \, g_A$.

(10) The importance of hyperfine effects and nucleon MDT in the analysis of total muon capture rates in odd-proton nuclei is pointed out. These have to be kept in mind when discussing evidence for the vanishing of the Cabibbo angle.

(11) The effect of (V+A) admixture, as envisaged by a class of weak interaction theories, in the muon capture Hamiltonian is

studied by analysing singlet and triplet capture rates in Hydrogen. A possible lower limit on the massof the right handed gauge bosons is deduced as $M_W^R > 420$ GeV in agreement with gauge theoretic estimates.

(12) The six hadronic form factors have been given a phenomenological Generalised Meson Dominance description and in particular, the connection between g_T and a particular decay mode of the τ-lepton has been analysed.

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PART I

CHAPTER I

GENERAL THEORY OF MUON CAPTURE

1. Introduction

The muon capture process is essentially a four-fermion weak interaction process, and the elementary reaction is given by

$$\mu^- + p \longrightarrow n + \nu_\mu \tag{1}$$

It differs from the well known β-decay process

$$n \longrightarrow p + e^- + \bar{\nu}_e \tag{2}$$

in two important respects: (i) Due to the larger rest mass of the muon ($\sim$100 MeV) as compared to the electron (0.5 MeV) the four-momentum transfer q_γ, is of the order of 100 MeV/c, whereas $q_\gamma \sim 0$ in β-decay; (ii) the effect of induced couplings due to the strong interactions (SI) of the nucleons is important in muon capture, especially the induced pseudoscalar coupling. Hence the bare (V-A) weak interaction Hamiltonian is modified by the induced couplings in muon capture, and in this chapter we discuss the construction of a Hamiltonian which on non-relativistic reduction yields the Fujii-Primakoff [1] Hamiltonian for μ^- capture. This chapter gives a brief review of the standard works available in literature [2] and is presented for the purpose of providing the general background for ensuing chapters. We also review briefly the recent developments [3] pertaining to Meson exchange corrections (MEC) to the Impulse Approximation (IA) calculations.

2. The Universal Fermi Interaction

In 1958, Feynman and Gell-Mann [4] proposed that the weak interaction Hamiltonian might be expressed simply and elegantly by means of the " current-current Interaction" according to which, the Hamiltonian is given by

$$\mathcal{H} = \frac{G}{\sqrt{2}} J_\mu J_\mu^+ \tag{3}$$

where in terms of 'bare nucleon' spinors,

$$J_\mu = \bar{\psi}_p \gamma_\mu (1+\gamma_5)\psi_n + \bar{\psi}_{\nu_e} \gamma_\mu (1+\gamma_5)\psi_e + \bar{\psi}_{\nu_\mu} \gamma_\mu (1+\gamma_5)\psi_\mu \tag{4}$$

and

$$J_\mu^+ = \bar{\psi}_n \gamma_\mu (1+\gamma_5)\psi_p + \bar{\psi}_e \gamma_\mu (1+\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu \gamma_\mu (1+\gamma_5)\gamma_{\nu_\mu} \tag{5}$$

In eqs. (4) and (5) γ_μ's are the Dirac matrices, $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ and $\psi_n, \psi_p, \psi_e, \psi_{\nu_e}, \psi_\mu$ and ψ_{ν_μ} are Dirac spinors for neutron, proton, electron, electron-neutrino, muon and muon neutrino respectively. The above (V-A) form was first suggested by Marshak and Sudarshan [5] on the basis of chiral invariance and also by Sakurai [6] on the basis of mass reversal invariance. The cross terms in eqn. (3) can be identified to represent the strangeness conserving weak processes like β-decay (without strong interaction effects),

$$\mathcal{H}_\beta = \frac{G}{\sqrt{2}} (\bar{\psi}_p \gamma_\mu (1+\gamma_5) \psi_n) (\bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_{\nu_e}) \quad (6)$$

and μ^- capture (without strong interaction effects),

$$\mathcal{H}_\mu = \frac{G}{\sqrt{2}} (\bar{\psi}_n \gamma_\mu (1+\gamma_5) \psi_p) (\bar{\psi}_{\nu_\mu} \gamma_\mu (1+\gamma_5) \psi_\mu) \quad (7)$$

To the lowest order in weak interaction, the S-matrix element for muon capture is given by

$$S = i (2\pi)^4 \delta(n+\nu-p-\mu) M \quad (8)$$

where n, ν, p and μ are the four momenta for neutron, neutrino, proton and muon respectively and M is the matrix element of the process. (From now on, the muon neutrino is referred to as the neutrino itself as this is always associated with muon in this thesis). The matrix element M can be written as

$$M = \bar{u}_\nu (1-\gamma_5) i \gamma_\mu \gamma_5 u_\mu \langle n | A_\mu | p \rangle + \bar{u}_\nu (1-\gamma_5) \gamma_\mu u_\mu \langle n | V_\mu | p \rangle \quad (9)$$

In the absence of strong interaction effects A_μ and V_μ are given by

$$A_\mu = f_A \bar{\psi}_n i \gamma_\mu \gamma_5 \psi_p$$
$$V_\mu = f_V \bar{\psi}_n \gamma_\mu \psi_p \quad (10)$$

where f_A and f_V are the unrenormalized axial vector and vector coupling constants respectively. However, nucleons cannot be considered as point like; as a consequence strong interaction effects will modify the form for A_μ and V_μ and in the process of modification, the various muon capture form factors which are dependent on the four momentum transfer of the process, are introduced. On the other hand leptons can be considered as point like particles, and the leptonic couplings in (V-A) sector are not renormalised since we treat the weak interaction to the lowest order and neglect electromagnetic effects.

The fundamental coupling constant G in eqn. (3) is found to be the same for muon capture, β-decay, muon decay and pion decay. It is this fact which is responsible for the universality of weak interactions.

3. <u>The Fujii-Primakoff Hamiltonian for Muon Capture:</u>

The polar vector matrix element must be of the form [7]

$$\langle n \mid V_\mu(o) \mid p \rangle = \langle U_n \mid O_\mu \mid U_p \rangle \tag{11}$$

where U_n and U_p are the neutron and proton spinors respectively. O_μ is an operator, to be constructed from Dirac γ-matrices and the four momenta p and n, such that $\langle U_n \mid O_\mu \mid U_p \rangle$ transforms as a polar vector under Lorentz transformations. Thus we may expect O_μ to be a linear combination of the following four-vectors,

$$p_\mu,\ n_\mu,\ \gamma_\mu,\ \sigma_{\mu\rho}\, p_\rho \ \text{ and } \ \sigma_{\mu\rho}\, n_\rho$$

where

$$\sigma_{\mu\rho} = \frac{1}{2}(\gamma_\mu \gamma_\rho - \gamma_\rho \gamma_\mu)$$

and p_μ and n_μ are the four momenta of the proton and neutron respectively. Defining

$$q_\mu = p_\mu - n_\mu \quad \text{(4-momentum transfer)}$$

and

$$P_\mu = p_\mu + n_\mu$$

O_μ can be expressed as a linear combination of

$$q_\mu,\ p_\mu,\ \gamma_\mu,\ \sigma_{\mu\rho},\ q_\rho \text{ and } \sigma_{\mu\rho} P_\rho .$$

Noting that the only available scalar is $q^2 = q_\mu q_\mu$ which is the square of four momentum transfer, the polar vector matrix element can be written as

$$\langle n | V_\mu | p \rangle = \bar{u}_n \Big\{ f_1(q^2) q_\mu + f_2(q^2) P_\mu + f_3(q^2)\gamma_\mu + f_4(q^2)\,\sigma_{\mu\rho} P_\rho + f_5(q^2)\,\sigma_{\mu\rho} q_\rho \Big\} u_p \tag{12}$$

where f_1, f_2, f_3, f_4 and f_5 are form factors which are functions of q^2. Further, by using the Dirac equation (for on mass shell fermions) and neglecting proton-neutron mass difference, it can be shown easily that among the five terms in (12) only three are linearly independent. Thus

$$\bar{u}_n \sigma_{\mu\rho} q_\rho u_p = \frac{1}{2} \bar{u}_n (4 M\gamma_\mu - 2P_\mu) u_p$$

and

$$\bar{u}_n \sigma_{\mu\rho} P_\rho u_p = - \bar{u}_n q_\mu u_p .$$

Eliminating the P_μ and $\sigma_{\mu\rho} P_\rho$ terms from (12), we obtain the general form for the polar vector matrix element as

$$\langle n | V_\mu | p \rangle = \bar{u}_n (C\gamma_\mu - i D \sigma_{\mu\rho} q_\rho + i F q_\mu) u_p \tag{13}$$

where for convenience C,D and F are introduced in place of f's. In an entirely similar way the axial vector matrix element can be written as

$$\langle n | A_\mu | p \rangle = \bar{u}_n \left\{ A\, i\gamma_\mu \gamma_5 - B\, q_\mu \gamma_5 + E\, \sigma_{\mu\rho} q_\rho \gamma_5 \right\} u_p \quad (14)$$

In eqns. (13) and (14), the D,F,B and E terms represent strong interaction effects on the bare (V-A) vector and axial vector vertex. Introducing a factor $\sqrt{2}$ from eqn. (3) the complete matrix element for muon capture following Tolhoek [8], is

$$M = \frac{1}{\sqrt{2}} \Big[(\bar{u}_\nu (1-\gamma_5) i\gamma_\mu \gamma_5 u_\mu) \Big\{ Ai\, \bar{u}_n \gamma_\mu \gamma_5 u_p - B(\bar{u}_n q_\mu \gamma_5 u_p) + E(\bar{u}_n \sigma_{\mu\rho} q_\rho \gamma_5 u_p) \Big\}$$
$$+ (\bar{u}_\nu (1-\gamma_5) \gamma_\mu u_\mu) \Big\{ C(\bar{u}_n \gamma_\mu u_p) - iD(\bar{u}_n \sigma_{\mu\rho} q_\rho u_p) + iF(\bar{u}_n q_\mu u_p) \Big\} \Big] \quad (15)$$

where A,B,C,D,E and F are the form factors which are functions of q^2, which are real if time reversal invariance holds. The following notation is introduced so that all the factors have the usual dimensions of the four-fermion coupling constant G.

$C = g_V$: Vector coupling constant

$A = g_A$: Axial vector coupling constant

$m_\mu B = g_P$ = Induced pseudoscalar coupling constant.

$2MD = g_M$ = Weak magnetism coupling constant

$2ME = g_T$ = Induced tensor coupling constant

$m_\mu F = g_S$ = Induced scalar coupling constant

where m_μ and M are the muon and nucleon masses respectively.

The non-relativistic reduction of eq.(15) consists in writing a two component wave function for the nucleons (nucleons move with non-relativistic velocities in nucleus) in analogy with the two component theory for the neutrino. In this scheme, the nucleon spinor becomes

$$u = \sqrt{\frac{E + M}{2E}} \begin{bmatrix} \chi \\ \phi \end{bmatrix}$$

with

$$\chi = - \frac{\vec{\sigma} \cdot \vec{p}}{E + M} \phi$$

where E and M are energy and mass of the nucleon and χ and ϕ are the two component Pauli spinors. As the neutrino is a massless particle we have

$$u_\nu = \frac{1}{\sqrt{2}} \begin{bmatrix} -\vec{\sigma} \cdot \vec{\nu} \, \phi_\nu \\ \phi_\nu \end{bmatrix}$$

where ϕ_ν is the 2-component wavefunction for the neutrino. As the muon is captured at rest from the atomic K-orbit (Weissenberg [24]) the muon spinor becomes

$$u_\mu = \begin{bmatrix} 0 \\ \phi_\mu \end{bmatrix}$$

where ϕ_μ is the muon wave function in the atomic K-orbit including finite nuclear size corrections. With the following convention for the Dirac γ-matrices

$$\bar{u} = u^\dagger \gamma_0$$

$$\gamma_0 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma_k = \begin{bmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{bmatrix},$$

$$k = 1,2,3.$$

We shall illustrate the technique of non-relativistic reduction of eq. (15), by choosing a typical term, viz, the A term.

Consider

$$A(\bar{u}_\nu\,(1-\gamma_5)\; i\gamma_\mu\,\gamma_5 u_\mu)\quad(\bar{u}_n\, i\gamma_\mu\,\gamma_5 u_p) \tag{16}$$

We shall evaluate the space and time components separately by putting $\mu = k$ $(k = 1,2,3)$ and $\mu = 4$ respectively in eqn. (16). The space and time components of the leptonic sector in eqn. (16) can be simplified to

$-(1-\vec{\sigma}\cdot\vec{\nu})\,\vec{\sigma}\,\phi_\nu^*\,\phi_\mu$ and $i(1-\vec{\sigma}\cdot\vec{\nu})\,\phi_\nu^*\,\phi_\mu$ respectively,

while the space and time components of the nucleonic sector can be reduced to $-\,\phi_n^*\,\vec{\sigma_i}\,\phi_p$ and $\phi_n^*\left[-\frac{i}{M}(\vec{\sigma_i}\cdot\vec{p_i}) + \frac{i}{2M}(\vec{\sigma_i}\cdot\hat{\nu})\,\nu\right]\phi_p$ respectively. So the A term is given by

$$g_A\left[\phi_\nu^*\,(1-\vec{\sigma_\ell}\cdot\hat{\nu})\phi_\mu\right]\left[\phi_n^*\left(\frac{\vec{\sigma_i}\cdot\vec{p_i}}{M} - \frac{\vec{\sigma_i}\cdot\hat{\nu}}{2M} + \vec{\sigma_\ell}\cdot\vec{\sigma_i}\right)\phi_p\right] \tag{17}$$

In eq. (17) $\vec{\sigma_\ell}$ and $\vec{\sigma_i}$ refer to lepton and nucleon Pauli spin operators respectively. Introducing isospin explicitly, eq. (17) can be written as

$$A\tau_\ell^{+}\,(1-\vec{\sigma_\ell}\cdot\hat{\nu})\;\sum_{i=1}^{A}\tau_i^{(-)}\left[\vec{\sigma_\ell}\cdot\vec{\sigma_i} - (\vec{\sigma_i}\cdot\hat{\nu})\frac{\nu}{2M} + \frac{\vec{\sigma_i}\cdot\vec{p_i}}{M}\right] \tag{18}$$

In a similar way all other terms can be reduced and the effective Hamiltonian for muon capture is derived. It is given by

$$\begin{aligned}\mathcal{H}_{eff.} = \frac{1}{\sqrt{2}}\,\tau_\ell^{+}\,\frac{(1-\vec{\sigma_\ell}\cdot\hat{\nu})}{\sqrt{2}}\;\sum_{i=1}^{A}\tau_i^{(-)}\Big[&G_V\,1_\ell\cdot 1_i + G_A\,\vec{\sigma_\ell}\cdot\vec{\sigma_i} \\ &- G_P\,(\vec{\sigma_\ell}\cdot\hat{\nu})(\vec{\sigma_i}\cdot\hat{\nu}) - \frac{g_V}{M}(\vec{\sigma_\ell}\cdot\hat{\nu})(\vec{\sigma_\ell}\cdot\vec{p_i}) \\ &- \frac{g_A}{M}(\vec{\sigma_\ell}\cdot\hat{\nu})(\vec{\sigma_i}\cdot\vec{p_i})\Big]\end{aligned} \tag{19}$$

where the <u>nucleon</u> part of (19) is to be taken between approximate <u>nuclear</u> states and the summation over i implies the use of impulse approximation. In eq. (19) the following effective coupling constants of muon capture, viz., G_V, G_A, and G_P are introduced.

$$\begin{aligned} G_V &= g_V\left(1 + \frac{\nu}{2M}\right) + g_S \\ G_A &= g_A - (g_V + g_M)\,\nu/2M \\ G_P &= (g_P + g_T - g_A - g_V - g_M)\,\nu/2M. \end{aligned} \tag{20}$$

The effective Hamiltonian derived above is known as the 'Fujii-Primakoff' Hamiltonian. The neutrino momentum which is also the momentum transfer of the process is given by

$$\nu \simeq m_\mu - (E_f - E_i) - \epsilon_\mu$$

where E_f and E_i are the final and initial energies of nuclear states and ϵ_μ is atomic binding energy. As remarked in the introduction ν is of the order of 100 MeV/c.

4. Muon Capture Form Factors.

As we have seen in the previous section, the muon capture Hamiltonian involves six form factors which are functions of the invariant four momentum transfer square (q_ν^2). It is to be noted here that the form factors in μ^- capture are evaluated at space like momentum transfer, i.e. the energy transfer to the final nucleus or nucleon ($\sim$10-15 MeV) is very much less than the three momentum transfer. This is because the neutrino carries away most of the energy (85 MeV) released in the μ^- capture reaction ($\sim$105 MeV). In this section we discuss the six form factors.

(a) Vector Coupling Constant.

The near equally between the Fermi coupling constant in β decay and μ decay suggests the universality of 4-fermion interaction. Extending this to μ-capture at zero momentum transfer, we have

$$g_V^{\mu}(0) = g_V^{\beta}(0) = 0.987\,G, \quad G = 1.02 \times 10^{-5}/M^2.$$

The q^2 dependence of g_V is not known and it is usually assumed to be nearly a constant independent of q^2. However, an estimate of the q^2 variation of $g_V(q^2)$ can be given within the framework of CVC theory of Feynman and Gell Mann [4]. As shown by Bernstein [25], it is possible to relate $g_V(q^2)$ to the elastic electron nucleon scattering form factors, $g_V^P(q^2)$ and $g_V^N(q^2)$, by the equation

$$g_V(q^2) = g_V^P(q^2) - g_V^N(q^2) \qquad (20a)$$

where P and N refer to proton and neutron respectively.

Expanding $g_V(q^2)$ in powers of q^2, it can be shown that

$$g_V(q^2) \approx 1 - \frac{0.03}{m_\pi^2}\,q^2 \qquad (20b)$$

Thus both for β-decay and μ^--capture, where the momentum transfers are $q^2 \sim m_e^2$ and $q^2 \sim m_\mu^2$ respectively, it is reasonable to ignore the q^2 dependence of the vector form factor. Also the CVC (Conserved Vector Current) theory of Feynman and Gell Mann [4] gives $g_V \sim 1$.

(b) Axial Vector Coupling Constant.

Experimental analysis of β decay 'ft' - values [9] gives the following relationship between the axial vector and vector coupling constants. The best current value for g_A, from neutron lifetime measurements, according to Wilkinson [20] is

$$g_A^{\beta}(o) = -(1.2507 \pm 0.0085)\, g_V^{\beta}(o)$$

This value has also been theoretically deduced by Adler [10] and Weisberger [11] on the basis of current algebra and low energy theorems. Studies on elastic neutrino scattering [12] show that the following q^2 dependence exists for the axial-vector form factor.

$$g_A^{\mu}(q^2) = g_A^{\mu}(o)\,(1 + q^2/m_A^2)^{-2}$$

where $m_A^2 \sim 0.71\ \mathrm{GeV}^2$. This is the so called <u>double pole</u> parametrization for the axial vector form factor. Since $q^2 \ll m_A^2$, $g_A^{\mu}(q^2) \simeq g_A^{\mu}(o)$. Further the PCAC hypothesis of Gell Mann and Levy [14] yields the Goldberger Treiman relation which at $q^2 = 0$ gives $g_A(o) = (g_\pi/M_p G)\, f_\pi$

(c) <u>CVC and the Weak Magnetism Coupling Constant.</u>

According to the Conserved Vector Current (CVC) hypothesis of Feymann and Gell-Mann [4], the weak vector current V_μ is identified as one of the components of the divergenceless isospin current of the strong interactions. More precisely, the weak charge raising and lowering vector currents (V_μ^+, V_μ^-) together with the isovector part of the non-leptonic electromagnetic current (V_μ^{el}) transform like the $I_z = +1, -1$, and 0 members of a single $I = 1$ triplet. Assuming this hypothesis, and the fact that V_μ^{el} is conserved, one expects V_μ also to be conserved (neglecting electromagnetic corrections)

$$\frac{\partial}{\partial x_\mu} V_\mu(x) = 0.$$

This is the statement of the CVC hypothesis. As a direct consequence of this, we have,

$$g_V^\mu(o) = (1)\,G \tag{21}$$

$$g_M^\mu(o) = (\mu_p - \mu_n)\, g_V^\mu(o) \tag{22}$$

$$g_S^\mu(o) = 0 \tag{23}$$

The first eqn. (21) shows that there is no renormalization of the vector form factor due to strong interaction effects, similar to the case where the electromagnetic form factor of the proton is unchanged by strong interaction effects. The next eqn. (22) shows that the coupling constant g_M is related to the electromagnetic processes via CVC. This form factor arises from the nucleon anomalous magnetic moment, by the virtual process in which the proton emits a π^+ meson which in turn is converted into a π^o meson after a $(\nu_\mu\,\mu^-)$ vertex, resulting in a final neutron. It is this mesonic cladding which gives rise to this 'weak magnetism', exactly analogous to the nucleon anamalous magnetic moment in the electromagnetic case represented by the Pauli form factor. The theoretical value of g_M coupling constant deduced by Feynman and Gell-Mann [4] is not in disagreement with experiments.

Finally, according to CVC, one of the two second class coupling constants, g_S, is identically zero. Nevertheless, as shown by Dominguez [23] using chiral symmetry breaking arguments, if $m_p \neq m_n$, then $\frac{g_S(o)}{g_V(o)} \sim 10^{-2}$, which is an order of magnitude larger than the naive expectation ($\frac{m_p - m_n}{M} \sim 10^{-3}$).

(d) PCAC and the Induced Pseudoscalar Coupling Constant.

In contrast to CVC, the axial vector current cannot be conserved, as this would lead to either the stable nature of π^- against decay [13] or an impossibly high value of g_P in β decay [2]. Gell-Mann and Levy [14] proposed the following partial conservation for A_μ

$$\frac{\partial}{\partial x_\mu} A_\mu(x) = a_\pi m_\pi^3 \phi_\pi(x),$$

where m_π is the pion mass, a_π is the pion decay constant (~ 0.94) and $\phi_\pi(x)$ is the pion field operator, obeying the Klein Gordon equation. The above PCAC hypothesis leads to the Goldberger-Trieman relation [15]

$$g_A M = g f_\pi \quad \text{(g is the pion-nucleon-nucleon coupling constant)},$$

derived on the basis of single pion pole dominance assumption, and is true within 6% . The Goldberger-Trieman (GT) relation predicts a q_ν^2 dependence for g_P in the form

$$\frac{g_P(q_\nu^2)}{g_A(q_\nu^2)} = \frac{2\,Mm_\mu}{q_\nu^2+m_\pi^2} \simeq 6.7 \text{ for } \mu^- \text{ - capture.}$$

A few remarks are in order here. Firstly, it has been recently claimed [16] that the 6% discrepancy in the Goldberger-Treiman relation can be explained by allowing for a 3% q_ν^2 variation from 0 to m_π^2 in the pion decay constant f_π , and a similar 3% variation in the π-nucleon-nucleon vertex, $K_{\pi NN}$. Secondly, the Goldberger-Treiman estimate for g_P seems to have been experimentally verified for the case of μ capture by hydrogen [17] . Thirdly, the quenching of g_P (due to meson exchanges and virtual pion scattering by other nucleons) has been established only for nuclear matter and for a finite nucleus surface effects become important and there is no clear and unambiguous understanding at present [18] . So, in our study we vary g_P/g_A over a range and study its effect on the muon capture process. We may note here that even though A_μ contains g_T term, PCAC says nothing about it.

(e) G-Parity Transformation:

G-parity transformation consists in the successive application of charge conjugation and rotation through π about I_2

axis in isospace

$$G = C\, e^{i\pi I_2} .$$

Weinberg [19] defines the currents which conserve G-parity as first class and currents violating G-parity as second class; defining the following currents

$$V_\mu^1 = g_V \gamma_\mu - i\frac{g_M}{2M}\, \sigma_{\mu\rho}\, q_\rho$$

$$V_\mu^2 = i\, \frac{g_S}{m_\mu}\, q_\mu$$

$$A_\mu^1 = i\, g_A\, \gamma_\mu \gamma_5 - \frac{g_P}{m_\mu}\, q_\mu \gamma_5$$

$$A_\mu^2 = g_T\, \sigma_{\mu\rho}\, q_\rho\, \gamma_5 , \quad V_\mu^1 + V_\mu^2 = V_\mu , \quad A_\mu^1 + A_\mu^2 = A_\mu$$

their transformation properties are given by

$$V_\mu^1 \xrightarrow{\quad G \quad} V_\mu^1$$

$$V_\mu^2 \xrightarrow{\quad G \quad} -V_\mu^2$$

$$A_\mu^1 \xrightarrow{\quad G \quad} -A_\mu^1$$

$$A_\mu^2 \xrightarrow{\quad G \quad} A_\mu^2$$

Here V_μ^2 and A_μ^2 are known as G-parity violating or second-class currents. While the CVC hypothesis tells us that the vector second class form factor $g_S(q^2)$ is zero, no such theoretical guidance is available for the axial second class form factor g_T. An excellent review of the present status of second class currents can be found in Wilkinson [20] .

(f) Induced Tensor Form Factor:

It is well known that in muon capture, g_P and g_T always occur in the combination $(g_P + g_T)$. This is due to the fact that the induced tensor term $\frac{g_T}{2M} \bar{u}_n \sigma_{\mu\rho} q_\rho \gamma_5 u_p$ reduced to the pseudoscalar form $-\frac{g_T}{2M} \bar{u}_n P_\lambda \gamma_5 u_p$ (where $P_\lambda = 2 p_\lambda - q_\lambda$) on using the free Dirac equation for on-mass-shell nucleons. Specifically, we obtain the following combination of g_P and g_T :

$$\frac{g_P}{m_\mu} q_\lambda + \frac{g_T}{2M} (2p_\lambda - q_\lambda) \quad \left[\bar{u}_n \gamma_5 u_p\right]$$

which on non-relativistic reduction yields the appropriate terms in the Fujii-Primakoff Hamiltonian. However, the above argument is true only for impulse approximation and does not hold for nucleons off the mass shell, in this case the vector and axial vector matrix elements consist of 12 bilinear covariants constructed out of the available vectors. Further details can be found in Bernstein [25] and a discussion of off-shell effects pertaining to second class currents in β-decay has been given by Kubodera, Delorme and Rho [33]. We have already noted the fact that the PCAC estimate for g_P in the elementary muon capture process seems to preclude any possibility of g_T [17] .

For the case of nuclear muon capture it has been shown in quite general terms by Castro and Dominguez [21] that the PCAC estimate for g_P is its upper bound in a nucleus. In absence of any compelling theoretical argument which forbids the existence of second class currents, and noting that the second class pseudotensor term is dependent on momentum transfer involved, there exists a possibility that the presence of g_T could be deduced in higher momentum transfer processes like muon capture and neutrino interactions, provided the nuclear physics part is either reasonably well understood or does not affect the observables concerned. In this thesis, we have studied the variation of $(g_P + g_T)$ with respect to observables which are insensitive to nuclear structure viz., the gamma-neutrino angular correlation coefficient, β_2, the average recoil nuclear polarization $P_{av.}$, and deduce a value for g_T by comparing with experiment. It is to be noted that the value of g_T so obtained is reasonably free from nuclear wavefunction uncertainties.

The following choice of numerical values of the coupling constants is made for calculations:

$$G = 1.02 \times 10^{-5}/M^2$$

$$g_V = 0.987\, G$$

$$g_M = 3.70\, g_V$$

$$g_S = 0$$

$$g_A = -1.25\, g_V$$

$$g_P^* = 7.5\, g_A$$

$$g_T^* = 0.$$

The starred quantities are varied and their effects are studied.

6. <u>Meson Exchange Corrections:</u>

In the Impulse Approximation (IA) approach, the interaction Hamiltonian responsible for the elementary process $\mu^- + p \longrightarrow n + \nu_\mu$, is taken over to the nuclear case, the summation index $\sum_{i=1}^{A}$ expressing the fact that the one-body operator is summed over A nucleons. However, the nucleus is not merely a collection of independent nucleons, but is bound together by strong interactions generated by various meson exchanges. It has been shown by Riska and Brown [22] that it is necessary to invoke meson exchange corrections in the case of $np \longrightarrow d\gamma$ electromagnetic process to remove the discrepancy between IA theory and experiment. Recently Kubodera, Delorme and Rho [3], have argued on the basis of soft pion theorems that only the time component of the two body mesonic amplitude is enhanced relative to the single particle operator. They have derived the explicit form for the two body mesonic amplitude in the non-relativistic limit, on the assumption that one-pion exchange process is dominant over other short-ranged processes such as multipion or heavier meson exchanges. In part II of the thesis, we indicate a method of incorporating the enhancement of the time part of the axial vector current in the one-body Fujii-Primakoff Hamiltonian in a phenomenological way, and evaluate its effect on the gamma neutrino angular correlation coeffecient β_2. It's effect on the average recoil nuclear polarization, $P_{av.}$, is considered in Part III of the thesis.

6. Elementary Particle Model (EPM).

To circumvent the problem of nuclear model uncertainties in the impulse approximation approach, Kim and Primakoff [26] suggested the 'Elementary Particle Model' approach to muon capture. In this approach nuclei are treated as elementary particles and the muon capture rates (and other observables) are written in terms of nuclear form factors. These form factors are determined by invoking general principles such as CVC and PCAC, and by appealing to related experiments involving weak and electromagnetic interactions. All nuclear physics complications reside in the form factors and the need for nuclear models does not arise since the form factors are determined from experiments. The matrix elements in the EPM aproach are similar in form to the vector and axial vector matrix elements in muon capture (see section 3) with the difference that the initial and final nucleus replace the nucleons. The CVC hypothesis of Feynman and Gell-Mann [4] relates the vector and weak magnetism form factors to the Dirac and Pauli charge form factor in the corresponding electromagnetic process such as inelastic electron scattering. The PCAC hypothesis of Gell-Mann and Levy [14] connects the induced pseudoscalar and axial vector coupling constants, while the axial vector coupling constant is determined from the corresponding β decay process. Calculations on allowed transitions in ^3He, ^{12}C and ^6Li have been performed by various authors [27] on the basis of Elementary Particle Model and good agreement with experiment has been obtained. However, this approach has many limitations:

(i) Much experimental input is needed. While β decay experiments are quite feasible, the inelastic electron scattering experiments are more difficult to perform and accounts for a sizeable fraction of uncertainity in the predicted muon capture rate. For an excited transition such $\mu^- + {}^{16}O(0^+) \longrightarrow {}^{16}N(0^-) + \nu_\mu$, the inelastic scattering experiment is not even possible.

(ii) The method is limited to partial capture rates at the most, since the nuclear matrix element depends on the angular momenta of the initial and final nuclear states.

(iii) The assumption that relations among nuclear form factors deduced from 'nuclear' CVC and PCAC (analogous to relations among nucleon form factors obtained from CVC and PCAC) are true beyond the impulse approximation is not justifiable.

7. The Foldy-Walecka Approach.

The closure approximation of Primakoff [1] has been used extensively in the calculation of total muon capture rates. In this approximation, the calculation of total capture rate is reduced to the evaluation of ground state expectation values of muon capture operators. In the case of doubly magic nuclei such as ^{16}O and ^{40}Ca, most of the capture takes place through the first forbidden dipole matrix elements due to the suppression of allowed transitions by Pauli principle. It has been shown by Luyten, Rood and Tolhoek [28] that in the single particle shell model, the following equality holds:

$$M_V^2 = M_A^2 = M_P^2$$

where M_V, M_A and M_P are the vector, axial vector and pseudoscalar matrix elements. On the assumption that the basic nucleon-nucleon interaction is of Wigner and Majorana type (spin-isospin independent forces), Foldy and Walecka [29] have shown that the above equalities hold true even when the effects of inter-particle forces on shell model states are taken into account. In such a case the Wigner supermultiplet [30] theory is applicable and ground states of nuclei ($A = 4n$) then belong to the identity representation of SU(4), i.e. they constitute a scalar supermultiplet, if the nuclear forces are short ranged and attractive. Foldy and Walecka then relatd the vector matrix element to an integral over photoabsorption cross-sections and the predicted capture rates are in good agreement with experiment.

Many authors [31] have considered supermultiplet symmetry breaking by spin-dependent nuclear forces and they conclude that spin dependent forces do not play a significant role. This question has also been considered by Parthasarathy [32] who showed that supermultiplet symmetry is broken in the muon capture process with emission of neutrons, when final state interaction is taken into account.

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CHAPTER II

DENSITY MATRIX METHODS

1. Introduction

The density matrix formalism [1] is a very convenient tool for the description of nuclear reactions, and in particular, the application of density matrix methods to the study of angular correlations has been extensively reviewed by Fraunfelder and Steffen [2] . In this chapter we give a brief review of the density matrix formalism with special reference to $\gamma - \nu$ angular correlations and then discuss recoil nuclear polarization taking into account the contribution due to excited states of the recoiling nucleus. We give here the general formalism for nuclear reactions in cascade and its application to the specific case of muon capture by ^{28}Si will be dealt with in part II of the thesis. We also discuss briefly the Fano's statistical tensors and quote a theorem which enables us to calculate the contribution of $^{12}B(1^-)$ state to the average recoil polarization of $^{12}B(1^+)$ state. An expression is derived here for the average expectation value of a set of statistical tensors which specify nuclear orientation and its application to the case of μ^- capture by ^{12}B will be considered in Part III of the thesis.

2. The Density Matrix.

In quantum mechanics, a pure state is characterised by the existence of an experiment that gives a result predictable with certainty when performed on a system in that state. It is represented as an eigenstate of an operator or as a superposition of eigenstates of an arbitrary operator. On the other hand, for a mixed state, there exists no experiment which gives a unique result predictable with certainity, and hence there is less than maximum information about the system. Such a mixed state can be represented by an incoherent superposition of pure states, the word incoherent implying that, to find the expectation value of an observable in the mixed state, one must first calculate the probability for each pure state and then take an average, attributing to each of the pure state an assigned weight.

A pure state can in general be written as

$$\psi = \sum_n a_n \mathcal{U}_n$$

where the $\mathcal{U}_n$'s are eigenvectors of some complete set of operators. The expectation value of an observable Q with respect to ψ is

$$\langle Q \rangle = \sum_m \sum_n a_m^* a_n Q_{nm} .$$

Now, a mixed state is a weighted superposition of the pure states ψ; calling the weights p_i, the average expectation value of Q is now given by

$$\langle \tilde{Q} \rangle = \sum_i p_i \langle Q \rangle_i$$

Defining

$$\rho_{nm} = \sum_i p_i \, a_m^{(i)*} a_n^{(i)} \qquad (1)$$

we obtain

$$\langle \tilde{Q} \rangle = \sum_{n,m} \rho_{mn} Q_{nm} = \mathrm{Tr}\,[Q\rho] \qquad (2)$$

The matrix ρ is called the " density matrix" for the mixed state. It is easily seen that the density matrix is Hermitian and that $\mathrm{Tr}\,[\rho] = 1$. Further details about the density matrix and its properties can be found in the review of Fano [1].

3. Angular Correlations:

Consider the sequence $|J_i M_i\rangle \xrightarrow{\mu^-} |J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle$

where the J's refer to the angular momenta and M's to their projections on the Z-axis. We now derive a relation between the final and initial state density matrices.

Suppose the system $|J_i M_i\rangle$ is initially not in a pure state but in a mixed state described by the density matrix element $(\rho_I)_{M_i,M_i'}$. If the final system $|J_f M_f\rangle$, to which the system evolves under the action of the operator H (in our case the μ^- capture Hamiltonian) is described by the density matrix element $(\rho_f)_{M_f M_f'}$, then we have

$$\langle M_f | \rho_f | M_f' \rangle = \sum_{M_i M_i'} \langle J_f M_f | H | J_i M_i \rangle \langle J_i M_i | \rho_I | J_i M_i' \rangle \langle J_f M_f' | H^{\dagger} | J_i M_i' \rangle$$

$$= \sum_{M_i M_i'} \langle J_f M_f | H | J_i M_i \rangle \langle J_i M_i | \rho_I | J_i M_i' \rangle \langle J_i M_i' | H | J_f M_f' \rangle^* \tag{3}$$

The above equation expresses the final state density matrix (ρ_f) in terms of the initial state density matrix (ρ_I) and can also be applied to the second reaction in the cascade viz. $|J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle$. Denoting the interaction Hamiltonian for γ-decay as H_γ, we have

$$\langle M_F | \rho_F | M_F' \rangle = \sum_{M_f M_f'} \langle J_F M_F | H_\gamma | J_f M_f \rangle \langle J_f M_f | \rho_f | J_f M_f' \rangle \langle J_f M_f' | H_\gamma | J_F M_F' \rangle^* \tag{4}$$

where (ρ_F) is the density matrix of final nucleus after γ-emission. In the specific case of γ-ν angular correlations, we consider muon capture by spin zero nucleus viz. $^{28}Si(0^+)$; hence the nucleus is randomly oriented and the initial state density matrix $(\rho_I)_{M_i M_i'} = \frac{1}{2J_i + 1} \delta_{M_i M_i'}$. The density matrix element for the intermediate nucleus $(\rho_f)_{M_f M_f'}$ in eqn. (4) is obtained by expressing the muon capture Hamiltonian in spherical tensors and performing standard angular momentum algebra. The density matrix element $(\rho_F)_{M_F M_F'}$ for the final state after γ-emission is obtained by substituting for $(\rho_f)_{M_f M_f'}$ in eqn. (4) from eqn. (3) term by term, and carrying out the necessary angular momentum algebra. The full details of this procedure are presented in Part II of this thesis wherein we treat both unpolarized and polarized muon capture by $^{28}Si(0^+)$. The use of density matrix methods in the study of γ- ν angular correlations in ^{16}O with a more general form for H_γ has been carried out first by Devanathan and Subramaniam [6] .

4. <u>Recoil Nuclear Polarization.</u>

The importance of the average recoil nuclear polarization $P_{av.}$ in nuclear muon capture was first pointed out by Devanathan, Parthasarathy and Subramanian [3] who showed using density matrix methods that $P_{av.}$ is insensitive to nuclear structure and hence is eminently suitable for obtaining information about the induced pseudoscalar coupling g_p. In a recent experiment, Possoz et. al. [4] have determined the average recoil nuclear polarization of $^{12}B(1^+)$ in the reaction $^{12}C(\mu, \nu_\mu)^{12}B$, taking into account the contribution from the 1^- branch. In part III of the thesis we present in detail our calculations of the contribution to the average recoil polarization of $^{12}B(1^+)$ from the gamma decay of the $^{12}B(1^-)$ level. In this section we discuss the Fano's statistical tensor and a theorem due to Rose [5] the use of which enables us to calculate the contribution from the 1^- level.

4a. Fano's Statistical Tensor

The Fano's statistical tensor is defined by:

$$G_\nu(J) = \sum_M (-1)^{J-M} P_M \; C(JJ\nu ; M -M 0) \qquad (5)$$

where P_M denotes the population of the magnetic sublevel M, and ν is the rank of the tensor. These statistical tensors determine the effect of the initial emitting state on the angular distribution and polarization of the emitted radiation.

The following special cases are of interest [5]

(i) For unoriented nuclei, if the populations are normalized such that $\sum_M P_M = 1$, and $P_M = \frac{1}{2J+1}$, then it is easily shown that

$$G_\nu(J) = \frac{1}{2J+1} \delta_{\nu 0}$$

and

$$G_0(J) = \frac{1}{2J+1} \sum_M P_M \qquad (6)$$

This shows that $G_0(J)$ represents total population.

(ii) When $\nu = 1$, and $\sum_M M P_M \neq 0$, then

$$G_1(J) = \sqrt{\frac{3}{2J+1}} \frac{1}{\sqrt{J(J+1)}} \sum_M M P_M \qquad (7)$$

and we say that the nucleus is polarized.

(iii) when $\nu = 2$ and $\sum_M P_M \left\{ (3M^2 - J(J+1) \right\} \neq 0$, then

$$G_2(J) = \frac{\sqrt{5}}{\left(J(J+1)(2J-1)\ (2J+1)(2J+3)\right)^{1/2}} \sum_M P_M \left\{ (3M^2 - J(J+1) \right\} \tag{8}$$

then the nucleus is aligned.

(iv) By using orthogonality of Clebsch-Gordon coefficient $C(J\,J\,\nu\ ,\ M - M\ 0)$, it can easily be shown that P_M and $G_\nu(J)$ are transforms of one another i.e.,

$$G_\nu(J) = \sum_M (-1)^{J-M} P_M\, C(JJ\nu;\ M\text{-}M0)$$

and

$$P_M = \sum_\nu (-1)^{J-M} G_\nu(J)\ \ C(J\ J\ \nu\ ,\ M - M\ 0). \tag{9}$$

We now enunciate a theorem due to Rose [5] which is directly relevant to our purpose of calculating the average recoil nuclear polarization of $^{12}B(1^+)$, taking into account the contribution from the 1^- branch.

Theorem: If a nuclear system is initially in a state of orientation given by a statistical tensor of rank λ and if it makes a transition to a final state whose orientation is given by a statistical tensor of rank λ', then $\lambda' = \lambda$ if the transition is parity conserving, and $\lambda' = \lambda \pm 1$ if the transition is a parity violating one.

The proof of the above theorem is straighforward and can be found in Rose [5] . We now briefly comment upon the relevance of the above theorem to our calculation, deferring the complete details to Part III of the thesis. The process of muon capture by $^{12}C(0^+)$ leads predominantly to the $^{12}B(1^+)$ state; however there is also a small excitation of $^{12}B(1^-)$ state ($\sim$12%). The $^{12}B(1^-)$ state is polarized by muon capture (being non-parity conserving) and hence the rank of the statistical tensor describing the 1^- state is 1. Since it decays by γ-emission to the 1^+ state (being a parity conserving transition), the rank of the statistical tensor is unchanged in accordance with the above theorem. The details of the method by which we have calculated this additional polarization of the $^{12}B(1^+)$ level due to the γ gamma-decay of 1^- level is given in Part III of the thesis.

4b. <u>Nuclear Spin Orientation.</u>

The description of nuclear spin orientation requires the knowledge of the average expectation values of the spherical tensor parameters T_K^{μ}, where K is the rank of the tensor and μ is its projection. The section briefly reviews the method of obtaining the average expectation values, for the tensor parameters. The discussion is after Devanathan, Parthasarathy and Subramaniam [3] .

Consider a nuclear transition from an initial state $|J_iM_i\rangle \rightarrow |J_fM_f\rangle$ which is caused by the transition operator t. Denoting by ρ_i and ρ_f the density matrices for initial and final states respectively, we can write

$$(\rho_f)_{M_fM_f'} = \sum_{M_i\,M_i'} \langle J_fM_f|\, t \,|J_iM_i\rangle \; (\rho_i)_{M_iM_i'} \langle J_fM_f'|\, t^{\dagger} \,|J_iM_i'\rangle \quad \ldots\ldots \tag{10}$$

The density matrix ρ_f completely specifies the spin orientation of the final nucleus and can be conveniently represented by a set of tensor parameters, T_K^{μ}, whose expectation value is defined by

$$T_K^{\mu} = \frac{\text{Trace}\,(T_K^{\mu}\,\rho_f)}{\text{Trace}\,\rho_f} \quad \ldots\ldots\ldots \tag{11}$$

T_K^{μ} are spherical tensors of rank K in the spin space of the final nucleus, obeying the following orthonormality condition

$$\text{Trace}\left\{(T_K^{\mu^+})\;T_{K'}^{\mu'}\right\} = [J_f]^2\,\delta_{KK'}\,\delta_{\mu\mu'} \tag{12}$$

The transition operator t is written in spherical tensor form, viz.,

$$t = \sum_{\lambda,m_\lambda} O_\lambda^{m_\lambda}\,. \tag{13}$$

If the initial nucleus is in an unoriented state, the density matrix becomes

$$(\rho_I)_{M_i M_i'} = \frac{1}{(2J_i + 1)} \delta_{M_i M_i'} \quad (14)$$

Now, trace $(T_K^{\mu}\ \rho_f)$ is given by

$$\mathrm{Trace}\ (T_K^{\mu}\ \rho_f) = \frac{1}{(2J_i+1)} \sum_{M_f M_f'} \sum_{M_i} \langle J_f M_f' | T_K^{\mu} | J_f M_f \rangle$$

$$\langle J_f M_f | \sum_{\lambda m_\lambda} O_\lambda^{m_\lambda} | J_i M_i \rangle \langle J_f M_f' | \sum_{\lambda' m_\lambda'} O_{\lambda'}^{m_\lambda'} | J_i M_i \rangle^{*} \quad (15)$$

Using Wigner-Eckart Theorem and simplifying, we obtain

$$\mathrm{Trace}\ (T_K^{\mu}\ \rho_f) = \frac{1}{[J_i]^2} \sum_{m_\lambda m_\lambda'} C(\lambda\lambda' . K\ ,\ m_\lambda - m_\lambda' - \mu)(-1)^{\lambda - m_\lambda} \frac{[J_f]^3}{[K]}$$

$$W(\lambda J_i\ K J_f\ ;\ J_f\ \lambda') \quad \langle J_f \| T_K \| J_f \rangle \langle J_f \| O_\lambda \| J_i \rangle \langle J_f \| O_{\lambda'} \| J_i \rangle^{*} \quad (16)$$

These expressions have been derived by Devanathan, Parthasarathy and Subramanian [3] . By putting $K = \mu = 0$ in the above expression we obtain Trace (ρ_f). Thus knowing Trace $(T_K^{\mu}\ \rho_f)$ and Trace (ρ_f), $\langle T_K^{\mu} \rangle$ can be calculated. The above eqn. (16) is quite generally applicable to obtain the orientation of the final nucleus in any nuclear transition from an unoriented nuclei, and in Part III of the thesis we apply it to calculate the recoil polarization of $^{12}B(1^-)$ in muon capture by ^{12}C.

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PART II

C H A P T E R III

GAMMA-NEUTRINO ANGULAR CORRELATIONS IN MUON CAPTURE BY ^{28}Si *

1. Introduction.

In this chapter, we present a detailed account of our calculation of the gamma-neutrino angular coefficients in both unpolarized and polarized muon capture by ^{28}Si. The process of interest in

$$\mu^- + {}^{28}Si(0^+) \longrightarrow {}^{28}Al^*(1^+ ;\ 2202\ \text{KeV}) + \nu_\mu$$
$$\hookrightarrow {}^{28}Al(0^+, 973\ \text{KeV}) + \gamma \qquad (1)$$

For an allowed transition (as in the above case), the angular distribution of γ-rays with respect to the Z-axis (neutrino direction) is given by

$$I(\theta_{\gamma\nu}) = I(0)\Big[1 + \alpha P_2(\cos\theta_{\gamma\nu}) + \beta_1(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})$$
$$P_2(\cos\theta_{\gamma\nu}) + \beta_2(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})\Big] \qquad (2)$$

where α, β_1 and β_2 are the correlation coefficients, $\vec{P}$ is the muon polarization at the instant of capture ($|\vec{P}| \sim 16\%$ in ^{28}Si)

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*R.Parthasarathy and V.N.Sridhar, Phys. Rev. C18 (1978) 1796.
Phys. Rev. C23 (1981) 861.

and $\hat{\gamma}$ and $\hat{\nu}$ are the unit vectors along photon and neutrino momenta respectively. It is to be noted here that in the case of unpolarized muon capture ($|\vec{P}| = 0$), only one coefficient (α) survives in eqn. (2). The experimental determination of the correlation coefficients was carried out by Miller et. al. [1] following a suggestion by Grenacs [2] that the $\gamma - \nu$ angular correlation coefficients in muon capture can be measured by observing the Doppler broadening of γ-rays due to recoil of the nucleus.

The theory of $\gamma - \nu$ angular correlations was developed in a series of papers by Popov et. al. [3] in terms of multipole expansion of the weak hadronic operators in close analogy with the theory of orbital electron capture, and by Devanathan and Subramanian [4] using density matrix methods. The multipole theory of Popov was applied to the case of muon capture in ^{28}Si by Ciechanowicz [5] who obtained a range for the induced pseudoscalar coupling constant (g_P) as $-4.9 < g_P/g_A < 1.2$ by comparing with the experimental of Miller et. al. [1] and claimed that this result indicates a downward renormalization of the Goldberger-Treiman value for g_P, for the $A = 28$ system. This drastic downward renormalization of g_P in nuclei, as predicted by Ciechanowicz seems to be unlikely for two reasons:

(i) the downward renormalization of g_P has been established only for infinite nuclear matter. (See for example, Delorme et. al. [6] and Rho [7]). In the case of finite nuclei, surface effects play an important role and there is no clear understanding of the renormalization of g_p at present.

(ii) the renormalization of g_p in nuclei is basically due to many-body effects such as virtual pion scattering by other nucleons etc., while the calculation of Ciechanowicz is based on the impulse approximation, in which one ignores meson exchange effects. The treatment of Devanathan and Subramanian [4] consists in the use of density matrix methods and a general γ-decay Hamiltonian. They have studied the problem of γ- ν angular correlations in muon capture by ^{16}O for which no experimental measurements are available at present. We derive here an expression for $I(\theta_{\gamma\nu})$ for both unpolarized and polarized muon capture which can be compared directly with the experiment of William and Mary group [1] on muon capture by ^{28}Si.

In Section 2 we discuss the Fujii-Primakoff Hamiltonian for <u>nuclear</u> muon capture. In Section 3 we give details pertaining to the construction of density matrix for the intermediate nucleus after muon capture. The operator for gamma emission and construction of densitry matrix for the final nucleus after gamma decay is discussed in Section 4. In section 5 complete expressions for the gamma-neutrino angular correlation coefficients α, β_1 and β_2 are obtained. In Section 6, we deduce relations among α, β_1, β_2, P_N and P_L, where P_N and P_L are the average recoil polarization and longitudinal polarization respectively, in the Fujii-Primakoff Approximation and discuss their significance. In Sections 7 and 8, we review briefly the formalism for partial capture rate and recoil nuclear polarization respectively. Meson Exchange Correction (MEC) effects on allowed muon capture are duscussed in Section 9. In Section 10, the nuclear models used are discussed and in Section 11 numerical results for the γ-ν angular correlation coefficients are presented along with discussion. In Appendices I and II, we give details of angular momentum algebra techniques necessary to obtain the angular correlation coefficients. In Appendix III, we give expressions required for the calculation of partial capture rate. In Appendix IV, we rewrite nuclear matrix elements in particle-hole model and in Appendix V, reduced matrix elements are evaluated.

2. The transition operator for the nucleus

The Fujii-Primakoff Hamiltonian for muon capture is

$$H_{eff.} = \frac{1}{2} \tau_L^+ (1 - \vec{\sigma}_L \cdot \hat{\nu}) \sum_{n=1}^{A} \tau_n^- \Big[G_V 1_L \cdot 1_n + G_A \vec{\sigma}_L \cdot \vec{\sigma}_n - G_P (\vec{\sigma}_L \cdot \hat{\nu}) (\vec{\sigma}_n \cdot \hat{\nu}) - \frac{g_V}{M} (\vec{\sigma}_L \cdot \hat{\nu}) (\vec{\sigma}_L \cdot \vec{p}_n) - \frac{g_A}{M} (\vec{\sigma}_L \cdot \hat{\nu}) (\vec{\sigma}_n \cdot \vec{p}_n) \Big] \delta(\vec{r} - \vec{r}_n) \qquad (3)$$

where τ_L^+ is the isospin operator for leptons

τ_n^+ is the isospin operator for nucleons

1_L is the unit operator for leptons

1_n is the unit operator for nucleons

σ_L is the Pauli spin operator for leptons

σ_n is the Pauli spin operator for nucleons

$\hat{\nu}$ is the unit vector in the direction of neutrino momentum

$\overrightarrow{p_n}$ is the linear momentum operator for nucleons, and

g_V ; g_A, G_V, G_A, G_P are muon capture coupling constants. The summation in eqn. (3) implies the use of Impulse Approximation, according to which the individual particle operators are being summed over.

The effective muon capture coupling constants are defined in Chapter 1 and we repeat them here for convenience.

$$G_V = g_V(1 + \frac{\nu}{2M}) + g_S$$

$$G_A = g_A - (g_V + g_M)\,\nu/2M$$

$$G_P = (g_P + g_T - g_A - g_V - g_M)\,\nu/2M.$$

The numerical values of g_V, g_A, g_P, g_M, g_T and g_S which are used in the thesis are given in Chapter I. We have varied the values of g_P (the induced pseudoscalar coupling constant) and g_T (the induced tensor coupling constant) and their variation with the γ- ν angular correlation coefficients is studied in Section II.

Using this effective muon capture Hamiltonian, the matrix element for the process

$$\mu^- + {}^{28}Si(0^+) \longrightarrow {}^{28}Al^*(1^+ ; 2202 \text{ KeV}) + \nu_\mu$$

can be written as

$$Q = \langle U_\nu | \Omega | U_\mu \rangle \quad (4)$$

where U_ν and U_μ are the Dirac spinors for neutrino and muon respectively. Ω in (4) is given by

$$\Omega = \frac{1}{2}(1 - \vec{\sigma_L} \cdot \hat{\nu}) \left\{ \mathcal{M}_1 + \vec{\sigma_L} \cdot \vec{\mathcal{M}_2} \right\} \quad (5)$$

where $\mathcal{M}_1$ and $\mathcal{M}_2$ are nuclear matrix elements whose explicit forms are given below: (We follow the notation of Devanathan Parthasarathy and Subramanian [8]).

$$\mathcal{M}_1 = G_V M_1 - \frac{g_V}{M} (\hat{\nu} \cdot \vec{M_3}) \quad (6)$$

$$\vec{\mathcal{M}_2} = G_A \vec{M_2} - G_P(\hat{\nu} \cdot \vec{M_2}) - i \frac{g_V}{M} (\hat{\nu} \times \vec{M_3}) - \frac{g_A}{M} M_4 \hat{\nu} \quad (7)$$

where

$$M_1 = \left\langle f \left| \sum_{n=1}^{A} \tau_n^- \, e^{-i\hat{\nu}\cdot\vec{r}_n} \, \phi_\mu(r_n) \right| i \right\rangle$$

$$\vec{M}_2 = \left\langle f \left| \sum_{n=1}^{A} \tau_n^- \, e^{-i\hat{\nu}\cdot\vec{r}_n} \, \phi_\mu(r_n) \, \vec{\sigma}_n \right| i \right\rangle$$

$$\vec{M}_3 = \left\langle f \left| \sum_{n=1}^{A} \tau_n^- \, e^{-i\hat{\nu}\cdot\vec{r}_n} \, \phi_\mu(r_n) \, \vec{p}_n \right| i \right\rangle$$

$$M_4 = \left\langle f \left| \sum_{n=1}^{A} \tau_n^- \, e^{-i\hat{\nu}\cdot\vec{r}_n} \, \phi_\mu(r_n) \, \vec{\sigma}_n \cdot \vec{p}_n \right| i \right\rangle \qquad (8)$$

In the above equations, $|i\rangle$ and $|f\rangle$ refer to initial and final states respectively, $\phi_\mu(r_n)$ is the muon wavefunction and $e^{-i\hat{\nu}\cdot\vec{r}_n}$ is due to the plane wave description of the outgoing neutrino. Following Sens [9] , the muon wavefunction can be considered to be a constant over the nuclear volume and hence can be factored out. However, the finite size of the nucleus changes the muon wavefunction and hence a correction is applied to the value of muon wave function at the centre of the nucleus. This corrected value is

$$(\phi_\mu)^2_{av.} = \frac{1}{\pi} \left(\frac{m_\mu}{m_e}\right)^3 \left(\frac{Z}{a_o}\right)^3 R_\mu \qquad (9)$$

where m_μ is mass of the muon

m_e is mass of the electron

Z is the atomic number of capturing nucleus

a_0 is the Bohr radius of Hydrogen atom (0.529×10^{-8} cm.)

and R_μ is the correction factor for finite size of the nucleus. It is given by $R_\mu = \left(\frac{Z_{eff}}{Z}\right)^3$, where $Z_{eff.}$ is the effective nuclear charge as seen by the muon, a concept first introduced by Wheeler [10]. The approximation of the nucleus as a point charge breaks down at large $Z(\sim 30)$ when the muon orbit is actually inside the nucleus. The values for Zeff. obtained by Wheeler assuming harmonic oscillator wavefunctions is in agreement with the more reliable calculation of Sens [9] using X-ray and electron scattering data to determine nuclear charge distributions and muon wavefunctions. For the case of ^{28}Si, $R_\mu = 0.6653$.

3. Intermediate State density matrix after muon capture.

Consider the process $|J_i M_i\rangle \xrightarrow{\mu^-} |J_f M_f\rangle$. As discussed in Chapter II (Section 3), the density matrix element of the state $|J_f M_f\rangle$ denoted by $(\rho^{\mu c})_{M_f M_f'}$, is given by

$$\langle J_f M_f | \rho_f^{\mu c} | J_f M_f' \rangle = \sum_{M_i M_i'} \langle J_f M_f | H_{\mu c} | J_i M_i \rangle \langle J_i M_i | \rho_I | J_i M_i' \rangle \langle J_i M_i' | H_{\mu c} | J_f M_f' \rangle^* \tag{10}$$

where $(\rho_I)_{M_I M_I'}$ is the density matrix element of the initial state which is equal to $\frac{1}{2J_i+1}\,\delta_{M_i M_i'}$ if the initial state is unpolarized (as is the case with process (1)) and $H_{\mu c}$ is the muon capture Hamiltonian. In Section 2 of this Chapter, we have rewritten the muon capture Hamiltonian $H_{\mu c}$ in a convenient way (see eqns. (4) - (8)). It is easily seen from eqn. (4) that the square of the matrix element after summing and averaging over lepton spins is given by (for polarized muon capture)

$$|Q|^2 = \mathrm{Tr}\left\{\Omega\,\frac{(1+\vec{\sigma}\cdot\vec{P})}{2}\right\}\Omega^+ \tag{11}$$

where $\vec{P}$ is the muon polarization at the instant of capture and Ω is given by eqn. (5). The trace can be evaluated by using the fact that the trace of an odd number of σ's is zero and the identity

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\,\vec{\sigma}\cdot(\vec{A}\times\vec{B}).$$

Since the first step in the cascade $|J_i M_i\rangle \xrightarrow{\mu^-} |J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle$ is a muon capture process, we must pick those terms in eqn. (11) which contribute to the capture rate. Therefore, we may write

$$|Q|^2 = \mathcal{M}_1\mathcal{M}_1^* + \vec{\mathcal{M}}_2\cdot\vec{\mathcal{M}}_2^* - (\vec{P}\cdot\hat{\nu})(\mathcal{M}_1\mathcal{M}_1^* - \vec{\mathcal{M}}_2\cdot\vec{\mathcal{M}}_2^*) - (\hat{\nu}\cdot\vec{\mathcal{M}}_2)(\vec{P}\cdot\vec{\mathcal{M}}_2^*) - (\vec{P}\cdot\vec{\mathcal{M}}_2)(\hat{\nu}\cdot\vec{\mathcal{M}}_2^*) \tag{12}$$

Expressing $\mathcal{M}_1$ and $\vec{\mathcal{M}}_2$ in terms of M_1, M_2, M_3 and M_4 as given in eqns. (6) and (7), the square of the matrix element can neutrino directions. It is to be noted that eqn. (13) gives the density matrix $(\rho^{\mu c})_{M_f M_f'}$ of the intermediate state after muon capture (see eqn. (10)).

We now give the final form (after performing the angular momentum algebra) of the density matrix $(\rho^{\mu c})_{M_f M_f'}$. The explicit evaluation of the terms is given in Appendix I. We follow the notation of Rose [11] for angular momentum coefficients.

$$\left(\rho^{\mu c}_{M_f M_f'}\right) = \sum_J \Big\{ G_A^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'-J_f} [\ell][\ell'] \, C(\ell\ell'J;000)$$

$$W(J_f 1 J \ell'; \ell J_f) \, I(\ell 1 J_f; \ell' 1 J_f) + (G_P^2 - 2 G_P G_A)$$

$$\sum_{\ell\ell'} (i)^{\ell'-\ell} \frac{[\ell][\ell']}{[J_f]^2} \, C(\ell 1 J_f; 000) \, C(\ell' 1 J_f; 000)$$

$$C(J_f J_f J; 000) \, I(\ell 1 J_f, \ell' 1 J_f) + \frac{2}{M} (G_P - G_A) \, g_A$$

$$\sum_{\ell\lambda} (i)^{-J_f+\ell-1} (-1)^{\lambda - J_f} \frac{[\lambda][\ell]}{[J_f]^2} \, C(\ell 1 J_f; 000)$$

$$C(J_f J_f J, 000) \, G(\ell 1 J_f; J_f 1 \lambda 1 J_f) + \frac{2}{M} G_A \, g_V \sum_{\ell\ell'\lambda} \sqrt{2}$$

$$(i)^{\ell'-\ell+3} [J_f]^2 [1][\lambda][\ell][\ell'] \, C(\ell' 1 \lambda; 000)$$

$$C(\lambda \ell' J; 000) \, W(J_f 1 \lambda 1; \ell' 1) \, W(J_f \lambda J_f \ell'; 1 J)$$

written as

$$\left| Q \right|^2 = \frac{1}{2} (A + B) \tag{13}$$

where

$$A = G_V^2 M_1 M_1^* + G_A^2 \vec{M_2} \cdot \vec{M_2}^* + (G_P^2 - 2 G_P G_A) \left| \hat{\nu} \cdot M_2 \right|^2 - \frac{2 G_V g_V}{M} M_1 (\hat{\nu} \cdot \vec{M_3}^*) + 2 (G_P - G_A) \frac{g_A}{M} (\hat{\nu} \cdot \vec{M_2}) M_4^* + \frac{2 G_A g_V}{M} i \vec{M_2} \cdot (\hat{\nu} \times \vec{M_3}^*) \tag{14}$$

and

$$B = \vec{P} \cdot \Big[- G_V^2 M_1 M_1^* \hat{\nu} + G_A^2 \vec{M_2} \cdot \vec{M_2}^* \hat{\nu} - G_P^2 \left| \hat{\nu} \cdot M_2 \right|^2 \hat{\nu} - 2 (G_A - G_P) G_A (\hat{\nu} \cdot \vec{M_2}) \vec{M_2}^* - \frac{2}{M} G_P g_A (\hat{\nu} \cdot M_2) \vec{M_4}^* \hat{\nu} + \frac{2}{M} G_V g_V M_1 (\hat{\nu} \cdot \vec{M_3}^*) \hat{\nu} + \frac{2}{M} G_A g_A M_4 \vec{M_2}^* + \frac{2}{M} G_A g_V i \vec{M_2} \cdot (\hat{\nu} \times M_3^*) \hat{\nu} \Big] \tag{15}$$

In the above expressions we have split the square of the matrix element into two parts A and B for convenience. It is readily seen from eqns. (14) and (15) that the terms in A contribute to unpolarized muon capture whereas the terms in B contribute to polarized muon capture (~~whereas the terms in B contribute to polarized muon capture~~). For evaluating the capture rate, one has to integrate over neutrino directions; since we are interested in angular correlations wherein the angular identity of the neutrino is to be retained, we do not perform an integration over

$$\mathcal{G}(\ell 1 J_f; \ell' 1 J_f\, 0 J_f) \Big\} (-1)^{M_f} \frac{[J_f]^2}{\sqrt{4\pi}\,[J]}\, C(J_f J_f J; -M_f M_f' M_J)\, Y_J^{M_J}(\hat{\nu}) \tag{16}$$

$$\left(\rho^{\mu c}_{M_f M_f'}\right)^B = \sum_J \Bigg(Y_J^{M_J}(\hat{\nu})\, (\vec{P}\cdot\hat{\nu}) \Big\{ \sum_{\ell\ell'} (i)^{\ell'-\ell}\, [\ell][\ell'] \cdot$$

$$I(\ell 1 J_f; \ell' 1 J_f) \big[G_A^2 (-1)^{\ell - J_f}\, C(\ell\ell' J; 000)\, W(J_f 1 J \ell'; \ell J_f)$$

$$- G_P^2 \frac{1}{[J_f]^2} C(\ell 1 J_f; 000)\, C(\ell' 1 J_f; 000)\, C(J_f J_f J; 000)$$

$$- \frac{2}{M} G_A g_V \sum_{\ell\ell'\lambda} \sqrt{2}\, (i)^{\ell'-\ell+3} [J_f]^2 [\ell][\ell'][1][\lambda]$$

$$C(\ell' 1 \lambda; 000)\, C(\ell \lambda J; 000)\, W(J_f 1 \lambda 1; \ell' 1)\, W(J_f \lambda J \ell; 1 J)$$

$$\mathcal{G}(\ell 1 J_f; \ell' 1 J_f\, 0 J_f) - \frac{2}{M} G_P g_A \sum_{\ell\lambda} (i)^{-\ell + J_f - 1} (-1)^{\lambda - J_f}$$

$$\frac{[\ell][\lambda]}{[J_f]^2}\, C(J_f J_f J; 000)\, C(\ell 1 J_f; 000)\, \mathcal{G}(\ell 1 J_f, J_f 1 \lambda 1 J_f) \Big\}$$

$$+ \sum_{\mathcal{L}} \big[Y_{\mathcal{L}}(\hat{\nu}) \times Y_1(\hat{P}) \big]_J^{M_J} \frac{[J]}{[J_f]} \sqrt{\frac{4\pi}{3}} \Big[2 (G_P - G_A) G_A$$

$$\sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell']\, C(\ell 1 J_f; 000)\, C(J_f \ell' \mathcal{L}; 000)\, W(J_f \ell' J 1; \mathcal{L} J_f)$$

$$I(\ell 1 J_f; \ell' 1 J_f) + \frac{2}{M} G_A g_A \sum_{\ell\lambda} (i)^{-\ell + J_f + 1} [\ell][\lambda]\, C(\ell J_f \mathcal{L}; 000)$$

$$W(J_f 1 J_f \mathcal{L}; \ell J)\, \mathcal{G}(\ell 1 J_f; J_f 1 \lambda 1 J_f) \Big] \Bigg) (-1)^{M_f} \frac{[J_f]^2}{\sqrt{4\pi}\,[J]} \tag{17}$$

$$C(J_f J_f J, -M_f M_f' M_J)$$

In eqns. (16) and (17), we have omitted G_V terms as they do not contribute to process (1) which is a pure allowed Gammow-Teller transition. As we have split the density matrix of intermediate nucleus after muon capture into two parts A and B, eqns. (16) and (17) give the complete density matrix elements for <u>unpolarized</u> and <u>polarized</u> muon capture respectively. Further, it is seen from the above expressions, that for the case of unpolarized muon capture, there is a simple angular dependence on $Y_J^{M_J}(\hat{\nu})$ as compared with $Y_J^{M_J}(\hat{\nu})\ (\vec{p}\cdot\hat{\nu})$ and $\left[Y_{\ell}(\hat{\nu}) \times Y_1(\hat{p})\right]_J^{M_J}$ in the case of polarized muon capture. The terms $I(\ell 1 J_f, \ell' 1 J_f)$ and $\mathcal{G}(\ell 1 J_f; \ell' 1 J_f 0 J_f)$ represent nuclear reduced matrix elements which are given by the following expressions:

$$I(\ell n J_f; \ell' n J_f) = 16\pi^2 \langle J_f \| \sum_{n=1}^{A} \{Y_\ell(\hat{r}_n) \times \sigma_n\}_{J_f} j_\ell(\nu r_n) \| 0\rangle \, |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} \{Y_{\ell'}(\hat{r}_n) \times \sigma_n\}_{J_f} j_{\ell'}(\nu r_n) \| 0\rangle^* \tag{17a}$$

$$\mathcal{G}(\ell 1 J_f; J_f 1 \lambda 1 J_f) = 16\pi^2 \langle J_f \| \sum_{n=1}^{A} \{Y_\ell(\hat{r}_n) \times \sigma_n\}_{J_f} j_\ell(\nu r_n) \| 0\rangle \, |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} \left[\{Y_{J_f}(\hat{r}_n) \times \sigma_1\}_\lambda \times \sigma_n\right]_{J_f} j_{J_f}(\nu r_n) \| 0\rangle^*$$

(17b)

where $n = 0$ or 1, such that $\sigma_0 = 1$ and $\sigma_1 = \sigma$. In the above expressions we have taken an initial spin zero nucleus, as is the case with process (1).

4. Final State Density Matrix after gamma-emission:

We shall now consider the second part of the cascade $|J_i M_i\rangle \xrightarrow{\mu^-} |J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle$, namely $|J_f M_f\rangle \xrightarrow{\gamma} |J_F M_F\rangle$. The operator for gamma emission is taken to be $\vec{J}_N \cdot \vec{A}_p$ following Rose [12] where $\vec{J}_N$ is the nucleon current and $\vec{A}_p$ is the vector potential of the emitted γ-ray with circular polarization $p(\pm 1)$. As discussed in Chapter II, the density matrix element $(\rho_F)_{M_F M_F}$ of the final nucleus after gamma emission from $|J_f M_f\rangle$ is

$$(\rho_F)_{M_F M_F} = \sum_{M_f M_f'} \langle J_F M_F | H_\gamma | J_f M_f\rangle (\rho^{\mu c})_{M_f M_f'} \times \langle J_F M_F | H_\gamma | J_f M_f'\rangle^* \tag{18}$$

where $(\rho^{\mu c})_{M_f M_f'}$ is given by eqns. (16) and (17). Following Rose [12], we now perform a multipole decomposition of the vector potential $\vec{A}_p$ as follows:

$$\vec{A}_p = \sqrt{2\pi} \sum_{L=1}^{\infty} \sum_{M=-L}^{L} (i)^L \sqrt{2L+1}\; D^L_{Mp}(\phi\theta 0)\left[\vec{A}_L^M(m) + ip\,\vec{A}_L^M(e)\right] \tag{19}$$

where m and e refer to magnetic and electric multipole respectively and $D^L_{MP}(\phi\theta 0)$ is the rotation matrix. Since we are interested in a $1^+ \longrightarrow 0^+$ transition, we assume pure multipolarity (L = 1) for the gamma-ray which is an M1 decay in our case. Substituting eqn. (19) in eq. (18) and carrying out the straightforward angular momentum algebra yields

$$(\rho_F)_{M_F M_F} = |a_\tau|^2 \left|\langle J_F \| L(\tau) \| J_f\rangle\right|^2 \sum_{M_f M_f'} (\rho^{\mu c})_{M_f M_f'}$$

$$\sum_{\gamma=0}^{2L} (-1)^p (-1)^{J_f - J_F}$$

$$C(L\, L\, \gamma\, ,\, p\, -p\, 0)\, \sqrt{4\pi}\, \frac{[J_F]^2}{[J_f]}\, W(J_f\, L\, J_f\, L\, ,\, J_F\, \gamma)$$

$$C(J_f\, \gamma\, J_f,\, M_f\, M_\gamma\, M_f')\, \left[Y_\gamma^{M_\gamma}(\hat{\gamma})\right]^{*} \tag{20}$$

Equation (20) gives the final state density matrix element after γ-emission and we are now in a position to calculate the gamma-neutrino angular correlation coefficients, which is carried out in the next section. In the above equation, $|a(\tau)|^2$ is a constant factor depending on the nature of multipolarity, $|\langle J_F \| L(\tau) \| J_f\rangle|^2$ is the square of the gamma decay matrix element and τ stands for either an electric or magnetic multipole transition.

5. The Gamma-Neutrino Angular Correlation Coefficients.

In this section we shall obtain closed expressions for the three γ- ν angular correlation coefficients α , β_1 and β_2 . For sake of convenience we shall derive the unpolarized (α) and (β_1, β_2) polarized angular correlations coefficients separately.

5 (a). The Correlation Coefficient (α) for unpolarized muon Capture:

We substitute for ($\rho^{\mu c}_{M_f M_f'}$) from eqn. (16) term by term in eqn. (20) and after summing over $M_f M_f'$, we obtain $\delta_{\gamma J}\, \delta_{M_\gamma M_J}$ using the orthogonality condition for Clebsch-Gordan coefficients. The two spherical harmonics $Y_J^{M_J}(\hat{\nu})$ and $Y_J^{M_J}(\hat{\gamma})^*$ combine to give $P_J(\cos\theta_{\gamma\nu})$, where $\theta_{\gamma\nu}$ is the angle between the gamma and neutrino directions. We illustrate the procedure by an example.

Consider the G_A^2 term in eqn. (16). Substituting in eqn. (20), we obtain

$$\sum_{M_F} (\rho_F)_{M_F M_F} = G_A^2 \,|a(M1)|^2\, |\langle 0^+ \| M1 \| 1^+\rangle|^2$$

$$\sum_J \Big[\sum_{\ell\ell'} \sum_{M_f M_f'} \sum_{\gamma=0}^{2L} (-1)^p \,(-1)^{J_f - J_F - M_f}\, (i)^{\ell'-\ell}$$

$$(-1)^{\ell' - J_f} \frac{[\ell]\,[\ell']\,[J_F]^2\,[J_f]^2 \sqrt{4\pi}}{[J_f]\,[J]\,\sqrt{4\pi}}$$

$$C(LL\gamma;p-p0)\ C(\ell\ell'J;000)\ C(J_fJ_fJ;-M_f\,M_f'\,M_J)$$
$$C(J_f\gamma J_f;M_f\,M_\gamma\,M_f')\ W(J_f 1 J\ell';\ell J_f)\ W(J_f L J_f L;J_F\gamma)$$
$$Y_J^{M_J}(\hat{\gamma})^*\ Y_J^{M_J}(\hat{\nu})\ I(\ell 1 J_f;\ell' 1 J_f)$$

We now note the following:

$$\text{(i)}\quad C(J_f\gamma J_f;M_f\,M_\gamma\,M_f') = C(J_fJ_f\gamma;-M_f\,M_f'\,M_\gamma)\,\frac{[J_F]}{[\gamma]}\,(-1)^{J_f-M_f-\gamma}$$

From orthogonality of Clebsch-Gordan coefficients

$$\sum_{M_f\,M_f'} C(J_fJ_fJ;-M_f\,M_f'\,M_J)\ C(J_fJ_f\gamma;-M_f\,M_f'\,M_\gamma) = \delta_{\gamma J}\,\delta_{M_\gamma M_J}$$

$$\text{(ii)}\quad \sum_{M_J} Y_J^{M_J}(\hat{\nu})\left[Y_\gamma^{M_\gamma}(\hat{\gamma})\right]^* = \frac{[J]^2}{4\pi}\,P_J(\cos\theta_{\gamma\nu})$$

The other terms in eqn. (16) can be evaluated in similar fashion. (See Appendix II) . Since the circular polarization of the gamma ray is not observed, we sum over p (± 1) in the Clebsch-

Gordan coefficient $G(LLJ\ ,\ p - p0)$ to obtain

$$- \left[1 + (-1)^J \right] \quad C(11J\ ,\ 1 - 10)\ ,$$

where we have taken $L = 1$, as we are considering emission of pure multipole M1 radiation. Since the diagonal elements of the density matrix represent population of sublevels and we summing over M_F , the gamma-neutrino correlation function is given directly by

$$I(\theta_{\gamma\nu}) = \sum_{M_F} \langle M_F | \rho_F | M_F \rangle$$

The complete expresion for $I(\theta_{\gamma\nu})$ is

$$\begin{aligned} I(\theta_{\gamma\nu}) = & - |a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_J \Big\{ \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1} \\ & [\ell][\ell'][1]^2 \, C(\ell\ell'J;000) \, W(11J\ell';\ell 1) \, I(\ell 11;\ell' 11) \\ & + (G_p^2 - 2 G_p G_A) \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell'] \, C(\ell 11;000) \\ & C(\ell' 11;000) \, C(11J;000) \, I(\ell 11;\ell' 11) + \frac{2}{M} (G_p - G_A) g_A \\ & \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} [\lambda][\ell] \, C(\ell' 11;000) \, C(11J;000) \, \mathcal{G}(\ell 11; \\ & 11\lambda 11) + \frac{2}{M} G_A g_V \sum_{\ell\ell'\lambda} \sqrt{2} \, (i)^{\ell'-\ell+3} [\ell][\ell'][\lambda][1]^3 \\ & C(\ell' 1\lambda;000) \, C(\lambda\ell J;000) \, W(11\lambda 1;\ell' 1) \, W(1\lambda 1\ell';1J) \\ & \mathcal{G}(\ell' 11;1101) \Big\} \, [1 + (-1)^J] \, C(11J;1-10) \, P_J(\cos\theta_{\gamma\nu}) \end{aligned} \tag{21}$$

In the above equation, we have specialised to the case of $0^+ \longrightarrow 1^+ \longrightarrow 0^+$ transition, i.e. $J_i = 0$, $J_f = 1$ and $J_F = 0$ with $L = 1$. When the summation over J is carried out, the term $J = 0$ is angle independent and $J = 1$ term does not contribute due to summation over p. (This is easily seen from the Clebsch-Gordan coefficient $\left(\left[1 + (-1)^J\right] C(11J\,,\,1\,-10)\right)$. Dividing the $J = 2$ part of eqn, (21) by the $J = 0$ part, the angular correlation function may be written as

$$I(\theta_{\gamma\nu}) = I(o)\left[1 + \alpha P_2(\cos\theta_{\gamma\nu})\right] \quad (22)$$

The angular correlation coefficient α is given by

$$\alpha = A/B$$

where A and B are obtained from eqn. (21) by putting $J = 2$ and $J = 0$ respectively. The numerical evaluation of α is carried out in Section 11. We give below the complete expression for α.

$$\alpha = A/B \quad (23)$$

Where

$$A = \frac{1}{\sqrt{6}}\Big[G_A^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1} [\ell][\ell'][1]^2 \, C(\ell\ell'2;000)\, W(112\ell';\ell 1)\, I(\ell 11;\ell' 11) + (G_P^2 - 2G_P G_A) \sqrt{\frac{2}{3}} \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell'] \, C(\ell 11;000)\, C(\ell' 11;000)$$

$$I(\ell 11;\ell' 11) + \frac{2}{M}\left(G_P - G_A\right) g_A \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} [\lambda]$$

$$[\ell]\, C(\ell 11;000)\, \mathcal{G}(\ell 11;11\lambda 11) \sqrt{\frac{2}{3}}$$

$$+ \frac{2}{M} G_A g_V \sum_{\ell\ell'\lambda} \sqrt{2}\, (i)^{\ell'-\ell+3} [\lambda][\ell][\ell'][1]^3$$

$$C(\ell' 1\lambda;000)\, C(\lambda\ell 2;000)\, W(11\lambda 1;\ell' 1)\, W(1\lambda 1\ell;12)$$

$$\mathcal{G}(\ell 11;\ell' 1101)$$

(23a)

and

$$B = \frac{1}{\sqrt{3}}\Big[-G_A^2 \sum_{\ell} [1]\, I(\ell 11;\ell 11) - (G_P^2 - 2G_P G_A)$$

$$\sum_{\ell\ell'} \frac{[\ell][\ell']}{[1]} C(\ell 11;000)\, C(\ell' 11;000)\, I(\ell 11;\ell' 11) - \frac{2}{M}(G_P - G_A)$$

$$g_A \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} \frac{[\lambda][\ell]}{[1]} C(\ell 11;000)\, \mathcal{G}(\ell 11;11\lambda 11)$$

$$+ \frac{2}{M} G_A g_V \sqrt{2} \sum_{\ell\ell'} (i)^{\ell'-\ell+3} [\ell][1]^2\, C(\ell' 1\ell;000)\, W(11\ell 1;\ell' 1)\, \mathcal{G}(\ell 11;\ell' 1101) \quad (23b)$$

5(b). <u>The Correlation Coefficients</u> (β_1, β_2) <u>for Polarized Muon Capture:</u>

The γ-ν angular correlation coefficients β_1 and β_2 for polarized muon capture are obtained by following the same procedure as that of obtaining α; we substitute eqn. (17) in eqn. (20) term by term and carry out the angular momentum algebra. However, due to different angular dependence exhibited by the terms in eqn. (17) viz., $Y_J^{M_J}(\hat{\nu})(\vec{P}\cdot\hat{\nu})$ and $\left[Y_{\mathcal{L}}(\hat{\nu})\times Y_1(\hat{P})\right]_J^{M_J}$ the spherical harmonic coupling is more involved than the simple spherical harmonic addition theorem employed in extracting α. We choose the following Kinematics which is convenient for our present purpose: gamma direction is chosen to be the Z-axis and an integration over the unphysical azimuthal angle ϕ_ν of the neutrino is carried out using the following relations due to Devanathan and Subramanian [4]:

$$\int_0^{2\pi}(\vec{P}\cdot\hat{\nu})\,P_J(\cos\theta_{\gamma\nu})\,d\phi_\nu = 2\pi\sum_L C(J1L;000)^2(\vec{P}\cdot\vec{\gamma})\,P_L(\cos\theta_{\gamma\nu}) \tag{24}$$

$$\int_0^{2\pi}\sqrt{\frac{4\pi}{3}}\,Y_L^0(\hat{\gamma})\left[Y_{\mathcal{L}}(\hat{\nu})\times Y_1(\hat{P})\right]_L^0 d\phi_\nu = \frac{[L][\mathcal{L}]}{2}\,C(\mathcal{L}1L;000)\,(\vec{P}\cdot\hat{\gamma})\,P_{\mathcal{L}}(\cos\theta_{\gamma\nu}) \tag{25}$$

With help of these relations we may combine the spherical harmonics occuring in eqns. (17) and (20) and obtain the distribution of γ-rays with respect to neutrino direction, denoted by $I(\theta_{\gamma\nu})$. We illustrate the procedure by two examples:

(i) G_A^2 term :

Substituting the G_A^2 term from eqn. (17) in eqn. (20), we obtain

$$G_A^2 \left\{ \sum_{\ell\ell'} \sum_{J} \sum_{M_f M_f'} \sum_{\gamma=0}^{2L} (i)^{\ell'-\ell} (-1)^{\ell'-J_f-M_f} (-1)^{p} (-1)^{J_f-J_F} \right.$$

$$\frac{[J_F]^2}{[J_f]} \frac{[\ell][\ell'][J_f]^2 \sqrt{4\pi}}{\sqrt{4\pi}\,[J]} C(\ell\ell'J;000)\, W(J_f lJ\ell';\ell J_f)$$

$$C(J_f J_f J;-M_f M_f' M_J)\, C(LL\gamma;p-p0)\, W(J_f L J_f L; J_F \gamma)$$

$$\left. C(J_f \gamma J_f; M_f M_\gamma M_f') \left[Y_\gamma^{M_\gamma}(\hat{\gamma})\right]^* Y_J^{M_J}(\hat{\nu})(\vec{p}\cdot\hat{\nu}) \right\} \tag{26}$$

Using the orthogonality and symmetry properties of Clebsch-Gordan coefficients, we obtain

$$(-1)^{\gamma} C(J_f \gamma J_f; M_f M_\gamma M_f') = C(J_f J_f \gamma; -M_f M_f' M_\gamma) \frac{[J_f]}{[\gamma]} (-1)^{J_f-M_f}$$

$$\sum_{M_f M_f'} C(J_f J_f \gamma; -M_f M_f' M_\gamma)\, C(J_f J_f J; -M_f M_f' M_J) = \delta_{\gamma J}\, \delta_{M_\gamma M_J} \tag{27}$$

Applying the spherical harmonic addition theorem

$$\sum_{M_J} \left[Y_J^{M_J}(\hat{\gamma})\right]^* Y_J^{M_J}(\hat{\nu}) = \frac{[J]^2}{4\pi} P_J(\cos\theta_{\gamma\nu}) \tag{28}$$

We now integrate over the unphysical azimuthal angle ϕ_ν using relation (24) to obtain

$$\int_0^{2\pi} \frac{[J]^2}{4\pi} P_J(\cos\theta_{\gamma\nu}) (\vec{P}\cdot\hat{\nu})\, d\phi_\nu = \frac{2\pi}{4\pi} [J]^2 \sum_L C(J1L;00)^2 (\vec{P}\cdot\hat{\gamma}) P_L(\cos\theta_{\gamma\nu}) \qquad (29)$$

Combining eqns. (26) - (29) and specialising to a $0^+ \longrightarrow 1^+ \longrightarrow 0^+$ transition ($J_i = J_F = 0$, $J_f = 1$, $L = 1$), eqn. (26) reduces to

$$G_A^2 |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell' JL} (i)^{\ell'-\ell} (-1)^{\ell'-1} (-1)^p [\ell][\ell'] C(\ell\ell' J;000)\, C(11J;p\,{-p}\,0)\, W(11J\ell';\ell 1) C(J1L;000)^2 (\vec{P}\cdot\hat{\gamma}) P_J(\cos\theta_{\gamma\nu})\, I(\ell 11;\ell' 11) \qquad (30)$$

We may rewrite the above equation using the relation

$$\sum_L C(J1L;000)^2 P_L(\cos\theta_{\gamma\nu}) = \left[C(J1J{+}1;000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J\, C(J1J{-}1;000)^2 P_{J-1}(\cos\theta_{\gamma\nu})\right]$$

where $\eta_J = 1$ for $J > 0$ and 0 for $J = 0$. Further summation over p (the circular polarization of the gamma ray is not observed) gives us

$$(-1)^p C(11J;p\,{-p}\,0) = -[1 + (-1)^J]\, C(11J;1\,{-1}\,0)$$

With the above mentioned simplifications, eqn. (30) may be written as

$$
\begin{aligned}
& -G_A^2 \, |a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_J \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell'] \\
& C(\ell\ell'J;000) \, W(11J\ell';\ell 1) \, (\vec{P}\cdot\hat{\gamma}) \left[C(J1J+1;000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) \right. \\
& \left. + \eta_J \, C(J1J-1;000)^2 \, P_{J-1}(\cos\theta_{\gamma\nu}) \right] [1+(-1)^J] \, C(11J;1-10)
\end{aligned}
\tag{31}
$$

(ii) <u>$2(G_P - G_A)\, G_A$ term:</u>

Substituting this term from eqn. (17) in eqn. (20), we have

$$
\begin{aligned}
& \sum_{\ell\ell'J} \sum_{\mathcal{L}} \sum_{M_f M_f'} \sum_{\gamma=0}^{2L} (-1)^p \, (-1)^{J_f - J_F} \, (i)^{\ell'-\ell} \, (-1)^{M_f} \, \frac{[J_F]^2}{[J_f]} \\
& \frac{[\ell][\ell'][J_f]^2 \sqrt{4\pi}}{\sqrt{4\pi}\,[J]} \, \frac{[J]}{[J_f]} \, C(\ell 1 J_f;000) \, C(J_f \ell' \mathcal{L};000) \, C(LL\gamma; p-p_0) \\
& W(J_f \ell' J 1;\mathcal{L} J_f) \, W(J_f L J_f L; J_F \gamma) \, C(J_f \gamma J_f; M_f M_\gamma M_f') \\
& C(J_f J_f J; -M_f \, M_f' \, M_J) \, [Y_\gamma^{M_\gamma}(\hat{\gamma})]^* \, [Y_{\mathcal{L}}(\hat{\nu}) \times Y_1(\hat{p})]_J^{M_J} \\
& I(\ell 1 J_f; \ell' 1 J_f)
\end{aligned}
\tag{32}
$$

Now we observe that the orthogonality and symmetry properties of Clebsch-Gordan coefficients as exhibited in eqns. (27) give $\delta_{\gamma J}\,\delta_{M_\gamma M_J}$. Choosing γ direction as Z-axis and integrating over $d\phi_\nu$, we can apply eqn. (25) and the result is (after putting $J_f = 1$, $J_F = 0$, $L = 1$)

$$-2(G_P - G_A)\, G_A \sum_{\mathcal{L}} \sum_{\ell\ell' J} (i)^{\ell'-\ell}\, [\ell]\,[\ell']\,[1]\,[\mathcal{L}]$$

$$C(\ell 1 1;000)\; C(\ell' 1 \mathcal{L};000)\; C(\mathcal{L} 1 J;000)\; P_{\mathcal{L}}(\cos\theta_{\gamma\nu})\,(\vec{P}\cdot\hat{\gamma})$$

$$[1+(-1)^J]\; C(11J;1-10)\; I(\ell 11;\ell' 11)\, |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+\rangle|^2 \tag{33}$$

In similar fashion, all other terms in eqn. (17) can be reduced and we now give the complete expression for the gamma ray angular distribution with respect to neutrino direction for polarized muon capture:

$$I(\theta_{\gamma\nu}) = \frac{1}{6\pi}\, |a(M1)|^2\, |\langle 0^+ \| M1 \| 1^+\rangle|^2 \sum_J \Big(\sum_{\ell\ell'} (i)^{\ell'-\ell}$$

$$I(\ell 11;\ell' 11) \Big\{ P_J(\cos\theta_{\gamma\nu})\,(3G_A^2\,(-1)^{\ell'-1}\, C(\ell\ell' J;000)$$

$$W(11J\ell';\ell 1) + (G_P^2 - 2G_P G_A)\, C(\ell 11;000)\; C(\ell' 11;000)$$

$$C(11J;000) + (\vec{P}\cdot\hat{\gamma})\big[C(J1J+1;000)^2\, P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J$$

$$C(J1J-1;000)^2\, P_{J-1}(\cos\theta_{\gamma\nu})\big]\big(3G_A^2\,(-1)^{\ell'+1}\, C(\ell\ell' J;000)$$

$$
\begin{aligned}
& W(11J\ell';\ell 1) - G_P^2\, C(\ell 11;000)\, C(\ell' 11;000)\, C(11J;000) - 2(G_A - G_P) \\
& G_A \sum_{\mathcal{L}} [1][\mathcal{L}]\, C(\ell 11;000)\, C(\ell' 1\mathcal{L};000)\, W(\mathcal{L}111;\ell' J)\, C(\mathcal{L}1J;00) \\
& P_{\mathcal{L}}(\cos\theta_{\gamma\nu})\, (\vec{P}\cdot\hat{\gamma})\Big)\Big\} + \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} [\ell][\lambda]\, C(\ell 11;000) \\
& C(11J;000)\, G(\ell 11;11\lambda 11) \Big\{ 2(G_P - G_A)\frac{g_A}{M} P_J(\cos\theta_{\gamma\nu}) - \frac{2G_P}{M} \\
& g_A \big[C(J1J+1;000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J\, C(J1J-1;000)^2 P_{J-1}(\cos\theta_{\gamma\nu}) \\
& (\vec{P}\cdot\hat{\gamma})\Big\} + \frac{2}{M} G_A g_V \sum_{\ell\ell'\lambda} \sqrt{2}\, (i)^{\ell'-\ell+3} [\ell][\ell'][1]^3[\lambda] \\
& C(\ell' 1\lambda;000)\, C(\ell\lambda J;000)\, W(11\lambda 1;\ell' 1)\, W(1\lambda 1\ell;1J)\, G(\ell' 11; \\
& \ell' 1101) \Big\{ P_J(\cos\theta_{\gamma\nu}) + \big[C(J1J+1;000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J \\
& C(J1J-1;000)^2 P_{J-1}(\cos\theta_{\gamma\nu})\big](\vec{P}\cdot\hat{\gamma}) + \frac{2G_A g_A}{M} \sum_{\ell'\lambda\mathcal{L}} (i)^{\ell'-2} \\
& (-1)^{\lambda-\mathcal{L}} [\lambda][\mathcal{L}][\ell'][1]\, C(1\ell'\mathcal{L};000)\, W(1\ell' J1;1\mathcal{L}) \\
& C(1\mathcal{L}J;000)\, G(\ell' 11;11\lambda 11)\, P_{\mathcal{L}}(\cos\theta_{\gamma\nu})\, (\vec{P}\cdot\hat{\gamma})\Big)\, \frac{[1+(-1)^J]}{C(11J;1-10)}
\end{aligned}
\qquad (34)
$$

where $\eta_J = 1$ for $J > 0$ and 0 for $J = 0$ and the nuclear reduced matrix elements have been defined in eqns. (17a) and (17b).

From eqn. (34) it can be seen that in summation over J, the term $J = 1$ does not contribute due to the presence of the term $\left[1 + (-1)^J\right]$. The terms with $J = 0$ and independent of $\vec{P}$ (muon polarization vector) become independent of $\theta_{\gamma\nu}$ and can be factored out as the $I(o)$ part of eqn. (2). The terms with $J = 2$ and muon polarization yield $(\vec{P} \cdot \hat{\gamma})(\hat{\gamma} \cdot \hat{\nu}) P_2(\text{Cos}\, \theta_{\gamma\nu})$ and $(\vec{P} \cdot \hat{\gamma})(\hat{\gamma} \cdot \hat{\nu})$, the coefficients of which determine β_1 and β_2 respectively (see eqn.(2)). As an example, consider the term

$$(\vec{P}\cdot\hat{\gamma})\left[C(J1J+1;000)^2 P_{J+1}(\text{Cos}\,\theta_{\gamma\nu}) + \eta_J\, C(J1J-1;00)^2 P_{J-1}(\text{Cos}\,\theta_{\gamma\nu})\right]$$

in the $\{\ldots\}$ part of eqn. (34). Putting $J = 2$, we obtain

$$(\vec{P}\cdot\hat{\gamma})\left[C(213;000)^2 P_3(\text{Cos}\,\theta_{\gamma\nu}) + C(211;00)^2 P_1(\text{Cos}\,\theta_{\gamma\nu})\right]$$

Expressing P_3 in terms of P_2 and P_1 by means of a recurrence relation among Legendre Polynomaisls

$$P_3(\text{Cos}\,\theta_{\gamma\nu}) = \frac{5}{3}\text{Cos}\,\theta_{\gamma\nu}\, P_2(\text{Cos}\,\theta_{\gamma\nu}) - \frac{2}{3} P_1(\text{Cos}\,\theta_{\gamma\nu}) \tag{34a}$$

we obtain

$$(\vec{P}\cdot\hat{\gamma})\left[\frac{3}{5}\left\{\frac{5}{3}(\hat{\gamma}\cdot\hat{\nu}) P_2(\text{Cos}\,\theta_{\gamma\nu}) - \frac{2}{3} P_1(\text{Cos}\,\theta_{\gamma\nu})\right\} + \frac{2}{5} P_1(\text{Cos}\,\theta_{\gamma\nu})\right]$$

Thus we can isolate the coefficients of $(\vec{P} \cdot \hat{\gamma})(\hat{\gamma} \cdot \hat{\nu}) P_2(\text{Cos}\,\theta_{\gamma\nu})$ and $(\vec{P} \cdot \hat{\gamma})(\hat{\gamma} \cdot \hat{\nu})$ to obtain expressions for the correlation coefficients β_1 and β_2 respectively. We now give below the complete closed expressions for β_1 and β_2 in polarized muon capture.

The correlation coefficient β_1 associated with angular dependence $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})\,P_2(\cos\theta_{\gamma\nu})$ is given by

$$\beta_1 = \frac{C}{D} \tag{35}$$

where

$$\begin{aligned}
C = \frac{1}{\sqrt{6}}\Big[\, & G_A^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1} [\ell][\ell']\, C(\ell\ell'2;000)\, W(112\ell';\ell 1) \\
& I(\ell 11;\ell' 11) - G_P^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell']\, C(\ell 11;000)\, C(\ell' 11;000) \sqrt{\tfrac{2}{3}} \\
& I(\ell 11;\ell' 11) + 2(G_A - G_P)\, G_A \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell'][1][3] \\
& C(\ell 11;000)\, C(\ell' 13;000)\, W(3111;\ell' 2)\, \tfrac{5}{3}\sqrt{\tfrac{3}{7}} - \tfrac{2}{M}\, G_P\, g_A \sum_{\ell\lambda} (i)^{\ell-2} \\
& (-1)^{\lambda-1} [\lambda][\ell]\, C(\ell 11;000)\, G(\ell 11;11\lambda 11) \sqrt{2/3} + \tfrac{2}{M}\, G_A\, g_V \sum_{\ell\ell'\lambda} \\
& \sqrt{2}\, (i)^{\ell'-\ell+3} [\lambda][\ell][\ell'][1]^3\, C(\ell 1\lambda;000)\, C(\lambda\ell' 2;000) \\
& W(11\lambda 1;\ell 1)\, W(1\lambda 1\ell';12)\, G(\ell 11;\ell' 1101) - \tfrac{2}{M}\, G_A\, g_A \sum_{\ell'\lambda} (i)^{\ell'-2} \\
& [\lambda][\ell'][1][3]\, (-1)^{\lambda-3}\, C(\ell' 13;000)\, W(1\ell' 21;13) \\
& \sqrt{\tfrac{3}{7}}\cdot\tfrac{5}{3}\; G(\ell' 11;11\lambda 11)
\end{aligned} \tag{36}$$

and

$$D = \frac{1}{\sqrt{3}} \Big[- G_A^2 \sum_{\ell} [1]\, I(\ell 11;\ell 11) + (G_P^2 - 2G_P G_A) \sum_{\ell\ell'} (i)^{\ell'-\ell}$$

$$\frac{[\ell][\ell']}{[1]}\, C(\ell 11;000)\, C(\ell' 11;000)\, I(\ell 11;\ell' 11) - \frac{2}{M}(G_P - G_A)$$

$$g_A \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} \frac{[\lambda][\ell]}{[1]}\, C(\ell 11;000)\, \mathcal{G}(\ell 11;11\lambda 11) +$$

$$\frac{2}{M} G_A g_V \sum_{\ell\ell'} \sqrt{2}\, (i)^{\ell'-\ell+3} [\ell][\ell'][1]\, C(\ell 1 \ell';000)$$

$$W(11\ell' 1;\ell 1)\ \mathcal{G}(\ell 11;\ell' 1101) \Big] \qquad (37)$$

The correlation coefficient β_2 associated with angular dependence $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})$ is given by

$$\beta_2 = E/D \qquad (38)$$

where

$$E = \frac{1}{\sqrt{3}} \Big[- G_A^2 \sum_{\ell} I(\ell 11;\ell 11)[1] + G_P^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell]$$

$$\frac{[\ell']}{[1]} I(\ell 11;\ell' 11) + 2(G_A - G_P) G_A \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'} [\ell]$$

$$\frac{[\ell']}{[1]} C(\ell 11;000)\, C(\ell' 11;000)\, I(\ell 11;\ell' 11) + \frac{2}{M} G_P g_A \sum_{\ell\lambda} (i)^{\ell-2}$$

$$(-1)^{\lambda-1} \frac{[\lambda][\ell]}{[1]}\, C(\ell 11;000)\, \mathcal{G}(\ell 11;11\lambda 11) + \frac{2}{M} G_A g_V$$

$$\sum_{\ell\ell'} \sqrt{2}\, (i)^{\ell'-\ell+3} [\ell][1]^2\, C(\ell 1\ell';000)\, W(11\ell' 1;\ell 1)$$

$$\mathcal{G}(\ell' 11;\ell 1101) - \frac{2 G_A g_A}{M} \sum_{\ell'\lambda} (i)^{\ell'-2} (-1)^{\lambda-1} \frac{[\lambda][\ell']}{[1]}$$

$$
\begin{aligned}
& C(\ell'11;000)\, \mathcal{G}(\ell'11;\ell 11\lambda 1)\Big] + \frac{1}{\sqrt{6}}\Big[+2(G_P - G_A)\,G_A \sum_{\ell\ell'} (i)^{\ell'-\ell} \\
& [\ell][\ell'][1]^2\, C(\ell 11;000)\, C(\ell'11;000)\, W(1111;\ell' 2)\sqrt{\tfrac{2}{3}}\, I(\ell 11;\ell' 11) \\
& -2(G_A - G_P)\,G_A \sum_{\ell\ell'} (i)^{\ell'-\ell}\, [\ell][\ell'][1]^2\, C(\ell 11;000)\, C(\ell' 13; \\
& 000)\, W(3111;\ell' 2)\,(2/3)\, I(\ell 11;\ell' 11) + \frac{2}{M} G_A g_A \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} \\
& [\lambda][\ell][1]^2 \sqrt{\tfrac{2}{3}}\, C(\ell 11;000)\, W(1\ell 21;11)\, \mathcal{G}(\ell 11;11\lambda 11) + \frac{2G_A g_A}{M} \\
& \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-3} \left(\tfrac{2}{3}\right) [\lambda][\ell][1]^2 [3]\, C(\ell 13;000) \\
& \qquad W(1\ell 21;13)\, \mathcal{G}(\ell 11;11\lambda 11)
\end{aligned}
\tag{39}
$$

and D is the same as in the denominator of eqn. (37). In the above equations, $[K] = \sqrt{2K+1}$ and we follow Rose [11] for angular momentum coefficients. It is seen from the above equations that the expression for denominator D is exactly the same as that of eqn. (23b) for unpolarized muon capture. This is because we are always dividing by $J = 0$ and $\vec{P} = 0$ angle independent part.

The numerical evaluation of β_1 and β_2 is carried out in section 11 of this Chapter.

6. Relations between α, β_1, β_2, P_L and P_N :

In this section, we shall derive relations between α, β_1, β_2, P_N and P_L, where P_N and P_L denote the average recoil polarization and the longitudinal polarization respectively.

Equations (23), (35) - (39) give the γ-neutrino angular correlation coefficients in terms of reduced matrix elements including relativistic terms in the Fujii-Primakoff Hamiltonian. If we now neglect relativistic $(1/M)$ terms and confine ourselves to S-wave neutrinos ($\ell = \ell' = 0$) known as the Fujii-Primakoff Approximation (FPA), the reduced matrix elements in the expressions for α, β_1 and β_2 cancel out resulting in the following simple expressions for the correlation coefficients:

$$\alpha = \frac{2\, G_P\, G_A - G_P^2}{3G_A^2 + G_P^2 - 2\, G_P\, G_A} \tag{40}$$

$$\beta_1 = \frac{G_P^2}{3G_A^2 + G_P^2 - 2\, G_P\, G_A} \tag{41}$$

$$\beta_2 = \frac{3\, G_A^2 - G_P^2}{3\, G_A^2 + G_P^2 - 2\, G_P\, G_A} \tag{42}$$

Under the same approximation (FPA), it was shown by Wolfenstein [13] that the longitudinal polarization (P_L) of the final nucleus in muon capture is given by

$$P_L = - \frac{2\, G_A^2}{3\, G_A^2 + G_P^2 - 2\, G_P\, G_A} \tag{43}$$

The FPA expression for P_N due to Devanathan, Parthasarathy and Subramanian [8] is

$$\frac{\overrightarrow{P_N}}{\overrightarrow{P}} \simeq \frac{2\, G_A^2 - 4/3\ G_P\, G_A}{3\, G_A^2 + G_P^2 - 2\, G_P\, G_A} \tag{44}$$

Comparing eqns. (40) - (42) with eqns. (43) and (44) we arrive at the following equalities:

$$-\alpha = 1 + \frac{3}{2} P_L \tag{45a}$$

$$\beta_1 = 1 - \frac{3}{2} \frac{\overrightarrow{P_N}}{\overrightarrow{P}} \tag{45b}$$

$$\beta_2 = -1 + \frac{3}{2} \frac{\overrightarrow{P_N}}{\overrightarrow{P}} - \frac{3}{2} P_L \tag{45c}$$

$$\beta_1 + \beta_2 = 1 + \alpha \tag{45d}$$

where $\overrightarrow{P}$ is the muon polarization at the instant of capture. Relation (45a) is implied in equation (60) of Devanathan and Subramanian [4] through $\xi = 1 + 2 P_L$, where ξ is the asymmetry coefficient of recoil nucleus, while the other relations are new.

The above relations are independent of nuclear models and muon capture coupling constants, and are valid even after taking into account relativistic terms and higher order partial waves for the neutrino. The method of derivation is similar to that of Devanathan and Subramanian [14] who show algebraically that the relation $\xi - 2P_L = 1$ is independent of nuclear structure and muon capture coupling constants; in our case we compare the complete expressions for α, β_1 and β_2 with those of P_L and P_N given by Wolfenstein [13] and Devanathan, Parthasarathy and Subramanian [8] respectively, and relate the appropriate nuclear matrix elements. Relations (45) have also been derived in a different way by Bernabeu [15] on the basis of helicity formalism. It is to be stressed here that our derivation of eqns. (45) is independent in the sense that, we start from the explicit muon capture Hamiltonian, derive complete expressions for α, β_1 and β_2 and compare them with those of P_N and P_L.

We now discuss the information which can be extracted from relations (45). It has been shown by Bernabeu [16] that the limits for P_L on the basis of time reversal invariance are 0 and -1 and any deviation from the above limits is claimed to be an indication of violation of time reversal invariance in muon capture. The above limits for P_L imply through relation (45a) that

$$-1 \leqslant \alpha \leqslant 0.5 \qquad (46)$$

While the experimental determination of P_L is difficult, the correlation coefficient α can be measured by using highly efficient Y-ray detectors (so as to observe Doppler broadening).

Any deviation of α from the above limits can be construed as an indication of violation of time reversal invariance in muon capture. The only measurement of α by Miller et. al. [1] gives results compatible with eqn. (46).

Relation (45b) provides us with an estimate of the average recoil nuclear polarization (P_N) of the intermediate nucleus, viz. $^{28}Al^*$ (1^+, 2202 KeV); using the measured value of β_1 by Miller et. al. [1], $P_N(^{28}Al^*(1^+)) \sim 0.6533$ which can be verified by an independent measurement.

Combining eqns. (45b) and (45c), we obtain

$$P_L = -\frac{2}{3}(\beta_1 + \beta_2) \tag{47}$$

which yields $P_L = -(0.7599 \pm 0.085)$ on substituting experimental data for β_1 and β_2 [1]. This value of P_L shows a $\sim 15\%$ enhancement over the value of $P_L = -\frac{2}{3}$ for a pure Gamow-Teller transition, indicating the importance of strong interaction induced effects. Relation (45d) shows that all the three correlation coefficients are not independent and the experimental values of Miller [1] satisfy the relation within the quoted experimental uncertainties. Lastly, using the bound on P_L (0 and -1) derived by Bernabeu [16] and on P_N($-\frac{1}{3}$ and $\frac{2}{3}$) by Rao, Kaliaperumal and Parthasarathy [16a], we find

$$0 < \beta_1 < 1.5 \tag{48a}$$

$$-1.5 < \beta_2 < 1.5 \tag{48b}$$

Similar bound for α , β_1 and β_2 have been obtained by Oziewicz [17] and the experimental values of Miller [1] for process (1) satisfy these bounds, within the quoted experimental uncertainties.

7. Partial Capture Rate.

We start with the Fujii-Primakoff Hamiltonian which may be written as

$$H_{eff.} = \frac{1}{2} \tau_L^{+} (1 - \vec{\sigma_L} \cdot \hat{\nu}) \sum_{n=1}^{A} \tau_n^{(-)} \Big[G_V 1_L \cdot 1_n + G_A \vec{\sigma_L} \cdot \vec{\sigma_n} - G_P (\vec{\sigma_L} \cdot \hat{\nu}) (\vec{\sigma_n} \cdot \hat{\nu}) - \frac{g_V}{M} (\vec{\sigma_L} \cdot \hat{\nu}) (\vec{\sigma_L} \cdot \vec{P_n}) - \frac{g_A}{M} (\vec{\sigma_L} \cdot \hat{\nu}) (\vec{\sigma_n} \cdot \vec{P_n}) \Big] \delta(\vec{r} - \vec{r_n})$$

where the various quantities in the above equation have already been defined in Section 2 of the present chapter. The matrix element for the muon capture process can be written as

$$Q = \langle u_\nu \mid \Omega \mid u_\mu \rangle$$

with Ω is defined by eqn. (5). After summing and averaging over lepton spins, we arrive at an expression for $|Q|^2$ in terms of nuclear matrix elements M_1, M_2, M_3 and M_4 which is given below [18] .

$$|Q|^2 = \frac{1}{2}\Big[G_V^2 \, M_1 M_1^* + G_A^2 \, \vec{M_2}\cdot\vec{M_2}^* + (G_P^2 - 2G_P G_A) \, |\hat{\nu}\cdot\vec{M_2}|^2 - \frac{2G_V g_V}{M} M_1(\hat{\nu}\times\vec{M_3}^*) + \frac{2}{M}(G_P - G_A) g_A \, (\hat{\nu}\cdot\vec{M_2}) M_4^* + \frac{2G_A g_V}{M} \, i \, \vec{M_2}\cdot(\hat{\nu}\times\vec{M_3}^*) \Big] \tag{49}$$

Since capture rate is a scalar observable, it is independent of the muon polarization vector $\vec{P}$. The detailed evaluation of the nuclear matrix elements occurring in eqn. (49) can be carried out along the same lines as discussed in Appendix I. However, since we are now interested in the calculation of capture rate for the process

$$\mu^- + {}^{28}Si(0^+) \longrightarrow {}^{28}Al^*(1^+,\ 2202\ \text{KeV}) + \nu_\mu \tag{50}$$

an integration over neutrino directions is to be carried out using the relation

$$\int Y_J^{M_J}(\hat{\nu})\ d\Omega_\nu = \sqrt{4\pi}\ \delta_{J0}\ \delta_{M_J 0} \tag{51}$$

The detailed expressions for nuclear matrix elements (after performing the angular momentum algebra) are given in Appendix III and numerical results for the partial capture rate for process (50) are given in Section 11 along with discussion.

8. Recoil Nuclear Polarization.

We now give a brief review of the formalism for the calculation of recoil nuclear polarization following Devanathan, Parthasarathy and Subramanian [8] .

In muon capture, one has to distinguish between two kinds of polarization: (i) the longitudinal recoil polarization (P_L) due to the definite helicity of the neutrino which results in the polarization of recoil nucleus along its direction of flight (opposite to the direction of neutrino momentum). This polarization is a manifestation of parity violation in muon capture. (ii) Due to muon polarization at the instant of capture ($\vec{P}$), the recoil nucleus has a polarization $P_{av.}$ along $\vec{P}$, this is called average polarization because the recoil directions (or equivalently the neutrino momentum direction in the rest frame of the nucleus) are averaged over. This polarization is essentially due to $\vec{P}$ and would exist in muon capture irrespective of whether parity is violated or not.

As discussed in Chapter II, the spin orientation of the final nucleus after muon capture can be conveniently represented by a set of tensor parameters T_K^{μ}, whose expectation value is defined by

$$T_K^{\mu} = \frac{\text{Trace}\,(T_K^{\mu}\,\rho_f)}{\text{Trace}\,(\rho_f)} \tag{52}$$

where ρ_f is the density matrix of the final nucleus. After expressing the muon capture operators in spherical tensor form and carrying out the angular momentum algebra, we obtain an expression for Trace $(T_K^{\mu}\,\rho_f)$ as given in eqn. (16) of Chapter II. The density matrix ρ_f can be constructed by squaring the matrix element Q in eqn. (4) including the muon polarization. However, we now pick those terms which contribute to $\mathrm{Tr}(T_K^{\mu}\,\rho_f)$. These terms involve muon capture couplings in a different combination as compared to partial capture rate and are given below:

$$-\mathcal{M}_1(\hat{\nu}\cdot\vec{\mathcal{M}}_2^{\,*}) - \mathcal{M}_1^{*}(\hat{\nu}\cdot\vec{\mathcal{M}}_2) - i\hat{\nu}\cdot(\vec{\mathcal{M}}_2\times\vec{\mathcal{M}}_2^{\,*}) + \mathcal{M}_1(\vec{P}\cdot\vec{\mathcal{M}}_2^{\,*}) + (\vec{P}\cdot\vec{\mathcal{M}}_2)\mathcal{M}_1^{*} - i\vec{P}\cdot(\vec{\mathcal{M}}_2\times\vec{\mathcal{M}}_2^{\,*}) \tag{53}$$

where $\mathcal{M}_1$ and $\vec{\mathcal{M}}_2$ are given by eqns. (6) and (7) of Section 2. We can express the above equation in terms of nuclear matrix elements M_1, $\vec{M_2}$, $\vec{M_3}$ and M_4; due to integration over neutrino direction ($\int d\Omega_\nu$), there remains only one vector $\vec{P}$, whose direction specifies the polarization vector of recoil nucleus, yielding the condition δ_{K1}. Choosing $\vec{P}$ along Z-axis, we now give the expression for recoil polarization ($\vec{P_N}$) following Devanathan, Parthasarathy and Subramanian [8]:

$$T_1^0 = A/B \tag{54}$$

where

$$A = -iG_A^2\,\vec{P}\cdot(\vec{M_2}\times\vec{M_2})^* - 2G_p G_A i\,\vec{P}\cdot(\hat{\nu}\times\vec{M_2})$$

$$(\hat{\nu}\cdot\vec{M_2}^{\,*}) - \frac{2G_A g_A}{M}\, i\,\vec{P}\cdot(\hat{\nu}\times\vec{M_2})M_4^*$$

$$+\frac{2G_A g_V}{M}(\vec{P}\cdot\hat{\nu})(\vec{M_2}\cdot\vec{M_3})^* + 2(G_p - G_A)\frac{g_V}{M}$$

$$(\hat{\nu}\cdot\vec{M_2})(\vec{P}\cdot\vec{M_3}^{\,*}) \tag{54a}$$

and

$$B = G_A^2\,\vec{M_2}\cdot\vec{M_2}^{\,*} + 2(G_p - G_A)\frac{g_A}{M}(\hat{\nu}\cdot\vec{M_2})M_4^*$$

$$+(G_p^2 - 2G_p G_A)\,|\hat{\nu}\cdot\vec{M_2}|^2 +$$

$$\frac{2G_A g_V}{M}\, i\,\vec{M_2}\cdot(\hat{\nu}\times\vec{M_3}^{\,*}) \tag{54b}$$

The detailed evaluation of the nuclear matrix elements occurring in eqns. (54a) and (54b) is carried out in Ref. [8] and we do not repeat it here. The recoil polarization $(\vec{P_N})$ is now given by

$$\vec{P_N} = \sqrt{\tfrac{2}{3}}\,\langle T_1^0\rangle\,\vec{P} \tag{55}$$

The importance of this observable to obtain reliable information about the induced pseudoscalar coupling (g_p) in muon capture was first pointed out by Devanathan, Parthasarathy and Subramanian [8]. This is mainly due to the fact that $\vec{P_N}$ is almost free from nuclear model uncertainties, in FPA the nuclear matrix elements cancel and one obtains

$$\overrightarrow{P_N} \simeq 0.61 \ \overrightarrow{P} \qquad (56)$$

The effect of nucleon momentum dependent terms and higher order neutrino partial waves can be shown to be negligible and hence P_N is almost nuclear model insensitive. We present numerical values for the recoil polarization of $^{28}Al^*(1^+$, 2202 KeV) in Section 11 along with discussion.

9. Meson Exchange Corrections on Allowed Muon Capture*

9a. Introduction

In this Section, we indicate a method of incorporating meson exchange current (MEC) effects into the effective Fujii-Primakoff Hamiltonian for muon capture and thereby evaluate the gamma-neutrino angular correlation coefficient (β_2) and the recoil polarization ($\overrightarrow{P_N}$). Since the nuclear force between nucleons is mediated by the exchange of virtual mesons possessing electric charge, their transfer between nucleons give rise to currents called meson exchange currents. As a result, in both electromagnetic and weak interactions in nuclei, a portion of the observed phenomena is due to meson exchange currents. In the Impulse Approximation (IA) calculations, MEC effects are ignored and hence a question naturally arises as to what extent will the IA results be affected by MEC effects.

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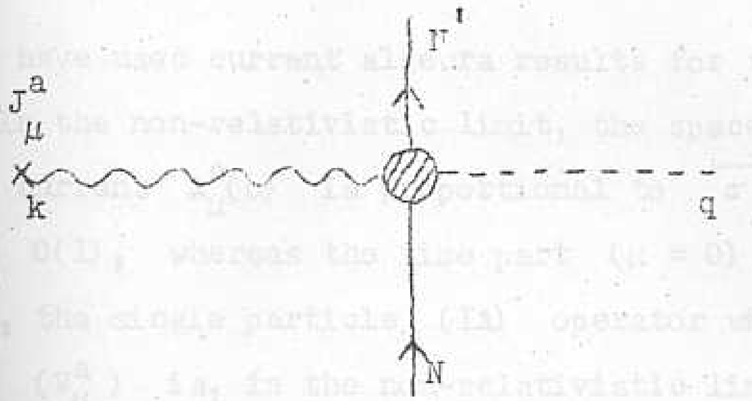
*R.Parthasarathy and V.N.Sridhar, Phys.Lett. 106B (1981) 363.

It was shown by Riska and Brown [19] that the discrepancy between IA theory and experiment in the $np \longrightarrow d\gamma$ electromagnetic process is removed only after including meson exchange corrections, besides the role of $\Delta(1232)$ isobar. Meson Exchange current calculations have been instrumental in clearing up a number of discrepancies between IA theory and experiment, in the case of ^3He and ^3H magnetic moments and the Gamow-Teller matrix element of ^3H beta decay. In this context, soft pion theorems play a crucial role in providing a model independent description of the dominant one pion exchange (OPE) current, as shown in an extensive review by Chemtob and Rho [20]. Recently Kubodera, Delorme and Rho (KDR) [21] have shown that soft pion theorems break down for the space part of axial vector current due to the role of $\Delta(1232)$ isobar and short-range correlations; they have argued further that it is the time part of axial vector current which is amenable to treatment by soft pion theorems. In the next subsection, we briefly review the arguments of Kubodera, Delorme and Rho [21], who show that the time part of axial current is measurably enhanced by MEC effects.

9(b). MEC effects on Axial Vector Current.

Among the various mesons (ρ , ω, σ , etc.) that can mediate between two nucleons, the pion is the lightest and hence one can expect the one pion exchange (OPE) current to be dominant over heavier meson exchange currents, which are suppressed due to short-range correlations between nucleons. We now follow Rho [22] and consider the following diagram:



which shows a current J_μ of four momentum k, producing a pion of four momentum q, N and N' represent nucleons with 4-momenta p and p' respectively, and the blob represents the unknown vertex. With neglect of heavier meson exchanges, the above diagram is known as the <u>seagull diagram.</u> From the soft-pion point of view, the amplitude for the above process can be written as [22]

$$M^{ab}_\mu \, (J^a_\mu) = \text{a piece due to IA} + \frac{1}{F_\pi} \, u(p') \left[J^a_\mu(o),\ Q^b_5(o) \right] u(p) \tag{57}$$

where $Q^b_5(o)$ is the axial charge

$$Q^b_5(o) \;=\; \int d^3x \;\; A_o^b(\vec{x}, o)$$

and F_π is the pion decay constant. For the case of vector current $V^a_\mu(o)$ we have

$$\left[V^a_\mu(o),\ Q^b_5(o) \right] \;=\; i\,\epsilon_{abc}\, A^c_\mu(o) \tag{58}$$

where we have used current algebra results for the above commutator. In the non-relativistic limit, the space part ($\mu = 1,2,3$) of axial current $A^c_\mu(o)$ is proportional to $\overrightarrow{\sigma}$ (spin operator) which is O(1), whereas the time part ($\mu = 0$) is O(p/M). By contrast, the single particle (IA) operator which is the vector operator (V^a_μ) is, in the non-relativistic limit, O(1) for time component ($\mu = 0$) and O(p/M) for space components (μ=1,2,3). Thus the space components of vector operator are enhanced relative to the single particle operator. For the axial current, where $J^a_\mu = A^a_\mu$, we have the commutator

$$\left[A^a_\mu(o),\ Q^b_5(o)\right] = i\,\epsilon_{abc}\, V^c_\mu(o) \qquad (59)$$

In this case, while the single particle operator A^a_μ is O(1) for space components ($\mu = 1,2,3$) and O(p/M) for time component ($\mu = 0$), in the non-relativistic limit, the two body vectorial part is O(1) for time component ($\mu = 0$) and O(p/M) for space components ($\mu = 1,2,3$). Thus it is the time component of the axial current which is essentially enhanced relative to the single particle operator. Kubodera, Delorme and Rho [21] have suggested angular correlation measurements in β decay as a possible testing ground for the enhancement of the time part of axial current. In the context of muon capture, Guichon et. al. [22] have studied partial capture rates in the reaction $\mu^- + {}^{16}O(0^+) \longrightarrow {}^{16}N(0^-) + \nu_\mu$

which is sensitive to the time part of axial vector current, and found a significant enhancement (albeit controversial) of the matrix element of the time part of axial current in the particle-hole model. In the next subsection, we discuss the modification of the Fujii-Primakoff Hamiltonian to incorporate the enhancement of the time part of axial current.

9(c). Modification of the Fujii-Primakoff Hamiltonian:

Starting with the Fujii-Primakoff Hamiltonian

$$H_{eff} = \frac{1}{2} \tau_L^{+} (1 - \vec{\sigma_L} \cdot \hat{\nu})) \sum_{i=1}^{A} \Big[G_V 1_L \cdot 1_i + G_A \vec{\sigma_L} \cdot \vec{\sigma_i} - G_P(\sigma_L \cdot \hat{\nu})$$

$$(\vec{\sigma_i} \cdot \hat{\nu}) - \frac{g_V}{M} (\vec{\sigma_L} \cdot \hat{\nu}) (\vec{\sigma_L} \cdot \vec{p_i}) - \frac{g_A}{M} (\vec{\sigma_L} \cdot \hat{\nu}) (\vec{\sigma_i} \cdot \vec{p_i})$$

$$\delta(Y - Y_i)$$

we may write the matrix element for muon capture process as

$$Q = \langle u | \Omega | u_\mu \rangle$$

where u_ν and u_μ are the Dirac spinors for muon neutrino and muon respectively and

$$\Omega = \frac{1}{2} (1 - \vec{\sigma_L} \cdot \hat{\nu}) \left\{ \mathcal{M}_1 + \vec{\sigma_L} \cdot \vec{\mathcal{M}_2} \right\}$$

$\mathcal{M}_1$ and $\vec{\mathcal{M}_2}$ are given by

$$\mathcal{M}_1 = G_V M_1 - \frac{g_V}{M} (\hat{\nu} \cdot \vec{M_3}) \qquad (60)$$

$$\vec{\mathcal{M}_2} = G_A \vec{M_2} - \check{G}_P(\hat{\nu} \cdot \vec{M_2}) - i \frac{g_V}{M} (\hat{\nu} \times \vec{M_3}) - \frac{g_A}{M} M_4 \hat{\nu} \qquad (61)$$

We now rewrite eqns. (60) and (61) in an explicit form wherein we exhibit space and time components of the weak hardonic bare vector and axial currents as follows:

$$\mathcal{M}_1 = g_V \int 1_i^o + g_V \frac{\nu}{2M} \int 1_i^s - \frac{g_V}{M} \hat{\nu} \cdot \int p_i^s \tag{62a}$$

$$\begin{aligned} \vec{\mathcal{M}}_2 = {} & g_A \int \sigma_i^s - g_V \frac{\nu}{2M} \int \sigma_i^s - g_M \hat{\nu} \cdot \int p_i^s - \frac{\nu}{2M} \Big(g_P \hat{\nu} \int \hat{\nu} \cdot \sigma_i^s \\ & - g_A \hat{\nu} \int \hat{\nu} \cdot \sigma_i^o - g_V \hat{\nu} \int \hat{\nu} \cdot \sigma_i^s - g_M \hat{\nu} \int \hat{\nu} \cdot \sigma_i^s \Big) \\ & - i \frac{g_V}{M} \int \hat{\nu} \cdot p_i^s - \frac{g_A}{M} \hat{\nu} \int (\sigma_i \cdot p_i)^o \end{aligned} \tag{62b}$$

In writing eqns. (62a) and (62b) we have used the following :

$$G_V = g_V \left(1 + \frac{\nu}{2M}\right)$$

$$G_A = g_A - (g_V + g_M) \frac{\nu}{2M}$$

$$G_P = (g_P - g_A - g_V - g_M + g_T) \frac{\nu}{2M} \ .$$

In the above equations, 1_i, σ_i and p_i are the nucleon unit, Pauli spin and momentum operators and the superscripts 0 and S stand for contributions from time and space part of bare vector and axial vector currents

In view of the arguments given in Section (9b), the MEC effects on the space part of axial vector current (the $g_A \int \sigma_i^s$ term in eqn. (62b)) is very small. In fact, the calculations of Rho [22] and Towner and Khanna [24] show that the space part of axial current is related to the π-nuclear scattering amplitude and the result of their analysis is that g_A is redefined as $g_A/1+\alpha$, where α is the polarizability parameter. Though there is substantial quenching in nuclear matter ($\sim 25\%$), the calculations of Towner and Khanna [24] reveal a very small amount of quenching of g_A ($\sim 1\%$) in light nuclei. Therefore we neglect MEC effects on the space part of axial current.

We denote the nuclear matrix elements of time component of IA current (one body operator) and meson exchange axial current (two body operator) by $\langle A^o_{IA} \rangle$ and $\langle A^o_{MEC} \rangle$ respectively. Then the ratio $\langle A^o_{MEC} \rangle / \langle A^o_{IA} \rangle$ (denoted by F) is a measure of the effect of MEC corrections to Impulse Approximation. The evaluation of F consists in evaluating $\langle A^o_{MEC} \rangle$ with specific nuclear wave-functions. The reason why the $^{16}O(0^+)(\mu^-, \nu_\mu)^{16}N(0^-)$ partial capture rate is sensitive to MEC effects is that, the partial capture rate is sensitive to momentum dependent terms i.e. the term $\frac{g_A}{M} \hat{\nu} \int (\vec{\sigma}_i \cdot \vec{p}_i)^o$ in eqn. (62b). It is precisely this term which is affected by meson exchange corrections. The importance of nucleon momentum dependent terms in the $0^+ \longrightarrow 0^-$

partial capture rate was first pointed out by Rood [25] in an entirely different context. Thus we may conclude that any observable sensitive to momentum dependent terms will be a good candidate for detecting the MEC effects. Having added meson exchange corrections, we may drop the superscripts 0 and s, absorb F in the coupling constants and redefine $\vec{\mathcal{M}}_2$ as

$$\vec{\mathcal{M}}_2 = G_A \int \vec{\sigma}_i - G_P' \hat{\nu} \int \hat{\nu} \cdot \vec{p}_i - i \frac{g_V}{M} \hat{\nu} \times \int \vec{p}_i - \frac{g_A'}{M} \hat{\nu} \int \vec{\sigma}_i \cdot \vec{p}_i \tag{63}$$

where $G_P' = (g_P - Fg_A - g_V - g_M)\,\nu/2M$ and $g_A' = Fg_A$. We calculate the effect of F on muon capture observables, such as partial capture rate, recoil polarization $(\vec{P}_N)$ and the gamma-neutrino angular correlation coefficient (β_2), by varying F from 0 to 0.5 (corresponding to 50% MEC). In view of the fact that MEC effects on the space part of axial vector current are small, our calculations should be viewed as corrections to the impulse approximation rather than an indication for MEC effects. Numerical results are presented in Section 11.

10. Nuclear Models.

In eqns. (23), (35) and (38), the angular correlation coefficients α, β_1 and β_2 are expressed in terms of nuclear reduced matrix elements and muon capture coupling constants. We have evaluated α , β_1 and β_2 in the pure shell model (PSM) and the particle hole model of Donnelly and Walker (DW) [26] . The justification for using DW wave-functions to describe the $^{28}Al^{*}(1^{+}$, 2202 KeV) state is as follows:

It has been pointed out by Uberall [27] that the 1^{+} states of the final nucleus in muon capture are analogous to the M1 excitation states in inelastic electron scattering. The process of inelastic electron scattering leading to 1^{+} final nuclear levels has been extensively studied by Donnelly and Walker [26] using the Serber-Yukawa residual interaction and they find that the excitation of 1^{+} at 13.67 MeV in ^{28}Si is dominant at a momentum transfer of 100 MeV/c. Comparing this with the experimental studies of Miller [1] which indicate that the 2202 KeV 1^{+} level of $^{28}Al^{*}$ is the dominant transition in muon capture (at the same momentum transfer of 100 MeV/c), we have used the DW wave-functions to evaluate the correlation coefficient in process (1).

10(a). Particle Hole Formalism:

The particle-hole (hereafter referred to as p-h) wave-function of the final state can be written as

$$|J_f M_f\rangle = \sum_{ph} \sum_{m_p\, m_h} X^{J_f}_{ph} (-1)^{j_h+m_h} C(j_p j_h J_f \,,\, m_p m_h M_f)\, a^{+}_{p m_p} a_{h-m_h} |0\rangle \quad (64)$$

where p(particle) is used to denote a typical unoccupied state, characterised by a set of quantum numbers (n_p, l_p, j_p) , h (hole) is used to denote a typical occupied state, characterised by a set of quantum numbers (n_h, l_h, j_h), the ket $|0\rangle$ is the Hartree-Fock ground state and $X_{p,h}$ are the configuration mixing coefficients associated with the p-h configurations with normalization

$$\sum_{ph} \left| X^{J_f}_{ph} \right|^2 = 1 \tag{65}$$

In equation (64) $a^+(a)$ denote creation (annihilation) operators specified by the particle and hole labels on them. They satisfy the following anticommutation relations for fermions :

$$\left\{ a^+_\alpha , a^+_\beta \right\} = \left\{ a_\alpha, a_\beta \right\} = 0$$

$$\left\{ a^+_\alpha , a_\beta \right\} = \delta_{\alpha\beta} \tag{66}$$

and when acting on the Hartree-Fock ground state, have the following properties

$$a^+_h \; |0\rangle = 0$$

$$a_p \; |0\rangle = 0 \tag{67}$$

For a general nuclear transition operator expressible in spherical tensor form,

$$\sum_{i=1}^{A} \left(\sum_{\lambda\, m_\lambda} t_\lambda^{m_\lambda} \right)_i \tag{68}$$

In second quantized formalism, we have

$$\sum_{i=1}^{A} \left(\sum_{\lambda m_\lambda} t_\lambda^{m_\lambda} \right)_i = \sum_{\alpha\beta} \left\langle \alpha \left| \sum_{\lambda, m_\lambda} t_\lambda^{m_\lambda} \right| \beta \right\rangle a_\alpha^+ a_\beta \tag{69}$$

where α and β represent single particle states. The corresponding matrix element (ME) may be written as

$$\text{ME} = \left\langle J_f M_f \left| \sum_{i=1}^{A} \sum_{\lambda m_\lambda} (t_\lambda^{m_\lambda})_i \right| 00 \right\rangle \tag{70}$$

This is the form in which all the matrix elements in eqns. (28), (35) and (38) occur. The matrix element in second quantized form can now be written as

$$\text{ME} = \sum_{ph} \sum_{m_p m_h} \sum_{\alpha\beta} X_{ph}^{J_f} (-1)^{j_h+m_h} C(j_p j_h J_f , m_p m_h M_f)$$

$$\left\langle \alpha \left| \sum_{\lambda m_\lambda} t_\lambda^{m_\lambda} \right| \beta \right\rangle \left\langle 0 \left| a_{h-m_h}^+ a_{pm_p} a_\alpha^+ a_\beta \right| 0 \right\rangle$$

In the above equation we do not include isospin coupling of the particle and hole, as we will be dealing only with $T = 1$ final states and hence the coupling of isospin to $T_f = 1$ understood. Using the anticommutation relations for a's and a^+'s given in eqns. (66), we obtain

$$\text{ME} = \sum_{ph} \sum_{m_p m_h} \sum_{\lambda m_\lambda} X_{ph}^{J_f} (-1)^{j_h+m_h} C(j_p j_h J_f , m_p m_h M_f)$$

$$\left\langle p\, m_p \left| t_\lambda^{m_\lambda} \right| h - m_h \right\rangle$$

Applying Wigner-Eckart theorem [11]

$$ME = \sum_{ph} \sum_{m_p m_h} \sum_{\lambda m_\lambda} X_{ph}^{J_f} (-1)^{j_h+m_h} C(j_p j_h J_f , m_p m_h M_f)$$

$$\langle p \| t_\lambda \| h \rangle \quad C(j_h \lambda j_p , -m_h m_\lambda m_p) \qquad (71)$$

Using the orthogonality and symmetry properties of Clebsch-Gordan coefficients [11], eqn. (71) can be simplified to

$$ME = \sum_{ph} \sum_{\lambda m_\lambda} \frac{[j_p]}{[J_f]} X_{ph}^{J_f} \langle j_p \| t_\lambda \| j_h \rangle \delta_{J_f \lambda} \delta_{M_f M_\lambda} \qquad (72)$$

This is the final result which is used to rewrite the matrix elements in eqns. (23), (35) and (38) in the p-h model. It may be mentioned here that the pure shell model (PSM) wave function may be recovered from the above formalism by putting all the X_{ph}'s equal to 1, corresponding to a pure configuration, which in our case is the particle-hole configuration $(1d_{3/2})(1d_{5/2})^{-1}$. The explicit expressions for typical terms is given in Appendix IV. The radial wavefunctions are taken to be the harmonic oscillator wave functions with the oscillator strength parameter

$$b = 1.80 \text{ fm.} \qquad (73)$$

The radial integrals occuring in the nuclear matrix elements and which are defined in Appendix V are of the form (for momentum independent terms)

$$\langle j_\ell(\nu r)\rangle_{ph} = \int_0^\infty R_{n_p l_p}(r)\, j_\ell(\nu r)\, R_{n_h l_h}(r)\, r^2\, dr \qquad (74)$$

with the R_{nl} being the harmonic-scillator radial wavefunctions

and $\nu = m_\mu - (E_f - E_i)$

where E_i is the energy of the initial nuclear state.

E_f is the energy of the final nuclear state.

and m_μ is the muon mass.

The radial integrals for momentum dependent nuclear matrix elements are given by

$$F_- = \int_0^\infty R_{n_p l_p}(r)\, j_\ell(\nu r) \left[\frac{d}{dr} - \frac{l_h}{r}\right] R_{n_h l_h}(r)\, r^2\, dr \qquad (75)$$

and

$$F_+ = \int_0^\infty R_{n_p l_p}(r)\, j_\ell(\nu r) \left[\frac{d}{dr} + \frac{l_h+1}{r}\right] R_{n_h l_h}(r)\, r^2\, dr \qquad (76)$$

The above radial integrals ane analytically evaluated using the method of De Forest and Walecka [28] :

$$\int_0^\infty R_{n'l'}(r)\, j_L(qr)\, R_{nl}(r)\, r^2\, dr = \frac{2^L}{(2L+1)!!}\, e^{-y}((n'-1)!\,(n-1)!)^{1/2}$$

$$\left(\Gamma(n'+\ell'+1/2)\; \Gamma(n+\ell+1/2)\right)^{1/2} \sum_{m'=0}^{n'-1} \sum_{m=0}^{n-1} \frac{(-1)^{m'+m}}{m'!\; m!} \; \frac{1}{(n'-m'-1)!\,(n-m-1)!}$$

$$\frac{\Gamma\left(\frac{1}{2}(\ell'+\ell + 2m' + 2m + L + 3)\right)}{(m' + \ell' + 3/2) \qquad (m+\ell+3/2)} \; F\left(\tfrac{1}{2}(L-\ell'-\ell-2m'-2m);\; L + 3/2;\; y\right) \tag{77}$$

with $y = \frac{b^2q^2}{4}$ (where q is the momentum transfer of the process)

$$F(\alpha, \beta, y) = 1 + \frac{\alpha}{\beta} y + \frac{\alpha}{\beta}\frac{\alpha+1}{\beta+1}\frac{y^2}{2!} + \ldots\ldots$$

Γ is the gamma function.

For the momentum dependent radial integrals, we may reduce the derivatives to the above form using the following relations:

$$\left(\frac{d}{dr} - \frac{\ell}{r}\right) R_{1\ell} = -\frac{1}{b}\,(\ell+3/2)^{1/2}\, R_{\ell+1} \tag{78}$$

$$\left(\frac{d}{dr} + \frac{\ell+1}{r}\right) R_{1\ell} = \frac{1}{b}\,(2(2\ell+1))^{1/2}\, R_{1\ell-1} - \frac{1}{b}\,(\ell+3/2)^{1/2}\, R_{1\ell+1} \tag{79}$$

11. Numerical Results and Discussion :

(a) Numerical Results:- In Table I, we present numerical values for the correlation coefficients α, β_1 and β_2 in the particle-hole model of Donnelly and Walker [26] . Models I and II are without and with momentum dependent terms respectively. It is to be noted that the relation $1 + \alpha = \beta_1 + \beta_2$ is satisfied almost exactly in Table 1.

TABLE I

$(g_P+g_T)/g_A$	α		β_1		β_2	
	Model I	Model II	Model I	Model II	Model I	Model II
-10.0	-0.07799	0.02441	0.0042	0.00581	0.91759	1.01850
- 7.5	-0.02030	0.08465	0.00032	0.00962	0.97938	1.0750
- 5.0	-0.03934	0.14566	0.00119	0.01969	1.03810	1.12590
- 2.5	0.10026	0.20645	0.00799	0.03694	1.09220	1.16950
0	0.1616	0.26581	0.02170	0.0622	1.13980	1.20350
2.5	0.2222	0.32231	0.04325	0.09629	1.17890	1.2260
5.0	0.28070	0.37435	0.07349	0.13963	1.2072	1.2347
7.5	0.33569	0.42026	0.11309	0.19243	1.2246	1.2278
10.0	0.38531	0.45839	0.16243	0.25449	1.2228	1.2038
12.5	0.42789	0.48721	0.22150	0.32514	1.2063	1.1620
15.0	0.46178	0.50552	0.28986	0.40327	1.1719	1.1022
17.5	0.48553	0.51248	0.36656	0.48735	1.11890	1.0251
20.0	0.49810	0.5078	0.45018	0.57551	1.04790	0.93288

In Table II, a comparison of α, β_1 and β_2 in FPA and Models I and II, alongwith other theoretical estimates and experimental data for the PCAC estimate of g_P, is presented. Models I and II are without and with nucleon momentum dependent terms:

TABLE II

Correlation coefficients	Ciechanowicz's values given by Mukhopadhyay [39]	FPA	Model I	Model II	Expt. [1]
α	0.4	0.2925	0.3357	0.4203	0.15 ± 0.25 0.29 ± 0.3
β_1	0.88	0.0809	0.11309	0.19243	0.02 ± 0.03
β_2	0.53	1.2115	1.2246	1.2278	1.12 ± 0.10

In Table III, we display numerical values for the partial capture rate in the process $\mu^- + {}^{28}Si(0^+) \longrightarrow {}^{28}Al^*(1^+ ;\ 2202\ KeV) + \nu_\mu$ in the particle-hole model of Donnelly and Walker [26] with and without MEC effects. The values have been rescaled by ξ^2 where ξ is the " amplitude reduction factor' (see section (b) for discussion of ξ).

TABLE III

$(g_P+g_T)/g_A$	Impulse Approximation no MEC (units of 10^5 sec^{-1})	with 50% MEC (units of 10^5 sec^{-1})	
-10.0	0.6792	0.6872	
- 7.5	0.6406	0.6482	
- 5.0	0.6056	0.6124	
- 2.5	0.5742	0.5802	
0.0	0.5464	0.5518	Expt. [1]
2.5	0.5222	0.5255	(0.484 ± 0.086)
5.0	0.5017	0.5055	$\times 10^5$ Sec^{-1}
7.5	0.4845	0.4877	
10.0	0.4712	0.4735	
12.5	0.4613	0.4629	
15.0	0.4548	0.4559	
17.5	0.4523	0.4526	
20.0	0.4532	0.4528	

In Table IV, numerical values for recoil polarization of $^{28}Al^{*}(1^{+}$, 2202 KeV) in the process $\mu^{-} + {}^{28}Si(0^{+}) \longrightarrow {}^{28}Al^{*}(1^{+};$ 2202 KeV) $+ \nu_{\mu}$ are given in (i) Independent Particle Model (IPM) (ii) particle-hole model of Donnelly and Walker [26] with and without MEC effects.

TABLE IV

$\frac{(g_P+g_T)}{g_A}$	IPM		DW	
	IA	with 50% MEC	IA	with 50% MEC
-10.0	0.6784	0.6784	0.6789	0.6789
- 7.5	0.6759	0.6787	0.6769	0.6777
- 5.0	0.6692	0.6709	0.6706	0.6723
- 2.5	0.6576	0.6570	0.6595	0.6621
0	0.6405	0.6442	0.6427	0.6466
2.5	0.6175	0.6226	0.6199	0.6250
5.0	0.5881	0.5945	0.5906	0.5970
7.5	0.5522	0.5598	0.5547	0.5624
10.0	0.5099	0.5188	0.5121	0.5212
12.5	0.4616	0.4717	0.4635	0.4737
15.0	0.4081	0.4192	0.4095	0.4207
17.5	0.3504	0.3623	0.3511	0.3630
20.0	0.2898	0.3021	0.2898	0.3022

(b). Discussion:

(i) Gamma-Neutrino Angular Correlation Coefficients:

It is seen from Table I that the relation $1 + \alpha = \beta_1 + \beta_2$ is satisfied almost exactly testifying to the correctness of our numerical calculations.

From Table II, a comparison of the numerical values of α, β_1 and β_2 in FPA and Model I shows that higher order neutrino partial waves contribute significantly to α and β_1, but not so much to β_2. Similarly comparing α, β_1 and β_2 in FPA and Model II, it can be seen that nucleon momentum dependent terms enhance α and β_1 but not β_2. In FPA, the nuclear matrix elements in α, β_1 and β_2 cancel and any deviation of the calculated values of α, β_1 and β_2 from the FPA estimate can be taken as an indication of nuclear physics effects through higher order neutrino partial waves. The exact values of α and β_1 (0.4203 and 0.19243 at $g_P = 7.5\ g_A$) are widely different from their FPA values (0.2925 and 0.0809), while β_2 is nearly the same (1.2278 in exact calculation and 1.2115 in FPA). Hence we conclude that only β_2 is truly insensitive to nuclear physics uncertainties and therefore can be used to obtain a value for $(g_P+g_T)/g_A$ by comparison with experiment. This is in contradiction to the conclusion of Popov et. al. [3] that all the correlation coefficients are nuclear model insensitive.

By comparing our value for β_2 with the experiment of Miller et. al. [1] , we obtain

$$(g_P + g_T) / g_A = (13.5 \; {}^{+3.5}_{-5.5}) \; g_A$$

which is to a large extent free from nuclear wavefunction uncertainties. This is in agreement with our analysis of $^{12}B(1^+, g.s.)$ recoil polarization (see next chapter) and with that of Kobayashi et. al. [29] , who find $(g_P + g_T) g_A = (10.3 \pm 2.7)$. It is in contradiction with the value of Ciechanowicz [5] who finds

$$-4.9 < (g_P + g_T) / g_A < 1.2$$

In Figure 1, we display graphically the effect of meson exchange corrections on β_2, calculated along the lines discussed in Section 9. It is clear from the graph that MEC effects are quite small for an allowed transition dominated by the space part of axial current. However, MEC effects decrease the numerical value of β_2 upto $g_P \sim 10 \; g_A$ and then enhance it uniformly. By comparing with experiment [1] , we find two sets of g_P/g_A values:

Set I : (-6.65 ± 4.3) in IA and (-9.1 ± 3.1) with 50% MEC

Set II : (12.5 ± 5) in IA and (12.9 ± 3.9) with 50% MEC .

Set I obviously contradicts PCAC and by comparing with the value of g_P/g_A obtained from our analysis of recoil nuclear polarization $(g_P/g_A = 13.62 \pm 2.17)$, we choose set II to obtain a final value for $(g_P + g_T) / g_A$ as

$$(g_P + g_T) / g_A = (13.3 \pm 3)\ g_A$$

a value to a large extent free from nuclear wavefunction uncertainties. The MEC effects to the space part of the IA axial vector current do not change our conclusions. This value of $(g_P + g_T)/g_A$ indicates a remote possibility of quenching of g_P in the $A = 28$ system and with $g_P/g_A = 7.5$, we obtain $g_T/g_A = (5.8 \pm 3)$.

The above value of g_T can at best be taken as a qualitative indicator of the induced tensor form factor in muon capture; this is because, as has been pointed out by Wilkinson [30], g_T is only an effective form factor deduced from Lorentz invariance arguments and one has to extract information from g_T using microscopic models in which second class currents are identified with specific meson exchanges, NN ρ vertices etc. However, in view of recent experiments on $P_{av.}/P_L$ by Roesch et. al. [31] and on ^{12}B alignment by Roesch et. al. [32], which seem to show conclusively the absence of SCC, our analysis can be interpreted as $g_P \simeq (13.3 \pm 3)\, g_A$, which is consistent with the values of Kobayashi [29] and the recent Argonne National Laboratory measurement [33] in β decay of $^{16}N(0^-)$. On the other hand, assuming $g_P = 7.5\ g_A$, our results indicate an <u>upper bound</u> for g_T as $(5.8 \pm 3)\, g_A$ to be compared with the upper limit obtained by Bardin and Zavattini [34] , namely $g_T \sim (0.3 \pm 1.9)$.

In conclusion, we give below a table, wherein we compare our value of g_P/g_A with values of g_P/g_A obtained from various processes in muon capture.

TABLE V

No.	Observable used in muon capture	Nuclei	Ref.	Range for g_P/g_A .
1	Recoil polarization	^{12}B	[35]	7.1 ± 2.7
2	β-decay and μ-capture	^{16}O	[33]	10.0 ± 2.5
3	Alignment	^{12}B	[32]	9.4 ± 1.7
4	$P_{av.}/P_L$	^{12}B	[31]	9.0 ± 1.7
5	Capture Rate	Hydrogen	[34]	8.7 ± 1.9
6	Recoil Polarization	^{12}B	[36]	15.0 ± 4.0
7	γ- ν angular correlation	^{28}Si	[5]	$-4.9 < g_P/g_A < 1.2$
8	γ- ν angular correlation	^{28}Si	present work	$(13.5 {}^{+3.5}_{-5.5})$
9	Recoil polarization	^{12}B	present work	(13.62 ± 2.1).

(ii) Partial Capture Rate:

From Table III, it is seen that the partial capture rate λ is not very much affected by MEC effects, since we are considering an allowed transition dominated by space part of the axial vector current. The value obtained by Ciechanowicz [5] is: $\lambda(1^+) = 8.16 \times 10^5\ sec^{-1}$ at $g_P = 5\ g_A$ in disagreement with the experiment of Miller et. al. [1] which yields $\lambda(1^+) = (0.484 \pm 0.086) \times 10^5\ sec^{-1}$. We obtain a value of $\lambda(1^+) = 3.964 \times 10^5\ sec^{-1}$ and $4.1036 \times 10^5\ sec^{-1}$ for $g_P = 7.5\ g_A$ and $g_P = 5\ g_A$ respectively, using Donnelly Walker [26] wavefunctions, obtaining considerable improvement over the values of Ciechanowicz. The particle-hole model of Donnelly and Walker can further be improved by taking into account the effect of many particle many hole wavefunctions through the introduction of "amplitude reduction factor" ξ. This factor was introduced by Donnelly and Walecka [37] for the $A = 12$ system ; they have studied a variety of semi-leptonic weak interactions in nuclei and the reduction factor ξ was introduced to scale the TDA particle-hole amplitudes purely by comparison with experiments. The value of $\xi = 2.27$ deduced, for example, by comparing β decay rate with experiment was found to account for other processes such as muon capture rates etc. By comparing our $\lambda(1^+)$ at $g_P = 7.5 g_A$

with experiment [1] , we obtain $\xi = 2.86$ in conformity with the value of Donnelly and Walecka [37] and the value for $\lambda(1^+)$ given in Table III are rescaled (divided) by ξ^2.

(iii) Recoil Nuclear Polarization:

In Table IV, we present numerical values for the average recoil polarization ($P_{av.}$) of $^{28}Al^*(1^+; 2202$ KeV) in the Independent Particle Model (IPM) and the particle hole model of Donnelly and Walker. It is clear from the table that $P_{av.}$ is to a large extent insensitive to the choice of nuclear wavefunctions and that MEC effects are negligible due to the dominance of Gamow-Teller operator (space part of axial vector current) in an allowed $0^+ \longrightarrow 1^+$ transition. We note here that the introduction of ξ (amplitude reduction factor) does not affect $P_{av.}$, because $P_{av.}$ involves ratio of reduced matrix elements.

While the average recoil polarization of ^{12}B can be measured by the known β decay asymmetry of the recoiling ^{12}B nucleus, the same method is not applicable to the case of $^{28}Al^*(1^+)$ since it decays by γ-emission. One way of measuring $P_{av.}$ would be to look for the circular polarization of emergent gamma-rays, which is related to $P_{av.}$ through the relation (P_c is the circular polarization of γ-rays)

$$P_c = -3/2\, P_{av.} \cos\theta$$

as shown by Parthasarathy [38] .

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APPENDIX I

Evaluation of $(\rho^{\mu c})_{M_f M_f'}$:

In this appendix, we derive explicitly a few terms in eqns. (14) and (15) to illustrate techniques of angular momentum algebra involved in the evaluation of $(\rho^{\mu c})_{M_f M_f'}$. For convenience, we shall treat the cases of unpolarized and polarized muon capture separately.

(i) Unpolarized Muon Capture:

$G_A^2 \; M_2 \cdot M_2^*$ term : The Matrix element $\vec{M_2}$ is

$$\vec{M_2} = \langle J_f M_f | \sum_{i=1}^{A} e^{-i\hat{\nu}\cdot\vec{r_i}} \phi_\mu(r_i)\, \vec{\sigma_i}\, |00\rangle \tag{A1}$$

Expressing $e^{-i\hat{\nu}\cdot\vec{r_i}}$ in partial waves and $\vec{\sigma_1}$ in spherical basis,

$$e^{-i\hat{\nu}\cdot\vec{r_i}} = 4\pi \sum_{lm} (i)^{-l}\, Y_l^{-m}(\hat{\nu})\,(-1)^m\, Y_l^m(r_i)\, j_l(\nu r_i) \tag{A2}$$

$$\vec{\sigma} = \sum_\mu \sigma_1^\mu (-1)^\mu \hat{\xi}_1^{-\mu} \tag{A3}$$

eqn. (A1) can now be written as

$$\vec{M}_2 = \langle J_f M_f | \sum_{lm\mu\lambda} 4\pi (i)^{-l} (-1)^{m+\mu} Y_l^{-m}(\hat{\nu}) \hat{\xi}_1^{-\mu} j_l(\nu r_i)$$

$$C(l 1 \lambda; m \mu m_\lambda) [Y_l^m(\hat{r}_i) \times \sigma_1^\mu]_\lambda^{m_\lambda} |00\rangle$$

where we have coupled $Y_l^m(\hat{r}_i)$ and σ_1^μ. We next apply Wigner-Eckart theorem to the above expression to obtain the condition $\delta_{\lambda J_f} \delta_{m_\lambda M_f}$. Therefore,

$$\vec{M}_2 = \sum_{lm\mu} 4\pi (i)^{-l} (-1)^{M_f} Y_l^{-m}(\hat{\nu}) \hat{\xi}_1^{-\mu} C(l 1 J_f; m \mu M_f)$$

$$\langle J_f \| [Y_l(\hat{r}_i) \times \sigma_1]_{J_f} j_l(\nu r_i) \| 0 \rangle \tag{A4}$$

Similarly $\vec{M}_2^*$ can be reduced to

$$\vec{M}_2^* = \sum_{l'm'\mu'} 4\pi (i)^{l'} Y_{l'}^{m'}(\hat{\nu}) C(l' 1 J_f; m' \mu' M_f')$$

$$\langle J_f \| [Y_{l'}(\hat{r}_i) \times \sigma_1]_{J_f} j_{l'}(\nu r_i) \| 0 \rangle^* \tag{A5}$$

Combining (A4) and (A5) and noting that

$$(-1)^{\mu} \xi_1^{-\mu} \cdot \xi_1^{\mu'} = \delta_{\mu\mu'} \tag{A6}$$

we obtain

$$\vec{M}_2 \cdot \vec{M}_2^{\,*} = \sum_{lm\mu} \sum_{l'm'} 16\pi^2 (i)^{l'-l} (-1)^{M_f} Y_l^{-m}(\hat{\nu}) Y_{l'}^{m'}(\hat{\nu})$$

$$C(l1J_f; m\mu M_f)\, C(l'1J_f; m'\mu M_f')$$

$$\langle J_f \| [Y_l(\hat{r}_i) \times \sigma_1]_{J_f}\, j_l(\nu r_i) \| 0 \rangle$$

$$\langle J_f \| [Y_{l'}(\hat{r}_i) \times \sigma_1]_{J_f}\, j_{l'}(\nu r_i) \| 0 \rangle^* \tag{A7}$$

Combining the two spherical harmonics using the relation

$$Y_{J_1}^{M_1}(\hat{\nu})\, Y_{J_2}^{M_2}(\hat{\nu}) = \sum_J \frac{[J_1][J_2]}{\sqrt{4\pi}\,[J]} C(J_1 J_2 J; 000)$$

$$C(J_1 J_2 J; M_1 M_2 M)\, Y_J^M(\hat{\nu}) \tag{A8}$$

we obtain

$$\vec{M}_2 \cdot \vec{M}_2^{\,*} = \sum_{lm\mu} \sum_{l'm'J} 16\pi^2 (i)^{l'-l} (-1)^{M_f} \frac{[l][l']}{\sqrt{4\pi}\,[J]}$$

$$\underline{C(l1J_f; m\mu M_f)}\; \underline{C(l'1J_f; m'\mu M_f')}$$

$$\underline{C(ll'J; m m' M_J)}\; C(ll'J; 000)\; Y_J^{M_J}(\hat{\nu})$$

$$\langle J_f \| [Y_l(\hat{r}_i) \times \sigma_1]_{J_f}\, j_l(\nu r_i) \| 0 \rangle$$

$$\langle J_f \| [Y_{l'}(\hat{r}_i) \times \sigma_1]_{J_f}\, j_{l'}(\nu r_i) \| 0 \rangle^*$$

Combining the three underlined Clebsch-Gordan coefficients into a Racah and Clebsch-Gordan coefficient, we finally obtain,

$$G_A^2 \overrightarrow{M_2} \cdot \overrightarrow{M_2}^* = G_A^2 \sum_{\ell \ell' J} 16\pi^2 (i)^{\ell'-\ell} (-1)^{\ell'-J_f} (-1)^{M_f}$$

$$\frac{[\ell][\ell'][J_f]^2}{\sqrt{4\pi}\,[J]} \, C(\ell\ell'J;000) \, C(J_f J_f J; -M_f M_f' M_J)$$

$$W(J_f 1 J \ell'; \ell J_f) \, Y_J^{M_J}(\hat{\nu}) \, |\phi_\mu|^2_{av.}$$

$$\langle J_f \| [Y_\ell(\hat{r}_i) \times \sigma_1]_{J_f} \, j_\ell(\nu r_i) \| 0 \rangle$$

$$\langle J_f \| [Y_{\ell'}(\hat{r}_i) \times \sigma_1]_{J_f} \, j_{\ell'}(\nu r_i) \| 0 \rangle^* \qquad \text{(A9)}$$

In the above expression, we have factored the muon wavefunction (ϕ_μ) out of the matrix element by assuming an average value over the entire nucleus. This applies to all the subsequent evaluation of matrix elements.

$(G_P^2 - 2\, G_P\, G_A) \quad |\hat{\nu} \cdot M_2|^2$ term:

In this case, expanding $\hat{\nu}$ in spherical basis

$$\hat{\nu} = \sqrt{\frac{4\pi}{3}} \sum_\eta Y_1^\eta(\hat{\nu}) \, (-1)^\eta \, \xi_1^{-\eta} \qquad \text{(A10)}$$

Combining with expression for $\overrightarrow{M_2}$ in (A 4) and using eqns. (A3) and (A8), we obtain

$$(\hat{\nu}\cdot\overrightarrow{M_2}) = \sum_{l m \mu \lambda} 4\pi\sqrt{\frac{4\pi}{3}}\,(i)^{-l}\,(-1)^{M_f}\,C(l1\lambda;\,m\mu m_\lambda)$$

$$C(l1J_f;\,m\mu M_f)\;C(l1\lambda;\,000)\,\frac{[l][1]}{\sqrt{4\pi}\,[\lambda]}\,Y_\lambda^{-m_\lambda}$$

$$\langle J_f\,\|\,[Y_l(r_i)\times\sigma_1]_{J_f}\,j_l(\nu r_i)\,\|\,0\rangle$$

Using orthonormality of Clebsch-Gordan coefficients, we obtain the condition $\delta_{\lambda J_f}\;\delta_{m_\lambda M_f}$. Therefore,

$$(\hat{\nu}\cdot\overrightarrow{M_2}) = \sum_{l} 4\pi\,\frac{[l]}{[J_f]}\,(i)^{-l}\,(-1)^{M_f}\,C(l1J_f;\,000)\,Y_{J_f}^{-M_f}(\hat{\nu})$$

$$\langle J_f\,\|\,[Y_l(\hat{r}_i)\times\sigma_1]_{J_f}\,j_l(\nu r_i)\,\|\,0\rangle \tag{A11}$$

Similarly,

$$(\hat{\nu}\cdot\overrightarrow{M_2})^* = \sum_{l'} 4\pi\,(i)^{l'}\,\frac{[l']}{[J_f]}\,C(l'1J_f;\,000)\,Y_{J_f}^{M_f'}(\hat{\nu})$$

$$\langle J_f\,\|\,[Y_{l'}(\hat{r}_i)\times\sigma_1]_{J_f}\,j_{l'}(\nu r_i)\,\|\,0\rangle^*$$

Using eqn. (A8), the final expression may be written as

$$\left(G_P^2 - 2G_P G_A\right) |\hat{\nu}\cdot\vec{M}_2|^2 = \left(G_P^2 - 2G_P G_A\right) \sum_{\ell\ell' J} 16\pi^2 (i)^{\ell'-\ell}$$

$$(-1)^{M_f} \frac{[\ell][\ell']}{\sqrt{4\pi}\,[J]} \; C(\ell 1 J_f; 000)\; C(\ell' 1 J_f; 000)$$

$$C(J_f J_f J; 000)\; C(J_f J_f J; -M_f M_f' M_J)\; Y_J^{M_J}(\hat{\nu})$$

$$\langle J_f \| [Y_\ell(\hat{r}_i)\times\sigma_1]_{J_f}\, j_\ell(\nu r_i) \| 0\rangle$$

$$\langle J_f \| [Y_{\ell'}(\hat{r}_i)\times\sigma_1]_{J_f}\, j_{\ell'}(\nu r_i) \| 0\rangle^* \tag{A12}$$

$2\,(G_P - G_A)\,\frac{g_A}{M}\,(\hat{\nu}\cdot\vec{M}_2)\,M_4^*$ <u>term</u> :

The M_4 matrix element is

$$M_4 = \langle J_f M_f | \sum_{i=1}^{A} e^{-i\hat{\nu}\cdot\vec{r}_i}\, \vec{\sigma}_i\cdot\vec{P}_i | 00\rangle \tag{A13}$$

Using eqns. (A2), (A3) and (A6), expanding $\vec{p}_i$ in spherical basis

$$\vec{p}_i = \sum_\lambda (i)^{-1}\, \vec{\nabla}_1^{\lambda}\, (-1)^\lambda\, \xi_1^{-\lambda} \tag{A14}$$

we obtain

$$M_4 = \sum_{l m \mu \lambda \epsilon} 4\pi (i)^{l-1} (-1)^{m+\mu} C(l1\lambda;\, m-\mu\ m_\lambda)\, Y_l^{-m}(\hat{\nu})$$

$$C(\lambda 1 \epsilon;\, m_\lambda\, \mu\, m_\epsilon)\, \left[\{Y_l(\hat{r}_i) \times \nabla_1\}_\lambda \times \sigma_1\right]_\epsilon^{m_\epsilon} j_l(\nu r_i)$$

where we have combined $Y_l(\hat{r}_i)$, ∇ and σ by means of Clebsch Gordan coefficients. Using Wigner-Eckart theorem and orthogonality conditions for Clebsch-Gordan coefficients gives the conditions $\delta_{\epsilon J_f}$ $\delta_{m_\epsilon M_f}$ and $\delta_{l J_f}$ $\delta_{m M_f}$ respectively. Hence, M_4 reduces to

$$M_4 = \sum_{\lambda} 4\pi (i)^{-J_f+1} (-1)^{J_f+M_f-\lambda} \frac{[\lambda]}{[J_f]} Y_{J_f}^{-M_f}(\hat{\nu})$$

$$\langle J_f \| [Y_l(r_i) \times \nabla_1]_{J_f}\, j_l(\nu r_i) \| 0 \rangle$$

(A15)

Combining (A15) and (A11) and using eqn. (A8), we obtain finally,

$$\frac{2}{M}(G_P - G_A)\, g_A\, (\hat{\nu}\cdot\vec{M}_1)\, M_4^* = \frac{2}{M}(G_P - G_A)\, g_A$$

$$\sum_{l\lambda J} (i)^{-J_f+l-1} (-1)^{\lambda - J_f - M_f} \frac{[\lambda][l]}{\sqrt{4\pi}\,[J]} C(l1J_f;000)$$

$$C(J_f J_f J;\, 000)\, C(J_f J_f J;\, -M_f\, M_f'\, M_J)\, Y_J^{M_J}(\hat{\nu})\, j_l(\nu r_i)$$

(A16)

$$\langle J_f \| [\{Y_{l'}(\hat{r}_i) \times \nabla_1\}_\lambda \times \sigma_1]_{J_f}\, j_{l'}(\nu r_i)] \| 0\rangle^* \langle J_f \| [Y_l \times \sigma_1]_{J_f}^{j_l(\nu r)} \| 0 \rangle$$

$\frac{2}{M} G_A g_V i \vec{M_2} \cdot (\hat{\nu} \times \vec{M_3}^*)$ term:

The matrix element $\vec{M_3}$ is

$$\vec{M_3} = \langle J_f M_f | \sum_{i=1}^{A} e^{-i\hat{\nu}\cdot\vec{r_i}} \vec{p_i} \phi_\mu(r_i) | 00 \rangle \quad \text{(A17)}$$

Expanding $e^{-i\hat{\nu}\cdot\vec{r_i}}$ in partial waves and $\vec{p_i}$ in spherical basis as given in (A2) and (A14), and applying Wigner-Eckart theorem

$$\vec{M_3} = \sum_{\ell m \lambda} 4\pi (i)^{\ell-1} (-1)^{m+\lambda} \hat{\xi}_1^{-\lambda} c(\ell 1 J_f; m \lambda M_f) Y_\ell^{-m}(\hat{\nu})$$

$$\langle J_f \| [Y_\ell(\hat{r}_i) \times \nabla_1]_{J_f} j_\ell(\nu r_i) \| 0 \rangle \quad \text{(A18)}$$

Therefore

$$\vec{M_3}^* = \sum_{\ell' m' \lambda'} 4\pi (i)^{-\ell'+1} Y_{\ell'}^{m'}(\hat{\nu}) \hat{\xi}_1^{\lambda'} c(\ell' 1 J_f; m' \lambda' M_f')$$

$$\langle J_f \| [Y_{\ell'}(\hat{r}_i) \times \nabla_1]_{J_f} j_{\ell'}(\nu r_i) \| 0 \rangle^* \quad \text{(A19)}$$

Using eqn. (A10) for $\hat{\nu}$, we can evaluate $(\hat{\nu} \times \vec{M_3}^*)$ with the help of the following relation for spherical basis unit vectors:

$$\hat{\xi}_1^{-\eta} \times \hat{\xi}_1^{\lambda'} = i\sqrt{2} \left(c(111; -\eta\ \lambda'\ -(\eta-\lambda')\right) \hat{\xi}_1^{-(\eta-\lambda')} \quad \text{(A20)}$$

$$(\hat{\nu} \times \overrightarrow{M_3}^*) = \sum_{l' m' \lambda' \eta} 4\pi \sqrt{\frac{4\pi}{3}} \sqrt{2}\, (i)^{l'-2} (-1)^{\eta} \hat{\xi}_1^{-(\eta-\lambda')}$$

$$Y_1^{\eta}(\hat{\nu})\, Y_{l'}^{m'}(\hat{\nu})\, C(l' 1 J_f; m' \lambda' M_f')\, C(111; -\eta\, \lambda'\, -(\eta-\lambda'))$$

$$\langle J_f \| [Y_{l'}(r_i) \times \nabla_1]_{J_f}\, j_{l'}(\nu r_i) \| 0 \rangle^* \tag{A21}$$

Combinding the two spherical harmonics according to (A8) and performing standard angular momentum algebra yields,

$$(\hat{\nu} \times \overrightarrow{M_3}^*) = \sum_{l' \alpha m_\alpha} 4\pi \sqrt{2}\, (i)^{-l'+2} (-1)^{l'-\alpha} [l'][1]$$

$$C(l' 1 \alpha; 000)\, C(\alpha 1 J_f; m_\alpha\, M_f' - \alpha\, M_f')\, W(\alpha 1 J_f 1; l' 1)$$

$$Y_\alpha^{m_\alpha}(\hat{\nu})\, \langle J_f \| [Y_{l'}(r_i) \times \nabla_1]_{J_f}\, j_{l'}(\nu r_i) \| 0 \rangle^* \tag{A22}$$

Combining (A4) and (21), we obtain

$$i \overrightarrow{M_2} \cdot (\hat{\nu} \times \overrightarrow{M_3}^*) = \sum_{l m \mu} \sum_{l' \alpha m_\alpha} 16\pi^2 \sqrt{2}\, (i)^{l'-l+3} (-1)^{M_f}$$

$$W(\alpha 1 J_f 1; l' 1)\, C(l' 1 \alpha; 000)\, C(l 1 J_f; m\, \mu\, M_f)\, Y_\alpha^{m_\alpha}(\hat{\nu})$$

$$C(\alpha 1 J_f; m_\alpha\, (M_f' - m_\alpha)\, M_f')\, \hat{\xi}_1^{-\mu} \cdot \hat{\xi}_1^{(M_f' - m_\alpha)}\, Y_l^{-m}(\hat{\nu})$$

$$\langle J_f \| [Y_l(\hat{r}_i) \times \sigma_1]_{J_f}\, j_l(\nu r_i) \| 0 \rangle\, \langle J_f \| [Y_{l'}(\hat{r}_i) \times \nabla_1]_{J_f}\, j_{l'}(\nu r_i) \| 0 \rangle^*$$

Using (A8) and carrying out the standard angular momentum algebra yields the final expression:

$$\frac{2G_A g_V}{M}\, i\,\vec{M_2}\cdot(\hat{\nu}\times\vec{M_3}^{\,*}) = \frac{2G_A g_V}{M}\sum_{\ell\ell'\lambda J} 16\pi^2\sqrt{2}\,(i)^{\ell'-\ell+3}$$

$$(-1)^{M_f}\ \frac{[\ell][\ell'][\lambda][1]}{\sqrt{4\pi}\ [J]}\ C(\ell 1\lambda;000)\ C(\lambda\ell' J;000)$$

$$W(J_f 1\lambda 1;\ell 1)\ W(J_f\lambda J_f \ell';1J)\ C(J_f J_f J;-M_f' M_f M_J)\ Y_J^{M_J}(\hat{\nu})$$

$$\langle J_f\|[Y_\ell(\hat{r}_i)\times\sigma_i]_{J_f}\, j_\ell(\nu r_i)\|0\rangle\ \langle J_f\|[Y_{\ell'}(\hat{r}_i)\times\nabla_i]_{J_f}\, j_{\ell'}(\nu r_i)\|0\rangle^* \tag{A23}$$

(ii) <u>Polarized Muon Capture:</u>

From eqn. (17), it is seen readily that the matrix elements of four terms viz.,

$$G_A^2\,\vec{M_2}\cdot\vec{M_2}^{\,*}\,(\vec{P}\cdot\hat{\nu}),\quad G_P^2\,|\hat{\nu}\cdot\vec{M_2}|^2\,(\vec{P}\cdot\hat{\nu}),\quad \frac{2G_P g_A}{M}\,(\hat{\nu}\cdot\vec{M_2})\,M_4^*\,(\vec{P}\cdot\hat{\nu})$$

and $\frac{2G_A g_V}{M}\, i\,\vec{M_2}\cdot(\hat{\nu}\times M_3^*)\,(\vec{P}\cdot\hat{\nu})$ are the same as in the previous expressions for unpolarized muon capture, except that they are multiplied by an extrafactor $(\vec{P}\cdot\hat{\nu})$. Therefore, we turn our attention to the next two terms:

<u>$2(G_A - G_P)\,G_A\,(\hat{\nu}\cdot\vec{M_2})\,(\vec{P}\cdot\vec{M_2}^{\,*})$ term :</u>

From (A11), we have

$$(\hat{\nu}\cdot\vec{M_2}) = \sum_{\ell} 4\pi (i)^{-\ell} (-1)^{M_f} \frac{[\ell]}{[J_f]} C(\ell 1 J_f; 000)$$

$$Y_{J_f}^{-M_f}(\hat{\nu}) \langle J_f \| [Y_\ell(\hat{r}_i) \times \sigma_1]_{J_f} j_\ell(\nu r_i) \| 0 \rangle$$

We exp$_a$nd the muon polarization vector $\vec{P}$ in spherical basis.

$$\vec{P} = P\sqrt{\frac{4\pi}{3}} \sum_{\beta} (-1)^\beta Y_1^\beta(\hat{P})\, \hat{\xi}_1^{-\beta} \tag{A24}$$

Combining (A5) and (A22),

$$(\vec{P}\cdot\vec{M_2}^*) = \sum_{\ell' m' \mu'} 4\pi \sqrt{\frac{4\pi}{3}} (i)^{+\ell'} Y_{\ell'}^{m'}(\hat{\nu}) Y_1^{\mu'}(\hat{P})$$

$$C(\ell' 1 J_f; m' \mu' M_f') \langle J_f \| [Y_{\ell'}(\hat{r}_i) \times \sigma_1]_{J_f} j_\ell(\nu r_i) \| 0 \rangle^* \tag{A25}$$

Therefore

$$(\hat{\nu}\cdot\vec{M_2})(\vec{P}\cdot\vec{M_2}^*) = \sum_{\ell\ell' m' \mu'} 16\pi^2 \sqrt{\frac{4\pi}{3}} (-1)^{M_f} (i)^{\ell'-\ell}$$

$$C(\ell 1 J_f; 000)\, C(\ell' 1 J_f; m' \mu' M_f')\, Y_{J_f}^{-M_f}(\hat{\nu})\, Y_{\ell'}^{m'}(\hat{\nu})$$

$$Y_1^{\mu'}(\hat{P}) \langle J_f \| [Y_\ell(\hat{r}_i) \times \sigma_1]_{J_f} j_\ell(\nu r_i) \| 0 \rangle$$

$$\langle J_f \| [Y_{\ell'}(\hat{r}_i) \times \sigma_1]_{J_f} j_{\ell'}(\nu r_i) \| 0 \rangle^*$$

Combining the two spherical harmonics according to (A8),

$$(\hat{\nu}\cdot\vec{M_2})(\vec{P}\cdot\vec{M_2}^*) = \sum_{\ell\ell' m'\mu'\mathcal{L}} \sum_{J} \frac{16\pi^2}{\sqrt{3}} (i)^{\ell'-\ell} (-1)^{M_f}$$

$$\frac{[\ell][\ell']}{[\mathcal{L}]} C(\ell 1 J_f;000)\ C(J_f \ell' \mathcal{L};000)\ \underline{C(\ell' 1 J_f; m'\mu' M_f')}$$

$$\underline{C(J_f \ell' \mathcal{L}; -M_f m' M_{\mathcal{L}})}\ \underline{C(\mathcal{L} 1 J; M_{\mathcal{L}} \mu' M_J)}$$

$$[Y_{\mathcal{L}}^{M_{\mathcal{L}}}(\hat{\nu}) \times Y_1^{\mu'}(\hat{P})]_J^{M_J} \langle J_f \| \cdots \| 0\rangle \langle J_f \| \cdots \| 0\rangle^*$$

Combining the three underlined Clebsch-Gordan coefficients and summing over μ' yields the final expression:

$$2(G_A - G_P)\, G_A\, (\hat{\nu}\cdot\vec{M_2})(\vec{P}\cdot\vec{M_2}^*) = 2(G_A - G_P)\, G_A$$

$$\sum_{\ell\ell'} \sum_{\mathcal{L}J} \frac{1}{\sqrt{3}} (i)^{\ell'-\ell} (-1)^{M_f} [\ell][\ell'][J_f]\ C(\ell 1 J_f;000)$$

$$C(J_f \ell' \mathcal{L};000)\ W(J_f \ell' J 1; \mathcal{L} J_f)\ C(J_f J_f J; -M_f M_f' M_J)$$

$$\langle J_f \| [Y_\ell(\hat{r}_i)\times\sigma_1]_{J_f}\, j_\ell(\nu r_i) \| 0\rangle \langle J_f \| [Y_{\ell'}(\hat{r}_i)\times\sigma_1]_{J_f}\, j_{\ell'}(\nu r_i) \| 0\rangle^* \quad (A26)$$

In similar fashion, combining (A15) and (A23) the nucleon momentum dependent term $\frac{2}{M} G_A\, g_A\, M_4\, (\vec{P}\cdot\vec{M_2}^*)$ can be evaluated and the final result is given below:

$$\frac{2}{M} G_A g_A M_4 (\vec{P} \cdot \vec{M_2}^*) = \frac{2}{M} G_A g_A \sum_{l \lambda J} (i)^{-l+J_f+1}$$

$$(-1)^{M_f} [l][\lambda] \; C(l J_f \mathcal{L}; 000) \; W(J_f 1 J_f \mathcal{L}; 1J)$$

$$[Y_L(\hat{\nu}) \times Y_1(\hat{P})]^{m_{\mathcal{L}}}_{\mathcal{L}} \langle J_f \| [\{Y_l(\hat{r}_i) \times \nabla_i\}_\lambda \times \sigma_1]_{J_f} \, j_l(\nu r_i) \|0\rangle$$

$$\langle J_f \| [Y_{l'}(\hat{r}_i) \times \sigma_1]_{J_f} \, j_{l'}(\nu r_i) \|0\rangle^* \qquad \text{(A27)}$$

APPENDIX II.

As stated in Section 5, the $\gamma - \nu$ angular correlation coefficients are obtained by substituting for $(\rho^{\mu c})_{M_f M_f'}$ from eqns. (16) and (17) in eqn. (20). We give below the resulting expressions for each term after simplification.

(i) Unpolarized Muon Capture:

G_A^2 term :

$$|a(M1)|^2 |\langle 0^+ \| M1 \| 1^+\rangle|^2 \sum_{l l' J} B(J) \, (i)^{l'-l} \, (-1)^{l'-J_f}$$

$$[l][l'][J_f]^2 \, C(l l' J; 000) \, W(J_f 1 J l'; l J_f) \, I(l 1 J_f; l' 1 J_f) \qquad \text{(A1)}$$

where

$$B(J) = -\left[1 + (-1)^J\right] C(LLJ; 1-10)\, W(J_f L J_f L; J_F J) \qquad (A2)$$

Putting $J = 2,\ J_f = L = 1,\ J_F = 0$ in (A1), we obtain the contribution of G_A^2 term to the numerator of the expression for α given in eqn. (23)

J = 2 part :

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1} [\ell][\ell'] [1]^2\ C(\ell\ell'2; 000)\ W(112\ell'; \ell 1)\ I(\ell 11; \ell' 11) \qquad (A2)'$$

where we have used the fact that $B(2) = -2\,C(112\,;\,1-10)$

$W(1111\,,\,02) = -\dfrac{2}{3\sqrt{6}}$.

J = 0 part : $J_f = L = 1,\ J_F = 0.$

Noting that $B(o) = 2\,C(110,\ 1-10)$ $W(1111,\ 00) = -\dfrac{2}{3\sqrt{3}}$,

we obtain

$$-|a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+ \rangle|^2\ \frac{2}{3\sqrt{3}} \left\{ -\sum_{\ell} [1]\ I(\ell 11; \ell 11) \right\} \qquad (A3)$$

which gives the contribution of G_A^2 term to the denominator of α . In deriving (A3) we made use of the following:

$$C(\ell \ell' 0; 000) = \frac{(-1)^{\ell}}{[\ell]} \delta_{\ell\ell'} \quad \text{and} \quad W(110\ell'; \ell 1) = \frac{1 \cdot \delta_{\ell\ell'}}{[1][\ell]}$$

In similar fashion, we now give the J = 2 (numerator) and J = 0 (denominator) parts (with $J_f = L = 1$, $J_F = 0$) of other terms which contribute to α.

$(G_P^2 - 2\, G_P\, G_A)$ term:

$$|a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell' J} B(J)\, (i)^{\ell'-\ell} \, [\ell][\ell']$$
$$C(\ell 1 J_f; 000)\, C(\ell' 1 J_f; 000)\, C(J_f J_f J; 000)\, I(\ell 1 J_f, \ell' 1 J_f) \qquad (A4)$$

J = 2 part:

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} \, [\ell][\ell']$$
$$C(\ell 11; 000)\, C(\ell' 11; 000) \sqrt{\frac{2}{3}}\, I(\ell 11; \ell' 11) \qquad (A5)$$

J = 0 part :

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \left\{ - \sum_{\ell\ell'} (i)^{\ell'-\ell} \frac{[\ell][\ell']}{[1]} \right.$$
$$C(\ell 11; 000)\, C(\ell' 11; 000)\, I(\ell 11; \ell' 11) \qquad (A6)$$

In deriving the above equations, we have used the following:

$$C(112\ ;\ 000) = \sqrt{\tfrac{2}{3}} \quad \text{and} \quad C(110;\ 000) = \frac{(-1)}{[1]}$$

$\frac{2}{M}(G_P - G_A)\,g_A$ term :

$$|a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+\rangle|^2 \sum_{\ell\lambda J} B(J)\,(i)^{-J_f+\ell-1}\,(-1)^{\lambda-J_f}$$

$$[\lambda][\ell]\ C(\ell 1 J_f;000)\ C(J_f J_f J;000)\ \mathcal{G}(\ell 1 J_f; J_f 1 \lambda 1 J_f) \tag{A7}$$

J = 2 part:

$$-\frac{2}{3\sqrt{6}}\,|a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+\rangle|^2 \left\{-\sum_{\ell\lambda}(i)^{\ell-2}(-1)^{\lambda-J_f}\right.$$

$$[\lambda][\ell]\ C(\ell 11;000)\ \mathcal{G}(\ell 11;11\lambda 11)\sqrt{\tfrac{2}{3}} \tag{A8}$$

J = 0 part

$$-\frac{2}{3\sqrt{3}}\,|a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+\rangle|^2 \left\{-\sum_{\ell\lambda}(i)^{\ell-2}(-1)^{\lambda-J_f}\right.$$

$$\frac{[\lambda][\ell]}{[1]}\ C(\ell 11;000)\ \mathcal{G}(\ell 11;11\lambda 11) \tag{A9}$$

$\frac{2}{M} G_A g_V$ term :

$$|a(M1)|^2 \, |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell' \lambda J} B(J) \sqrt{2} \, (i)^{\ell' - \ell + 3}$$

$$[\lambda][\ell][\ell'][1][J_f]^2 \; c(\ell 1 \lambda; 000) \; c(\lambda \ell' J; 000)$$

$$W(11\lambda 1; \ell 1) \; W(1\lambda 1 \ell'; 1J) \; \mathcal{G}(\ell 11; \ell' 101)$$

(A10)

J = 2 part:

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell' \lambda} \sqrt{2} \, (i)^{\ell' - \ell + 3}$$

$$[\lambda][\ell][\ell'][1]^3 \; c(\ell 1 \lambda; 000) \; c(\lambda \ell' J; 000)$$

$$W(11\lambda 1; \ell 1) \; W(1\lambda 1 \ell'; 12) \; \mathcal{G}(\ell 11; \ell' 101) \qquad \text{(A11)}$$

J = 0 part:

Noting that

$$c(\lambda \ell' 0; 000) = \frac{(-1)^{\ell'}}{[\ell']} \delta_{\lambda \ell'} \quad \text{and}$$

$$W(1\lambda 1 \ell'; 10) = \frac{(-1)^{\ell'}}{[1][\ell']} \quad \text{, we obtain}$$

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell'} \sqrt{2} \, (i)^{\ell' - \ell + 3}$$

$$[\ell][1]^2 \; c(\ell 1 \ell'; 000) \; W(11\ell' 1; \ell 1) \; \mathcal{G}(\ell 11; \ell' 101) \qquad \text{(A12)}$$

(ii) Polarized Muon Capture:

G_A^2 term : The expression derived in the text was

$$|a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell' J} B(J) (i)^{\ell'-\ell} (-1)^{\ell'-1}$$

$$[\ell][\ell'][1]^2 \, c(\ell \ell' J; 000) \; W(11J\ell'; \ell 1) \; (\vec{P} \cdot \hat{\gamma})$$

$$\left[c(J1J+1; 000)^2 \, P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J \, c(J1J-1; 000)^2 \, P_{J-1}(\cos\theta_{\gamma\nu}) \right]$$

$$I(\ell 11; \ell' 11) \tag{A13}$$

with B(J) defined as in A(2). Putting J = 2 and expressing P_3 in terms of P_2 and P_1 (as given in eqn. (34a) of the text),

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1} [\ell]$$

$$[\ell'][1]^2 \, c(\ell \ell' 2; 000) \; W(112\ell'; \ell 1) \; (\vec{P} \cdot \hat{\gamma}) \left[\frac{3}{5} \left\{ \frac{5}{3} \right. \right.$$

$$\left. \left. (\hat{\gamma} \cdot \hat{\nu}) \, P_2(\cos\theta_{\gamma\nu}) - \frac{2}{3} P_1(\cos\theta_{\gamma\nu}) \right\} + \frac{2}{5} P_1(\cos\theta_{\gamma\nu}) \right]$$

$$I(\ell 11; \ell' 11) \tag{A14}$$

From the above equation, the coefficient of $(\vec{P} \cdot \hat{\gamma})(\hat{\gamma} \cdot \hat{\nu}) \, P_2(\cos\theta_{\gamma\nu})$ is seen to be

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell \ell'} (i)^{\ell'-\ell} (-1)^{\ell'-1}$$

$$[\ell][\ell'][1]^2 \, c(\ell \ell' 2; 000) \; W(112\ell'; \ell 1) \, I(\ell 11; \ell' 11) \tag{A15}$$

which is the first term in the numerator of β_1. For J = 2, the coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})$ is seen to be zero

$$(\vec{P}\cdot\hat{\gamma})\left[-3/5 \cdot 2/3\, P_1(\cos\theta_{\gamma\nu}) - \frac{2}{5} P_1(\cos\theta_{\gamma\nu})\right] = 0$$

However, putting J = 0 in eqn. (A13) yields the $(\vec{P} \cdot \hat{Y})$ $(\hat{Y} \cdot \hat{\nu})$ coefficient, since $\eta_J = 0$ for $(J-1) < 0$. So the coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})$, which contributes to the numerator β_2 is

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \left[-\sum_{\ell} [1]\, I(\ell 11; \ell' 11)\right] \qquad \text{(A16)}$$

G_P^2 term :

$$|a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell' J} B(J)\,(i)^{\ell'-\ell}\, [\ell][\ell']\, c(\ell 11; 000)\, c(\ell' 11; 000)\, c(11J; 000)\, I(\ell 11; \ell' 11)\, (\vec{P}\cdot\hat{\gamma}) \left[c(J1J+1; 000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J\, c(J1J-1; 000)^2 P_{J-1}(\cos\theta_{\gamma\nu})\right] \qquad \text{(A17)}$$

J = 2 part :

The coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})\, P_2(\cos\theta_{\gamma\nu})$ is

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell][\ell']\, c(\ell 11; 000)\, c(\ell' 11; 000) \sqrt{\frac{2}{3}}\, I(\ell 11; \ell' 11) \qquad \text{(A18)}$$

This is the G_P^2 term in the numerator of β_1.

J = 0 part :

The coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})$ is

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \left\{ - \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell] \right.$$

$$\frac{[\ell']}{[1]} c(\ell 1 1; 000) \; c(\ell' 1 1; 000) \; I(\ell 1 1; \ell' 1 1)$$

(A19)

which is represents the G_P^2 term in the numerator of β_2.

$\frac{2}{M} G_A g_V$ term :

$$|a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'\lambda J} B(J) \sqrt{2} (i)^{\ell'-\ell+3}$$

$$[\ell][\ell'][1]^3 [\lambda] \; c(\ell 1 \lambda; 000) \; c(\ell' \lambda J; 000) (\vec{P} \cdot \hat{Y})$$

$$W(1 1 \lambda 1; \ell 1) \; W(1 \lambda 1 \ell'; 1 J) \; \mathcal{G}(\ell 1 1; \ell' 1 1 0 1) \left\{ c(J 1 J+1; \right.$$

$$\left. 000)^2 P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J \, c(J 1 J-1; 000)^2 P_{J-1}(\cos\theta_{\gamma\nu}) \right]$$

(A20)

J = 2 part :

The coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu}) P_2(\cos\theta_{\gamma\nu})$ is

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'\lambda} \sqrt{2} \; (i)^{\ell'-\ell+3}$$

$$[\ell][\ell'][1]^3 [\lambda] \; c(\ell 1 \lambda; 000) \; c(\lambda \ell' 2; 000)$$

$$W(1 1 \lambda 1; \ell 1) \; W(1 \lambda 1 \ell'; 1 2) \; \mathcal{G}(\ell 1 1; \ell' 1 1 0 1) \qquad \text{(A21)}$$

which is the $\frac{2}{M}$ G_A g_V term in the numerator of β_1.

J = 0 part :

The coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})$ is

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'} \sqrt{2}\,(i)^{\ell'-\ell+3} [\ell]$$

$$[1]^2\, c(\ell 1 \ell'; 000)\, W(11\ell'1; \ell 1)\, \mathcal{G}(\ell'11; \ell 1101) \qquad \text{(A22)}$$

which is the $\frac{2}{M}$ G_A g_V term in the numerator of β_2 .

$\frac{2}{M}$ G_P g_A term :

$$|a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\lambda J} B(J)\,(i)^{-\ell+J_f-1} (-1)^{\lambda-J_f}$$

$$[\ell][\lambda]\; c(J_f J_f J; 000)\; c(\ell 1 J_f; 000)\; \mathcal{G}(\ell 1 J_f; J_f 1 \lambda 1 J_f)$$

$$(\vec{P} \cdot \hat{Y}) \left[c(J 1 J+1; 000)^2\, P_{J+1}(\cos\theta_{\gamma\nu}) + \eta_J\, c(J 1 J-1; \right.$$

$$\left. 000)^2\, P_{J-1}(\cos\theta_{\gamma\nu}) \right] \qquad \text{(A23)}$$

J = 2 part :

The coefficient of $(\vec{P} \cdot \hat{Y})(\hat{Y} \cdot \hat{\nu})\, P_2(\cos\theta_{\gamma\nu})$ is

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1}$$

$$[\ell][\lambda] \sqrt{\frac{2}{3}}\; c(\ell 11; 000)\; \mathcal{G}(\ell 11; 11\lambda 11)$$

(A24)

representing the $\frac{2}{M} G_P g_A$ term in the numerator of β_1.

J = 0 part :

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \left\{ - \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} \frac{[\ell][\lambda]}{[1]} \; C(\ell 11;000) \; G(\ell 11; 11\lambda 11) \right\} \tag{A25}$$

representing the $\frac{2}{M} G_P g_A$ term in the numerator of β_2.

$2(G_P - G_A)\, G_A$ term :

The expression derived in the text is

$$|a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'\mathcal{L}} B(J)\, (i)^{\ell'-\ell} [\ell][\ell'] [1][\mathcal{L}] \; C(\ell 11;000)\, C(\ell' 1 \mathcal{L};000)\, W(\mathcal{L} 111; \ell' J) \; C(\mathcal{L} 1 J;000)\; I(\ell 11; \ell' 11)\; P_{\mathcal{L}}(\cos\theta_{\gamma\nu}) \; (\vec{P}\cdot\hat{\gamma}) \tag{A26}$$

Putting J = 2, we find from the parity Clebsch $C(\mathcal{L}1J;000)$ that $\mathcal{L}$ can take values 1 and 3. Therefore

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 \; |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell] [\ell'] [1] \; C(\ell 11;000)\, I(\ell 11; \ell' 11) \Big[[1]\, C(112;000)$$

$$C(\ell'11;000)\ W(1111;\ell'2)\ P_1(\cos\theta_{\gamma\nu})\ (\vec{P}\cdot\hat{\gamma}) + [3]$$

$$C(\ell'13;000)\ W(3111;\ell'2)\ C(312;000)\ (\vec{P}\cdot\hat{\gamma})$$

$$\left\{\frac{5}{3}(\hat{\gamma}\cdot\hat{\nu})\ P_2(\cos\theta_{\gamma\nu}) - \frac{2}{3}\ P_1(\cos\theta_{\gamma\nu})\right\}\Bigg]$$

(A27)

Where we have expressed P_3 in terms of P_2 and P_1. Noting that $C(112;000) = \sqrt{2/3}$ and $C(312;000) = \sqrt{3/7}$, the coefficient of $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})\ P_2(\cos\theta_{\gamma\nu})$ is

$$-\frac{2}{3\sqrt{6}}\ |a(M1)|^2\ |\langle 0^+\|M1\|1^+\rangle|^2\ \sum_{\ell\ell'}(i)^{\ell'-\ell}\ [\ell][\ell']$$

$$[1]\ [3]\ C(\ell 11;000)\ C(\ell'13;000)\ W(3111;\ell'2)\sqrt{3/7}$$

$$(5/3)\ I(\ell 11;\ell'11)$$

(A28)

giving the $2(G_P - G_A)\ G_A$ term in the numberator of β_1.

It is seen from eqn. (A27), that there are two terms which are coefficients of $(\vec{P}\cdot\hat{\gamma})(\hat{\gamma}\cdot\hat{\nu})$ and contribute to the numerator of β_2. These are given below:

$$-\frac{2}{3\sqrt{6}}\ |a(M1)|^2\ |\langle 0^+\|M1\|1^+\rangle|^2\ \sum_{\ell\ell'}(i)^{\ell'-\ell}\ [\ell][\ell']$$

$$\sqrt{\frac{2}{3}}\ [1]^2\ C(\ell 11;000)\ C(\ell'11;000)\ W(1111;\ell'2)\ I(\ell 11;\ell'11)$$

(A29)

and

$$-\frac{2}{3\sqrt{6}}\ |a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \Big\{ \sum_{\ell\ell'} (i)^{\ell'-\ell} [\ell]$$

$$[\ell'][1]^2 C(\ell'13;000)\ C(\ell11;000)\ W(3111;\ell'2)\ (\tfrac{2}{3})$$

$$I(\ell 11;\ \ell' 11) \tag{A30}$$

In addition to the above two terms, there is a further contribution to the numerator of β_2, which is obtained by putting $J = 0$ in eqn. (A26), this gives $\mathcal{L} = 1$, and the final expression is

$$-\frac{2}{3\sqrt{3}} \Big\{ -\sum_{\ell\ell'} (i)^{\ell'-\ell} (-1)^{\ell'} \frac{[\ell][\ell']}{[1]}\ C(\ell11;000)\ C(\ell'11;$$

$$000)\ I(\ell 11;\ \ell' 11) \Big\} \tag{A31}$$

$\frac{2\,G_A\,g_A}{M}$ term : -

$$|a(M1)|^2\ |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \quad \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-\mathcal{L}} [\lambda][\ell]$$

$$[1]\ [\mathcal{L}]\ C(\ell' 1 \mathcal{L};000)\ W(1\ell' J_f 1;\ 1\mathcal{L})\ C(\mathcal{L} 1 J_f;000)$$

$$P_{\mathcal{L}}(\cos\theta_{\gamma\nu})\ (\vec{p}\cdot\hat{\gamma}) \tag{A32}$$

As explained above for the $2(G_A - G_P)G_A$ term, putting $J = 2$ in the eqn.(A 32), we obtain one term in the numerator of β_1 and two in the numerator of β_2. Further, the $J = 0$ part of eq.(32) accounts for the third term of the numerator of β_2.

We give below the relevant expressions:

$$-\frac{2}{3\sqrt{6}} \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-3} [\lambda] [\ell] [1] [3] \, c(\ell 13;00)$$

$$W(1\ell' 21;13) \left(\sqrt{\tfrac{3}{7}} \cdot \tfrac{5}{3}\right) \mathcal{G}(\ell 11;11\lambda 11) \tag{A33}$$

which contributes to β_1.

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-3} [\lambda]$$

$$[\ell] [1] [3] \, c(\ell' 13;000) \, W(1\ell' 21;13) \cdot \frac{2}{3}\sqrt{\frac{3}{7}}$$

$$\mathcal{G}(\ell 11;11\lambda 11) \tag{A34}$$

and

$$-\frac{2}{3\sqrt{6}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1}$$

$$[\lambda] [\ell] [1]^2 \sqrt{\frac{2}{3}} \, c(\ell' 11;000) \, W(1\ell' 21;11)$$

$$\mathcal{G}(\ell 11;11\lambda 11) \tag{A35}$$

with $J = 0$ we obtain

$$-\frac{2}{3\sqrt{3}} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \left\{ -\sum_{\ell\lambda} (i)^{\ell-2} (-1)^{\lambda-1} \right.$$

$$\frac{[\lambda][\ell]}{[1]} \, c(\ell' 11;000) \, \mathcal{G}(\ell 11;11\lambda 11) \tag{A36}$$

Equations (A34), (A35) and (A36) contribute to the numerator of β_2.

APPENDIX III.

Matrix elements for Partial Capture Rate:

The matrix elements for partial capture rate can be evaluated in the same way as shown in Appendix I. However, an integration over neutrino directions should be carried out using the following eqn.

$$\int Y_J^{M_J}(\hat{\nu})\, d\Omega_\nu = \sqrt{4\pi}\ \delta_{J0}\ \delta_{M_J 0}\ .$$

The final expressions for the matrix elements occurring in eqn. (49) of the text are given below:

$$M_1 M_1^* = 16\pi^2 [J_f]^2 \langle J_f \| \sum_{n=1}^{A} Y_{J_f}(\hat{r}_n)\, j_{J_f}(\nu r_n) \| 0 \rangle \langle J_f \| \sum_{n=1}^{A} Y_{J_f}(\hat{r}_n)\, j_{J_f}(\nu r_n) \| 0 \rangle^* |\phi_\mu|^2_{av.}$$

$$\vec{M_2} \cdot \vec{M_2}^* = 16\pi^2 \sum_{\ell} [J_f]^2 |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} [Y_\ell(\hat{r}_n) \times \sigma_1(n)]_{J_f}\, j_\ell(\nu r_n) \| 0 \rangle \langle J_f \| \sum_{n=1}^{A} [Y_\ell(\hat{r}_n) \times \sigma_1(n)]_{J_f}\, j_\ell(\nu r_n) \| 0 \rangle^*$$

$$(\hat{\nu}\cdot\overrightarrow{M_2})(\hat{\nu}\cdot\overrightarrow{M_2}^{*}) = 16\pi^2 \sum_{\ell\ell'} [\ell][\ell'] \, C(\ell 1 J_f;000)$$

$$C(\ell' 1 J_f;000)\, |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} [Y_\ell(\hat{r}_n)\times\sigma_1(n)]_{J_f}$$

$$j_\ell(\nu r_n)\|0\rangle \langle J_f \| \sum_{n=1}^{A} [Y_{\ell'}(\hat{r}_n)\times\sigma_1(n)]_{J_f} j_{\ell'}(\nu r_n)|0\rangle^{*}$$

$$M_1\cdot(\hat{\nu}\cdot\overrightarrow{M_3}^{*}) = 16\pi^2 \sum_{\ell} [J_f][\ell](i)^{-\ell-1-J_f}(-1)^{J_f}$$

$$C(\ell 1 J_f;000)\, |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} Y_{J_f}(\hat{r}_n)\, j_{J_f}(\nu r_n)\|0\rangle$$

$$\langle J_f \| \sum_{n=1}^{A} [Y_\ell(\hat{r}_n)\times\nabla_1(n)]_{J_f} j_\ell(\nu r_n)\|0\rangle^{*}$$

$$i\overrightarrow{M_2}\cdot(\hat{\nu}\times\overrightarrow{M_3}^{*}) = 16\pi^2 \sum_{\ell\ell'} (i)^{\ell'-\ell+1}\sqrt{2}\,[J_f]^2[\ell'][1]$$

$$C(\ell' 1\ell;000)\, W(J_f 1\ell 1;\ell' 1)\, |\phi_\mu|^2_{av.} \langle J_f \| [Y_\ell(\hat{r}_n)\times$$

$$\sigma_1(n)]_{J_f}\|0\rangle \langle J_f \| \sum_{n=1}^{A} j_{\ell'}(\nu r_n)[Y_{\ell'}(\hat{r}_n)\times\nabla_1(n)]_{J_f}\|0\rangle$$

$$M_4\cdot(\hat{\nu}\cdot\overrightarrow{M_2}^{*}) = 16\pi^2 \sum_{\ell\lambda} (i)^{-\ell+J_f+1}(-1)^{\lambda-J_f}[\lambda][\ell]$$

$$C(\ell 1 J_f;000)\, |\phi_\mu|^2_{av.} \langle J_f \| \sum_{n=1}^{A} [Y_{\ell'}(\hat{r}_n)\times\sigma_1(n)]_{J_f}$$

$$j_{\ell'}(\nu r_n)\|0\rangle^{*} \langle J_f \| \sum_{n=1}^{A} j_{J_f}(\nu r_n)\, [(Y_{J_f}(\hat{r}_n)\times\nabla_1(n))_\lambda \times\sigma_1(n)]_{J_f}\|0\rangle^{*}$$

APPENDIX IV

Nuclear Matrix Elements in Particle-Hole Model:

The basic relation used to rewrite the nuclear matrix elements in particle hole (p-h) model is

$$\langle J_f \| \sum_{j=1}^{A} \sum_{\lambda} (O_\lambda)_j \| 0 \rangle = \sum_{ph} \frac{[j_p]}{[J_f]} X_{ph}^{J_f} \langle j_p \| O_\lambda \| j_h \rangle \delta_{\lambda J_f} \delta_{m_\lambda M_f} \qquad \text{(IV.1)}$$

For example the $G_A^2 \overrightarrow{M_2} \cdot \overrightarrow{M_2}^*$ can be written in p-h model as

$$G_A^2 \overrightarrow{M_2} \cdot \overrightarrow{M_2}^* = \sum_{\ell\ell' J} \sum_{ph} \sum_{p'h'} 16\pi^2 (i)^{\ell'-\ell} (-1)^{\ell'-J_f-M_f} \frac{[\ell][\ell'][j_p][j_{p'}]}{\sqrt{4\pi}\,[J_f]^2 [J]} X_{ph}^{J_f} X_{p'h'}^{J_f} C(\ell\ell' J;000)$$

$$W(J_f 1 J \ell';\ell J_f)\, C(J_f J_f J; -M_f M_f' M_J)\, Y_J^{M_J}(\hat{\nu})\, |\phi_\mu|^2_{av.}$$

$$\langle j_p \| \{Y_\ell(\hat{r}_i) \times \sigma_1\}_{J_f} \| j_h \rangle \langle j_{p'} \| \{Y_{\ell'}(\hat{r}_i) \times \sigma_1\}_{J_f} \| j_{h'} \rangle$$

$$\langle j_\ell(\nu r_i) \rangle_{ph} \langle j_{\ell'}(\nu r_i) \rangle_{p'h'} \qquad \text{(IV.2)}$$

In the above equation the $j_\ell(\nu r)$ refer to particle hole radial integrals discussed in Section 10, while the evaluation of reduced matrix elements is taken up in Appendix V.

A P P I N D I X V

Evaluation of Reduced Matrix Elements:

The two reduced matrix elements used in our calculation are

$$(1) \quad \langle j_p \| (Y_\ell(\hat{r}) \times \sigma_n)_{J_f}\, j_\ell(\nu r) \| j_h \rangle$$

and

$$(2) \quad \langle j_p \| j_\ell(\nu r) [(Y_\ell(\hat{r}) \times \nabla_1)_\lambda \times \sigma_n]_\Lambda \| j_h \rangle$$

where n can take values 0 or 1 and σ_0 is an unit operator in spin space. The first reduced matrix element can be evaluated by decoupling the states in angular and spin parts and the result is

$$(1) = \begin{Bmatrix} \ell_h & 1/2 & j_h \\ \ell & n & \lambda \\ \ell_p & \frac{1}{2} & j_p \end{Bmatrix} \frac{[j_h][\ell_h][\lambda][1/2][n][\ell]}{\sqrt{4\pi}} \; c(\ell_h \ell \ell_p; 000) \qquad (V.1)$$

Regarding the second matrix element, we first separate the angular and spin parts as follows:

$$(2) = \langle R_{n_p l_p}(r); \ell_p \tfrac{1}{2} j_p \| j_L(\nu r) [(Y_\ell(\hat{r}) \times \nabla_1)_\lambda \times \sigma_n]_\lambda \| R_{n_h l_h}(r)\, \ell_h \tfrac{1}{2} j_h \rangle = \begin{Bmatrix} \ell_h & \frac{1}{2} & j_h \\ \lambda & n & \Lambda \\ \ell_p & \frac{1}{2} & j_p \end{Bmatrix} [j_h]$$

$$[\tfrac{1}{2}][\Lambda][\ell_p] \langle R_{n_p l_p}(r) \| j_\ell(\nu r) (Y_\ell(r) \times \nabla_1)_\lambda \| R_{n_h l_h}(r); \ell_h \rangle \qquad (V.2)$$

where the R_{nl} are the harmonic oscillator wave functions. The angular part can be evaluated using the gradient formula, so as to yield,

$$\langle u_{n_p l_p}(r);\, l_p \,\|\, j_l(\nu r)\,(Y_l(\hat{r}) \times \nabla_1)_\lambda \,\|\, u_{n_h l_h}(r);\, l_h \rangle = (-1)^{l+1-\lambda}\, \frac{[l]\,[\lambda]}{\sqrt{4\pi}\;[l_p]} \Big\{ \sqrt{l_h+1}\;[l_h+1]\; c(l_h+1\; l\; l_p;\, 000)\; W(l_h 1 l_p l;\, l_h+1\; \lambda)\, F_- - \sqrt{l_h}\;[l_h-1]\; c(l_h-1\; l\; l_p;\, 000)\; W(l_h 1 l_p l;\, l_h-1\, \lambda)\, F_+ \Big\}$$

(V.3)

where F_- and F_+ are given in eqns. (75) and (76) of Chapter III and $\langle \frac{1}{2} \| \sigma_n \| \frac{1}{2} \rangle = [n]$.

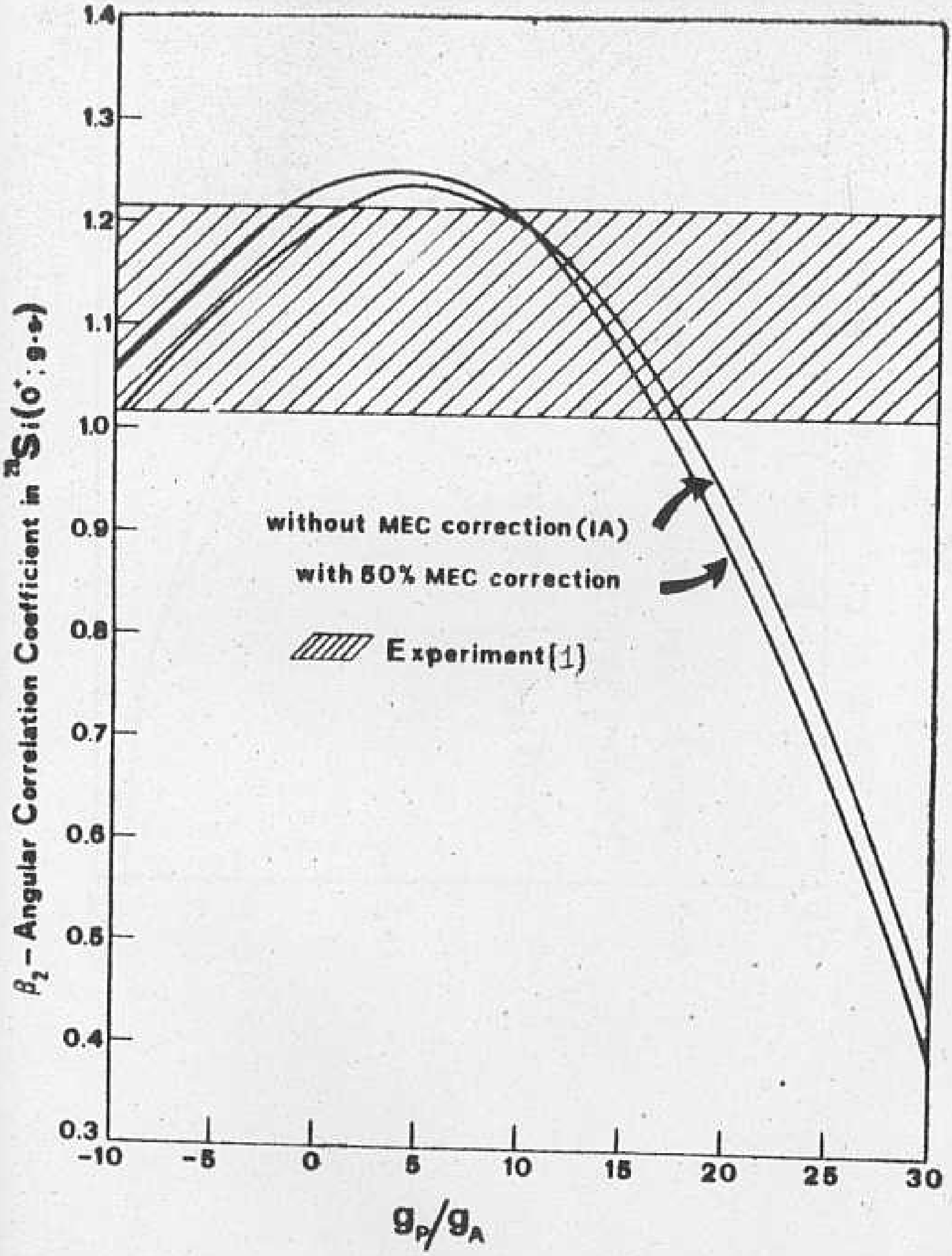
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
-10
-5
0
5
10
15
20
25
30
β_2 – Angular Correlation Coefficient in $^{28}Si(0^+; g\text{-}s)$
g_P/g_A
without MEC correction (IA)
with 50% MEC correction
Experiment [1]

Fig. 1

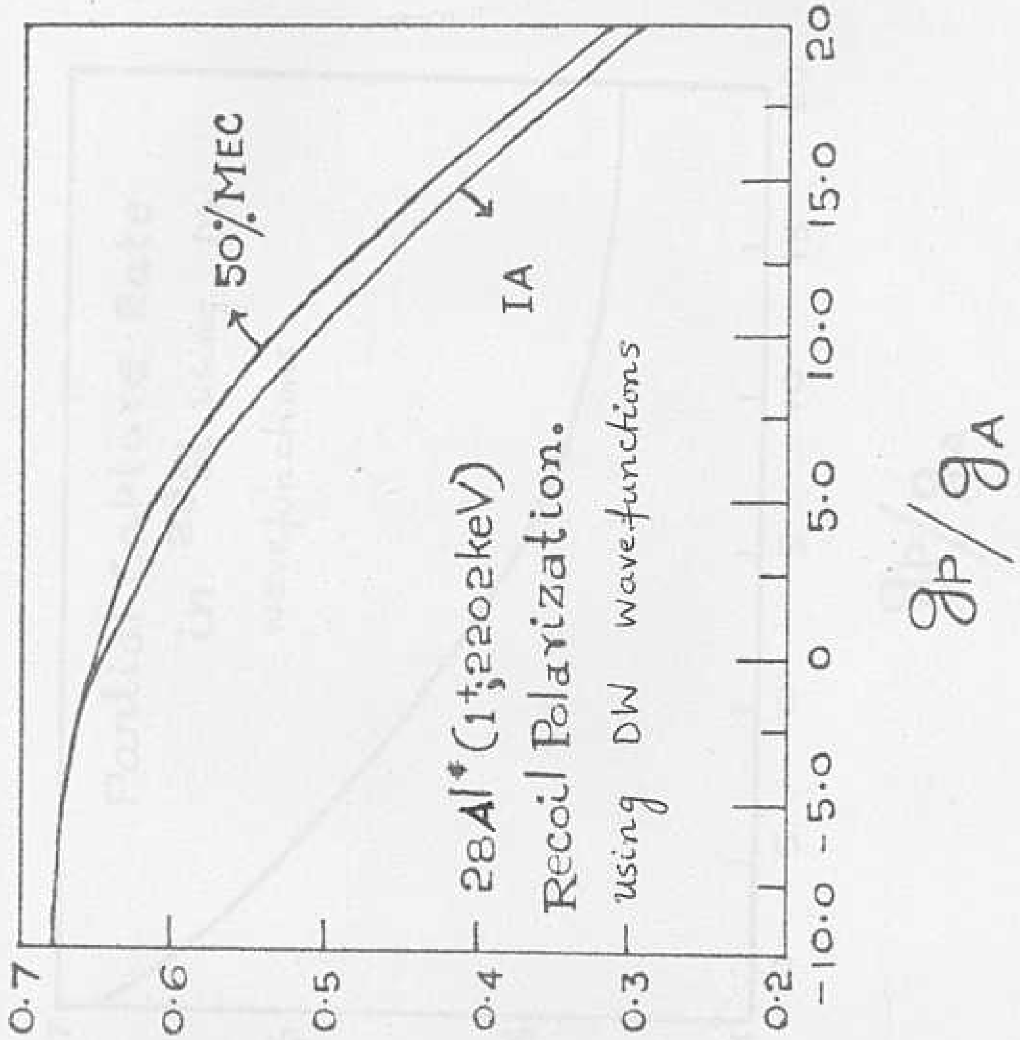
0.7
0.6
0.5
0.4
0.3
0.2
-10.0
-5.0
0
5.0
10.0
15.0
20
50% MEC
IA
28Al* (1+; 2202 keV)
Recoil Polarization.
using DW wavefunctions
g_P/g_A

Fig. 2

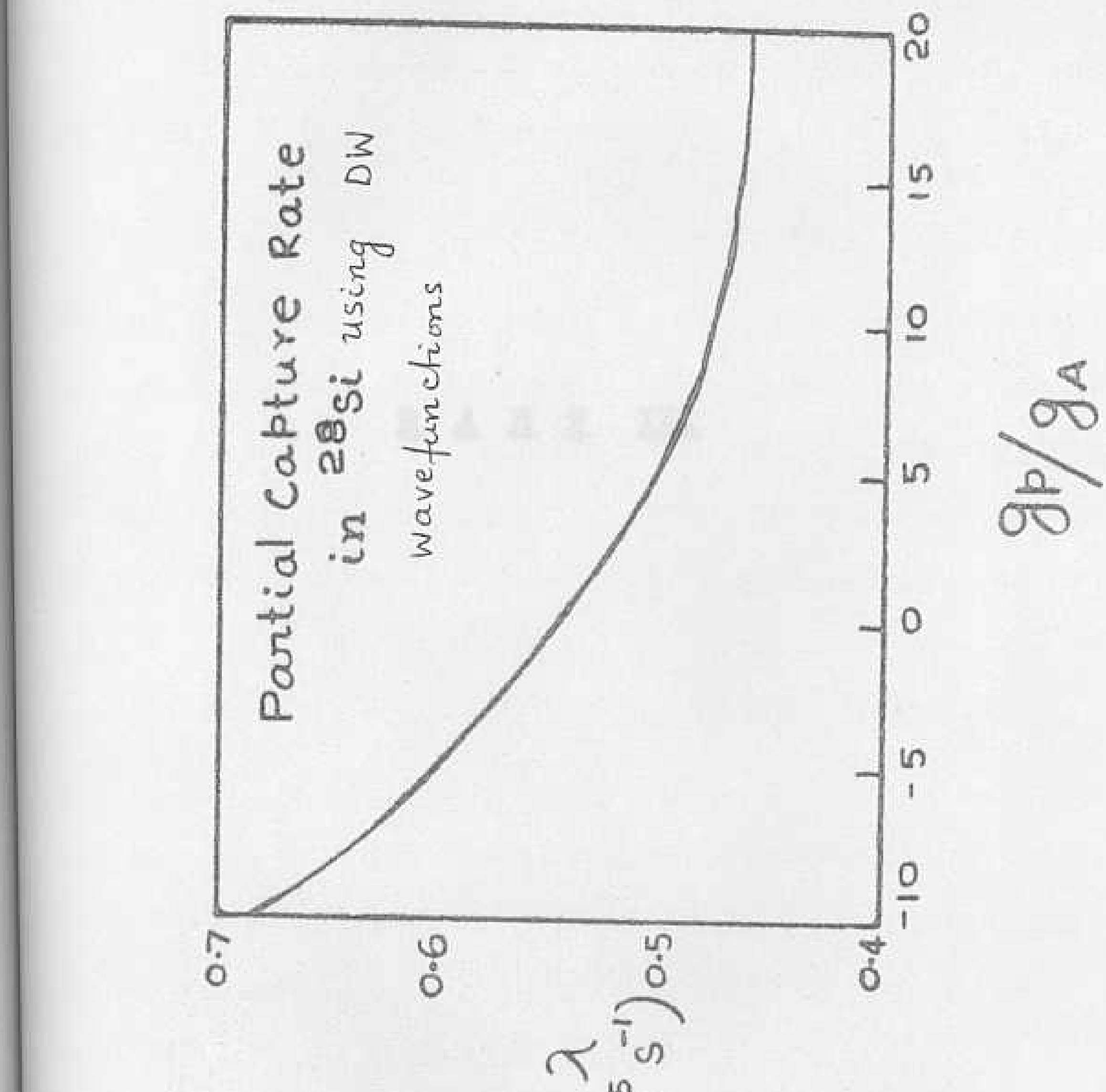
Partial Capture Rate
in 28Si using DW
wavefunctions
0.7
0.6
0.5
0.4
λ (10⁵ s⁻¹)
-10
-5
0
5
10
15
20
gP/gA

Fig. 3

P A R T III

C H A P T E R IV

RECOIL NUCLEAR POLARIZATION OF $^{12}B(1^+)$ IN $\mu^- + {}^{12}C(0^+;\ g.s.) \longrightarrow {}^{12}B(1^+;\ g.s.) + \nu_\mu$ *

1. Introduction:

In this Chapter, we discuss the average recoil polarization of $^{12}B(1^+ ;\ g.s.)$ in the process

$$\mu^- + {}^{12}C(0^+) \longrightarrow {}^{12}B(1^+,\ g.s.) + \nu_\mu \qquad (1)$$

taking into account contributions from the excited states of ^{12}B. Although the muon capture process populates mostly the $^{12}B(1^+)$ level, there is also a significant excitation of other levels, particularly the 1^- level of ^{12}B. The partial capture rates (λ) to various excited states of ^{12}B have been measured by Miller [1] as

$\lambda(1^+) = (6.0 \pm 0.4) \times 10^3\ s^{-1}$, $\lambda(1^-) = (0.89 \pm 0.10) \times 10^3\ s^{-1}$, $\lambda(2^+) = 0$, and $\lambda(2^-) \preceq 0.41 \times 10^3\ s^{-1}$. More recently, Roesch et. al. [2] have remeasured the above mentioned capture rates and their results are as follows: $\lambda(1^+) = (6.28 \pm 0.29) \times 10^3\ s^{-1}$, $\lambda(1^-) = (0.38 \pm 0.10) \times 10^3\ s^{-1}$, $\lambda(2^+) = 0.27 \pm 0.1) \times 10^3\ s^{-1}$ and $\lambda(2^-) = (0.12 \pm 0.08) \times 10^3\ s^{-1}$.

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* R.Parthasarathy and V.N.Sridhar, Phys. Lett. 82B (1979) 167
Phys. Lett. 106B (1981) 363.

The importance of average recoil nuclear polarization ($P_{av.}$) in muon capture was first pointed out by Devanathan, Parthasarathy and Subramanian [3], who showed that the recoil polarization of $^{12}B(1^+ ; g.s.)$ in process (1) is insensitive to nuclear structure and hence can be used to draw reliable, model independent conclusions regarding the induced pseudoscalar coupling in muon capture. The experimental measurement of $P_{av.}$ of $^{12}B(1^+ ; g.s.)$ in process (1) was carried out by Possoz et. al. [4], employing selective implantation techniques to preserve the polarization of nuclei recoiling into forward and backward hemispheres. From the measured value of $P_{av.}(^{12}B(1^+ ; g.s.)) = 0.452 \pm 0.042$, they have deduced a value for the sum of the induced pseudoscalar (g_P) and induced tensor couplings g_T as $(g_P + g_T)/g_A = 7.1 \pm 2.7$. Adopting the PCAC estimate for g_P, this leads to $g_T/g_A = (1.0 \pm 2.7)$ which is compatible with zero. In this connection it may be mentioned that similar conclusions have been obtained by Roesch et. al. [5], who measured the ratio $R = P_{av.}/P_L$ (where P_L is the longitudinal polarization), which is free from nuclear wavefunction uncertainties and is very sensitive to g_P.

However, the excited states of ^{12}B are polarized in muon capture and hence will contribute to the observed polarization of $^{12}B(1^+ ; g.s.)$. The correction due to the 1^- state of ^{12}B was estimated by Ciechanowicz [6] to be -0.25 using the generalised Helm model for $^{12}B(1^-)$ state and the corrected $P_{av.}(1^+)$ obtained by Possoz et. al. [4] was 0.532 ± 0.049. Such a large correction as obtained by Ciechanowicz [6] must be viewed with a degree of caution ; it is well known that the generalised Helm model employed by Ciechanowicz for the calculation of partial capture rates and polarization, gives poor agreement with experiment. This fact was also noted by Devanathan et. al. [7] in their study of partial muon capture rates for ^{12}B using Helm model. The calculation of Ciechanowicz has been criticised by Kobayashi [8] on the grounds that the muon capture matrix elements in generalised Helm model are parametrised and their values are obtained from inelastic electron scattering data, which do not seem to be sufficient to derive definite values for these parameters. Hence, as pointed out by Kobayashi et. al. [8] and Truttman et. al. [9], it is doubtful to use the results of Ciechanowicz [6] to correct the measured polarization of $^{12}B(1^+ ; g.s.)$, as was done by Possoz et. al. [4] .

The general formalism for the partial capture rate and recoil nuclear polarization is given in Section 7 and 8 of Chapter III and we do not repeat it here. In Section 2, we state a theorem due to Rose [10a] and outline its proof. In Section 3, the theorem is applied to estimate the correction due to the $^{12}B(1^-)$ state. The formalism of particle-hole models has already been discussed in Section 10 of Chapter III and in Section 4, we present numerical results for the corrected polarization of $^{12}B(1^+ ; g.s.)$ including the effect of $^{12}B(1^-)$ state, along with discussion.

2. THEOREM

If the nuclear system is initially in a state of orientation described by a statistical tensor of rank λ and if it makes a transition to a final state whose orientation is described by a statistical tensor of rank λ', then $\lambda' = \lambda$ if the transition is a parity conserving one, and $\lambda' = \lambda \pm 1$ if the transition is parity violating.

Proof. For convenience, we shall divide the proof into parts A and B for parity conserving and parity non-conserving cases, respectively. The following discussion is after Rose [10a].

PART A :

Parity Conserving Case:- We have shown in Section (4a) of Chapter II that P_m (population of sublevels) and $G_\nu(j)$ (Fano's statistical tensor) are transforms of each other, i.e.

$$G_\nu(j) = \sum_M (-1)^{J-M} P_M \, C(JJ\nu ; M - M0) \qquad (2)$$

It is now useful to introduce another statistical tensor α_ν, by expanding P_M as a polynomial of degree $2J$ in M. Since the Clebsch-Gordan coefficient $C(J\,\lambda\,J\,;\,M\,0\,M)$ is also a polynomial of degree λ in M, we may write

$$P_M = \sum_{\lambda=0}^{2J} \alpha_\lambda(J)\; C(J\,\lambda\,J\,,\,M\,0\,M) \tag{3}$$

Substituting eqn. (3) in eqn. (2) and using orthogonality properties,

$$G_\nu(j) = \frac{2J+1}{2\nu+1}\,\alpha_\nu \tag{4}$$

which may serve as a definition of α_ν. It may be seen from eqn. (4) that α_ν is a simple multiple of G_ν, the Fano's statistical tensor.

Consider now the transition from an initial state $|jm\rangle$, whose orientation is described by the statistical tensor α_ν, to a final state $|j_1 m_1\rangle$ whose orientation is to be determined. Let the parity conserving interaction be chosen as $H = M \cdot M^*$, where $\vec{M}$ can be represented by a spherical tensor. Denoting the populations of initial nucleus by P_m, the diagonal element of the density matrix of the final state is

$$\rho_{m_1 m_1} = \sum_m P_m\, \vec{M}\cdot\vec{M}^* \tag{5}$$

Writing $P_m = \sum_{\lambda} \alpha_\lambda(j)\ C(j\lambda j\ ;\ m\ o\ m)$, expressing $\vec{M}$ in spherical tensors and applying the Wigner-Eckart theorem [10.b]

$$\rho_{m_1 m_1} = \sum_{\lambda m} \alpha_\lambda(j)\ C(j\lambda j;\ mom)\ C(j\nu j_1; m m_\nu m_1)^2\ |\langle j_1 \| M_\nu \| j\rangle|^2 \qquad (6)$$

Summing over m, the three Glebsch-Gordan coefficients can be arranged to yield

$$\rho_{m_1 m_1} = |\langle j_1 \| M_\nu \| j\rangle|^2 \sum_\lambda \alpha_\lambda(j)\ (-1)^{j_1 - j - \nu - \lambda}$$

$$W(j j j_1 j_1; \lambda\nu)\ [j][j_1]\ C(j_1\lambda j_1; m_1\, o\, m_1) \qquad (7)$$

Now define

$$\alpha_\lambda(j_1) = \alpha_\lambda(j)\,(-1)^{j_1 - j - \nu - \lambda}\ W(j j j_1 j_1; \lambda\nu)\ [j][j_1] \qquad (8)$$

so that

$$\rho_{m_1 m_1} = |\langle j_1 \| M_\nu \| j\rangle|^2 \sum_\lambda \alpha_\lambda(j_1)\ C(j_1\lambda j_1; m_1\, o\, m_1)$$

$$= |\langle j_1 \| M_\nu \| j\rangle|^2\ P_{m_1} \qquad (9)$$

Thus we see that $\alpha_\lambda(j_1)$ plays the same role of $\alpha_\lambda(j)$ for the initial state. Since α_λ defines the orientation, we see from eqn. (8) that the parity conserving interaction $\vec{M} \cdot \vec{M}^*$ carries $\alpha_\lambda(j)$ to $\alpha_\lambda(j_1)$, that is, the rank of the tensor α_λ is unchanged. In particular, if the initial state $|jm\rangle$ is polarised $(\lambda = 1)$ it remains polarized $(\lambda = 1)$ in the final state $|j_1 m_1\rangle$.

We now derive a relationship between the initial and final state polarization. The polarization $(\vec{P_N})$ of a state $|JM\rangle$ may be written as

$$P_N(J) = \frac{1}{J} \sum_M M P_M \Big/ \sum_M P_M \tag{10}$$

Expressing $\sum_M M P_M$ and $\sum_M P_M$ in terms of the Fano's statistical tensors G_ν as given in eqns. (6) and (7) of Section (4a), Chapter II and using eqn. (4) to express G_ν in terms of α_ν we obtain

$$P_N(J) = \frac{1}{3} \sqrt{\frac{J+1}{J}} \; \frac{\alpha_1}{\alpha_0} \tag{11}$$

From the above equation, we have

$$\frac{P_N(j_1)}{P_N(j)} = \sqrt{\frac{j(j_1+1)}{j_1(j+1)}} \; \frac{\alpha_1(j_1)}{\alpha_1(j)} \; \frac{\alpha_0(j)}{\alpha_0(j_1)} \tag{12}$$

From eqn. (8), putting $\lambda = 0$ and using the properties of the Racah coefficients, we can easily show that

$$\alpha_0(j_1) = \alpha_0(j) \; .$$

Hence, we have

$$\frac{P_N(j_1)}{P_N(j)} = \sqrt{\frac{(j_1+1)\,j}{j_1(j+1)}} \; (-1)^{j_1-j-\nu-1} \, W(jjj_1j_1, 1\nu) \, [j][j_1] \tag{13}$$

For a pure dipole transition $(\nu = 1)$, the above relation reduces to

$$\frac{P_N(j_1)}{P_N(j)} = j(j+1) + j_1(j_1+1) - 2 \Big/ 2j_1(j+1) \tag{14}$$

PART B:

Parity non-conserving Case

Let a typical parity non-conserving transition be represented as

$$H = \frac{-i\vec{p}}{E} \cdot (\vec{M} \times \vec{M}^{*}) \tag{15}$$

where

$$M = \sum_{\mu} \left\langle j_1 m_1 \middle| \sigma_1^{\mu} \middle| jm \right\rangle \xi_1^{-\mu} (-1)^{\mu} \tag{16}$$

for the sake of illustration ($\xi_1^{-\mu}$ are the spherical basis vectors). The diagonal element of the final state density matrix can be written as

$$\rho_{m_1 m_1} = - i\vec{p} \cdot \sum_{m} p_m (\vec{M} \times \vec{M}^{*}) \tag{17}$$

Using Wigner-Eckart theorem and the following rule for the cross product of two spherical vectors

$$\hat{\xi}_1^{\mu} \times \hat{\xi}_1^{\mu'} = i \sqrt{2}\ C(111 , \mu\, \mu'\, \mu + \mu') \hat{\xi}_1^{(\mu+\mu')} \tag{18}$$

we can evaluate $(\vec{M} \times \vec{M}^{*})$ and the expression for the density matrix element becomes

$$\rho_{m_1 m_1} = \sqrt{2} \left| \langle j_1 \| \sigma_1 \| j \rangle \right|^2 \sum_{m} (-1)^{m_1 - m} p_m\, C(j1j , m\, m_1 - m)^2 \frac{p_z}{E}$$

$$C(111 , m - m_1\, m_1 - m) \tag{19}$$

Expanding p_m as given in eqn. (3), we obtain

$$\rho_{m_1 m_1} = v_2\sqrt{2}\ |\langle j_1 \| \sigma_1 \| j \rangle|^2\ S(m_1) \qquad (20)$$

where

$$S(m_1) = \sum_{m\lambda} \alpha_\lambda(j)\,(-1)^{m_1-m}\, c(j\lambda j; m0m)\; c(111; m-m_1\ m_1-m\ 0)\, c(j1j_1; m\ \ m_1-m)^2 \qquad (21)$$

Comparing eqns. (21) and (9), we see that $S(m_1)$ has the same status as P_{m_1}, the population of sublevel m_1. Hence $S(m_1)$ may be given the following polynomial form:

$$S(m_1) = \sum_{\nu=0}^{2j_1} \beta_\nu\ c(j_1\nu j_1; m_1 0 m_1) \qquad (22)$$

and thus β_ν has the same role as α_λ, in determining the orientation of the final nucleus. The eqn. (22) can be inverted to give

$$\beta_\nu = \frac{[\nu]}{[j_1]} \sum_{m_1} S(m_1)\ c(j_1\nu j_1; m_1 0 m_1) \qquad (23)$$

showing that β_ν and S are transforms of one another. Now substituting for $S(m_1)$ from eqn. (21) and carrying out the angular momentum algebra, we obtain

$$\beta_\nu = [1][\nu][j][j_1] \sum_\lambda \alpha_\lambda(j)\ c(\lambda 1\nu; 000) \sum_x W(\nu j_1 j 1; j x)\quad W(11j_1 j; 1x)\, W(j\lambda x 1; j\nu)\ (-1)^{j-x-1}\ [x]^2 \qquad (24)$$

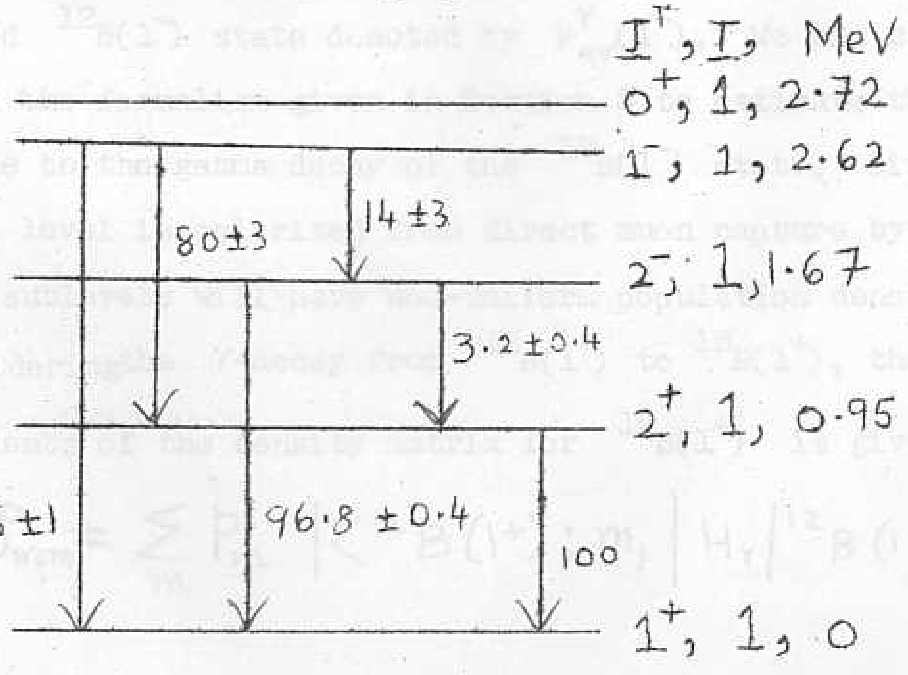
From the above eqn. it is seen that the rank of the orientation of the final nucleus ν, is related to λ, the rank of the orientation of the initial nucleus through the parity Clebsch-Gordan coefficient $C(\lambda 1 \nu, 000)$ which gives

$$\nu = \lambda \pm 1 \tag{25}$$

This completes the proof of the theorem.

3. Estimate of the Contribution from $^{12}B(1^-)$ State:

In this section, we apply the above theorem for parity conserving case, to estimate the contribution of the Y-decay of the $^{12}B(1^-)$ state to the average recoil polarization of the $^{12}B(1^+)$ ground state. The energy level diagram of ^{12}B is given below following Olness and Warburton [11].



In the above figure, the numerical values in gamma transitions refer to branching ratios. The contribution from each excited state to the polarization of $^{12}B(1^+ ; g.s.)$, is proportional to the capture rate and the branching ratio for gamma decay. Since the capture rate to the $^{12}B(1^-)$ level has been measured by Miller et. al. [1] as $\lambda(1^-) = (0.89 \pm 0.10) \times 10^3 \ s^{-1}$, we take into account only this excited state for calculating corrections to the $P_{av.}$ of $^{12}B(1^+)$, the capture rates to the other excited levels being $\lambda(2^+) = 0$, $\lambda(2^-) = 0.12 \times 10^3 \ s^{-1}$ and hence can be considered negligible compared with $\lambda(1^-)$.

The resultant polarization of $^{12}B(1^+; g.s.)$ can be written as a statistical sum of (1) direct polarization resulting from muon capture by $^{12}C(0^+)$ denoted by $P^{\mu}_{av.}(1^+)$ and (2) indirect polarization of $^{12}B(1^+ , g.s.)$ resulting from the gamma decay of the excited $^{12}B(1^-)$ state denoted by $P^{\gamma}_{av.}(1^+)$. We can carry over directly the formalism given in Section 2 to estimate the correction due to the gamma decay of the $^{12}B(1^-)$ state. Since the $^{12}B(1^-)$ level is polarized from direct muon capture by ^{12}C, its magnetic sublevels will have non-uniform population denoted by p_m. Considering the γ-decay from $^{12}B(1^-)$ to $^{12}B(1^+)$, the diagonal elements of the density matrix for $^{12}B(1^+)$ is given by

$$\rho_{m_1 m_1} = \sum_{m} P_m \left| \langle {}^{12}B(1+) ; m_1 \right| H_\gamma \left| {}^{12}B(1-) m \rangle \right|^2$$

where m_1 denotes the sublevels of $^{12}B(1^+)$ and $H_\gamma = \vec{j}_N \cdot \vec{A}_p$ is the Hamiltonian for gamma decay, which is a parity conserving transition. From the arguments of Section 2, it is now obvious that the $^{12}B(1^+)$ is polarized due to the $^{12}B(1^-)$ polarization; using eqn. (14) we now conclude that

$$P^{\gamma}_{av.}(\,^{12}B(1^+)) = 0.5 \; P^{\mu}_{av.}(\,^{12}B(1^-)) \tag{26}$$

Denoting the partial capture rates to the $^{12}B(1^+)$ and $^{12}B(1^-)$ states by $\lambda(1^+)$ and $\lambda(1^-)$ respectively, the resultant $^{12}B(1^+)$ average polarization ($P^{res.}_{av.}$) can now be written as

$$P^{res.}_{av.}(1+) = \frac{\lambda(1+)}{\lambda(1+)+\lambda(1-)} P^{\mu}_{av.}(1+) + \frac{\lambda(1-)}{\lambda(1+)+\lambda(1-)} P^{\gamma}_{av.}(1+) \tag{27}$$

with $P^{\gamma}_{av.}(1^+)$ given by eqn. (26).

4. Numerical Results and Discussion.

(a) Numerical Results:

In Table 1, numerical values for $\lambda(1^+)$ and $P^{\mu}_{av.}(1^+)$ are given for various values of $(g_P + g_T) / g_A$ for the following nuclear models: (i) Independent Particle Model (IPM) (ii) particle-hole model of Gillet and Vih Mau [12] which includes $2\hbar\omega$ excitations and (iii) particle-hole model of Donnelly and Walker [13] wherein the two body residual Serber-Yukawa force is used to diagonalise the shell-model Hamiltonian in 1p-1h basis.

TABLE 1

$\frac{g_P+g_T}{g_A}$	IPM $\lambda(1^+)$ $(10^3\ s^{-1})$	IPM $P_{av}(1^+)$	GV $\lambda(1^+)$ $(10^3\ s^{-1})$	GV $P_{av}(1^+)$	DW $\lambda(1^+)$ $10^3 s^{-1}$	DW $P_{av}(1^+)$
-10.00	47.609	0.6763	45.669	0.6763	9.403	0.6764
- 7.5	44.638	0.6775	42.825	0.6781	8.816	0.6765
- 5.0	41.932	0.6766	40.237	0.6757	8.821	0.6756
- 2.5	39.494	0.6689	37.905	0.6686	7.789	0.6689
0.0	37.322	0.6566	35.831	0.6557	7.370	0.6566
2.5	25.417	0.6379	34.012	0.6365	6.990	0.6379
5.0	33.779	0.6123	32.450	0.6102	6.668	0.6123
7.5	32.407	0.5792	31.145	0.5765	6.400	0.5792
10.0	31.303	0.5386	30.096	0.5352	6.181	0.5386
12.5	30.466	0.4907	29.304	0.4866	6.016	0.4907
15.0	29.893	0.4361	28.768	0.4315	5.896	0.4361
17.5	29.589	0.3760	28.489	0.3708	5.838	0.3760
20.0	29.551	0.3118	28.466	0.3062	5.835	0.3118
22.5	29.779	0.2452	28.669	0.2394	5.880	0.2452
25.0	30.275	0.1780	29.189	0.1721	5.997	0.1780

Exp. 6.0 ± 0.4 [1]
6.3 ± 0.3 [14]
6.75 ± 0.30 [15]

In Table 2, numerical values for $\lambda(1^-)$ and $P^{\mu}_{av.}(1^-)$ are given for the above mentioned nuclear models. $\lambda(1^-)$ and $P^{\mu}_{av.}(1^-)$ are independent of $(g_P + g_T) / g_A$. Our results are compared with experiment and other theories.

TABLE 2

Nuclear Model		$\lambda(1^-)$ in 10^3 s^{-1}	$P^{\mu}_{av.}(1^-)$
IPM		1.927	0.6285
GV		1.423	0.6664
DW		0.593	0.6523
Other theories:			
Kobayashi et. al.	[8]	1.4 (a)	0.431 (a)
		0.877 (b)	0.607 (b)
		1.22 (c)	0.533 (c)
		2.78 (d)	0.657 (d)
		9.40 (e)	-0.332 (e)
Ciechanowicz	[6]	0.23	-0.25.

(a) Cohen-Kurath (CK) model I

(b) CK model II

(c) CK model III

(d) Single particle jj coupling shell model (i)

(e) Single particle jj coupling shell model (ii).

In Table 3, numerical values for the resultant average polarization $P_{av.}^{res.}(1^+)$ using eqn. (27), are given for various values of $(g_P + g_T) / g_A$ in the above mentioned nuclear models. We also give numerical values for $P_{av.}^{res.}(1^+)$ including 50% MEC corrections, using Gillet-Vinh Mau [12] wavefunctions.

TABLE 3

$\frac{g_P + g_T}{g_A}$	$P_{av.}^{res}\left({}^{12}B(1^+)\right)$ IPM	GV	DW	50% MEC GV
-10.0	0.6619	0.6645	0.6515	0.6533
- 7.5	0.6621	0.6655	0.6511	0.6539
- 5.0	0.6594	0.6625	0.6478	0.6511
- 2.5	0.6521	0.6548	0.6401	0.6440
0.0	0.6396	0.6448	0.6275	0.6315
2.5	0.6212	0.6229	0.6092	0.6134
5.0	0.5965	0.5973	0.5851	0.5885
7.5	0.5649	0.5650	0.5545	0.5565
10.0	0.5267	0.5257	0.5177	0.5189
12.5	0.4819	0.4797	0.4748	0.4741
15.0	0.4310	0.4277	0.4263	0.4278
17.5	0.3752	0.3706	0.3732	0.3673
20.0	0.3157	0.3099	0.3166	0.3081

In Fig. 1 we display graphically MEC effects on the $P^{res.}_{av.}(1^+)$ of $^{12}B(1^+ ; g.s.)$.

(b) Discussion:

From Tables 1 and 2, it is seen that the partial capture rates $\lambda(1^+)$ and $\lambda(1^-)$ are model dependent, while the average recoil polarization $P^{\mu}_{av.}(1^+)$ is almost model independent. This can be traced to the fact that the expression for $P_{av.}$ as given in eqn. (55) of Chapter III involves ratio of reduced matrix elements which cancel in FPA, while the effect of nucleon momentum dependent terms and higher order neutrino partial waves, is small. From Table 2., it is seen that our values for $P_{av.}(1^-)$ are in good agreement with that of Kobayashi et. al. [8] and experiment. It is contradictory to the value obtained by Ciechanowicz [6] which is $P_{av.}(1^-) = -0.25$. As mentioned in Section 1 of this Chapter, the negative value for $P_{av.}(1^-)$ obtained by Ciechanowicz could be due to the generalised Helm model employed in the calculation, whose inadequacies were noted by Kobayashi et. al. [8] . From Table 3 , it is clear that $P^{res.}_{av.}(1^+)$ differs from uncorrected $P^{\mu}_{av.}(1^+)$ by a small amount (4%) ; this is due to the circumstance that $\lambda(1^-) \ll \lambda(1^+)$, so that the statistical factor $\frac{\lambda(1^-)}{\lambda(1^+) + \lambda(1^-)}$ is small as compared to $\frac{\lambda(1^+)}{\lambda(1^+) + \lambda(1^-)}$.

Since $P^{res.}_{av.}(1^+)$ is nuclear model insensitive, it can be safely used for extracting values of $(g_P + g_T) / g_A$. Comparing with the experiment of Possoz et. al. [4], we obtain

$$(g_P + g_T) / g_A = (13.3 \pm 1.8)\ g_A .$$

In what follows, we correct the above Impulse Approximation estimate for $(g_P + g_T) / g_A$ by including MEC corrections.

The transition $^{12}C(0^+ ; g.s.) \longrightarrow {}^{12}B(1^+ ; g.s.)$ is an allowed process dominated by the Gamow-Teller operator and hence MEC effects (which enhance only the time component of the axial vector current) are expected to be negligible. On the other hand, the transition $^{12}C(0^+ ; g.s.) \longrightarrow {}^{12}B(1^- ; 2.62\text{ MeV})$ is a first forbidden process; it is independent of $(g_P + g_T) / g_A$ and the time component of the axial current so that MEC effects are again negligible. We have evaluated the resultant average recoil polarization of $^{12}B(1^+ , g.s.)$ using the wave functions of Gillet and Vinh Mau [12], for various values of F (which is a measure of MEC effects, see Section 9 of Chapter III). In figure 1, we display the variation of $P^{res.}_{av.}(1^+)$ with g_P / g_A without and with 50% MEC corrections. By comparing with experiment, [4] we find that

$$(g_P + g_T) / g_A = (13.62 \pm 2.1)\, g_A \qquad (28)$$

a value nearly independent of MEC corrections. Combining this with our value of $(g_P + g_T) / g_A = (12.9 \pm 3.9)$ obtained from the analysis of $\gamma - \nu$ angular correlation coefficient β_2, we conclude that

$$(g_P + g_T) / g_A = (13.3 \pm 3) g_A \tag{29}$$

a value to a large extent free from nuclear wavefunction uncertainties. This value is in conformity with the value of Kobayashi et. al. [8] who obtains

$$(g_P + g_T) / g_A = (10.3 \pm 2.7) g_A \tag{30}$$

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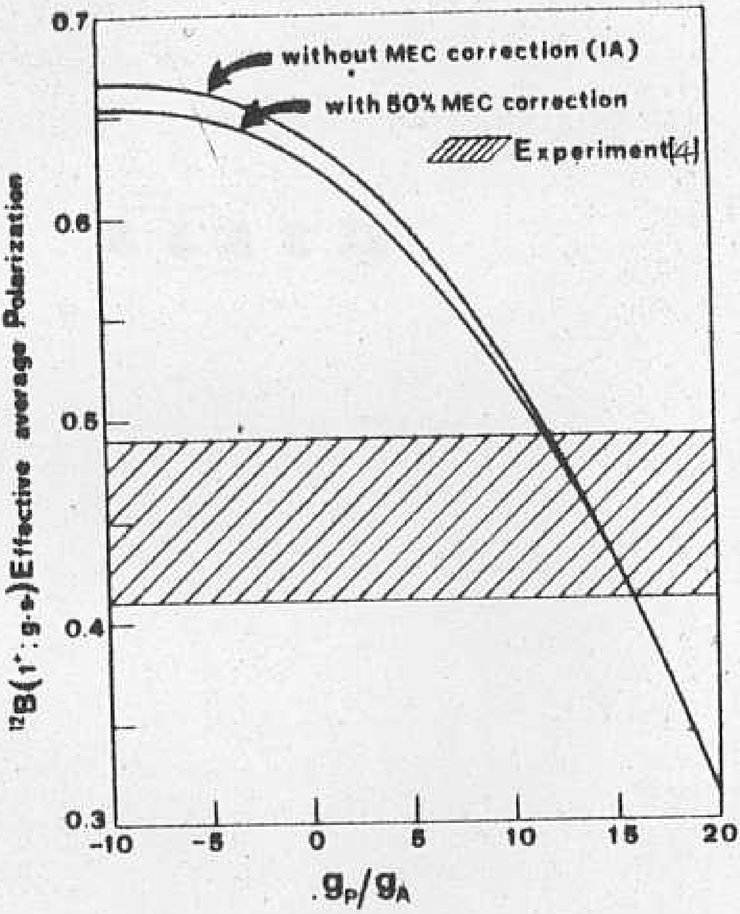

without MEC correction (IA)
with 50% MEC correction
Experiment[4]
$^{12}B(1^+; g\text{-}s)$ Effective average Polarization
0.7
0.6
0.5
0.4
0.3
-10
-5
0
5
10
15
20
g_P/g_A

Fig. 1

P A R T IV

CHAPTER V

QUENCHING OF CABIBBO ANGLE AND TOTAL MUON CAPTURE RATES*

1. Introduction

In this chapter, we discuss total capture rates in heavy-nuclei within the context of the Salam Strathdee theory [1] of vanishing of the Cabibbo angle at large electromagnetic fields. The critical magnetic field above which the Cabibbo angle (θ_c) could reduce to zero, was estimated by Salam and Strathdee [1] to be of the order of 10^{16}G. Using heuristic, qualitative arguments, Suranyi and Hedinger [2] suggested that such large magnetic fields could possibly be present in the interior of odd-proton nuclei. They have argued that, in the case of an odd-even nucleus which may be regarded as a combination of nuclear core and a single proton, the magnetic field generated by the proton is given by

$$H = (1/A) \times 10^{16} \text{ Gauss} \tag{1}$$

where 1 is the angular momentum of the single proton state. A more detailed calculation of electromagnetic fields in the interior of the nucleus (and the constituent nucleons) has been carried out by Lee and Khanna [3], using a single particle shell model; they find that the Lorentz invariant quantity

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* R.Parthasarathy and V.N.Sridhar, Can. J. Phys. 56 (1978) 1606.

$\mathcal{H} = B^2 - E^2/c^2$, where B and E refer to magnetic and electric fields respectively, is large and positive at the centre of the nucleus and negative in the rest of the nucleus.

In the context of total muon capture rates, it has been shown by Watson [4] that better agreement with experiment can be obtained in the case of $^{93}_{41}Nb$, which can be thought of as a core of $^{92}_{40}Zr$ and a single proton in the $\ell = 4$ state. Assuming magnetic fields of the order of 10^{17}G, and using the following formula of Primakoff [5] for total capture rates (Λ) ;

$$\Lambda = \gamma \times \Lambda(1,1) \; Cos^2 \theta_c \; Z^4_{eff.} \; (1 - \delta' \frac{A - Z}{ZA}) \tag{2}$$

where $\Lambda(1,1) = G_V^2 + 3\,G_A^2 + G_P^2 - 2\,G_P\,G_A$

γ is the capture rate in Hydrogen,

$Z_{eff.}$ is the effective nuclear charge as seen by the muon

δ' takes care of Pauli principle,

we may write

$$\frac{\Lambda(^{93}_{41}Nb)}{Cos^2 \theta_c} = \gamma \; \Lambda(1,1) \; Z^4_{eff.} \; (1 - \delta' \frac{A-Z}{ZA}) \tag{3}$$

Thus, a deviation of the observed total capture rate in ^{93}Nb from the Primakoff formula by a factor of $1/Cos^2 \theta_c$ can be taken to be an indication of vanishing of θ_c . However, we note that the

the normal and abnormal values of $\cos\theta_c$ (0.97 and 1.0) differ by 3% and total capture rates with and without $\theta_c = 0$, differ by 6%. Hence, in order to test the Salam Strathdee idea of vanishing of the Cabibbo angle, other corrections unrelated to θ_c and omitted by Watson must be taken into account. It must be mentioned here that recent studies by Suzuki [20], Wilcke [21] and Linde [23] have cast doubts on the ultra-high magnetic fields in nuclei required for the vanishing of θ_c, and further it seems that nuclear structure effects play an important role and must be kept in mind when comparing theory and experiment.

In Section 2, we review briefly the Salam-Strathdee theory leading to strangeness conservation in weak processes. In Section 3, we give the formulation of the total capture rate including recent improvements by Goulard and Primakoff [6]. In Sections 4 and 5 we discuss hyperfine effects and the effect of momentum dependent terms (MDT) and we present our results in Section 6 along with discussion.

2. A Brief Review of the Salam-Strathdee Theory.

The theory of symmetry restoration as propounded by Salam and Strathdee [1], is based on a formal analogy between spontaneously broken gauge theories and the phenomenological Ginzburg-Landau theory [7] of superconductivity. In spontaneously broken gauge theories, one starts with a Lagrangian which is locally invariant under the action of a Lie group of transformations such as SU(2) and SU(3) ; this local invariance gives rise to a finite number of massless bosons, which are equal to the number of generators of the group. The local symmetry is now broken 'spontaneously' by the introduction of Higgs scalar fields [8] which possess non-zero vacuum expectation values; the word 'spontaneously' meaning that the ground states of a system do not have the same symmetry as that of the Hamiltonian describing the system. These Higgs scalars then give masses to the various massless bosons, which are proportional to the vacuum expectation values of the Higgs scalars. Specifically, if one views the Cabibbo angle (θ_c) to be the mixing angle between the down (d or n) and strange (s or λ) quarks, then $\theta_c \neq 0$ implies the conservation of strangeness in weak interaction (or in other words 'strangeness symmetry') is violated, leading to a certain kind of order. Viewed in terms of the Higgs mechanism, the mass of the quark is proportional to the vacuum expectation value of the Higgs field ψ,

$$m_Q = g \langle \psi \rangle \tag{4}$$

where g is the coupling constant, coupling ψ to the quarks. The reason for such an involved procedure to generate masses is that the theory is not renormalizable if massive terms are included in the Lagrangian ; whereas it has been shown by t'Hooft [9] that spontaneously broken gauge theories are renormalizable if masses are generated by the Higgs mechanism [8] .

Salam and Strathdee now observe a formal analogy between Lagrangian theories of spontaneously broken symmetries and the free energy of a superconducting system in the theory of Ginzburg and Landau [7]. In this theory the free energy of the superconducting system, in the neighbourhood of a second order phase transition T_c, is expressed in terms of the order parameter , which is related to the density of Cooper pairs in the system and determines the degree of superconductivity (or order) of the system,

$$G_s = G_n + \alpha(T)\,|\phi|^2 + \frac{\lambda(T)}{2}\,|\phi|^4 + \cdots \qquad (5)$$

where G_s and G_n refer to the free energies of the superconducting and normal states of the system. By applying an external magnetic field which exceeds the critical strength,

$$B > B_c \approx |\phi|^2 \qquad (6)$$

the order (or superconductivity) is destroyed and the symmetry of the system is restored. One can now observe a formal similarity between eqn. (5) and the expression for the Higg's Lagrangian:

$$\mathcal{L}_{\text{Higgs}} = |\partial_\mu \psi|^2 - \mu^2 |\psi|^2 - \lambda |\psi|^4 \qquad (7)$$

with the Higgs field ψ playing the role of an order parameter.

As in the case of superconductivity, an external magnetic field exceeding a critical value could restore the symmetry ($\theta_c \sim 0$) leading to strangeness conservation in weak interactions.

Based on the above line of reasoning, Salam and Strathdee [1] have estimated the critical magnetic field (H_c) to be of order of 10^{16}G for θ_c to vanish.

3. Formalism for Total Muon Capture Rates.

In this section we discuss the formulation of the total muon capture rate taking into account the recent modified formula due to Goulard and Primakoff [6]

We start with the Fujii-Primakoff Hamiltonian

$$H_\mu^{eff.} = \frac{1}{2}\tau^+ (1-\vec{\sigma}_\ell\cdot\hat{\nu}) \sum_{i=1}^{A} \tau_i^{(-)} \Big[G_V\, 1\cdot 1_i + G_A\, \vec{\sigma}_\ell\cdot\vec{\sigma}_i - G_P (\vec{\sigma}_\ell\cdot\hat{\nu})(\vec{\sigma}_i\cdot\hat{\nu}) - \frac{g_V}{M}(\vec{\sigma}_\ell\cdot\hat{\nu})(\vec{\sigma}_\ell\cdot\vec{p}_i) - \frac{g_A}{M}(\vec{\sigma}_\ell\cdot\hat{\nu})(\vec{\sigma}_i\cdot\vec{p}_i)\Big]\,\delta(\vec{r}-\vec{r}_i) \tag{8}$$

where the various quantities in the above expression have been defined in Section 3 of Chapter I. The expressions for the partial capture rate using the above Hamiltonian has been given in Section 7 of Chapter III and we do not repeat them here. We recall here the definitions of the nuclear matrix elements M_I $(I = 1,2,3,4)$:

$$M_I = \langle J_f M_f | \sum_{i=1}^{A} \tau_i^{(-)} e^{-i \vec{\nu}_{ab} \cdot \vec{r}_n} \phi_\mu (r_n) \; O_I | J_i M_i \rangle \quad (9)$$

where O_I (1,2,3,4) is given by

$$O_1 = 1_i, \; O_2 = \sigma_i \; , \; O_3 = p_i \; , \; O_4 = \sigma_i \cdot p_i \; .$$

In eqn. (9), $\nu_{ab} = m_\mu c^2 - (E_b - E_a)$ is the momentum transfer for the partial transition $a \longrightarrow b$ and the muon wavefunction ϕ_μ can be averaged out of the matrix elements. The total capture rate Λ_t to all the energetically possible final levels is now given by the sum over all partial transitions:

$$\Lambda_t = \sum_b \Lambda_{\mu c} (a \longrightarrow b) \quad (10)$$

Since the sum over b cannot be evaluated in all its absoluteness, the following simplifying assumptions are usually adopted in the calculation of total muon capture rates:

(i) neglect of nucleon momentum dependent terms $(O(1/M))$ in the effective Hamiltonian for muon capture ;

(ii) identification of the operators appearing in M_I $(I = 1,2,4)$ as the generators of the Wigner supermultiplet (the spin-isospin SU(4) group) which then yields the relations due to Foldy and

Walecka [10]

$$\sum_b |M_1|^2 = \frac{1}{3} \sum_b |M_2|^2 = \sum_b |M_4|^2 \qquad (11)$$

(iii) replacement of the quantity $\nu_{ab} = m_\mu c^2 - (E_b - E_a) = m_\mu c^2 - \Delta E_{ba}$ by $m_\mu - \widetilde{\Delta E}$ independent of the final nuclear state. Since 90% of the total capture is due to the partial capture rate to the giant dipole (GDR) state as pointed out by Foldy and Walecka [10], $\widetilde{\Delta E}$ could be a representative value for the narrow band of energies where the GDR strength is concentrated and (iv) use of closure approximation, that is, the levels b of the final nucleus are assumed to form a complete set, so that $\sum_b |b\rangle\langle b| = 1$.

With these four assumptions the total capture rate becomes

$$\Lambda_t = \frac{\tilde{\nu}^2}{2\pi} |\phi_\mu|^2 \quad G^2(G_V^2 + 3\, G_A^2 + G_P^2 - 2\, G_P G_A) I \qquad (12)$$

where $$I = \sum_{ij} \langle a | \tau_i^+ \tau_j^- \exp\left[i \tilde{\nu} \cdot (\vec{r}_i - \vec{r}_j)\right] | a \rangle \qquad (13)$$

Thus we see that the evaluation of total capture rate is reduced to the calculation of the ground state expectation values of certain operators appearing in eqn. (13). The eqns. (12) and (13) can now be evaluated in various nuclear models; the calculation in the Fermi gas shell and statistical models has been carried out by Rood [11] and the evaluation of the total capture rate using the Unitary Model Operator Approach (UMOA) wave functions has been done by Parthasarathy and Waghmare [12].

The approach outlined above for the total capture rate using closure approximation is an example of a Non-Energy Weighted Sum Rule (NEWSR) : This is because in the $\sum_b$ in eqn. (10), we are pulling out the quantity ν_{ba} by assuming an average energy transfer $\tilde{\nu}$ according to assumption (3) stated above, and then the closure approximation is applied to sum over a complete set of final states. The main drawback of this approach is the uncertainty regarding $\tilde{\nu}$, the average neutrino energy, the value of which depends on physical intuition and guess work. There have been attempts to go beyond the framework of NEWSR by Bernabeu [13] who expands ν_{ab} as a Taylor series around the mean value $\tilde{\nu}$, and by Rosenfelder [14] who has developed systematic corrections to the closure approximation by introducing the notion of two or more mean excitation energies, the corrections depending on energy moments of distribution of transition strength. In the approach of Goulard and Primakoff [6], the ground state expectation value of the operator product $\theta^- \theta^+$, where

$$\theta_\pm = \sum_{i=1}^{A} e^{\pm \hat{\nu} \cdot \vec{r}_i} \tau_{i\pm} \tag{14}$$

is decomposed into its isoscalar, isovector, and isotensor parts, and the ground state expectation value of $\theta^- \theta^+$ becomes

$$\langle a | \theta^- \theta^+ | a \rangle = Z \left[1 + \frac{A}{2Z} \beta_0 - \left(\frac{A-Z}{2A} \right) + \frac{|A-2Z|}{8ZA} \right) \beta_2 \right] \tag{15}$$

where

$$\beta_0 = \langle a \| 2K_0 + K_2 \| a \rangle$$
$$\beta_2 = \langle a \| 4K_2 \| a \rangle \tag{16}$$

with the reduced matrix elements defined by

$$A\langle a | K_0 | a\rangle = \langle a | \sum_{i \neq j} e^{i\vec{\nu}\cdot\vec{r}_i} e^{i\vec{\nu}\cdot\vec{r}_j} \frac{2}{3}\vec{\tau}_i\cdot\vec{\tau}_j | a\rangle$$

and

$$\frac{1}{A}\left[3T_Z^2 - T_Z(T_Z+1)\right]\langle a \| K_2 \| a\rangle$$
$$= \langle a | \sum_{ij} e^{-i\vec{\nu}\cdot\vec{r}_i} e^{i\vec{\nu}\cdot\vec{r}_j} \frac{1}{3}(\vec{t}_i\cdot\vec{\tau}_j - 3t_{iz}\tau_{jz}) | a\rangle \tag{17}$$

Using eqn. (15) and the NEWSR for closure approximation, the total capture rate now becomes

$$\Lambda_t = \gamma\Lambda(1,1) Z_{eff.}^4 \left(\frac{\tilde{\nu}}{m_\mu}\right)^2 \left\{1 + \frac{A}{2Z}\beta_0 - \left(\frac{A-Z}{2A} + (|A-2Z|/8ZA)\right)\beta_2\right\} \tag{18}$$

However, the above formula still contains the parameter $\tilde{\nu}$ which has to be fixed either by physical intuition or by an appeal to experiment; to eliminate the dependence of the total capture rate on $\tilde{\nu}$, Goulard and Primakoff employ a combination of non-energy and energy weighted sum rules (NEWSR and EWSR) to arrive at the following expression for the total capture rate.

$$\Lambda_t = \gamma \Lambda(1,1)\, Z_{eff.}^4 \left[1 + \frac{A}{2Z}\beta_1 - \frac{A-2Z}{2Z}\beta_2 - \left\{ \frac{A-2Z}{2A} - \frac{|A-2Z|}{8ZA} \right\} \beta_2 \right] \tag{19}$$

The above expression , with β_1, β_2 and β_3 treated as constants independent of A and Z (taken to be the same for all the initial nuclear ground states), constitutes a three parameter fit to the experimental data on total muon capture rates. The values for the constants obtained by Goulard and Primakoff [6] are

$$\beta_1 = -0.03\ ,\ \beta_2 = -0.25\ ,\ \beta_3 = 3.24 \tag{20}$$

A microscopic calculation of the constants β_1, β_2 and β_3 has been carried out by Mekjian [15] who found that the introduction of long range correlation brings theory into better agreement with experiment.

We have evaluated total capture rates in ^{93}Nb, ^{235}U , ^{239}Pu, ^{232}Th and ^{92}Zr using eqn. (20) and results are presented in Section 6.

4. Hyperfine Effects

In this Section we discuss hyperfine effects in muon capture following Bernstein et. al. [16] . For non-zero nuclear spin I, the spin of the muon in the atomic Bohr orbit will couple with the nuclear spin I to form states of total spin I ± 1/2, known as hyperfine states. The average total muon capture rate is then the statistical sum of the two capture rates from the hyperfine states $I \pm 1/2$, denoted by λ_+ and λ_-. Thus we can write the average total muon capture rate ($\bar{\lambda}$) as

$$\bar{\lambda} = \left\{ (I+1)\,\lambda_+ + I\lambda_- \right\} \Big/ (2I+1) \tag{21}$$

The capture rates λ_+ and λ_- will in general be different due to two reasons: (1) There is in general a correlation between the spin of μ^- and I, and also between the proton spin and I, especially for odd Z nuclei. (2) The probability of a μ^- capture by a proton depends on their relative spin orientation. This is easily seen from the expression for muon capture rate in hydrogen, which is proportional to

$$\left\langle p \,\middle|\, a + b\,\sigma_p \cdot \sigma_\mu \,\middle|\, \mu^- \right\rangle \tag{22}$$

where

$$a = G_V^2 + 3G_A^2 + G_P^2 - 2G_P G_A$$

$$b = 2G_A^2 + 2G_V\, g_A - \tfrac{2}{3}\, G_V\, G_P + \tfrac{4}{3}\, G_P\, g_A\,.$$

A rough estimate of the difference between λ_+ and λ_- for captures from two hyperfine states $I + 1/2$ and $I - 1/2$ has been given by Bernstein et. al. [16] . For a nucleus with odd Z and A, they assume a spinless core of even number of protons and neutrons and an 'outside' proton which is regarded as free.

The difference between λ_+ and λ_- is then calculated according to the two effects mentioned above and their result can be written as follows:

$$\lambda_+ - \lambda_- = \begin{cases} \frac{b}{aZ'} (2I + 1)\, \bar{\lambda}/I & \text{for } I = L + 1/2 \\ -\frac{b}{aZ'} (2I + 1)\, \bar{\lambda}/(I + 1) & I = L - 1/2 \end{cases} \tag{23}$$

where L is the orbital angular momentum of the odd proton and $z' = (Z-1)\xi + 1$, ξ is the final state exclusion principle suppression factor. The constants a and b are given by the same expression as in eqn. (22). It is seen from eqn. (23) that the difference in hyperfine rates is proportional to $1/Z$ and hence negligible for heavy nuclei. For example, in the case of $^{93}Nb(Z = 41)$, $\lambda_+ - \lambda_- = 0.05\bar{\lambda}$.

Based on the arguments presented above, the effects of hyperfine capture rates are seen to be negligible. This can also be seen from another argument. We can imagine the nucleus ^{93}Nb to consist of a spinless ^{92}Zr $(I=0)$ core and an odd valence proton $(L = 4)$ with $j = 9/2$. Thus the total capture rate may be thought of as sum of contributions from the spinless core and the odd proton, the capture rate now closely resembles that of hydrogen with $j = 9/2$. Employing the formula

$$(H) = \frac{158\ s^{-1}}{g_V^{\beta\,2} + 3g_A^{\beta\,2}} \left\{ a + b \langle \vec{\sigma}_\mu \cdot \vec{\sigma}_p \rangle \right\} \tag{24}$$

where $\vec{\sigma}_\mu \cdot \vec{\sigma}_p = -\frac{11}{9}$ for F_- state and $= 1$ for F_+ state, and a and b are give by eqn. (22).

Substituting in the above equations, we find that

$$\Delta \bar{\lambda} = \Lambda(H,F_-) - \Lambda(H,F_+) = 170\ s^{-1}$$, which is very

small compared to the total capture rate $\sim 10^6\ s^{-1}$.

5. Effect of Momentum Dependent Terms (MDT):

In this section we discuss the effect of momentum dependent terms ($O(P/M)$) on total muon capture rates. These terms contribute essentially through the cross terms with momentum independent terms. Following Rood [11] , the change in the matrix element squared due to the influence of MDT can be expressed as

$$(\Delta M^2) = - G_V\, g_V\, M_1^2 - (G_A - G_P)\, g_A\, M_2^2 + G_A\, g_A\, M_3^2 \quad (25)$$

where

$$M_I^2 = \int \frac{d\Omega_\nu}{4\pi} \left[\langle a | \sum_{i,j} \tau_i^{(+)} \tau_j^{(-)} e^{i\vec{\nu}\cdot(\vec{r}_{ij})} O_I | a \rangle + c.c. \right] \quad (26)$$

with $I = 1,2,3$ and

$$O_1 = \hat{\nu}\cdot\vec{p}_j/M , \quad O_2 = (\vec{\sigma}_j \cdot \vec{p}_j)/M$$

$$O_3 = \vec{\sigma}_i \times \vec{p}_i/M$$

We can now separate the $i = j$ and $i \neq j$ terms in eqn. (26) and perform partial integrations.

$$M_1^2 = M_{11}^2 + M_{12}^2 + M_{13}^2 \tag{27}$$

where

$$M_{11}^2 = 2\int \frac{d\Omega_\nu}{4\pi} \langle a| \sum_i \tfrac{1}{2}(1+\tau_i^{(3)}) [(\vec{p}_i \cdot \hat{\nu})/M] |a\rangle \tag{28}$$

$$M_{12}^2 = \int \frac{d\Omega_\nu}{4\pi} \langle a| \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} e^{i\vec{\nu}\cdot\vec{r}_{ij}} [(\vec{p}_i + \vec{p}_j)\cdot \hat{\nu}/M] |a\rangle \tag{29}$$

$$M_{13}^2 = \frac{\nu}{M} \int \frac{d\Omega_\nu}{4\pi} \langle a| \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} e^{i\vec{\nu}\cdot\vec{r}_{ij}} |a\rangle \tag{30}$$

It has been shown by Rood [11] that due to the averaging over neutrino directions $\int \frac{d\Omega_\nu}{4\pi}$, M_{11}^2 and M_{12}^2 vanish. Hence only the third quantity M_{13}^2 remains, which when compared with the $i \neq j$ part of eqn. (13) can be written as

$$M_{13}^2 = -\left(\frac{\nu}{M}\right) Q \tag{31}$$

where

$$Q = \langle a| \sum_{i \neq j} \tau_i^{(+)} \tau_j^{(-)} e^{-i\hat{\nu}\cdot\vec{r}_{ij}} |a\rangle$$

We may similarly split M_2^2 into three parts,

$$M_2^2 = M_{21}^2 + M_{22}^2 + M_{23}^2 \tag{32}$$

and it can be shown that the angular integration $\int \frac{d\,\Omega_\nu}{4\pi}$ gives $M_{21}^2 = M_{22}^2 = 0$. Utilising the SU(4) relations

$$M_V^2 = \frac{1}{3}\, M_A^2 = M_P^2 \tag{33}$$

we may write M_{23}^2 as

$$M_{23}^2 = -\,(\nu/M)\; Q \tag{24}$$

where Q is defined as in eqn. (31). Further, if we assume a pure shell model for the state $|a\rangle$ (nuclear ground state), then it has been shown by Rood [11] that M_3^2 reduces to zero. Thus the correction due to the momentum dependent terms can be expressed as

$$(\Delta M^2) = \left[G_V\, g_V + (G_A - G_P)\, g_A\right] \frac{\nu}{M}\; Q \tag{35}$$

with

$$Q = \langle a| \sum_{i \neq j} \tau_i^{(+)}\, \tau_j^{(-)}\, e^{i\, \vec{\nu}\,\cdot\,\vec{r}_{ij}} |a\rangle \tag{36}$$

The quantity Q depends on the correlation between the nucleons, and it has been evaluated for various nuclear models by Rood; it takes a simple form in the Fermi gas model, which can be written as, following Bell and Loseveth [17],

$$Q = Z \left\{ 1 - 3/2 \, \nu/2k_F - \frac{1}{2} \left(\frac{\nu}{k_F} \right)^3 \right\} \tag{37}$$

where

$$k_F = \left(\frac{3}{2\pi^2} \right)^{1/2} \rho^{1/3}$$

k_F is the Fermi momentum, and ρ is the density of nucleons.

Thus the correction from MDT to the total capture rate is

$$(\Delta\Lambda) = \frac{\gamma^2}{2\pi} \left| \phi_\mu \right|^2_{av.} (\Delta M^2) \tag{38}$$

We present numerical results for $\Delta\Lambda$ in Section 6 along with discussion.

6. Numerical Results and Discussion :

In Table 1, relative contributions of momentum dependent terms to the total muon capture rate for various nuclei are presented. The value for ^{16}O and ^{40}Ca are taken from Rood [11] .

TABLE 1

Nuclei	$\Delta\lambda/\lambda$
^{16}O	0.1000
^{40}Ca	0.0900
^{92}Zr	0.0490
^{93}Nb	0.0420
^{232}Th	0.0104
^{235}U	0.0101
^{239}Pu	0.0088

From this table, we see that as Z increases, the momentum dependent term correction decreases. For ^{93}Nb the MDT correction is $\sim 4\%$, whereas for heavier nuclei such as ^{235}U, ^{239}Pu, it is $\sim 1\%$. Recalling that there is a difference of about 6% between normal and abnormal capture rates (with and without θ_c), we see that the corrections from MDT (unrelated to θ_c) are of the same order of magnitude for ^{93}Nb.

In Table 2, we present results for total capture rates along with experiment data of Johnson et. al. [18] and Eckhause et. al. [19] .

TABLE 2

Nucleus	without MDT	with MDT	Rescaled value (dividing by Cos θ_c)	Experiment
^{92}Zr	0.8244	0.867	-	0.851 ± 0.007
^{93}Nb	0.9500	0.992	1.0543	1.04 ± 0.014
^{232}Th	1.1100	1.1500	-	1.22 ± 0.03
^{235}U	1.2	1.2408	1.3187	1.29 ± 0.03
^{239}Pu	1.28	1.3208	1.4038	1.33 ± 0.04

From the table, we find that for even A nuclei, the inclusion of MDT brings theory into better agreement with experiment . For odd - A nuclei, even after taking into account the MDT corrections, there is a residual discrepancy which can be accounted for writhin the context of the Salam-Strathdee idea of the vanishing of the Cabibbo angle. Although the ^{93}Nb total muon capture rate could be accounted for by taking into account both MDT and vanishing of the Cabibbo angle, that in ^{235}U and ^{239}Pu can be accounted mainly by the Salam-Strathdee hypothesis since MDT corrections are negligible.

However, our results cannot be taken as an unambiguous indication of the vanishing of θ_c. Recently, Suzuki [20] has made an extensive study of total capture rates with improved experimental techniques, and his results for ^{93}Nb do show an anomalously large capture rate when compared with neighbouring nuclei. Also, Wileke et. al. [21] have shown that the large capture rates in Actinide nuclei, such as the ones we are considering, can be explained on the basis of the resonance model of Kozlowski and Zglinski [22]. Thus, it seems that the vanishing of θ_c is not the only explanation for the large capture rates; nuclear structure effects seem to play an equally important part. In absence of a clear indication of the ultra-high magnetic fields in nuclei required for the vanishing of θ_c [23] and in view of the importance of nuclear structure effects, our calculations show the importance of hyperfine effects and momentum dependent terms, which should be taken into account before drawing any conclusions about the vanishing of θ_c.

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C H A P T E R   VI

WEAK INTERACTION ASPECTS OF MUON CAPTURE*

PART V

1. Introduction

In this chapter we study some weak interaction aspects of muon capture, namely the intermediate vector boson (IVB) aspect of the weak interaction and some elementary particle aspects pertaining to the second class induced tensor form factor (g_T) on the basis of Generalised Meson Dominance (GMD) model of Igarishi et. al. [1].

Recently the gauge theory of weak interactions has emerged as a successful renormalizable theory following the pioneering work of Salam [2] and Weinberg [3]. This theory which unifies electromagnetic and weak interactions is based on the group $SU(2)_L \times U(1)$, and its prediction of neutral currents via the neutral Z boson has been confirmed by experiments [4]. However, there are many interesting features in an enlarged version, viz. the $SU(2)_L \times SU(2)_R \times U(1)$ model [5]. This model reduces to the $SU(2)_L \times U(1)$ theory at low energies since the right handed gauge bosons associated with the gauge group $SU(2)_R$ are believed to be heavier than the corresponding left handed gauge bosons of the gauge group $SU(2)_L$. One of the interesting features of the $SU(2)_L \times SU(2)_R \times U(1)$ model is that the weak interaction

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* R. Parthasarathy and V.N. Sridhar, Silver Jubilee Physics Symposium, BARC, BOMBAY, 1981

CHAPTER VI

WEAK INTERACTION ASPECTS OF MUON CAPTURE*

1. Introduction

In this chapter we study some weak interaction aspects of muon capture, namely the intermediate vector boson (IVB) aspect of the weak interaction and some elementary particle aspects pertaining to the second class induced tensor form factor (g_T) on the basis of Generalised Meson Dominance (GMD) model of Igarishi et. al. [1] .

Recently the gauge theory of weak interactions has emerged as a successful renormalizable theory following the pioneering work of Salam [2] and Weinberg [3] . This theory which unifies electromagnetic and weak inteactions is based on the group $SU(2)_L \times U(1)$, and its prediction of neutral currents via the neutral Z boson has been confirmed by experiments [4] . However, there are many interesting features in an enlarged version, viz. the $SU(2)_L \times SU(2)_R \times U(1)$ model [5] . This model reduces to the $SU(2)_L \times U(1)$ theory at low energies since the right handed gauge bosons associated with the gauge group $SU(2)_R$ are believed to be heavier than the corresponding left handed gauge bosons of the gauge group $SU(2)_L$. One of the interesting features of the $SU(2)_L \times SU(2)_R \times U(1)$ model is that the weak interaction

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*R.Parthasarathy and V.N.Sridhar, Silver Jubilee Physics Symposium, BARC, BOMBAY, 1981 is a mixture of (V-A) and (V+A), and becomes parity conserving (i.e. contains equal amounts of left and right handedness) at high energies ($\sim$300 GeV). In such theories, an analysis of neutrino interactions by Bajaj and Rajasekharan [6] and Rizzio and Sidhu [7] yield a value for the mass of the right hand vector boson $M_{W_R} > 300$ GeV. The concept of manifest left-right symmetry in the weak interaction Hamiltonian was put forward by Beg et. al. [16] who argued that parity non-conservation at low energies was due to the spontaneous symmetry breakdown and by comparing with existing low energy experimental data, they deduce that there could be a $\sim$13% (V+A) admixture in the weak Hamiltonian.

In this chapter, we introduce (V + A) admixture in the Fujii-Primakoff Hamiltonian for muon capture and use it to compute hyperfine singlet and triplet capture rates in muon capture by hydrogen. In sections 2 and 3, we give the formulation of capture rate without and with (V + A) admixture respectively. In section 4, a value for the mixing parameter is deduced and a qualitative estimate for the mass of right handed intermediate vector boson is given which is not in disagreement with the values obtained from gauge theories. In Section 5 we discuss the GMD model of Iganishi et.al. [1] and derive an expression for the ratio of the second class coupling constant (g_T) to the vector coupling constant (g_V) in terms of strong and weak couplings of mesons and their masses.

In Section 6, we obtain a value for f_B / f_ρ, the ratio of B meson lepton coupling to ρ meson lepton coupling, from our value of g_T deduced from the study of $\gamma - \nu$ angular correlations and average recoil nuclear polarization.

2. Muon Capture Rate in Hydrogen

The elementary process of interest is

$$\mu^- + p \longrightarrow n + \nu_\mu \tag{1}$$

which is described by the Fujii-Primakoff Hamiltonian (FPH)

$$H = \frac{1}{2}(1 - \vec{\sigma}_\ell \cdot \hat{\nu}) \left[G_V + G_A \vec{\sigma}_\ell \cdot \vec{\sigma}_p + G_P \sigma_p \cdot \hat{\nu} \right] \tag{2}$$

neglecting momentum dependent terms. The effective coupling constants G_V, G_A and G_P in the above equation have already been defined in Chapter I and we do not repeat it here. The initial $\mu^- p$ system can exist in two hyperfine states, viz., the triplet (spin 1) and the singlet (spin 0) states. The capture rates for the two hyperfine states are different as first pointed out by Bernstein et. al. [8] and we now proceed to calculate them following Konopinski [9].

From Fermi's golden rule, the capture rate (λ) for process (1) is given by

$$\lambda = \frac{\nu^2}{2\pi} \left| \langle n \nu_\mu | \frac{1}{2}(1 - \vec{\sigma}_\ell \cdot \hat{\nu})(G_V + G_A \vec{\sigma}_\ell \cdot \vec{\sigma}_p + G_P \vec{\sigma}_p \cdot \hat{\nu}) | p \mu^- \rangle \right|^2 \tag{3}$$

where ν is the neutrino momentum and other symbols have the usual meaning. The FPH may be conveniently written as

$$(A + \vec{\sigma}.\vec{B}) \tag{4}$$

where

$$A = G_V - G_A \vec{\sigma_p}\cdot\hat{\nu} + G_P \vec{\sigma_p}\cdot\hat{\nu} \tag{4a}$$

$$B = G_A\vec{\sigma_p} - G_V\hat{\nu} - G_P \hat{\nu}(\vec{\sigma_p}\cdot\hat{\nu}) + i\, G_A(\vec{\sigma_p} \times \hat{\nu}) \tag{4b}$$

We now sum over the final neutron and neutrino states (closure) but do not average over the initial state $|p\mu^-\rangle$ since we wish to retain its identity. As the neutrino is not observed, we integrate over the solid angle of the neutrino using the relations (terms odd in $\hat{\nu}$ vanish on averagingover neutrino directions):

$$\int (\vec{\sigma}\cdot\hat{\nu}) \frac{d\hat{\nu}}{4\pi} = 0 \tag{5}$$

$$\int (\vec{\sigma_\ell}\cdot\hat{\nu})(\vec{\sigma_p}\cdot\hat{\nu}) \frac{d\hat{\nu}}{4\pi} = \frac{1}{3}\vec{\sigma_\ell}\cdot\vec{\sigma_p} \tag{6}$$

The capture rate is then obtained as

$$\lambda = \frac{\nu^2}{2\pi}\Big[G_V^2 + G_P^2 + 3G_P^2 - 2G_PG_A - 4\big\{G_A (G_A - \tfrac{2}{3}G_P) - G_V(G_A - \tfrac{1}{3}G_P)\big\}\langle p\mu^-|\sigma.\sigma_p|p\mu^-\rangle\Big] \tag{7}$$

Since $\langle \sigma \cdot \sigma_p \rangle = -3$ for singlet state

$= +1$ for triplet state

the singlet (λ_s) and triplet (λ_t) rates may be written as

$$\lambda_s = \frac{\nu^2}{2\pi} \left[(G_V - 3\,G_A)^2 + G_P^2 + 2\,G_P\,G_V - 6\,G_P\,G_A \right] \quad (8)$$

$$\lambda_t = \frac{\nu^2}{2\pi} \left[(G_V + G_A)^2 + G_P^2 - 2/3\,G_P\,(G_A + G_V) \right] \quad (9)$$

3. (V + A) admixture

To introduce the (V + A) current into the Fujii-Primakoff Hamiltonian, we note that the (V + A) lepton current is $\bar{\psi}_{\nu_\mu}\,\gamma_\mu\,(1 + \gamma_5)\,\psi_\mu$ which reduces to $\frac{1}{2}(1 + \vec{\sigma} \cdot \hat{\nu})$ on performing the non-relativistic reduction. For the bare hadron current $\bar{\psi}_n\,\gamma_\mu\,(1 + \gamma_5)\,\psi_p$, we take $g_A = +\,1.25\,g_V$. With these changes the modified Hamiltonian may be written as

$$H = \frac{1}{2} \left[(1-\lambda)\,(1 - \vec{\sigma}_\ell \cdot \hat{\nu})\,(G_V + G_A\,\sigma_\ell \cdot \sigma_p + G_P\,\sigma_p \cdot \hat{\nu}) + \lambda(1 + \vec{\sigma}_\ell \cdot \hat{\nu}) \right.$$
$$\left. (G_V + G_A'\,\vec{\sigma}_\ell \cdot \vec{\sigma}_p + G_P'\,\vec{\sigma}_p \cdot \hat{\nu}) \right] \quad (10)$$

λ is the mixing parameter and the primes on G_A and G_P refer to the fact that we are putting $g_A = 1.25\,g_V$. Employing the same method of calculation as in the last section, we may write

$$H = \frac{1}{2}\left[(1-\lambda)\,(A + \sigma.B) \;+\; \lambda(C + \sigma.D)\right] \tag{11}$$

where

$$C \;=\; G_V + G_A'\,\sigma_p. \quad + G_P'\,\sigma_p.\,\hat{\nu} \tag{11a}$$

$$D \;=\; G_V\,\hat{\nu} - i\,G_A'\,\vec{\sigma_p}\times\hat{\nu} + \;G_P'\,\hat{\nu}(\vec{\sigma_p}.\;\hat{\nu}) + G_A'\,\vec{\sigma_p} \tag{11b}$$

and A and B are given by eqns. (4). It is interesting to note that the cross terms vanish in the calculation; this is easily seen by computing the term:

$\left\{(1 - \vec{\sigma_\ell}.\,\hat{\nu})\;(\vec{\sigma_\ell}.\,\vec{\sigma_p})\right\}^{\dagger}(1 + \vec{\sigma}.\,\hat{\nu})$ which is equal to zero.

The singlet and triplet capture rates can now be written as

$$\lambda_s \;=\; \frac{\nu^2}{2\pi}\left[\left\{(G_V - 3G_A)^2 + 2\,G_P(G_V - 3G_A) + G_P^2\right\}(1-\lambda)^2 + \lambda^2 \left\{(G_V - 3\,G_A')^2 + G_P'^2 - 2G_P'\,(G_V - 3G_A')\right\}\right] \tag{12}$$

$$\lambda_t \;=\; \frac{\nu^2}{2\pi}\left[\left\{(G_V + G_A)^2 + G_P^2 - \frac{2}{3}\,G_P\,(G_V + G_A)\right\}(1-\lambda)^2 + \lambda^2 \left\{(G_V + G_A')^2 + \frac{2}{3}\,G_P'\,(G_A' + G_V) + G_P'^2\right\}\right] \tag{13}$$

4. Numerical Results and Discussion

(a) Numerical Results:- Choosing the canonical values for the coupling constants as given in Chapter I, we obtain according to eqns. (8) and (9),

$$\lambda_s = 643.7 \ s^{-1} \tag{14}$$

$$\lambda_t = 12.87 \ s^{-1} \tag{15}$$

The world average of experiments as quoted by Mukhopadhyay [10] for the singlet capture rate is

$$\lambda_s = (661 \pm 48) \ s^{-1} \tag{16}$$

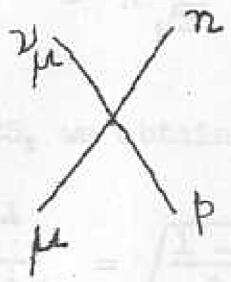
Due to its small value, the triplet capture rate λ_t is difficult to measure accurately, and only an upper bound exists at present:

$$\lambda_t < 103 \ s^{-1} \tag{18}$$

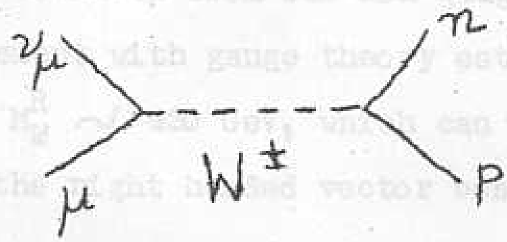
To obtain an estimate for λ, we drop λ^2 terms (λ is assumed to be small) and compare eqn. (12) with experiment to obtain $\lambda = 0.025$, indicating a small (V + A) admixture in the muon capture Hamiltonian.

(b) Discussion: The above theoretical calculations as well as the experimental values reveal that the muon capture interaction is predominantly (V-A) in character; for (V+A) interaction the rates are almost equal ($\lambda_s \sim \lambda_t$) contradicting experiment However, our value for λ, the mixing parameter, can be utilised to deduce a lower limit for the mass of right handed vector boson (M_{W_R}), the value of which is $\sim$ 300 GeV according to present day gauge theories.

In the intermediate vector boson (IVB) picture of weak interactions, the massive charged vector bosons $W^{\pm}$ mediate the interaction. Diagrammatically, instead of the usual contact interaction



we have the following diagram:



At low momentum transfers $q^2 \ll m_W^2$, the correspondence between the two pictures is given by [11]

$$\frac{g^2}{M_W^2} = \frac{G}{\sqrt{2}} \tag{19}$$

where g is the coupling constant for the $W^{\pm}$ meson and G is the weak coupling constant. Assuming that left and right IVB's mediate (V-A) and (V+A) interactions respectively, we may write

$$\frac{g^2}{M_W^{(V-A)\,2}} = (1-\lambda)G/\sqrt{2} \tag{20}$$

$$\frac{g^2}{M_W^{(V+A)\,2}} = \lambda \frac{G}{\sqrt{2}} \tag{21}$$

Putting $\lambda = 0.025$, we obtain

$$\frac{M_W^{V+A}}{M_W^{V-A}} = \sqrt{\frac{1-\lambda}{\lambda}} \simeq 6 \tag{22}$$

This naive estimate from our non-gauge theory calculation is not in disagreement with gauge theory estimates. Since $M_W^L \sim 70$ GeV [12] we obtain $M_W^R \sim 420$ GeV, which can be interpreted as the lower limit for the right handed vector boson mediating the (V+A) interaction.

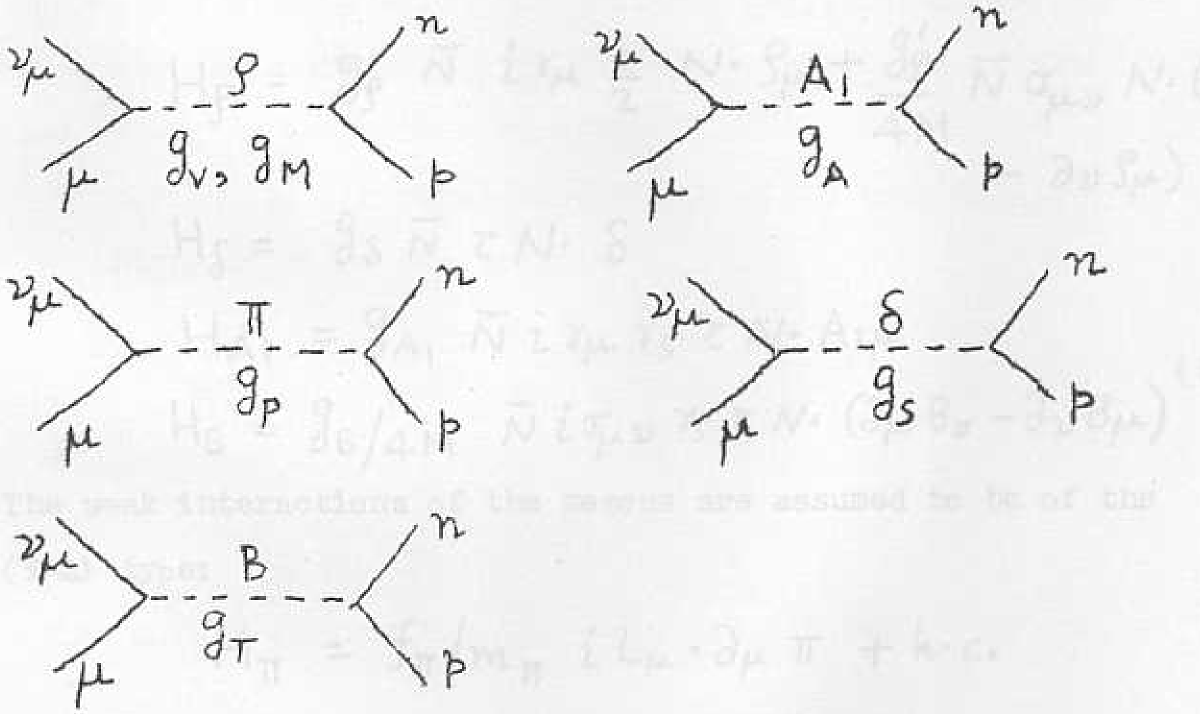
5. Generalised Meson Dominance (GMD) Model

The generalised meson dominance (GMD) model for weak interactions was proposed by Igarishi et. al. [1] as a natural extension of Yukawa's theory of β-decay. This model provides a more fundamental justification for the phenomenological formulation of Weinberg [17] for the six hadronic form factors. In this model, various strongly interacting mesons dominate the hadronic weak form factors, an approach similar in spirit to the one pion exchange diagram which gives rise to the induced pseudoscalar coupling in muon capture. Employing lowest order perturbation theory and both derivative and non-derivative couplings for the meson nucleon vertex, Igarishi et. al. express the weak hadronic form factors in terms of strong and weak couplings of mesons and their masses.

In the GMD model, the following isovector mesons contribute to weak form factors:

$$\pi(140), \quad \rho(760), \quad \delta(960), \quad A_1(1070), \quad B(1220)$$

where the numbers in the bracket refer to masses in MeV. Explicitly, we have the following diagrams:



where the form factors are defined by

$$\langle n | V_\mu | p \rangle = \bar{u}_n i \left[\gamma_\mu g_V + \sigma_{\mu\nu} q_\nu \frac{g_M}{2M} + i q_\mu \, g_S / m_\mu \right] u_p$$

$$\langle n | A_\mu | p \rangle = \bar{u}_n i \left[\gamma_\mu g_A + \sigma_{\mu\nu} q_\nu \frac{g_T}{2M} + i q_\mu g_P / m_\mu \right] \gamma_5 u_p \tag{23}$$

where the form factors are real assuming time reversal invariance and g_S and g_T are the second class form factors. In this model, the strong interactions of the intermediate vector mesons are assumed to be described by the following Hamiltonian:

$$\begin{aligned}
H_\pi' &= g_\pi \bar{N} i \gamma_5 \tau N \cdot \pi \\
H_\rho &= g_\rho \bar{N} i \gamma_\mu \frac{\tau}{2} N \cdot \rho_\mu + \frac{g_\rho'}{4M} \bar{N} \sigma_{\mu\nu} N \cdot (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) \\
H_\delta &= g_\delta \bar{N} \tau N \cdot \delta \\
H_{A1} &= g_{A_1} \bar{N} i \gamma_\mu \gamma_5 \tau N \cdot A_{1\mu} \\
H_B &= g_B / 4M \; \bar{N} i \sigma_{\mu\nu} \gamma_5 \tau N \cdot (\partial_\mu B_\nu - \partial_\nu B_\mu)
\end{aligned} \tag{24}$$

The weak interactions of the mesons are assumed to be of the (V-A) type:

$$H_\pi = f_\pi / m_\pi \, i L_\mu \cdot \partial_\mu \pi + h.c.$$

$$H_\rho = f_\rho i L_\mu \cdot \rho_\mu + h.c.$$

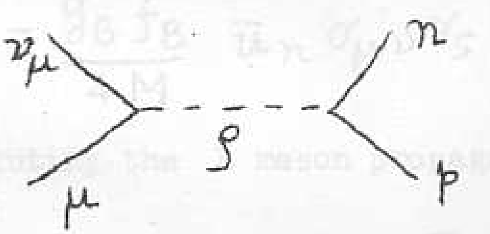
$$H_\delta = f_\delta / m_\delta \; L_\mu \cdot \partial_\mu \delta + h.c.$$

$$H_{A_1} = f_{A_1} \; i L_\mu \cdot A_{1\mu} + h.c.$$

$$H_B = f_B \; L_\mu \cdot B_\mu + h.c. \tag{25}$$

Using eqns. (24) and (25), the weak couplings F's in eqn. (23) can be expressed in terms of the strong and weak couplings of the mesons and their masses. In what follows, we shall be interested primarily in the ratio g_T / g_V i.e. the ratio of the induced tensor to the vector form factor .

To derive g_V in terms of the strong and weak coupling of the ρ meson, consider the diagram:



The matrix element for the above diagram may be written as

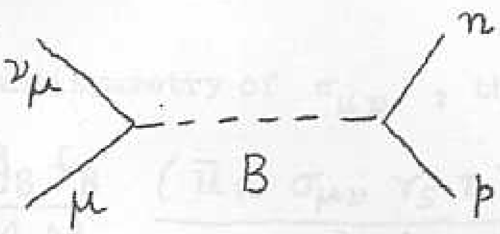
$$g_\rho f_\rho \left(\bar{u}_n \gamma_\mu \frac{\tau}{2} u_p\right) \frac{\left[\delta_{\mu\mu'} + q_\mu q_\nu / M_\rho^2\right]}{q^2 + M_\rho^2} \cdot L_{\mu'} \tag{26}$$

where we have used the expression for the vector boson propagator [11] . Since $q^2 \ll M_\rho^2$, the above expression reduces to

$$\frac{g_\rho f_\rho}{M_\rho^2} (\bar{u}_n \gamma_\mu \frac{\tau}{2} u_p) L_\mu$$

$$g_V = g_\rho f_\rho / 2 M_\rho^2 \tag{27}$$

To obtain g_T in terms of B-meson couplings, consider the following diagram:



The matrix element for the above diagram may be written as

$$- \frac{g_B f_B}{4M} \bar{u}_n \sigma_{\mu\nu} \gamma_5 u_p \cdot (q_\mu B_\nu - q_\nu B_\mu) B_{\mu'} L_{\mu'}$$

Substituting the B meson propagator

$$B_{\mu\mu'} = \left[\delta_{\mu\mu'} + \frac{q_\mu q_{\mu'}}{M_B^2} \right]$$

in the abo e expression, we obtain

$$-\frac{g_B f_B}{4M} \frac{(\bar{u}_n \sigma_{\mu\nu} \gamma_5 \tau u_p)}{q^2 + M_B^2} \left[q_\mu \left(\delta_{\nu\mu'} + \frac{q_\nu q_{\mu'}}{M_B^2} \right) - q_\nu \left(\delta_{\mu\mu'} + q_\mu q_{\mu'} / M_B^2 \right) \right] L'_\mu$$

Since $q^2 \ll M_B^2$, the above expression reduces to

$$-\frac{g_B f_B}{4M} \frac{(\bar{u}_n \sigma_{\mu\nu} \gamma_5 \tau u_p)}{M_B^2} (q_\mu L_\nu - q_\nu L_\mu)$$

Due to the antisymmetry of $\sigma_{\mu\nu}$, this leads to

$$2 . \frac{g_B f_B}{4M} \frac{(\bar{u}_n \sigma_{\mu\nu} \gamma_5 \tau u_p)}{M_B^2} L_\mu$$

From the above equation, we obtain an expression for g_T as

$$g_T = \frac{g_B f_B}{M_B^2} \tag{28}$$

Hence we obtain

$$g_T / g_V = 2 \left(\frac{M_\rho}{M_B} \right)^2 \left(\frac{g_B f_B}{g_\rho f_\rho} \right) \tag{29}$$

The strong couplings of the B and ρ mesons g_B and g_ρ are determined by comparing with experimental data on low energy strong interactions, as pointed out by Igarishi et. al. [1].

In particular, the B meson nucleon coupling has been determined to be [13]

$$g_B^2 / 4\pi = 72.84 \qquad (30)$$

while a comparison of the nucleon-nucleon one boson exchange potential (OBEP) constructed from the isovector mesons with experiment yields [14]

$$g_\rho^2 / 4\pi = 3.004 \qquad (31)$$

6. Second Class Currents

In the generalised Meson Dominance Model (GMD), the existence of the B meson with required couplings gives rise to a 'natural' existence of the axial second class current event at proton level. By analysing experimental data on the β decay ft - values of A = 12 system, Igarishi et. al. [1] obtains $g_T = -(1 \sim 0.2)\, g_A$, which leads to $\left|\frac{f_B}{f_\rho}\right| = \left(0.25 \sim 0.05\right)$ using eqn. (29). On the other hand, our analysis of two nuclear model insensitive observables, viz., the gamma-neutrino angular correlation coefficient β_2 and the average recoil nuclear polarization $(P_{av.})$ yields $g_T/g_A = (5.5 \pm 3)$ from which we obtain $\left|f_B / f_\rho\right| = (1.80 \pm 0.98)$.

Recently, Leroy and Pestieau [15] have raised the interesting possibility that the second class axial current could exist if the following decay mode of the τ meson is observed.

$$\tau^{\pm} \longrightarrow B^{\pm}(1220) + \nu_{\tau}, \qquad B^{\pm}(1220) \longrightarrow \omega + \pi^{\pm} \tag{32}$$

The following first class decay mode has already been observed

$$\tau^{\pm} \longrightarrow \rho^{\pm} + \nu_{\tau} \tag{33}$$

Comparing the rates for the above two processes, Leroy and Pestieau find that

$$\Gamma(\tau^{\pm} \longrightarrow B^{\pm} + \nu_{\tau}) \, / \, \Gamma(\tau^{\pm} \longrightarrow \rho^{\pm} + \nu_{\tau}) = 1.69 \, \frac{f_B^2}{f_\rho^2} \tag{34}$$

from which they conclude that there is a sizeable contribution of the axial second class current if $f_B/f_\rho \simeq 2.5$. Our value for $|f_B/f_\rho| = (1.8 \pm 0.98)$ deduced from an analysis of $\gamma - \nu$ angular correlations and average recoil nuclear polarization is not inconsistent with the predicted value of Leroy and Pestieau.

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