STUDIES IN
PHOTOPRODUCTION AND SCATTERING OF PIONS FROM LIGHT NUCLEI

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Preface

This thesis comprises the work done by the author during the years 1961-65 on the study of the photoproduction and scattering of pions from light nuclei under the guidance of Professor Alladi Ramakrishnan, previously Professor of Physics at the University of Madras and now the Director, MATSCIENCE, the Institute of Mathematical Sciences, Madras.

Five papers based on part of this work have been published and two more are in the process of publication. The available reprints have been attached to the thesis. Collaboration in some papers with my colleagues was necessitated by the nature and the range of the problems dealt with and due acknowledgment is made at the proper places.

The author is greatly indebted to Professor Alladi Ramakrishnan for his guidance and encouragement throughout the preparation of this work and very grateful to the University of Madras and the Institute of Mathematical Sciences for providing excellent facilities for research.

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STUDIES ON PHOTOPRODUCTION AND SCATTERING OF PIONS FROM LIGHT NUCLEI

Introduction

Among the collision processes in nuclear and elementary particle physics, two of the most important are the photoproduction and the scattering of pions. These are particularly useful in the study of the structure of light nuclei.

The principle aim of this thesis is the quantitative study of the structure of the following light nuclei; the deuteron, H^3, He^3 and He^4, through these processes. The choice of these nuclei has been made to focus attention on the problems relating to nuclear magnetic moments, quadrupole moments, charge independence and the charge symmetric nature of the nuclear forces without getting involved in the extraneous features like shell structure in more complex nuclei.

These nuclei offer the following special features :

(i) The deuteron : The non-availability of free neutron targets demands the search for available nuclear targets wherein the neutron is bound. The simplest of such targets is the deuteron which is equivalent to a free neutron target, especially for processes like negative pion production where the neutron alone takes part. The cross-section for processes involving single neutron can be obtained by extrapolation from the cross-sections for similar processes with deuterons. Also the deuteron is suitable for the study of pairwise interactions used for the analyses of many nucleon systems, especially for building a correct shell model structure.

(ii) H^3 and He^3 : The study of mirror nuclei H^3 and He^3 is useful to investigate in detail the the three-nucleon problem wherein all three nucleons interact strongly with one another. The series of nuclides H^1, H^2, H^3, He^3 and He^4 form an unrivalled set for the study of the changes arising from the addition or removal of a neutron or proton from a given nucleus. This allows the charge independent nature of nuclear forces to be investigated under 'carefully controlled conditions'. All this information when available will certainly help to throw light on the question of the existence or otherwise of an intrinsic three-nucleon force.[1)]

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1) H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker and M.R. Yearian, Phys. Rev. Letters, **11**, 132 (1963).

iii) He^4 : This smallest closed shell nucleus is strongly bound. The measure of its root mean square radius will indicate the amount by which the meson clouds of the physical nucleons will shrink due to nuclear binding.

Among the main results of the study are :

Both for the photoproduction and the scattering of pions from single nucleons the dispersion theoretic approach of Chew et al[1] is known to be successful. We show that the amplitudes of Chew et al can explain the experimental data[2] for the charged pion photoproduction even from deuterons if the impulse approximation[3] technique is used. Also it is shown that the inclusion of the hard core radius can explain the neutral pion photoproduction with slightly less percentage for the D state admixture than required by a similar calculation by Erickson and Shaerf.[4]

The study of neutral pion photoproduction from H^3 and He^3 indicates that the measurement of the differential cross-sections of neutral pion photoproduction from

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1) G.F.Chew, M.L.Goldberger, F.E.Low and Y. Nambu, Phys. Rev. 106 , 1345, (1957)
2) Beneventano, G. Bernardini, G.Stoppini and L.Tau , Nuovo Cimento, 10 , 1109 (1958).
3) G.F.Chew, Phys. Rev., 80 , 196 (1950)
4) G.F.Chew and G.C.Wick, Phys. Rev., 85 , 636 (1952)
G.F.Chew and M.L.Goldberger, Phys. Rev., 87 , 778 (1952)
4) B.F.Erickson and C.Shaerf, Phys. Rev. Letters , 11 , 432, (1963)

H^3 and He^3) will enable us to detect the amount of quenching of the nuclear magnetic moments of the bound nucleons, the result being practically independent of the S' state and $T = 3/2$ state admixtures.

It is shown that the measurement of the differential cross-sections for π^+ scattering by He^3 and π^- scattering by H^3 can be used to estimate the probabilities not only of the now familiar S' state but also the $T = 3/2$ state admixtures in the ground-state of the three-nucleon systems. The presence of the $T = 3/2$ state is extremely relevant[1] for the theoretical explanation of the experimentally measured electric charge form factors of the three nucleon systems as has been noted and stressed in a fundamental paper by Gibson and Schiff.[2].

The rather low value of the root mean square radius of the nucleus deduced from the elastic neutral pion photo-production cross-section shows that the nuclear binding shrinks the meson clouds of the physical nucleons.

These results are presented in detail in a sequence of ten chapters arranged for convenience into three parts dealing with the deuteron, the three nucleon systems and He^4 respectively.

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1) T.A.Griffy, Physics Letters, **11** , 155 (1964)
2) B.F.Gibson and L.I.Schiff, Phys. Rev., **138** , B 26 (1965)

In Chapter 1, the differential cross-sections for the photoproduction of charged pions from deuterons are estimated using the impulse approximation and a good fit[1] with the experimental data of Beneventano et al is found. The difference in the π^+ and the π^- cross-sections are analysed in detail.

In Chapter 2, the sensitivity of the differential cross-sections for the elastic photoproduction of neutral pions from deuteron, to the D state admixture, hard core radius and the two choices of the effective ranges, is analysed. It is found[2] that the D state admixture allowed by the calculations for the magnetic moment, quadrupole moment and the binding energy may explain the available experimental data for the neutral pion photoproduction if a large hard core radius r_c is taken.

Part II starts with Chapter 3, dealing with the modification of the anamolous magnetic moment of the nucleons i.e. quenching of the nucleon magnetic moments, when bound in nuclei is analysed. It is shown[3] that the differential cross-sections for the photoproduction of neutral pions

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1) V. Devanathan and K. Anantanarayanan, Nuovo Cimento, 32, 723, (1964)
2) K. Anantanarayanan and K. Srinivasa Rao (submitted to Nuovo Cimento)
3) G. Ramachandran and K. Anantanarayanan, Nuclear Physics, 52, 633, (1964).

from H^3 and He^3 are sensitive to quenching. Also it is proved that the differential cross-sections for the photoproduction of charged pions from single nucleons can be deduced from the differential cross-sections for the charged pions photoproduced from H^3 and He^3 if the nuclear form factors are known.

In the next Chapter, it is shown that the differential cross-sections for the charged pions scattered by H^3 and He^3 are sensitive to the S' state admixture at 90^0 for the scattering angle and it is suggested[1)] that the measurement of the cross-sections be used for an estimation of the S' state probability.

The most important conclusion[2)] of the thesis is relating to the $T = 3/2$ state admixture. We show in Chapter 5 that the differential cross-section in a direction perpendicular to the incident beam for the process $\pi^+ + He^3 \longrightarrow \pi^+ + He^3$ is considerably decreased by a small admixture of $T = 3/2$ state. If either $T = 3/2$ state or S' state alone is present then the admixture can be found from the experimentally measured differential cross-section for π^+ scattered by He^3 at

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1) G. Ramachandran and K. Anantanarayanan, Nuclear Physics, **64**, 632, (1965)

2) K. Anantanarayanan, Physics Letters, **18**, Number 3, 1965.

90° . On ~~x~~ the other hand, if an admixture of both the states exists, we suggest, the measurement of the differential cross-sections at 90° of π^{-} scattered by H^{3} which along with the measured differential cross-sections of π^{+} scattered by He^{3} at 90° will provide the percentage of admixture of both the states.

In Chapter 6 , we prove[1] that the conclusions of the Chapter 3 are still valid when we take an admixture of S' state and $T = 3/2$ state for the three nucleon systems.

In Chapter 7 , the estimates of the differential cross-sections for the photoproduction and scattering of pions using Irving and Irving-Gunn radial functions are made. It is suggested[1] that the choice of the proper radial functions can be made from the study of the differential cross-sections in the forward angles for the π^{+} scattered by H^{3} or He^{3}. The chosen function can be used to study the quenching effects.

The inclusion of P and D state admixtures are considered, in Chapter 8 , with reference to the scattering of pions from the three nucleon systems.

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1) K. Anantanarayanan (submitted to Nuclear Physics)

In the final Chapter of Part II an upperbound to the $T = 3/2$ state admixture in He^3 is found, using a perturbation calculation and taking into account only the Coulomb forces between the protons, for various choices of the radial functions. Necessary modifications are suggested when charge asymmetric nuclear forces are present.

Part III consisting of a single Chapter (Chapter 10) deals with the ground state of the He^4 nucleus. An estimate[1] of the root mean square radius for the He^4 nuclus is made from the experimental data[2] for the neutral pion photoproduction from He^4 .

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1) K. Anantanarayanan, Nuclear Physics (in print)

Chapter 1 *

1. The photoproduction of π mesons from deutrons has been discussed phenomenologically by many authors[1-5] using the impulse approximation[6] according to which "the meson production amplitudes from various nucleons in a nucleus are superposed linearly to form the production amplitude for the whole nucleus." This is an assumption analogous to the Born approximation in the scattering of X - rays by many electron systems or the Fermi approximation in the scattering of slow neutrons by crystals. Sufficient conditions for the validity[7] of the impulse approximation[@] are :

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* G. Ramachandran and K. Anantanarayanan (unpublished)

V. Devanathan and K. Anantanarayanan, Nuovo Cimento, 32, 723 (1964).

1) G.F.Chew and H.W.Lewis, Phys. Rev. 84, 779 (1951)
2) M.Lax and H.Feshbach, Phys. Rev. 88 , 500 (1952)
3) John Chappelear, Phys. Rev. 99 , 254 (1955)
4) S.Machida and T.Tamura, Prog. Theor. Phys. 6 , 572 (1951)
5) Y. Saito, Y.Watanabe and Y.Yamaguchi, Prog. Theor. Phys. 7 , 103 (1952)
6) G.F.Chew, Phys. Rev. 80 , 196 (1950)
7) G.F.Chew and G.C.Wick, Phys. Rev. 85 , 636 (1955).

@ see Appendix A .

(1) Small individual amplitudes in comparison to the distance between sources,

(2) Long mean free paths for both incoming and outgoing particles in comparison to the overall dimensions of the system,

(3) A "collision time" which is short compared to the period of the nuclear system.

These conditions are satisfied in our problem. The smallness of the fine structure constant guarantees the small amplitudes and long photon mean free path. The condition on the outgoing mesonm mean free path can, in the case of the deutron, be restated as a requirement that the meson-nucleon scattering cross section be small compared to the cross sectional area of deutron. The latter is $\sim 10^{-24} cm^2$ whereas the meson nucleon cross section is $\sim 10^{-26} cm^2$ only. The collision time seems certain to be sufficiently small, since the average extent to which energy conservation is violated in the intermediate states must be of the order of the meson rest energy. Also due to the weak binding of deutron the meson "clouds" surrounding the individual neutron and proton overlap only slightly. So, we can assume the validity of the approximation.

We shall discuss the photoproduction of pions from two and many body systems under impulse approximation, not in a phenomenological way as has been done by earlier authors,

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1) Chedester, Isaacs, Sachs and Steinberger, Phys. Rev., 82, 958 (1951).

but by using the Chew, Goldberger, Low and Nambu amplitudes[1] for the basic photoproduction processes

$$\gamma + N \rightarrow N + \pi \tag{1.1}$$

from a nucleon[2].

Denoting by $(\nu_0 = |\underset{\sim}{\nu}|, \underset{\sim}{\nu})$ the energy and momentum of the incident photon, $\underset{\sim}{\epsilon}$ the photon polarization, $(\mu_0 = \sqrt{\mu^2 + 1}, \underset{\sim}{\mu})$ the energy and momentum of the final meson in the barycentric system, the CGLN amplitudes can be written for the various processes as :

$$t(\gamma + p \rightarrow p + \pi^0) = f^{(+)} + f^{(0)} \tag{1.2}$$

$$t(\gamma + n \rightarrow n + \pi^0) = f^{(+)} - f^{(0)} \tag{1.3}$$

$$t(\gamma + p \rightarrow n + \pi^+) = \sqrt{2}(f^{(-)} + f^{(0)}) \tag{1.4}$$

$$t(\gamma + n \rightarrow p + \pi^-) = -\sqrt{2}(f^{(-)} - f^{(0)}) \tag{1.5}$$

where $f^{(+)}$, $f^{(-)}$ and $f^{(0)}$ are given with the neglect of secondary scattering corrections by

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1) G.F.Chew, M.L.Goldberger, F.E.Low and Y. Nambu, Phys. Rev. 106 , 1345 (1957). (Hereinafter referred to as CGLN amplitudes)

2) We shall use throughout the natural system of units where $\hbar = c = 1$ = pion mass.

$$f^{(+)} = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \Big[i\, \boldsymbol{\sigma} \cdot \boldsymbol{\mu} \times (\boldsymbol{\nu} \times \boldsymbol{\epsilon})\, \lambda\, h^{+-} + $$

$$+ i (\boldsymbol{\sigma} \cdot \boldsymbol{\mu}) \frac{(\boldsymbol{\mu} \cdot \boldsymbol{\epsilon})}{2 M \mu_0} + $$

$$+ \boldsymbol{\mu} \cdot (\boldsymbol{k} \times \boldsymbol{\epsilon})\, \lambda\, h^{++} \Big] \qquad (1.6)$$

$$f^{(-)} = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \Big[\frac{1}{(1 + \mu_0/M)} \Big[i\, \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} + \frac{2i}{(k^2+1)} (\boldsymbol{\sigma} \cdot \boldsymbol{k}) (\boldsymbol{\mu} \cdot \boldsymbol{\epsilon}) \Big] + $$

$$+ i\, \boldsymbol{\sigma} \cdot \boldsymbol{\mu} \times (\boldsymbol{\nu} \times \boldsymbol{\epsilon})\, \lambda\, h^{--} + $$

$$- i (\boldsymbol{\sigma} \cdot \boldsymbol{\mu}) \frac{(\boldsymbol{\mu} \cdot \boldsymbol{\epsilon})}{2 M \mu_0}$$

$$+ \boldsymbol{\mu} \cdot (\boldsymbol{\nu} \times \boldsymbol{\epsilon})\, \lambda\, h^{+-} \Big] \qquad (1.7)$$

and

$$f^{(0)} = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \Big[- i (\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon})\, \alpha\, \mu_0^2 + $$

$$- i \alpha\, \boldsymbol{\sigma} \cdot \boldsymbol{\mu} \times (\boldsymbol{\nu} \times \boldsymbol{\epsilon}) + $$

$$+ i (\boldsymbol{\sigma} \cdot \boldsymbol{\mu}) \frac{(\boldsymbol{\mu} \cdot \boldsymbol{\epsilon})}{2 M \mu_0} \Big] \qquad (1.8)$$

where $e^2 = 1/137$ is the fine structure constant and pion nucleon coupling constant f is taken to be $\sqrt{0.08}$,

$$\underset{\sim}{k} = \underset{\sim}{\nu} - \underset{\sim}{\mu}, \tag{1.9}$$

$$\lambda = \frac{\mu_p - \mu_n}{4Mf^2}, \tag{1.10}$$

$$\alpha = \frac{\mu_p + \mu_n}{2M\mu_0}, \tag{1.11}$$

the nucleon mass $M = 6.717$, μ_p and μ_n, the magnetic moments of proton and neutron, and

$$h^{+-} = -\frac{2}{3\mu^3} e^{i\delta_{33}} \sin\delta_{33} \tag{1.12}$$

$$h^{--} = \frac{1}{3\mu^3} e^{i\delta_{33}} \sin\delta_{33} \tag{1.13}$$

$$h^{++} = \frac{4}{3\mu^3} e^{i\delta_{33}} \sin\delta_{33} \tag{1.14}$$

where only the dominant pion nucleon phase shifts δ_{33} in the $(3/2, 3/2)$ state are taken into account.

The amplitudes $f^{(+)}$, $f^{(-)}$ and $f^{(0)}$ are so normalized that the differential cross-section in the centre of momentum frame is given by

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \mu \mu_0 \, |\langle final \,|\, T \,|\, initial \rangle|^2 \tag{1.15}$$

2. The existence of a positive electric quadrupole moment for the deuteron and the deviation of the deuteron magnetic moment (μ_D) from the simple sum of the proton magnetic moment (μ_P) and neutron magnetic moment (μ_n) imply the existence of an admixture of D-state to the predominant ground state (S- state) wave function of the deutron. The existence of D- state in turn necessitates the existence of a "tensor force".

Generalised Pauli principle requires the antisymmetry of the overall wave function Ψ with respect to the interchange of nucleons and parity conservation does not allow the admixture of even and odd orbital angular momentum states. The choices of the overall wave function Ψ which satisfies these requirements are

$$\Psi = \frac{1}{r} \sum_{L=0,2} u_L(r)\, \Phi_L(\theta, \varphi)\, \eta_0 \tag{1.16}$$

$$\Psi = \frac{1}{r}\, u_1(r)\, \Phi_1(\theta, \varphi)\, \eta_0 \tag{1.17}$$

where $\underset{\sim}{r} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2$, $\underset{\sim}{r}_i$ being the positron coordinate of the i^{th} nucleon and (r, θ, φ) is the spherical polar coordinates of $\underset{\sim}{r}$, equation (1.16) represents the a linear combination of S and D states while (1.17) represents a pure P- state wave function, $\Phi_L(\theta, \varphi)$ is the spin-orbital wave function and η_0 is the iso-singlet wave function. Explicitly

$$\eta_0 = \frac{1}{\sqrt{2}} \{ p(1)\,n(2) - p(2)\,n(1) \} \tag{1.18}$$

and
$$\Phi_L(\theta,\varphi) = \sum_{L_z = J_z - 1}^{L_z = J_z + 1} C(L\,1\,1;\, L_z\, J_z - L_z)\, Y_{L,L_z}(\theta,\varphi)\, \chi_{J_z - L_z} \tag{1.19}$$

where $Y_{L,L_z}(\theta,\varphi)$ are the normalised spherical harmonics and χ 's are the triplet spin functions.

Pure P -state or pure D -state wave functions are ruled out for the following reason: Because of the centrifugal force due to angular momentum the attraction in a P or D state should be much stronger than in a S -state in order to make the corresponding energy lower than any possible S -level. Thus we have to choose a linear combination of S and D states $(1\cdot16)$ for the ground state of the deuteron.

Renaming the S state and D -state radial wave functions u_0 and u_2 as u and w , they are normalised as :

$$\int_0^\infty (u^2 + w^2)\, dr = 1 \tag{1.20}$$

The equations for the radial wave functions are solved approximately 1, 2) after making suitable assumptions about the tensor force. Reasonable deuteron radial wave functions may be constructed by assuming suitable functional

1) K.V.Laurikainen and E.K.Euranto, Ann. Univ. Turk, A 18 , 2 (1956).
2) K.V.Laurikainen, Ann. Acad . Sci. fenn. A 1 , Nr.208, (1956)

forms containing several parameters and adjusting those so as to fit the deuteron binding energy ε , the quadrupole moment Q , the D state probability P_D and in addition the triplet scattering length and the triplet effective range.

We introduce a change of variable $x = \alpha r$ where $1/\alpha = 4.316 \times 10^{-13}$ cm. [$\alpha^2 = EM/\hbar^2$ where $E = 2.226$ Mev is the binding energy of the deutron, M is the nucleon mass and $\hbar$ is the Planck's constant divided by 2π . $1/\alpha$ can be used as a measure of the size of the deutron. The deutron effective range $\rho(-E,-E)$ is taken to be either 1.704×10^{-13} cm or 1.734×10^{-13} cm . The radial wave functions are :

$$u(x) = N \cos \varepsilon_g \left[1 - e^{-\beta(x - x_c)} \right] e^{-x} \tag{1.21}$$

$$w(x) = N \sin \varepsilon_g \left[1 - e^{-\gamma(x - x_c)} \right]^2 \times$$

$$\times\ e^{-x} \left[1 + \frac{3}{x}\left(1 - e^{-\gamma x}\right) + \frac{3}{x^2}\left(1 - e^{-\gamma x}\right)^2 \right] \tag{1.22}$$

when $x = \alpha r \geqslant x_c$ and

$u = w = 0 , \quad x < x_c .$

x_c is the hard core radius. The values of the parameters β , γ and ε_g are given by Hedin and Conde[1)]

1) Handbuch der Physik, Vol. 39 , p. 92.

for $P_D = 3\%$; 4% and 5% ; $x_c = 0, 0.1$ and 0.15 ; $\rho = 1.704$ fm and 1.734 fm . The normalization factor is given by:

$$N^2 = \left\{ \frac{2}{1 - \alpha\rho(-\varepsilon,-\varepsilon)} \right\}$$

$$\text{or} \quad N^2 = 3.0347 \quad \text{for} \quad \rho = 1.704 \times 10^{-13}\ \text{cm}. \tag{1.23}$$

$$N^2 = 3.3433 \quad \text{for} \quad \rho = 1.734 \times 10^{-13}\ \text{cm}.$$

Tables 1 presents the values of the parameters β, γ, $\sin\varepsilon_g$ for various values of x_c, ρ and P_D as evaluated by Hendin and Conde .

Table I

The parameters in the deutron wave function fitted to all the triplet low energy data and a hard core radius x_c *.

x_c (4.316×10^{-13} cm)	$\rho(-\epsilon,-\epsilon)$ 10^{-13} cm.	P_D (%)	β	γ	$\sin \epsilon_g$
0.00	1.704	3	4.860	2.494	0.03232
		4	4.751	2.982	0.02923
		5	4.647	3.275	0.02754
	1.734	3	4.741	2.505	0.03193
		4	4.637	2.986	0.02891
		5	4.536	3.289	0.02720
0.10	1.704	3	8.237	3.155	0.02942
		4	7.961	3.798	0.02666
		5	7.699	4.346	0.02514
	1.734	3	7.933	3.175	0.02901
		4	7.675	3.814	0.02634
		5	7.431	4.364	0.02487
0.13	1.704	3	10.223	3.413	0.02873
		4	9.814	4.144	0.02611
		5	9.433	4.771	0.02471
	1.734	3	9.774	3.436	0.02832
		4	9.397	4.170	0.02577
		5	9.045	4.799	0.02438

* Numerical calculations by L.T.Hedin and P.H.L.Conde.

3. Now we study the problem of charged pion photoproduction from deuteron assuming pure S state. This is a reasonable first approximation since D- state probability is about 4 %. Further we find (see Chapter 2) that the form factors F_{SD} and F_{DD} which arise in S to D and D to D state transitions respectively are very small compared to F_{SS} which is the form factor for S to S state transitions.

In the next chapter, however, we study the problem of neutral pion photo production taking into account the small admixture of D- state along with the variations of the parameters x_c and $\rho(-\varepsilon,-\varepsilon)$.

4. The charged pion photoproduction from deuteron is necessarily inelastic as there are no bound states of a dineutron or diproton system. The nucleus breaks up into two nucleons. The T- matrix for the processes

$$\gamma + D \rightarrow \pi^+ + n + n \qquad (1.24)$$

$$\gamma + D \rightarrow \pi^- + p + p. \qquad (1.25)$$

can be written in the impulse approximation as

$$T = \left(t_1 e^{i \underset{\sim}{k}_1 \cdot \underset{\sim}{r}_1} \tau^{\pm} + t_2 e^{i \underset{\sim}{k}_2 \cdot \underset{\sim}{r}_2} \tau_2^{\pm} \right) \qquad (1.26)$$

where $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ are the positron vectors of the nucleons 1 and 2 , $\tau_{1,2}$ are the isotopic spin operators for the nucleons 1 and 2 , t_1 and t_2 are the basic photoproduction amplitudes for the nucleons 1 and 2 , $\tau^{\pm}$ are defned as

$$\tau_{\pm} = \frac{1}{2}\left(\tau_x \pm i\,\tau_y\right) \tag{1.27}$$

where τ_x , τ_y are the 2×2 Pauli matrices in isotopic spin space and $\underset{\sim}{k} = \underset{\sim}{\nu} - \underset{\sim}{\mu}$ is the momentum transferred to the hit nucleon.

If we take into account only the dominant $S-$ state, the initial state of the deutron can be written as

$$|i\rangle = (2\pi)^{-3/2} \frac{u(r)}{r} \, {}^{3}\chi_m \, \eta_0 \tag{1.28}$$

We denote the position coordinate corresponding to the centre of mass of the two nucleon system by

$$\underset{\sim}{R} = \frac{\underset{\sim}{r}_1 + \underset{\sim}{r}_2}{2} \tag{1.29}$$

using which we can write for the two possible final states -- spin triplet and spin singlet -- of the two nucleons :

$$|f\rangle_e = (2\pi)^{-3/2} \, u_{f,e}(\underset{\sim}{k}\cdot\underset{\sim}{r}) \, e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}} \; {}^{1}\chi_0 \, \eta^{1}_{\pm 1} \tag{1.30}$$

$$|f\rangle_o = (2\pi)^{-3/2}\, u_{f,o}(\underset{\sim}{k}\cdot\underset{\sim}{r})\; e^{i\underset{\sim}{k}\cdot\underset{\sim}{R}}\; {}^3\chi_m\, \eta^1_{\pm 1} \qquad (1.31)$$

where $u_{f,e}(\underset{\sim}{k}\cdot\underset{\sim}{r})$ and $u_{f,o}(\underset{\sim}{k}\cdot\underset{\sim}{r})$ are the symmetric and antisymmetric radial wave functions of the two nucleons in the final state.

The matrix elements for the symmetric and antisymmetric case reduce to:

$$Q_e = \frac{1}{\sqrt{2}} \langle {}^1\chi_0 | t_1 - t_2 | {}^3\chi_m \rangle E \qquad (1.32)$$

$$Q_o = \frac{1}{\sqrt{2}} \langle {}^3\chi_{m'} | t_1 + t_2 | {}^3\chi_m \rangle O \qquad (1.33)$$

where E and O are the overlap integrals.

$$E = \int u^*_{f,e}(\underset{\sim}{k}\cdot\underset{\sim}{r}) \cos(\underset{\sim}{k_0}\cdot\underset{\sim}{r}) \frac{u(r)}{r}\, d^3\underset{\sim}{r} \qquad (1.34)$$

$$O = \int u^*_{f,o}(\underset{\sim}{k}\cdot\underset{\sim}{r}) \sin(\underset{\sim}{k_0}\cdot\underset{\sim}{r}) \frac{u(r)}{r}\, d^3\underset{\sim}{r} \qquad (1.35)$$

To evaluate the matrix element (1.32) and (1.33) we neglect the motion of the nucleons in the deutron and choose for the operator t the expression (1.4) or (1.5) which for a particular state of polarization of the photon say ε_x, can be written in the convenient form

$$T = A_x\sigma_+ + B_x\sigma_- + C_x\sigma_z + D_x \qquad (1.36)$$

We chôose for our frame of reference the direction of propagation of the photon as the Z axis,and the plane which contains $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$ as the x-z plane and θ the angle between $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$.

After squaring, summing and averaging over initial spin states and photon polarizations, we obtain

$$\frac{1}{2}\sum_{m}|\langle{}^1\chi_0|\,t_1-t_2\,|{}^3\chi_m\rangle|^2 = \frac{2}{3}\left(|A|^2+|B|^2+2|C|^2\right) \qquad (1\cdot37)$$

$$\frac{1}{2}\sum_{m}\sum_{m'}|\langle{}^3\chi_{m'}|\,t_1+t_2\,|{}^3\chi_m\rangle|^2 = \frac{4}{3}\left(|A|^2+|B|^2+2|C|^2+3|D|^2\right) \qquad (1\cdot38)$$

and

$$|Q|^2 = |Q_e|^2 + |Q_o|^2$$

$$= \frac{1}{3}\left(|A|^2+|B|^2+2|C|^2\right)|E|^2 +$$

$$+\frac{2}{3}\left(|A|^2+|B|^2+2|C|^2+3|D|^2\right)|O|^2 \qquad (1\cdot39)$$

where

$$|A|^2 = \frac{1}{2}\left(|A_x|^2+|A_y|^2\right),$$

$$|B|^2 = \frac{1}{2}\left(|B_x|^2+|B_y|^2\right),$$

$$|C|^2 = \frac{1}{2}\left(|C_x|^2 + |C_y|^2\right)$$

$$|D|^2 = \frac{1}{2}\left(|D_x|^2 + |D_y|^2\right)$$

where the subscripts x and y correspond to the photon polarizations ε_x and ε_y. The differential cross-section is given by

$$\frac{d^2\sigma}{d\Omega\, d\mu_0} = (2\pi)^{-2}\mu\mu_0 \int |Q|^2\, \delta\left(\mu_0 - \nu_0 + \frac{k^2+k_0^2}{M} + \varepsilon\right) d\underset{\sim}{k} \qquad (1.40)$$

$|E|^2$ and $|O|^2$ have been evaluated by Lax and Feshbach choosing the Hulthen function

$$u(r) = \left[\frac{\alpha}{2\pi(1-\alpha\rho_t)}\right]^{1/2}\left(e^{-\alpha r} - e^{-\beta r}\right) \qquad (1.41)$$

for the initial radial wave function and

$$u_{f,e}(\underset{\sim}{k}\cdot\underset{\sim}{r}) = (2\pi)^{-3/2}\cos(\underset{\sim}{k}\cdot\underset{\sim}{r}) \qquad (1.42)$$

$$u_{f,o}(\underset{\sim}{k}\cdot\underset{\sim}{r}) = (2\pi)^{-3/2}\sin(\underset{\sim}{k}\cdot\underset{\sim}{r}) \qquad (1.43)$$

5. Neglecting the contributions from terms proportional to $1/M$ and terms of higher order, Devanathan and Ramachandran[1] have computed the cross-section for the photoproduction of π^+ from deutron at 320 Mev using the Lax and Feshback integrals. But they do not get a good fit with the experimental result of White et al.[2]

We therefore study the contribution to the cross-section $d^2\sigma / d\Omega\, d\mu_0$ due to terms of order $1/M$ and higher order. After summing over final spin states and averaging over initial photon polarizations and deutron spins we get

$$\frac{d^2\sigma}{d\Omega\, d\mu_0} = \frac{1}{2}(2\pi)^{-2}\mu\mu_0\left[|Q_1|^2 I_1 - \frac{|Q_1|^2 + 4|D|^2}{3} I_2\right] \qquad (1.44)$$

where

$$4|D|^2 = \frac{8\pi^2 e^2 f^2}{\mu_0 \nu_0}\left(\frac{8}{9\mu^4}\nu^2\lambda^2 \sin^2\delta_{33}\sin^2\theta\right) \qquad (1.45)$$

and I_1 and I_2 are the Lax and Feshbach[3] integrals given by

$$I_1 = \frac{2M\alpha k_\pi}{\pi(1-\alpha\rho)}\left[\frac{1}{P_\alpha^4 - 4k_\pi^2 k_0} + \frac{1}{P_\beta^4 - 4k_\pi^2 k_0^2} + \right.$$
$$\left. - \frac{1}{2k_\pi k_0(\beta^2-\alpha^2)}\log\frac{(P_\beta^2 - 2k_\pi k_0)(P_\alpha^2 + 2k_\pi k_0)}{(P_\alpha^2 + 2k_\pi k_0)(P_\alpha^2 - 2k_\pi k_0)}\right] \qquad (1.46)$$

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1) V. Devanathan and G. Ramachandran, Nucl. Phys. 22, 312 (1961)

2) R.S. White, M.J. Jacobson and A.G. Schulz, Phys. Rev., 88, 836 (1952)

3) M. Lax and H. Feshbach, Phys. Rev. 88, 509 (1952).

$$I_2 = \frac{M\alpha}{2\pi(1-\alpha\rho)} \frac{(\beta^2-\alpha^2)}{k_0(p_\alpha^2+p_\beta^2)} \times$$

$$\times \left[\frac{1}{p_\alpha^2} \log \frac{p_\alpha^2 + 2k_x k_0}{p_\beta^2 - 2k_x k_0} + \right.$$

$$\left. - \frac{1}{p_\beta^2} \log \frac{p_\beta^2 + 2k_x k_0}{p_\beta^2 - 2k_x k_0} \right] \qquad (1.47)$$

where

$$\left.\begin{aligned} k_x^2 &= M(\nu_0 - \mu_0 - \varepsilon) - k_0^2 \\ p_\alpha^2 &= M(\nu_0 - \mu_0 - \varepsilon) + \alpha^2 = M(\nu_0 - \mu_0) \\ p_\beta^2 &= M(\nu_0 - \mu_0 - \varepsilon) + \beta^2 = M(\nu_0 - \mu_0) + \beta^2 - \alpha^2 \end{aligned}\right\} (1.48)$$

The cross-section for π^+ is given by the expression (1.44) where $|Q_1|^2$ is now given by :

$$|Q_1|^2 = |Q_1|_0^2 - \alpha |Q_1|_1^2 + \alpha^2 |Q_1|_2^2 \qquad (1.49)$$

where

$$|Q_1|_0^2 = \frac{8\pi^2 e^2 f^2}{\mu_0 \nu_0}\left[\frac{2}{\left(1+\frac{\mu_0}{M}\right)^2}\left(1-\frac{2\mu^2 \sin^2\theta}{(k^2+1)^2}\right) + \frac{1}{\left(1+\frac{\mu_0}{M}\right)}\frac{4\nu\lambda}{3\mu}\left(\frac{\nu \sin^2\theta}{k^2+1}-\frac{\cos\theta}{\mu}\right)\cos\delta_{33}\sin\delta_{33} + \frac{2\nu^2\lambda^2}{9\mu^4}\left(1+\frac{3}{2}\sin^2\theta\right)\sin^2\delta_{33}\right] \qquad (1.50)$$

$$|Q_1|_1^2 = \frac{8\pi^2 e^2 f^2}{\mu_0 \nu_0}\left[\frac{1}{\left(1+\frac{\mu_0}{M}\right)}\left\{\frac{4\mu^2 \sin^2\theta}{k^2+1}(\nu^2-\mu_0^2) + 4(\mu_0^2-\mu\nu\cos\theta)\right\} + \frac{2\nu\lambda}{3\mu}\cos\delta_{33}\sin\delta_{33}\left\{\frac{\nu}{\mu}(1+\cos^2\theta)-\frac{2\mu_0^2}{\mu^2}\cos\theta\right\}\right] \qquad (1.51)$$

and

$$|Q_1|_2^2 = \frac{8\pi^2 e^2 f^2}{\mu_0 \nu_0}\left[2\mu_0^2(\mu_0^2-2\mu\nu\cos\theta) + \mu^2\nu^2(1+\cos^2\theta)\right] \qquad (1.52)$$

where θ is the angle between $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$.

The cross-section for negative meson production is again given by (1.44) with $|Q_1|^2$ given now by the following expression

$$|Q_1|^2 = |Q_1|_0^2 + \alpha |Q_1|_1^2 + \alpha^2 |Q_1|_2^2 + |Q_1|_a^2 \qquad (1.53)$$

where the additional term

$$|Q_1|_a^2 = \frac{8\pi^2 e^2 f^2}{\mu_0 \nu_0} \left\{ \frac{1}{(1+\frac{\mu_0}{M})} \frac{4\mu^2 \sin^2\theta}{M\mu_0(k^2+1)} (\mu^2 - \mu\nu \cos\theta) + \right.$$

$$\left. - \frac{2\mu_0 \mu^2}{M} \sin^2\theta + \frac{\mu^4 \sin^2\theta}{M^2 \mu_0^2} - \frac{2\mu^2 \sin^2\theta}{M\mu_0(1+\frac{\mu_0}{M})} \right\} \qquad (1.54)$$

In the reference[1)] numerical results have been reported at incident photon energy ν_0 = 320 Mev corresponding to the experimental results of White et al[23)] and the theoretical estimates were found to be above the experimental values. The contributions from the terms $-\alpha |Q_1|_1^2$ and $\alpha^2 |Q_1|_2^2$ which have to be added to the earlier estimates are now obtained numerically and are presented in Fig. 1 in which we plot $\log|\Delta|$ as a function of the meson energy for various angles θ, where Δ denotes the contributions to $d^2\sigma/d\Omega\, d\mu_0$ from $-\alpha_1 |Q_1|_1^2 + \alpha^2 |Q_1|_2^2$. We see that except at the forward angles, the cross-section for π^+ production is reduced though not to the extent desired by the experiment of White et al.

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1) V. Devanathan and G. Ramachandran, Nucl. Phys. 23, 312 (1961)

6. In the calculation described in section 3, the outgoing meson takes all possible energies permitted by the two particle kinematics. Further the impulse approximation calculation is made in the laboratory system where the recoil energy is considerable. The difficulty of the above method is due to the inability to perform the integration over the meson energy analytically. An alternate method is to do the impulse approximation calculation in the centre of momentum system where the recoil energy of the nucleus is considerably smaller. Dalitz and Yennie [1] have found that the static model is more reasonable in the centre of momentum system than in the laboratory system. We can now invoke the closure approximation according to which all possible relative momenta of the final states of the two nucleons may be taken into account without considering the relative energy involved. This is a useful approximation which considerably simplifies the numerical calculations.

Neglecting the binding energy of the deutron and invoking the closure approximation the integral $\int |Q|^2 \, d\underset{\sim}{k}_0$ can be easily evaluated since

$$\int |E|^2 \, d\underset{\sim}{k}_0 = \frac{1}{2}\,(1 + I) \qquad (1.55)$$

$$\int |O|^2 \, d\underset{\sim}{k}_0 = \frac{1}{2}\,(1 - I) \qquad (1.56)$$

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1) R.A. Dalitz and D.R. Yennie, Phys. Rev. 105, (1957) 1598

where

$$I = \int \cos(k r)\, |u_d(r)|^2\, d^3 r$$

$$= \frac{1}{1-\alpha\beta} \cdot \frac{\alpha}{k_0} \left\{ \tan^{-1}\left(\frac{k_0}{\alpha}\right) + \tan^{-1}\left(\frac{k_0}{\beta}\right) - 2\tan^{-1}\left(\frac{2k_0}{\alpha+\beta}\right) \right\} \quad (1.57)$$

Now the differential cross section for the charged pion photoproduction takes a simple form

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \frac{\mu \mu_0}{2} \left(Z^{\pm} - \frac{Z^{\pm} + 4|D|^2}{3} I \right) \quad (1.58)$$

where Z^+ corresponds to π^+ photoproduction and Z^- corresponds to π^- production. The quantities Z^+ and Z^- are given by

$$Z^+ = |Q_1|_0^2 - \alpha |Q_1|_1^2 + \alpha^2 |Q_1|_2^2 \quad (1.59)$$

$$Z^- = |Q_1|_0^2 + \alpha |Q_1|_1^2 + \alpha^2 |Q_1|_2^2 + |Q_1|_a^2 \quad (1.60)$$

The quantities $4|D|^2$, $|Q_1|_0^2$, $|Q_1|_1^2$, $|Q_1|_2^2$ and $|Q_1|_a^2$ were defined earlier in section 5.

Numerical calculations have been made using the expression (1.58) for the differential cross sections of the positive and negative pions from deuterons at photon energies of 230 Mev , 290 Mev and 320 Mev .

In the Figs. (1.2) and (1.3) the differential cross sections for π^+ and π^- at 230 Mev are presented along with the experimental results of Beneventano et al [1) for the same energy. A fairly good fit is obtained.

7. In view of the good experimental fit obtained at 230 Mev for both π^+ and π^- photoproduction cross section it is interesting to estimate the cross sections at slightly higher energies. It will be useful to study the effect of binding in the photoproduction cross sections.

We compare the cross section for charged pion photoproduction from the deutron with that from the single nucleons. Since it is only the proton or neutron, in the deuteron, which takes part in the production process, we can view the cross section for the deuteron as that equivalent to a single nucleon bound in the nucleus. Thus, the comparison of the cross sections for deuteron and single nucleons will give the differences ~~inx~~ the single nucleon cross sections when they are bound in the nucleus.

The binding effects are due to two reasons :

1) Pauli principle restricts the spin distributions of bound nucleons;
2) Due to the nuclear potential we have to introduce in the expression for the differential cross section,

1) Beneventano et al. Nuovo Cimento. 10, 1109, (1958)

a form factor (known as nuclear from factor) which is monotonically decreasing function of the scattering angle θ . ~~This results in the appreciable reduction of the differential cross sections for the backward angles.~~

We can compare the expression (1.58) with the corresponding expression for the single nucleon given below:

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \frac{\mu\mu_0}{2} \quad Z^{\pm} \tag{1.61}$$

The combined effect of the Pauli principle and the nuclear potential results in the decrease in the differential cross-section by

$$(2\pi)^{-2} \frac{\mu\mu_0}{6} \left(Z^{\pm} + 4 |D|^2 \right) I \tag{1.62}$$

I being a monotonically decreasing function of the scattering angle θ we can expect that the difference in the differential cross sections will be smaller in the backward angles.

In Fig. (1.4) we have plotted against the scattering angle θ, the differential cross section for the photo production of π^+ meson from a free proton along with the differential cross section for the π^+ meson photo production from deuteron at the laboratory energy 230 Mev for the incident photon.

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1) R.A.Dalitz and D.R.Yennie, Phys. Rev. 105 , (1957) 1598

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Numerical calculations have been made using the expression (1. 58) for the differential cross sections of the positive and negative pions from deuterons at photon energies of 230 Mev, 290 Mev and 320 Mev.

In the Figs. (1.2) and (1.3) the differential cross sections for π^+ and π^- at 230 Mev are presented along with the experimental results of Beneventano et al [1] for the same energy. A fairly good fit is obtained.

7. In view of the good experimental fit obtained at 230 Mev for both π^+ and π^- photoproduction cross sections, it is interesting to estimate the cross sections at slightly higher energies. It will be useful to study the effect of binding in the photoproduction cross sections.

We compare the cross section for charged pion photoproduction from the deutron with that from the single nucleons. Since it is only the proton or neutron, in the deuteron, which takes part in the production process, we can view the cross section for the deuteron as that equivalent to a single nucleon bound in the nucleus. Thus, the comparison of the cross sections for deuteron and single nucleons will give the differences in the single nucleon cross sections when they are bound in the nucleus.

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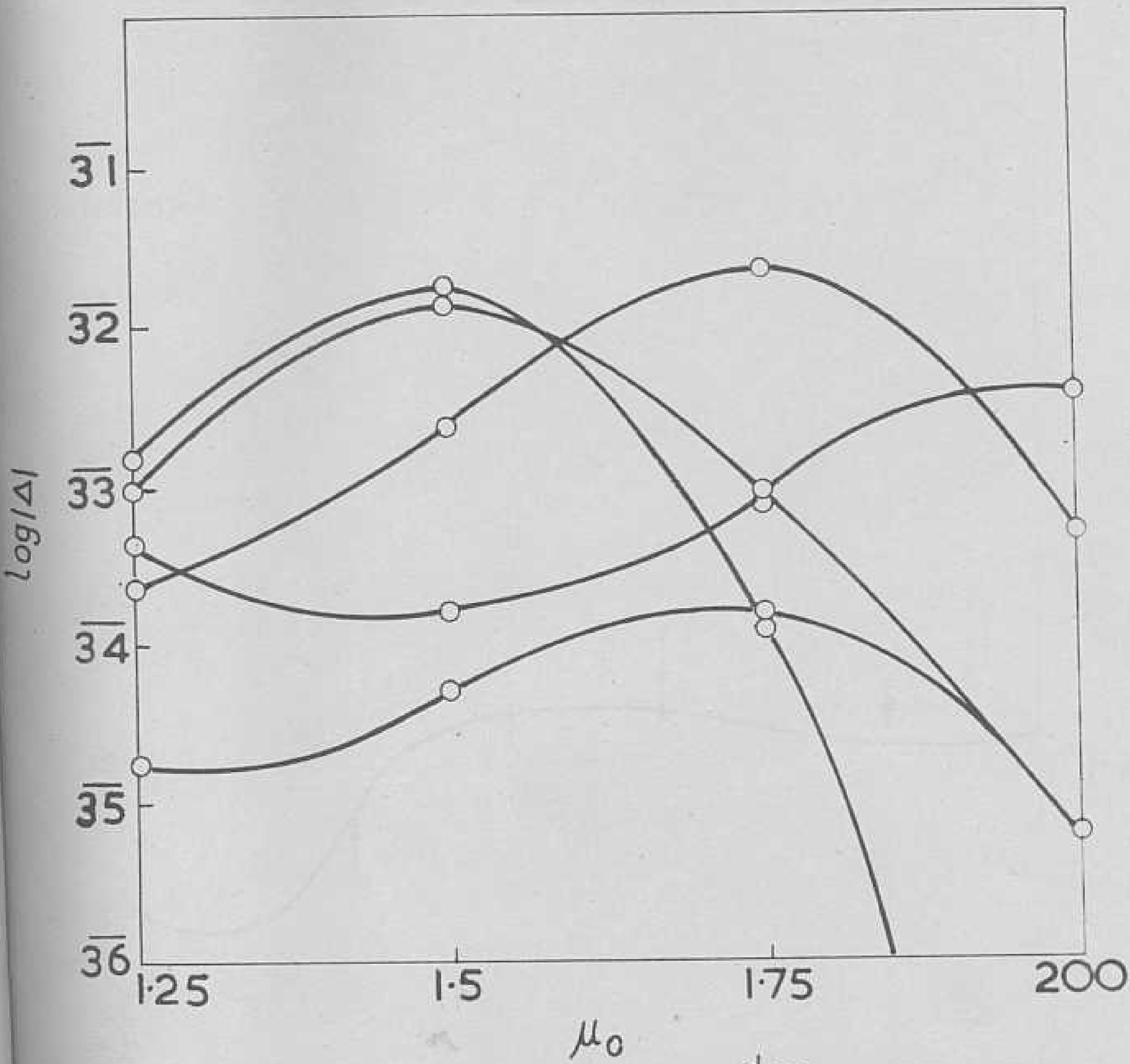
In Fig. (1.4) we have plotted against the scattering angle θ the differential cross section for the photo production of π^+ meson from a free proton along with the differential cross section for the π^+ meson photo production from deuteron at the laboratory energy 230 Mev for the incident photon.

In Fig. (1.5) we have plotted against the scattering angle θ, the differential cross section for the photo-production of π^+ meson from a free proton along with the differential cross section for the π^+ meson photo production from deuteron at the laboratory energy 290 Mev for the incident photon.

In Fig. (1.6) we have plotted against the scattering angle θ, the differential cross section for the photo-production of π^- meson from a free neuteron along with the differential ~~smr~~ cross-section for the π^- meson photoproduction from deuteron at the laboratory energy 230 Mev for the incident photon.

In Fig. (1.7) we have plotted against the scattering angle θ, the differential cross section for the photo-production of π^- meson from a free neutron along with the differential cross section for π^- photoproduction from deutron at the laboratory energy 290 Mev for the incident photon.

As expected we find that the differences are considerable in the forward angles but in the backward angles they are small.



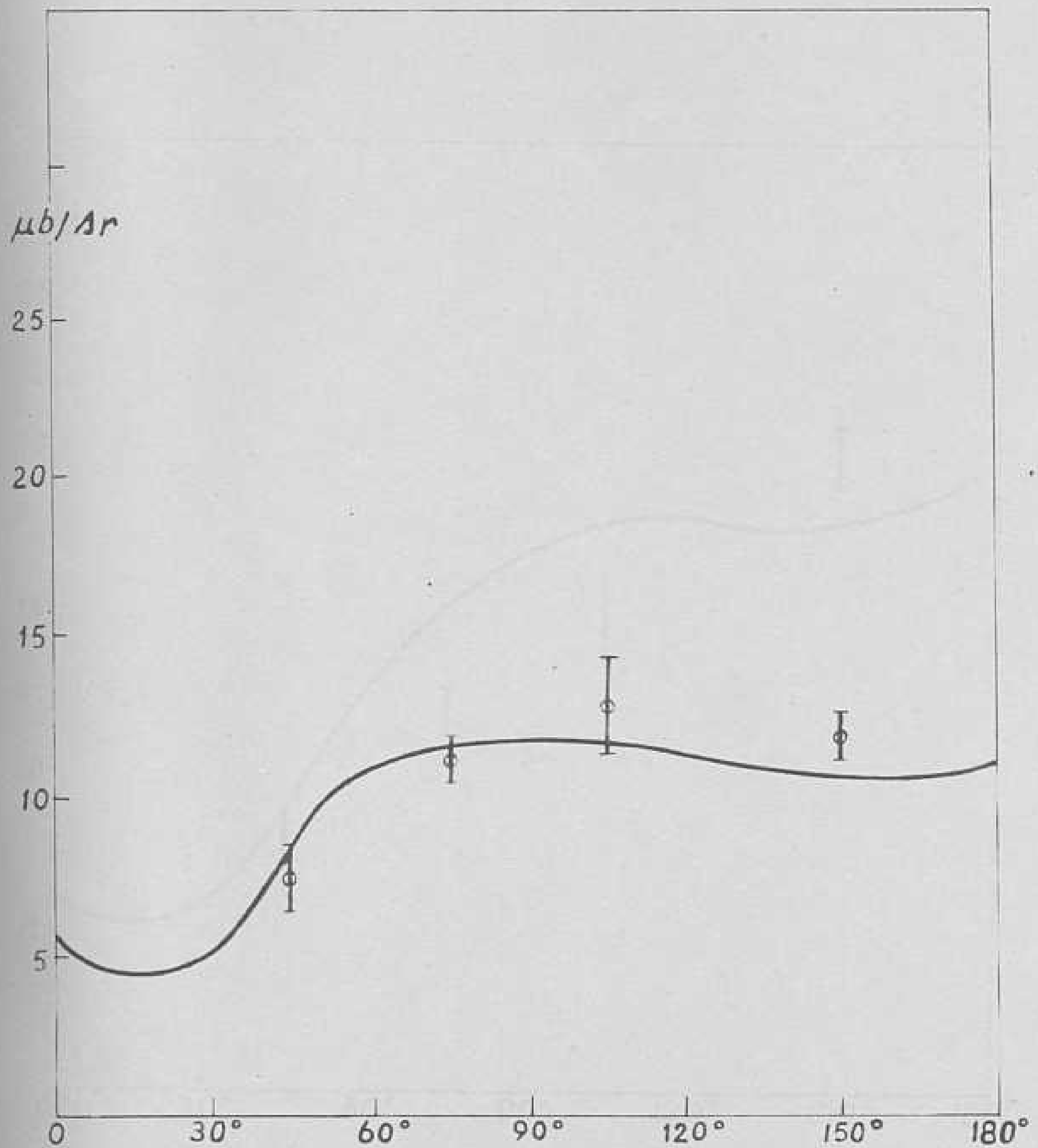
Δ = CONTRIBUTION TO $\frac{d\sigma}{d\Omega \, d\mu_0}$ FROM

$\frac{1}{M}$ TERMS $D > 0$ FOR 0°

< 0 FOR 45°, 90°, 135°

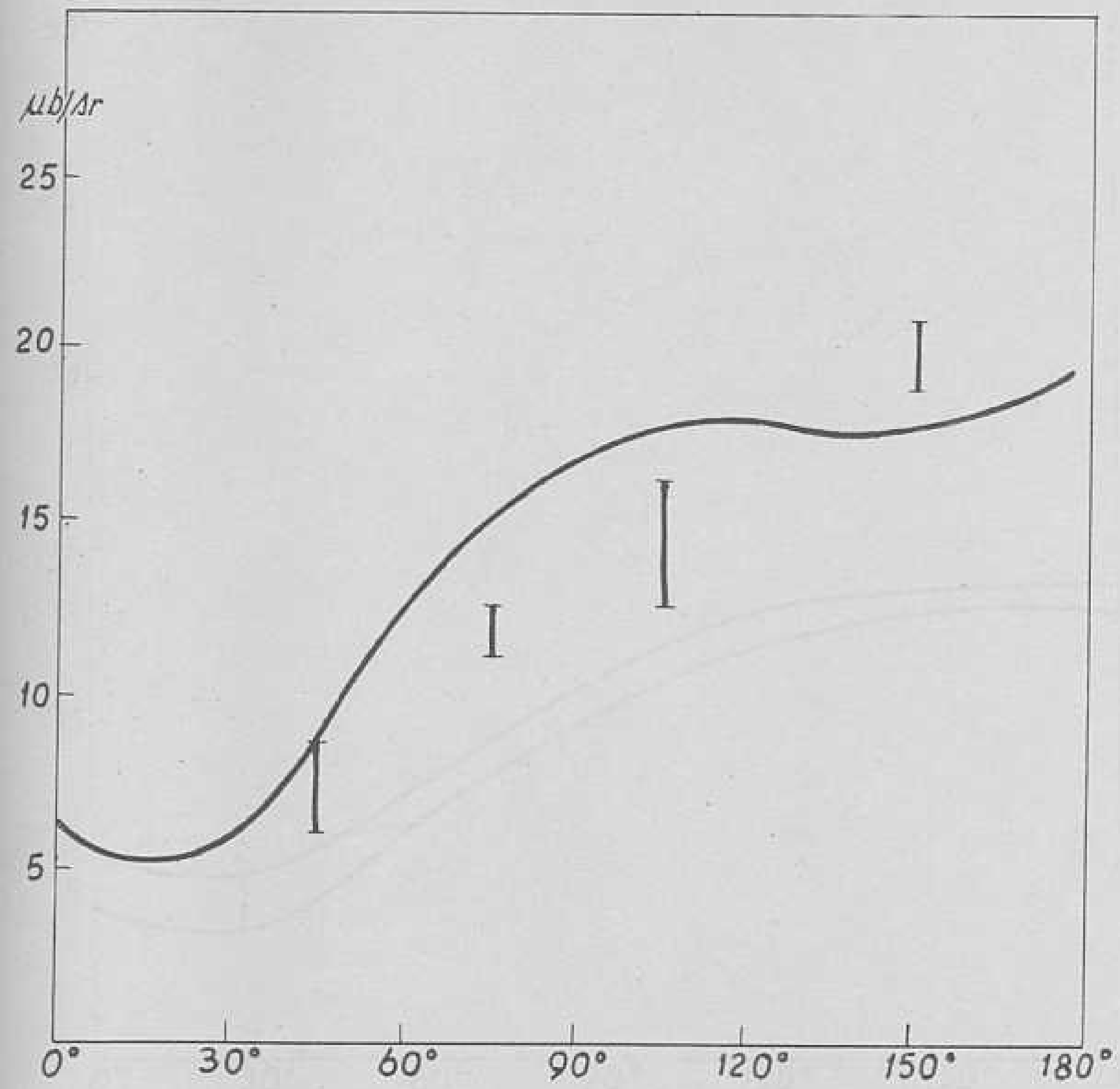
AND 180°

FIG. (1·1)

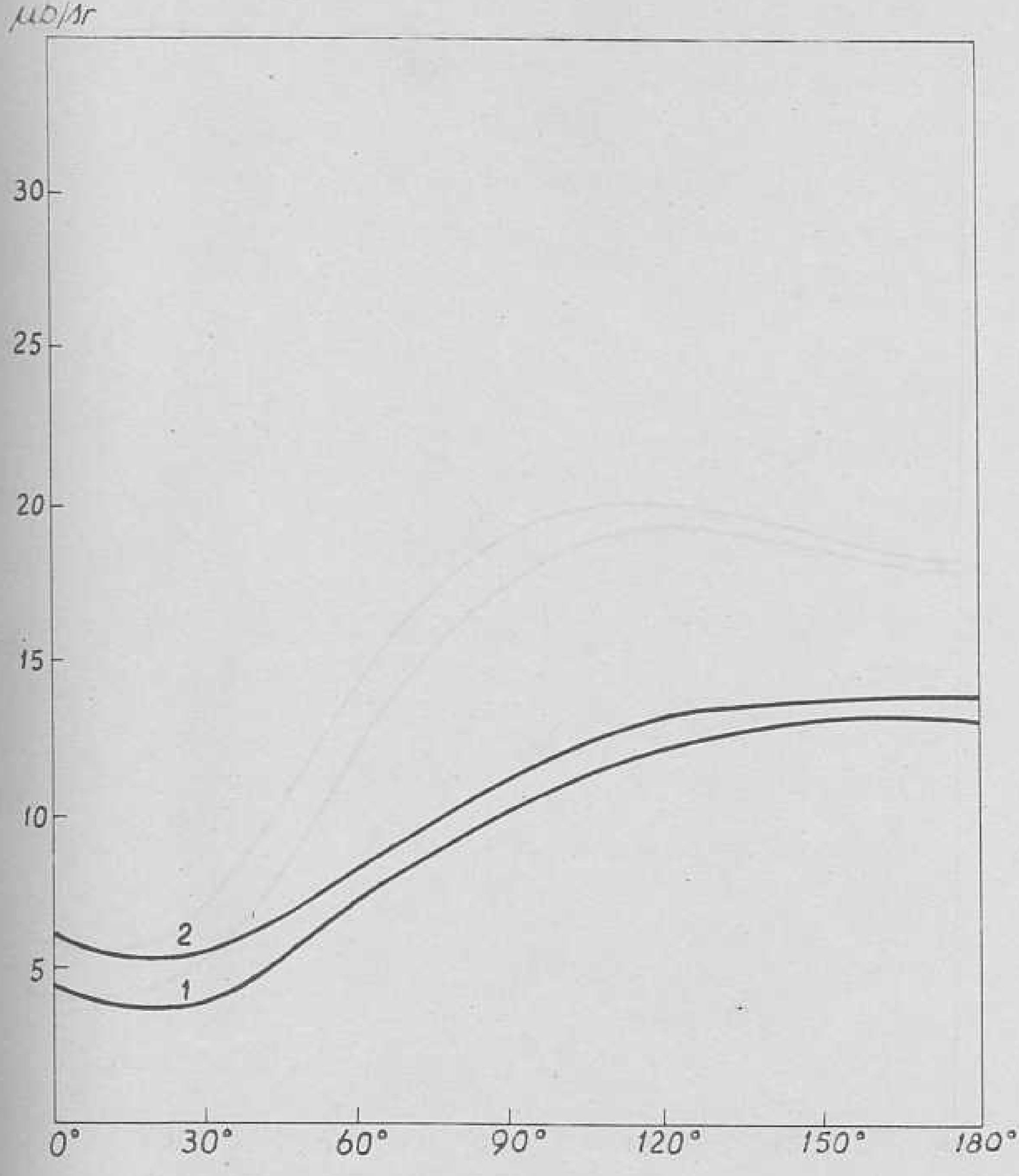


$\frac{d\sigma}{d\Omega}$ FOR $\gamma + D \rightarrow n + n + \pi^+$ AT E_γ^{LAB} = 230 mev

FIG. (1·2)

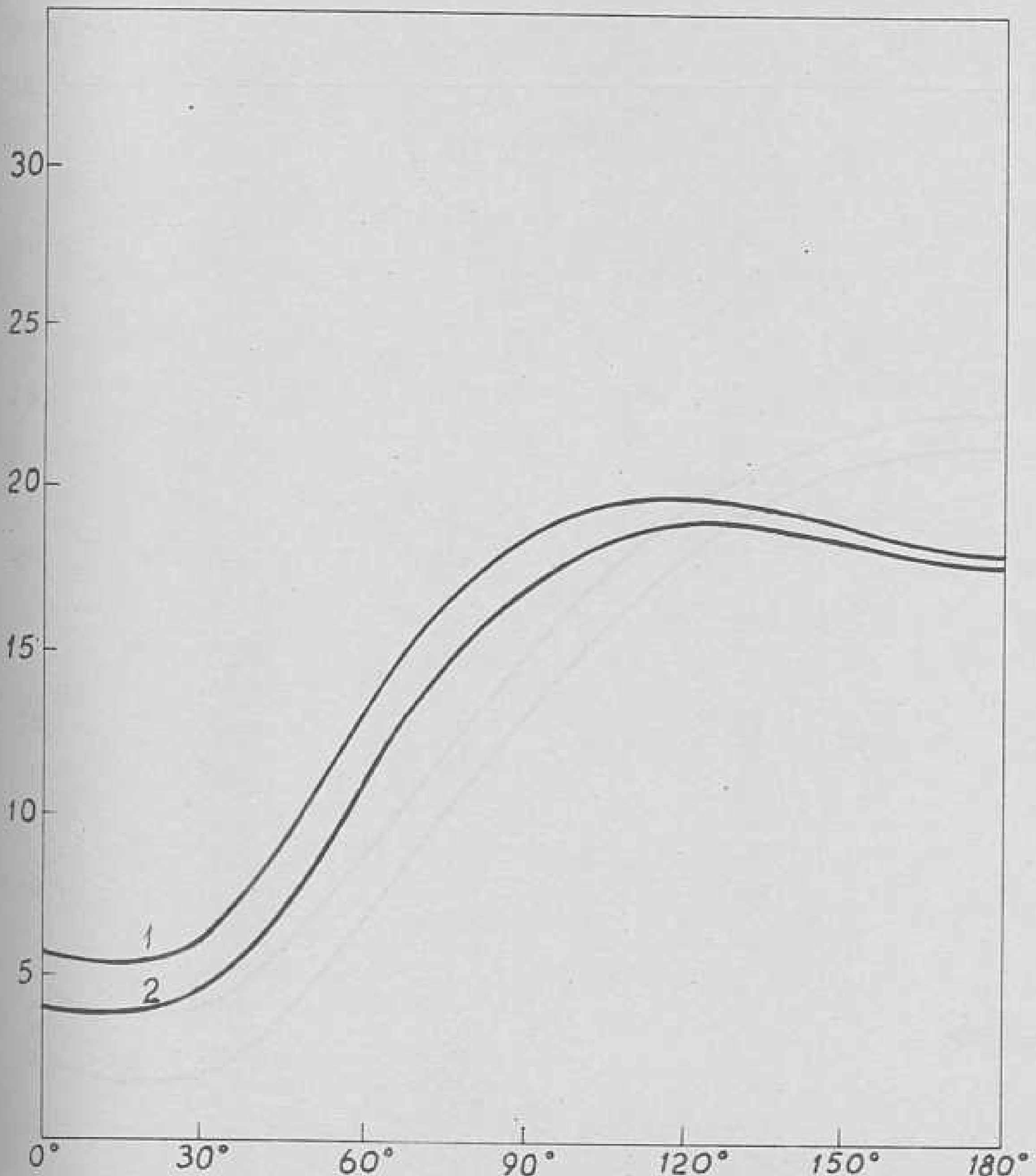


$\frac{d\sigma}{d\Omega}$ FOR $\gamma + D \rightarrow P + P + \pi^-$
AT $E_\gamma^{LAB} = 230$ mev.
FIG. (1·3)



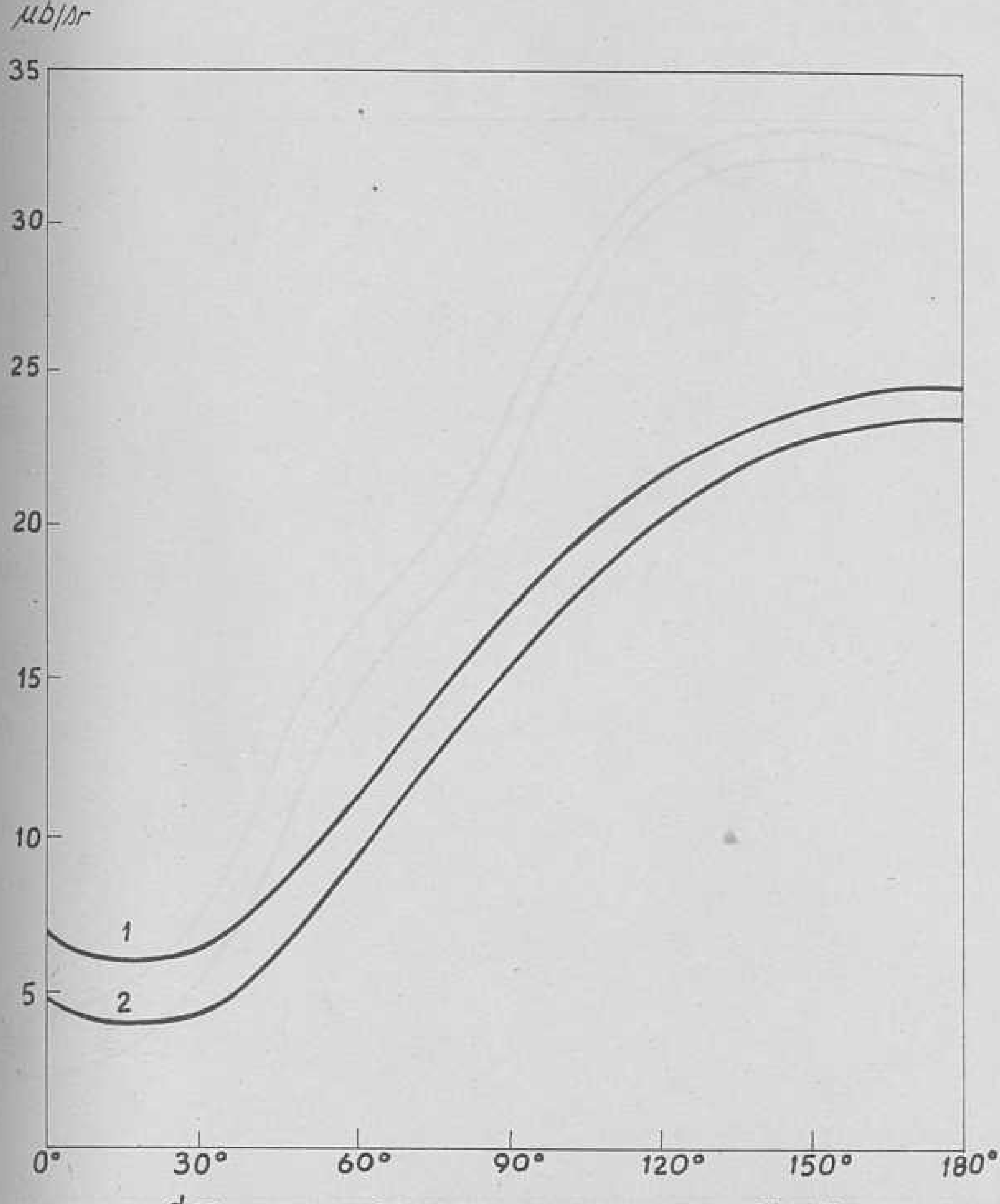
$\frac{d\sigma}{d\Omega}$ IN THE CENTRE OF MOMENTUM SYSTEM FOR 1) $\gamma + P \rightarrow n + \pi^+$
2) $\gamma + D \rightarrow n + n + \pi^+$

FIG. (1·4)



$\frac{d\sigma}{d\Omega}$ IN THE CENTRE OF MOMENTUM SYSTEM
AT $E_\gamma^{LAB} = 290$ mev.
FOR 1) $\gamma + P \rightarrow n + \pi^+$
2) $\gamma + D \rightarrow n + n + \pi^+$

FIG. (1·5)



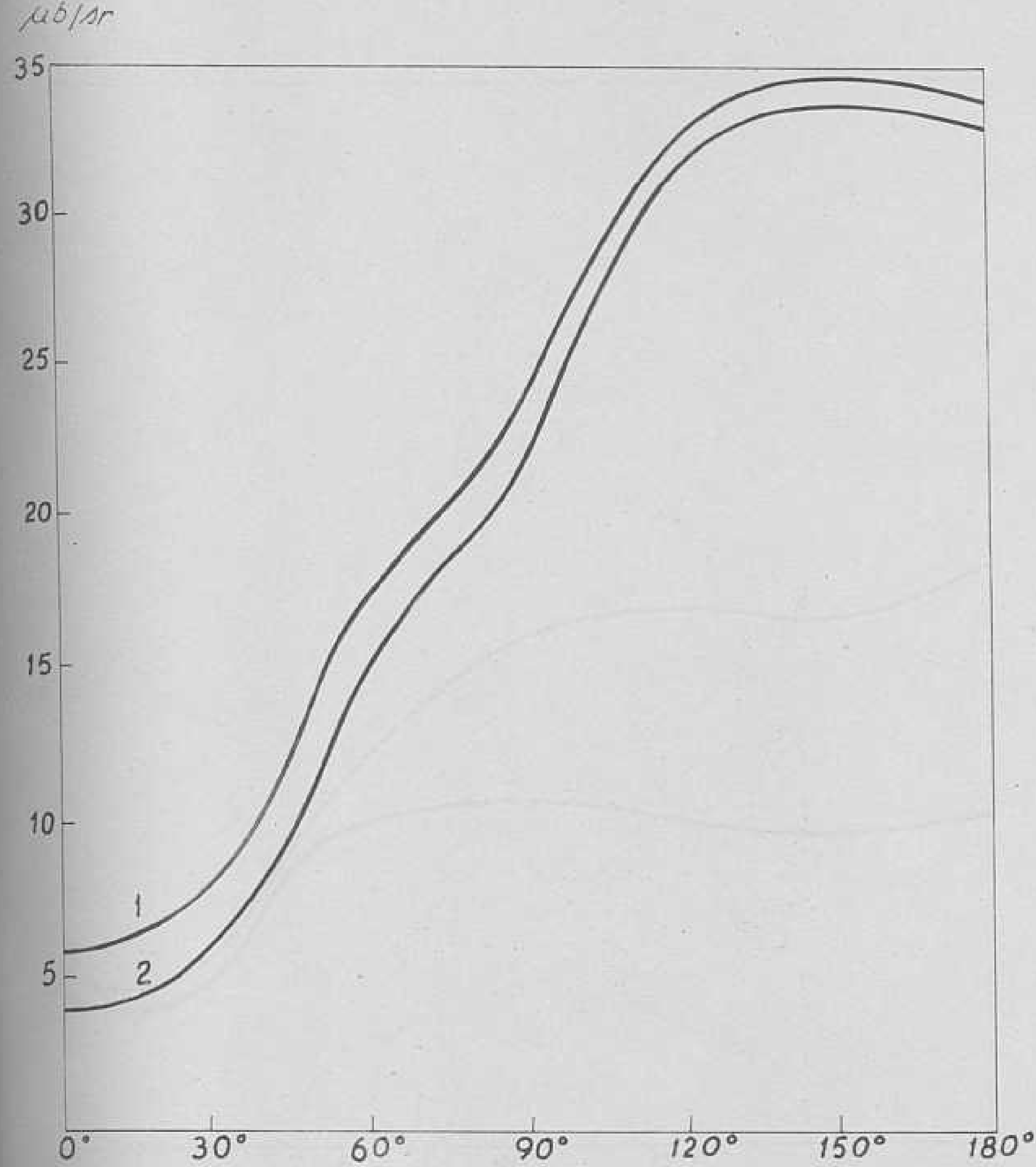
$\frac{d\sigma}{d\Omega}$ IN THE CENTRE OF MOMENTUM SYSTEM

AT $E_\gamma = 230$ mev.

FOR 1) $\gamma + n \rightarrow p + \pi^-$

2) $\gamma + D \rightarrow p + p + \pi^-$

FIG. (1·6)



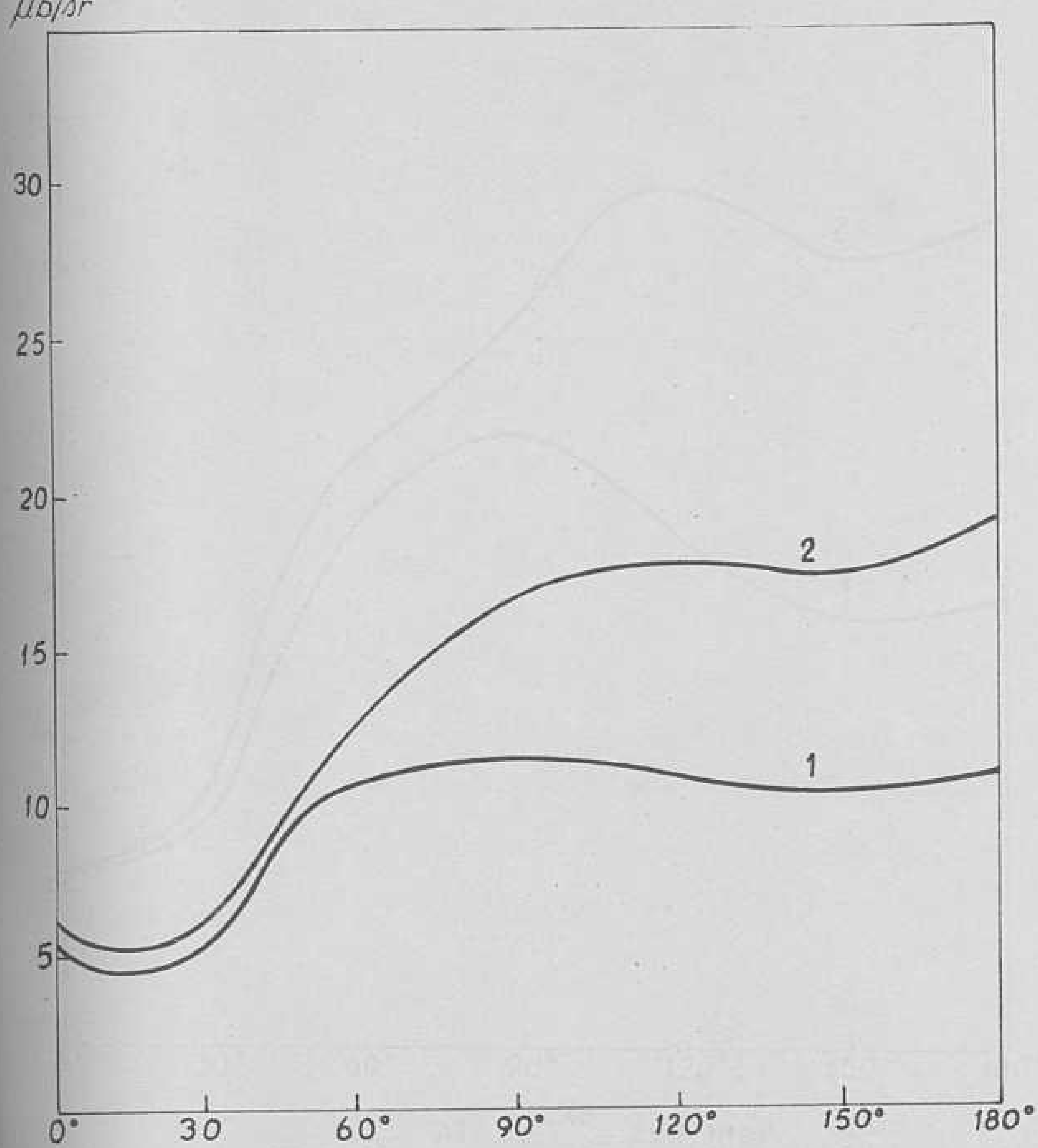
$\frac{d\sigma}{d\Omega}$ IN THE CENTRE OF MOMENTUM SYSTEM

AT $E_\gamma^{LAB} = 290$ mev.

FOR 1) $\gamma + n \rightarrow p + \pi^-$

2) $\gamma + D \rightarrow p + p + \pi^-$

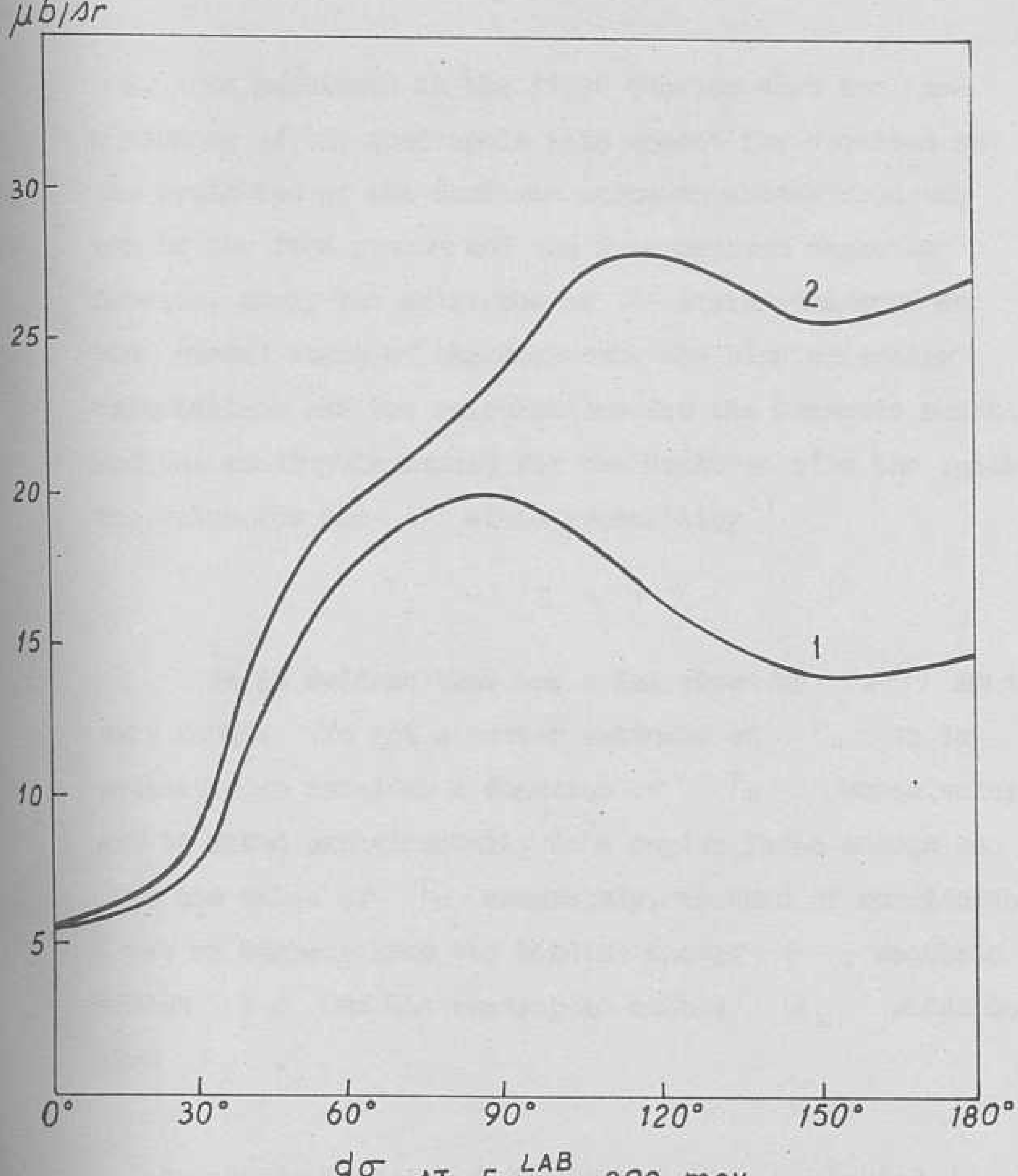
FIG. (1·7)



$\frac{d\sigma}{d\Omega}$ AT $E_{\gamma}^{LAB} = 230$ mev

FOR 1) $\gamma + D \rightarrow n + n + \pi^{+}$
2) $\gamma + D \rightarrow p + p + \pi^{-}$

FIG. (1·8)



$\frac{d\sigma}{d\Omega}$ AT $E_\gamma^{LAB} = 290$ mev

FOR 1) $\gamma + D \rightarrow \pi^+ + n + n$

2) $\gamma + D \rightarrow \pi^- + p + p$.

FIG. (1·9)

Chapter 2*

1. We mentioned in the first chapter that the non-vanishing of the quadrupole pole moment for deuteron and the deviation of the deuteron magnetic moment from the sum of the free proton and the free neutron magnetic moments, imply the existence of D- state admixture in the ground state of the deuteron. The binding energy calculations and the calculations for the magnetic moment and the quadrupole moment for the deuteron give the following value for the D state probability[1)]

$$P_D = 3 \pm 1 \% \qquad (2.1)$$

It is evident that the value given by (2.1) is very rough. To get a better estimate of P_D it is advisable to consider a function of P_D whose values can be found experimentally in a region large enough to give the value of P_D accurately, instead of considering a few mm numbers like the binding energy ε, magnetic moment μ_D and the quadrupole moment Q_D which depend upon P_D.

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* K. Ananthanarayanan and K. Srinivasa Rao. (Submitted to Nuovo Cimento)

1) Handbuch der Physik. Vol. 39;

The differential cross-section for the elastic photoproduction of pions offers itself for such an analysis. In the calculation[1] done earlier for the differential cross section of the neutral pion photo-production from deuteron at the energy 500 Mev for the incident photon, a good fit was obtained with the experimental data when a wave function with 7 % D-state was assumed for the deuteron ground state. From the experimental data they conclude that

$$P_D = (7.5 \pm 0.6)\% \qquad (2.2)$$

A latter calculation[2] also supports the value $P_D \simeq 7\%$.

From (2.1) one can say that the value given by (2.2) for P_D is too high to give good fits for ϵ, μ_D and Q_D. So it is interesting to analyse the differential cross-section for the neutral pion photo-production at a lower energy where the effects of the second pion resonance can be more reasonably neglected, using for P_D, the values allowed by (2.1). Also it is interesting to analyse the effects of the hard core radius r_c and the variation of the effective range $\rho(-\epsilon, -\epsilon)$ on the differential cross section.

1) B.F.Erickson and C.Shaerf, Phys. Rev. Letters, 11, 432, (1963)

2) J. Goldemberg and C.Shaerf, Phys. Rev. Letters, 12, 298, (1964)

Here it is found that the differential cross section for the elastic neutral pion photoproduction from deuteron is very sensitive to the D - state admixture but not so sensitive either to the hard core radius r_c or the effective range $\rho(-\epsilon, -\epsilon)$. The increase of P_D decreases the differential cross section so also the increase of r_c. So that the largest possible values of P_D and r_c viz., $P_D = 5\%$ and $r_c = 0.561$ fermi give the lowest cross section. As the experimental values for the differential cross sections are well below the theoretical values with $P_D = r_c = 0$, it is quite easy to see that the values $P_D = 5\%$ and $r_c = 0.561$ fermi should give the best available fit. Even these values ($P_D = 5\%$, $r_c = 0.561$ fm) do not decrease theoretical values for the cross section to give very good agreement with the experimental data. The inclusion of multiple scattering effects is known[1] to decrease the theoretical cross sections. Hence it is suggested that it is better to include this effect before trying to increase the D state probability to get a better fit with the experiment.

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1) John Chappelear, Phys. Rev. 99, 254 (1955)

2. When an admixture of D- state is assumed the ground state of the deuteron can be written as

$$\psi^{m} = {}^{3}\chi_{m}(1,2)\left(\frac{p(1)n(2)-p(2)n(1)}{\sqrt{2}}\right)\frac{u(r)}{r}(2\pi)^{-3/2} +$$

$$+ (2\pi)^{-3/2}\,\frac{w(r)}{r}\left(\frac{p(1)n(2)-p(2)n(1)}{\sqrt{2}}\right) \times$$

$$\times \sum_{m'} {}^{3}\chi_{m'}(1,2)\, C(121;m',m-m')\, Y^{2}_{m-m'}(\hat{\underset{\sim}{r}}) \qquad (2.3)$$

where $u(r)$ and $w(r)$ are respectively the S- state and D state wave functions given in the chapter 1. Other symbols are explained ~~that~~ there.

The matrix element of the transition amplitude for the photoproduction of pions from deuteron in the impulse approximation can be written as follows :

$$\langle f|T|i\rangle = \langle f|\sum_{i=1,2} t_i \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}_i)|i\rangle \qquad (2.4)$$

where $|i\rangle$ and $|f\rangle$ are the initial and final states of the deuteron and $\underset{\sim}{k} = \underset{\sim}{\nu} - \underset{\sim}{\mu}$, $\underset{\sim}{\nu}$ being the momentum of the incoming photon and $\underset{\sim}{\mu}$ the momentum of the outgoing pion, $\underset{\sim}{r}_i$ is the positron vector of the i^{th} nucleon in the nucleus. $|i\rangle$ and $|f\rangle$ are given by

$$|i\rangle = \psi^{m} \tag{2.5}$$

$$|f\rangle = \psi^{m'} \exp(i\underset{\sim}{k}\cdot\underset{\sim}{R}) \tag{2.6}$$

where $\underset{\sim}{R} = \frac{1}{2}(\underset{\sim}{r}_1 + \underset{\sim}{r}_2)$. The single nucleon amplitude t_i in (2.4) for the neutral pion photoproduction can be written in the iso-spin space as

$$t = \frac{1}{2}(t_p + t_n) + \frac{1}{2}(t_p - t_n)\,\tau_z \tag{2.7}$$

where t_p and t_n are the amplitudes for the process $\gamma + p \to p + \pi^0$ and $\gamma + n \to n + \pi^0$ respectively.

Using the antisymmetry of $|i\rangle$ and $|f\rangle$ for the simultaneous interchange of the nucleon indices in all the spin, iso-spin and the configuration spaces, We can write (2.4) as

$$\langle f|T|i\rangle = 2\langle f|\, t_2 \exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}_2)|i\rangle \tag{2.8}$$

Noting the vanishing of the matrix element of τ_z between states with $T=0$, we have

$$\langle f|T|i\rangle = \langle f|(t_p(2) + t_n(2))\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}_2)|i\rangle \tag{2.9}$$

Using the expressions (2.5) and (2.6) for $|i\rangle$ and $|f\rangle$ respectively and the following expansion for $\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r})$:

$$\exp(i\underset{\sim}{k}\cdot\underset{\sim}{r}) = 4\pi\sum_{l=0}^{\infty}\sum_{-m}^{+m} Y_m^l(\hat{\underset{\sim}{k}})\,Y_{-m}^l(\hat{\underset{\sim}{r}})\,j_l(kr) \qquad (2.10)$$

we have the following expression for $\langle f|T|i\rangle$:

$$\langle f|T|i\rangle = F_{SS}\, t_m^{m'} - F_{SD}\sum_{m_2} C(121;\, m_2, m-m_2)\,Y^2_{m-m_2}(\hat{\underset{\sim}{k}})\, t_{m_2}^{m'} +$$

$$- F_{SD}\sum_{m_2} C(121;\, m_2, m'-m_2)\,Y^2_{m'-m_2}(\hat{\underset{\sim}{k}})\, t_m^{m_2} +$$

$$+ F_{DD}\sum_{m_2} C(121;\, m_2, m-m_2)\, C(121;\, m_2, m'-m_2)\, t_{m_2}^{m'-m+m_2} \qquad (2.11)$$

where we have neglected the terms ~~in~~ proportional to the negligible elements $\int w^2\, j_l(\tfrac{1}{2}k\rho)\, d^3\underset{\sim}{\rho}$ for $l = 2$ and 4 and where

$$t_m^{m'} = \langle {}^3\chi_{m'}|(t_p(2) + t_n(2))|{}^3\chi_m\rangle \qquad (2.12)$$

$$F_{SS} = \int_0^{\infty} u^2(\rho)\, j_0\!\left(\frac{k\rho}{2}\right) d\rho \qquad (2.13)$$

$$F_{SD} = \int_0^{\infty} u\, w\, j_2\!\left(\frac{k\rho}{2}\right) d\rho \qquad (2.14)$$

$$F_{DD} = \int_0^\infty w^2 J_0\left(\frac{k\rho}{2}\right) d\rho \qquad (2.15)$$

and $$\hat{\underset{\sim}{k}} = \underset{\sim}{k} / |\underset{\sim}{k}|$$

The matrix elements $t_m^{m'}$ can be calculated by writing the following expression for $(t_p(2) + t_n(2))$:

$$(t_p(2) + t_n(2)) = A\sigma^+(2) + B\sigma^-(2) + C\sigma_z(2) + D \qquad (2.16)$$

3. The integrals F_{SS} , F_{SD} and F_{DD} were evaluated numerically using IBM 1620 computer * taking for u and w the expressions given in the chapter 1 for various values of ρ , x_c and P_D .

In Table 1 we give the values of F_{SS} , F_{SD} and F_{DD} for a set of values. It can be found that the error involved in neglecting F_{SD} , and F_{DD} is small. Hence we do not take into account the terms containing F_{SD} or F_{DD} .

4. Neglecting the terms containing F_{SD} and F_{DD} we have

$$\langle f | T | i \rangle = F_{SS} \langle {}^3\chi_{m'} | (t_p(2) + t_n(2)) | {}^3\chi_m \rangle \qquad (2.17)$$

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* The author is grateful to the authorities of The Engineering Fundamental Research Centre for facilities offered for using the Computer.

Using the expressions (1.2) and (1.3) of Chew et al for the amplitudes t_p and t_n we can write

$$(t_p + t_n) = i\,\underset{\sim}{\sigma}\cdot\underset{\sim}{K} + L \qquad (2.18)$$

where

$$\underset{\sim}{K} = \frac{4\pi e f}{\sqrt{\mu_0 \nu_0}}\left[\underset{\sim}{\mu}\times(\underset{\sim}{\nu}\times\underset{\sim}{\epsilon})\,\lambda h^{+-} + \frac{(\underset{\sim}{\mu}\cdot\underset{\sim}{\epsilon})\,\mu}{2M\mu_0}\right] \qquad (2.19)$$

and

$$L = \frac{4\pi e f}{\sqrt{\mu_0 \nu_0}}\;\underset{\sim}{\mu}\cdot(\underset{\sim}{\nu}\times\underset{\sim}{\epsilon})\,\lambda h^{++} \qquad (2.20)$$

with

$$h^{+-} = \frac{1}{3\mu^3}\left[e^{i\delta_{11}}\sin\delta_{11} + e^{i\delta_{13}}\sin\delta_{13} - 2e^{i\delta_{33}}\sin\delta_{33}\right] \qquad (2.21)$$

$$h^{++} = \frac{1}{3\mu^3}\left[e^{i\delta_{11}}\sin\delta_{11} + 4e^{i\delta_{13}}\sin\delta_{13} + 4e^{i\delta_{33}}\sin\delta_{33}\right] \qquad (2.22)$$

$$\lambda = \frac{\mu_p - \mu_n}{4Mf^2} \qquad (2.23)$$

Here δ_{11} , δ_{13} and δ_{33} are respectively the P wave phase shifts $(1/2, 1/2)$, $(1/2, 3/2)$ and $(3/2, 3/2)$.

Instead of neglecting the small phase shifts δ_{11} and δ_{13} as in chapter 1 , here we take them into account as we are interested even in small correction terms.

Squaring, summing over final states and averaging over initial spin states and the incident photon polarizations we obtain

$$|Q|^2 = \frac{1}{4} \sum_{m, m', \underset{\sim}{\epsilon}} |<f|T|i>|^2$$

$$= \frac{16\pi^2 e^2 f^2}{3 \mu_0} \Big\{ \mu^2 \lambda^2 \nu \Big[(1+\cos^2\theta) |h^{+-}|^2 + \frac{3}{2} \sin^2\theta |h^{++}|^2 \Big] + \frac{\mu^4 \sin^2\theta}{4 M^2 \mu_0^2 \nu_0} \Big\} |F_{SS}|^2 \qquad (2.24).$$

If we make the justifiable assumption that the recoil deuteron receives only momentum but no appreciable energy, the differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \mu \mu_0 |Q|^2 \qquad (2.25)$$

Though F_{SD} and F_{DD} are neglected, the D state probability exhibits itself through F_{SS} as the parameter β in F_{SS} is a function of P_D .

5. As the recoil energy is less in the centre of momentum system the static model calculation for $d\sigma/d\Omega$ is done in the centre of mass system and then the cross-section was transformed to the laboratory system using the well known transformation

$$\left(\frac{d\sigma}{d\Omega}\right)_{Lab} = \frac{(1+\gamma^2+2\gamma\cos\theta)^{3/2}}{1+\gamma\cos\theta}\left(\frac{d\sigma}{d\Omega}\right)_{c.m.} \qquad (2.26)$$

where θ is the centre of mass scattering angle and $\gamma = m_\pi/M$. The scattering angle θ_{Lab} in the laboratory system is related to $\theta_{c.m.}$ by the following x relation

$$\tan\theta_{Lab} = \frac{\sin\theta_{c.m.}}{\gamma+\cos\theta_{c.m.}} \qquad (2.27)$$

In Figs. (2.1) to (2.9) the differential cross section inthe laboratory system for the process $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{lab} = 280$ MeV is plotted against the scattering angle θ , for the various values of P_D, r_c and $\rho(-\epsilon,-\epsilon)$. Comparisons are made with experimental data provided by Davis and Corson[1] And Rosengren and Baron[2]

1) H.L.Davis and D.R.Corson, Phys. Rev. 99 , 273 (1955)
2) J.W.Rosengren and N.Baron, Phys. Rev. 101, 410 (1956)

6. In fig. (2.1) the differential cross section for the process $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{Lab} = 280$ Mev is plotted against the scattering angle θ in the laboratory system for $\rho_\phi = 1.704 \times 10^{-13}$ c.m.

$$r_c = 0 \quad \text{or} \quad x_c = 0.0.$$

and $P_D = 0\%,\ 3\%,\ 4\%$ and 5%

In fig. (2.2) the differential cross section for the process $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{Lab} = 280$ Mev is plotted against the scattering angle θ in the laboratory system for

$$\rho_\phi = 1.734 \times 10^{-13} \text{ c.m.}$$

$$r_c = 0 \quad \text{or} \quad x_c = 0.0$$

and $P_D = 0\%,\ 3\%,\ 4\%$ and 5%

The two theoretical curves for $P_D = 0\%$ are far from the experimental values. The theoretical curves for $P_D = 5\%$ are very near the experimental values whereas the curves for $P_D = 3\%$ and $P_D = 4\%$ are not very far from the experimental values.

So for $x_c \neq 0$ we do the calculations only for $P_D = 3\%$, 4%, and 5%.

7. In fig. (2.3) the differential cross section for the process $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{lab} = 280$ Mev

is plotted against the scattering angle θ in the laboratory system for

$$\rho_1 = 1.704 \times 10^{-13} \text{ cm},$$

$$r_c = 0.4316 \times 10^{-13} \text{ cm. or } x_c = 0.1,$$

$$\text{and } P_D = 3\%,\ 4\% \text{ and } 5\%.$$

In fig. (2.4) the differential cross-section for the process $\gamma + D \rightarrow D + \pi^{\circ}$ at $E_{\gamma}^{Lab} = 280$ MeV is plotted against the scattering angle θ in the laboratory system for

$$\rho_1 = 1.734 \times 10^{-13} \text{ cm},$$

$$r_c = 0.4316 \times 10^{-13} \text{ cm. or } x_c = 0.1,$$

$$\text{and } P_D = 3\%,\ 4\% \text{ and } 5\%$$

In both the cases $\rho_1 = 1.704$ fm and $\rho_1 = 1.734$ fm theoretical values for $P_D = 5\%$ are pretty close to the experimental values whereas for $\rho_1 = 1.734$ fm and $P_D = 5\%$ the fit is better than for $\rho_1 = 1.704$ fm and $P_D = 5\%$.

8. In fig. (2.5) the differential cross-section for the process $\gamma + D \rightarrow D + \pi^{\circ}$ at $E_{\gamma}^{Lab} = 280$ MeV is plotted against the scattering angle θ in the laboratory system for

$$\rho_\phi = 1.704 \times 10^{-13} \text{ cm.},$$

$$r_c = 0.561 \times 10^{-13} \text{ cm.} \quad \text{or} \quad x_c = 0.13,$$

$$\text{and} \quad P_D = 3\%,\ 4\% \text{ and } 5\%$$

In fig. (2.6) the differential cross section for the process $\gamma + D \to D + \pi^0$ at $E_\gamma^{Lab} = 280$ MeV is plotted against the scattering angle θ in the laboratory system for

$$\rho_\phi = 1.734 \times 10^{-13} \text{ cm},$$

$$r_c = 0.561 \times 10^{-13} \text{ cm} \quad \text{or} \quad x_c = 0.13$$

$$\text{and} \quad P_D = 3\%,\ 4\% \text{ and } 5\%.$$

In both the cases $\rho_\phi = 1.704$ fm and $\rho_\phi = 1.734$ fm the theoretical values are pretty close to the experimental values whereas for $\rho_\phi = 1.734$ fm and $P_D = 5\%$ the fit is better than for $\rho_\phi = 1.704$ fm and $P_D = 5\%$.

9. From the sections 6, 7 and 8 it can be found that the only reasonable value for P_D is 5 %. So we compare the values of the differential cross-section with $P_D = 5$ % for the three different values of r_c viz., 0, .4316 fm and .561 fm.

In fig. (2.7) we have plotted the differential cross-section for $\gamma + D \to D + \pi^0$ at $E_\gamma^{Lab} = 280$ MeV against the scattering angle θ in the laboratory system for

$$\rho_1 = 1.704 \times 10^{-13}\ \text{cm},$$

$$P_D = 5\%,$$

$$\text{and} \quad r_c = 0,\ 0.4316\ \text{fm and } 0.561\ \text{fm}.$$

$$\text{or} \quad x_c = 0\cancel{.0},\ 0.1\cancel{0} \text{ and } 0.13$$

In fig. (2.8) we have plotted the differential cross-section for $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{Lab} = 280$ MeV against the scattering angle θ in the laboratory system for

$$\rho_2 = 1.734 \times 10^{-13}\ \text{cm},$$

$$P_D = 5\%,$$

$$\text{and} \quad x_c = 0.0,\ 0.10 \text{ and } 0.13$$

We find that in both the cases $\rho_1 = 1.704$ fm and $\rho_2 = 1.734$ fm. the theoretical values for $x_c = 0.13$ or equivalently $r_c = 0.561$ fm are closer to the experimental values than with $x_c = 0.0$ or $x_c = 0.1$.

10. Taking for P_D and x_c the best values viz., $P_D = 5\%$, $x_c = 0.13$, we compare the theoretical values $\rho_1 = 1.704$ fm and $\rho_2 = 1.734$ fm. .

In fig. (2.9) the differential cross section for the process $\gamma + D \rightarrow D + \pi^0$ at $E_\gamma^{Lab} = 280$ MeV is plotted against the scattering θ in the laboratory ~~xxxxx~~ system taking for P_D , x_c and ρ_1 the set of

R

values

(1) $\rho_1 = 1.704$ fm , $P_D = 5\%$, and $x_c = 0.13$

(2) $\rho_1 = 1.734$ fm, $P_D = 5\%$, and $x_c = 0.13$

The theoretical values for $\rho_1 \frac{d\sigma}{d\Omega}$ with $\rho_1 = 1.734$ fm. are closer to the experimental values than with $\rho_1 = 1.704$ fm.

Hence we conclude that the best of the choices for P_D , x_c and ρ_1 are

$$\begin{aligned} P_D &= 5\%, \\ x_c &= 0.13, \\ \text{and } \rho_1 &= 1.734 \times 10^{-13} \text{ fm.} \end{aligned} \qquad (2.28)$$

11. The theoretical curve for the differential cross-section taking for P_D , x_c and ρ_1 the values given by (2.28) though very close to the experimental values do not cut all the vertical segments provided by the experiments.

It may be argued that a value for P_D higher than 5 % shall give the best desired fit . But as the value for P_D higher than 5 % will contradict the equation (2.1) , it is necessary to analyse whether there is anything other than P_D that decreases the differential cross section to a desired extent. It is well-known that the multiple scattering effects decrease the differential cross section for the neutral pion photoproduction.[1] Hence it is suggested that it is better to take into account the multiple scattering effects before increasing the value of P_D to more than 6 % .

It is a pleasure to acknowledge the authorities of the Centre of Engineering and Fundamental Research, Madras, India, for making the I.B.M. 1620 computor freely available for the numerical calculation of the deuteron form factors.

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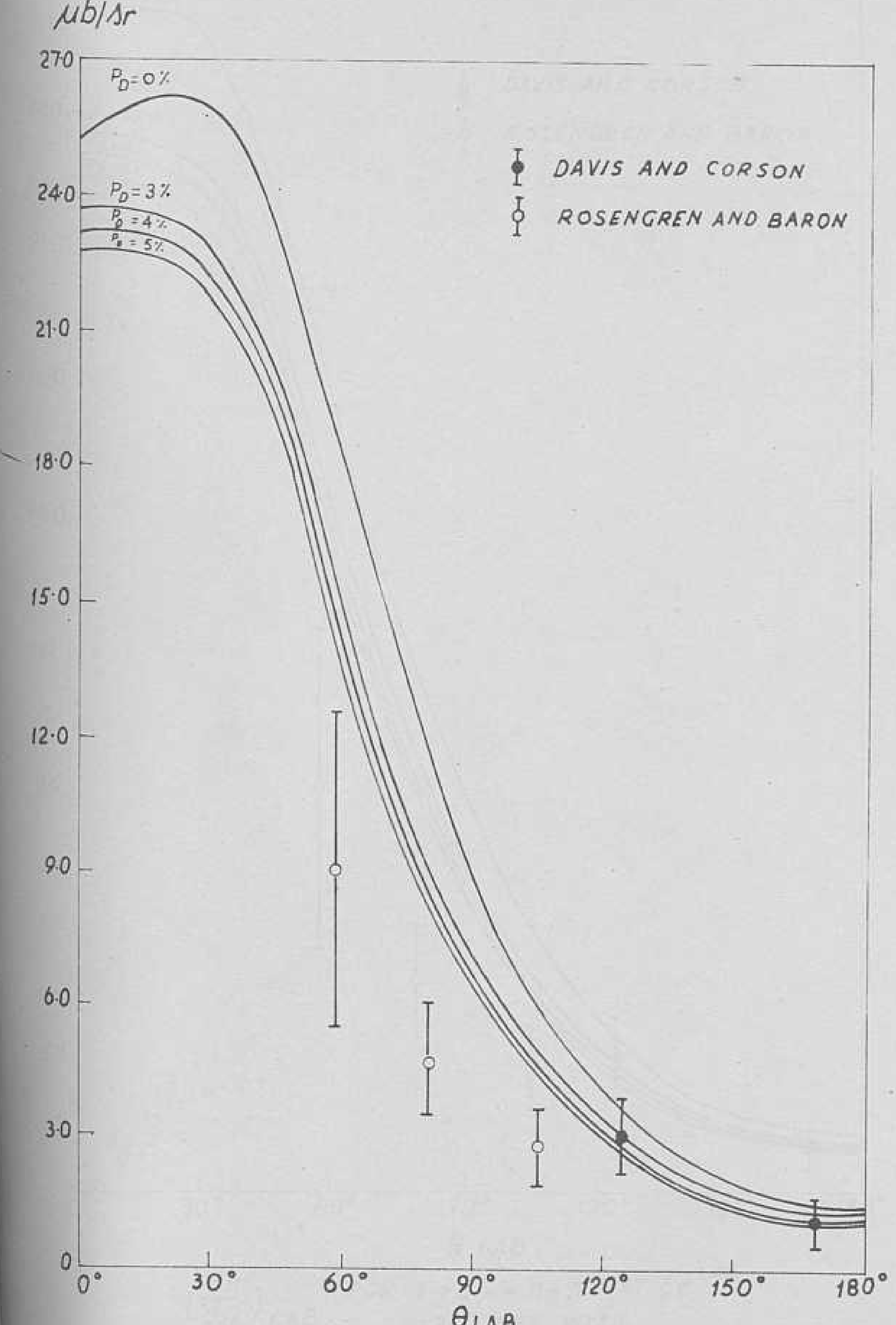
1) John Chappelear, Phys. Rev. 99 , 254 (1955)

Table 1

The nuclear form factors for deuteron as a function of the momentum transfer* (k) in the centre of mass system.

(k)	F_{SS}	F_{SD}	F_{DD}	$\frac{F_{SD}}{F_{SS}}$	$\frac{F_{DD}}{F_{SS}}$
0.2702	0.9482	0.2994×10^{-1}	0.3084×10^{-4}	0.3158×10^{-1}	0.3253×10^{-4}
1.467	0.5891	-0.3256×10^{-2}	0.279×10^{-4}	-0.5526×10^{-2}	0.353×10^{-4}
2.056	0.4012	-0.2767×10^{-2}	0.149×10^{-4}	-0.6895×10^{-2}	0.3715×10^{-4}
2.896	0.2705	-0.2016×10^{-2}	0.837×10^{-5}	-0.7452×10^{-2}	0.3094×10^{-4}

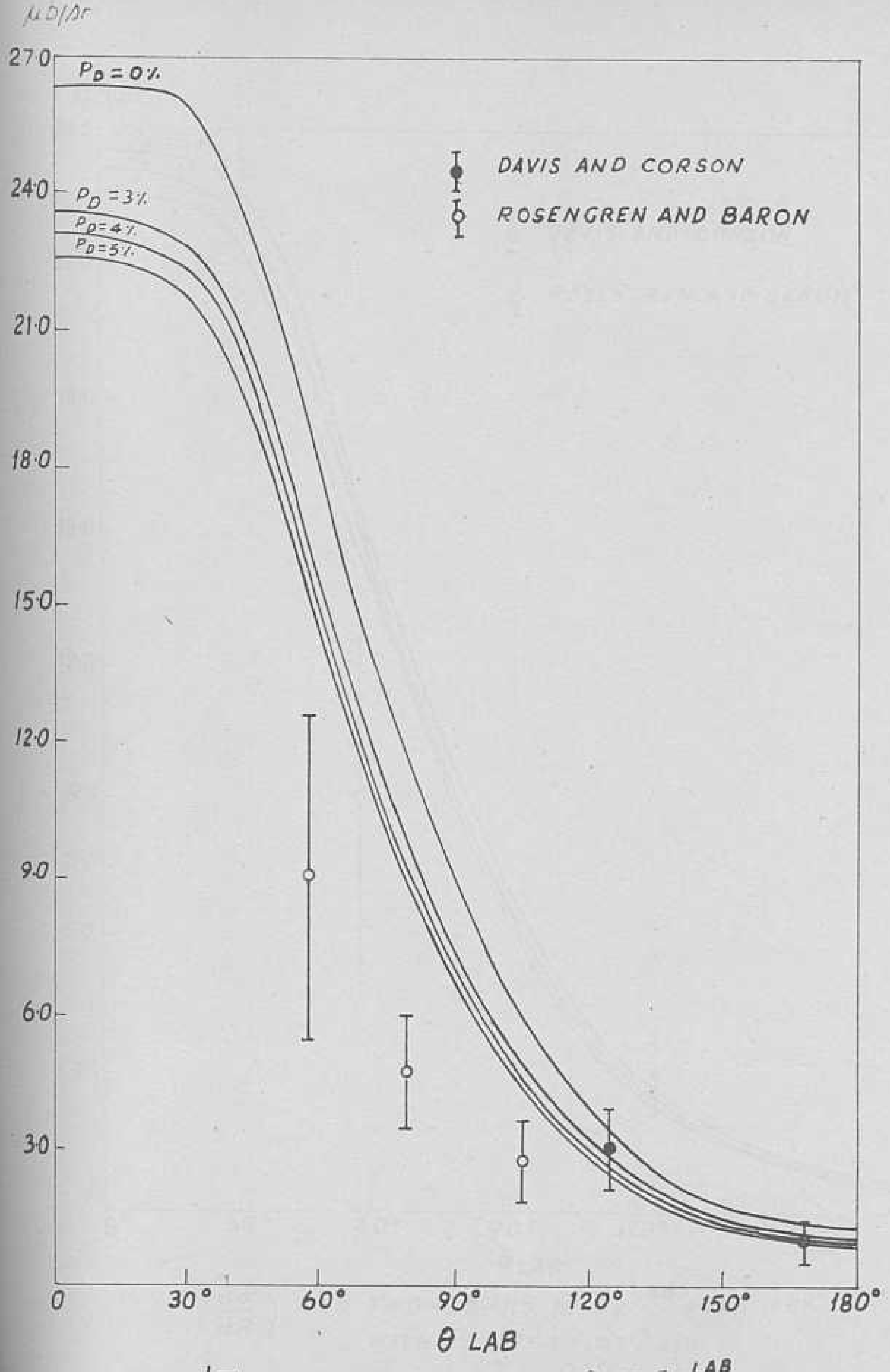
* Units $\hbar$ = c = pion mass = 1.



$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev

WITH $\rho_1 = 1{\cdot}704 \times 10^{-13}$; $x_c = 0$

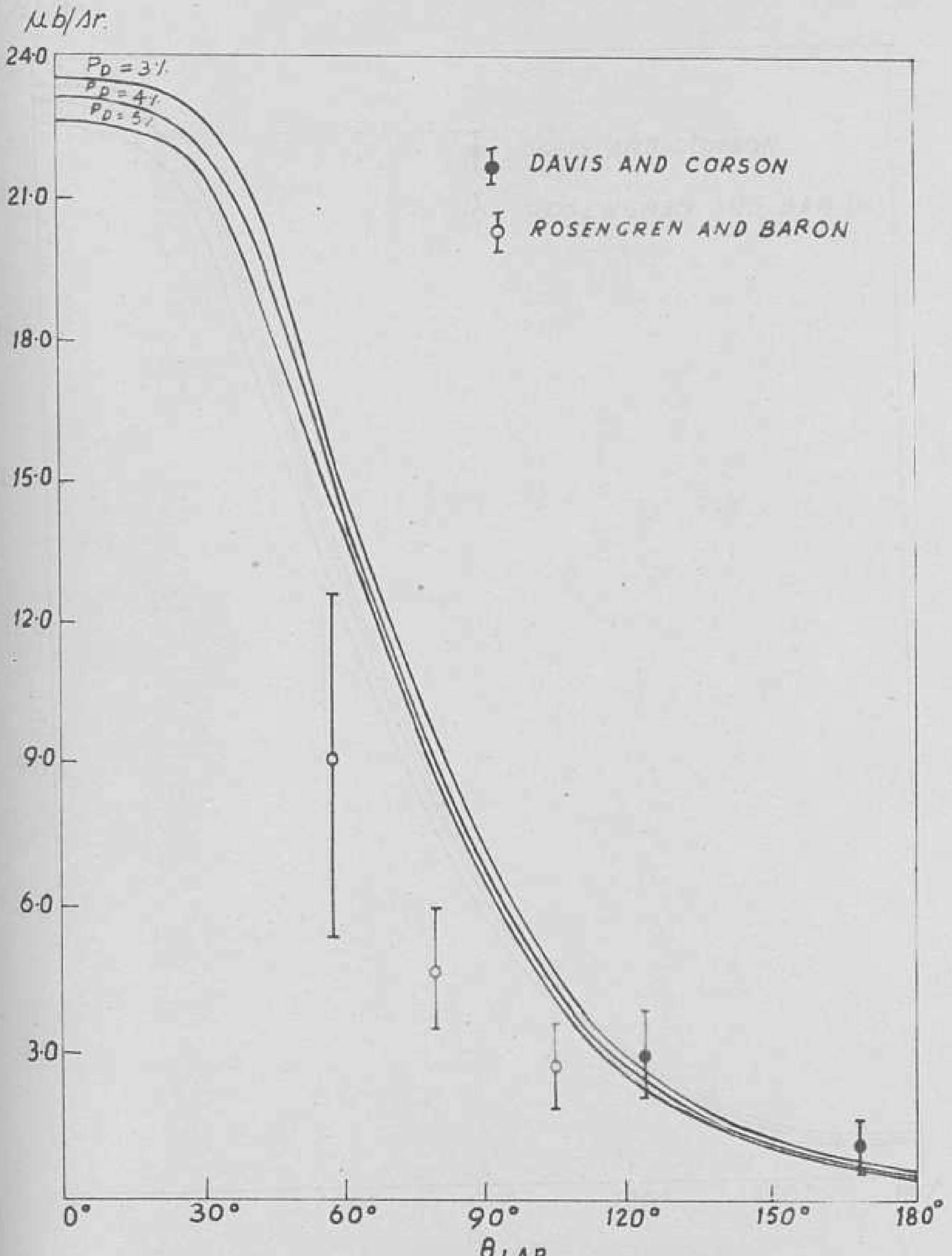
FIG.(2·1)



$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^{\circ}$ AT E_γ^{LAB} = 280 mev WITH

$\rho_1 = 1{\cdot}734 \times 10^{-13}$ cm. $x_c = 0$

FIG.(2·2)

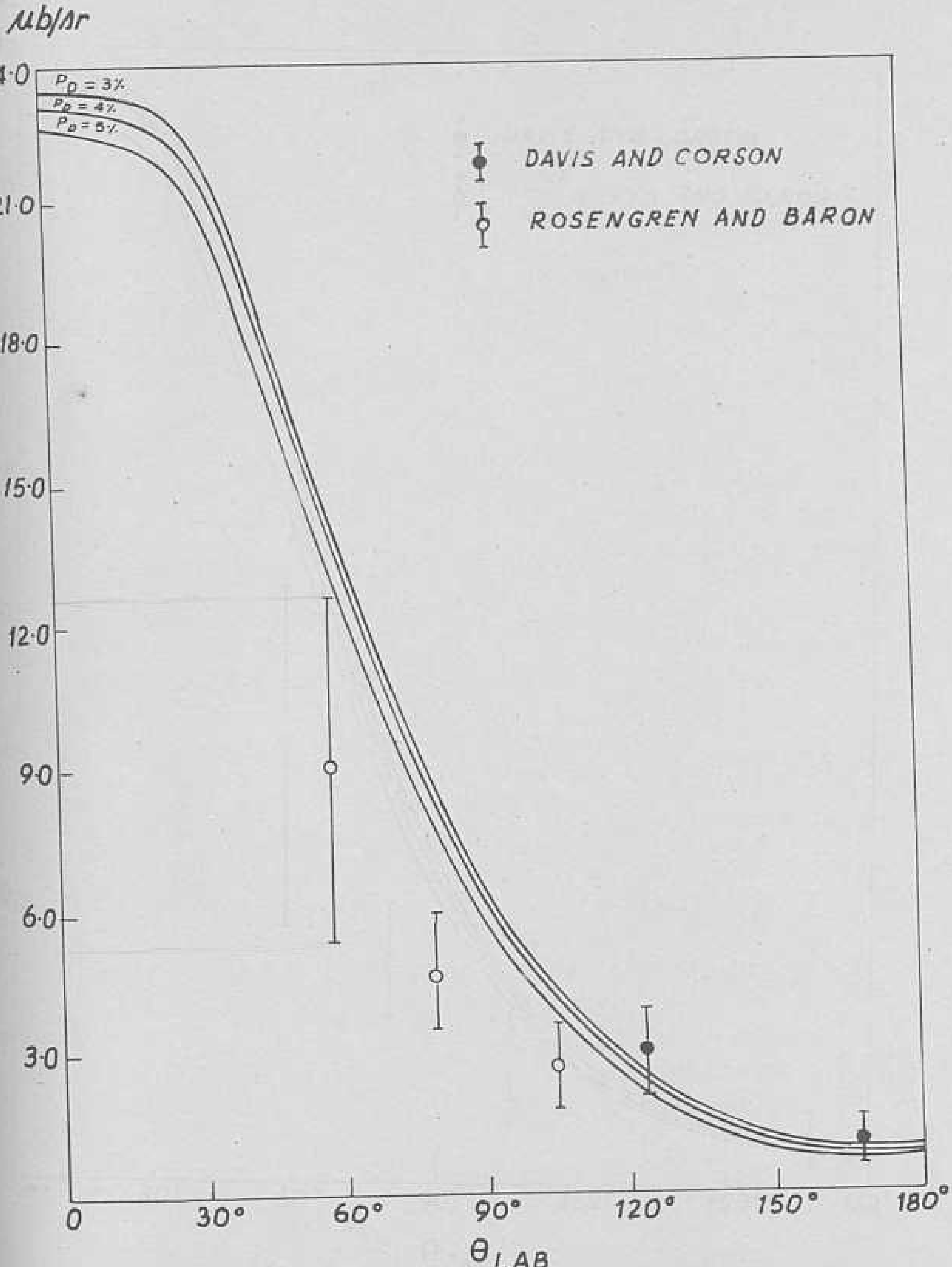


$\left(\frac{d\sigma}{d\Omega}\right)$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev

WITH $P_1 = 1.704 \times 10^{-13}$ cm.

$x_c = 0.10$

FIG. (2.3)

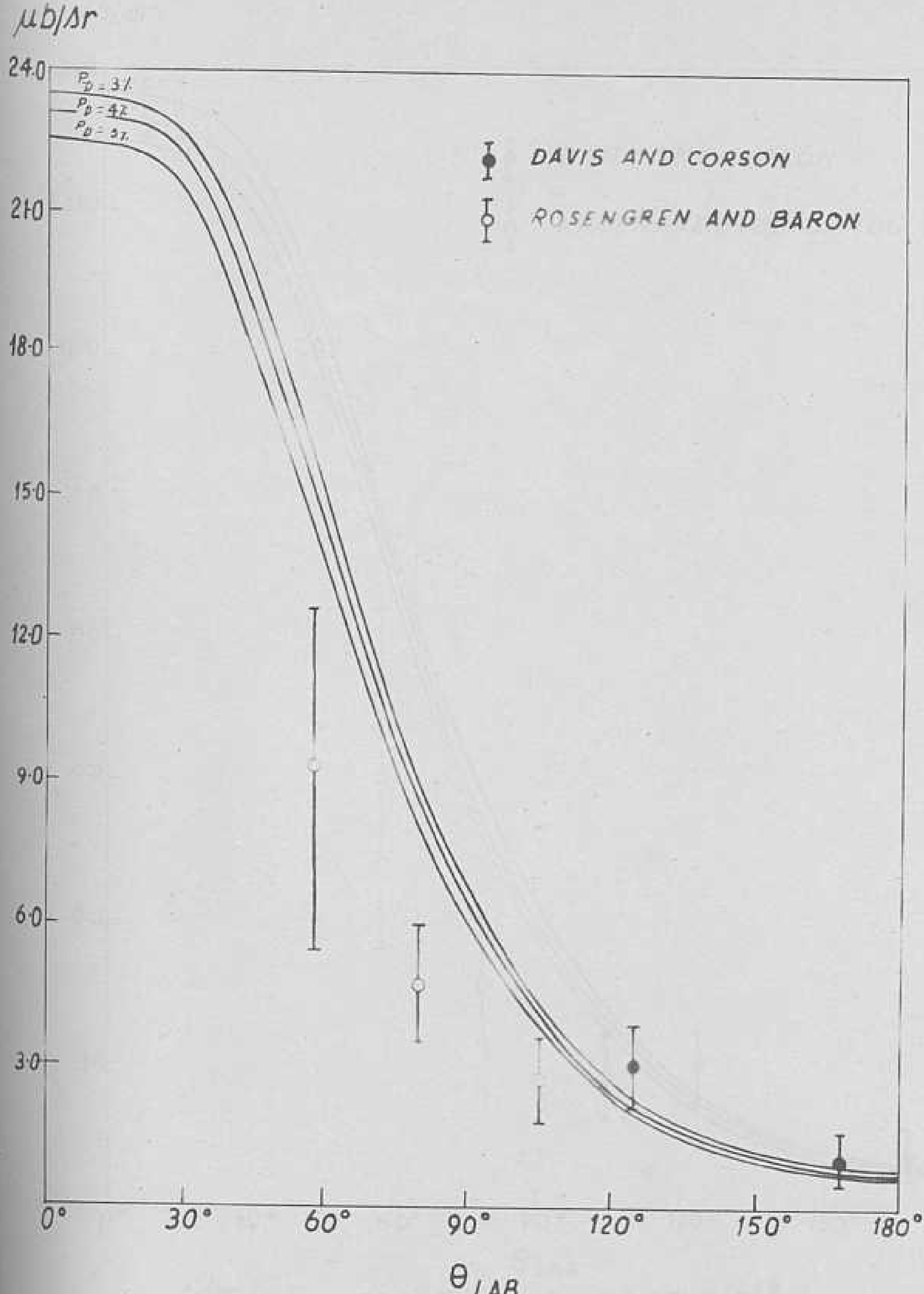


$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev

WITH $P_1 = 1.734 \times 10^{-13}$ cm.

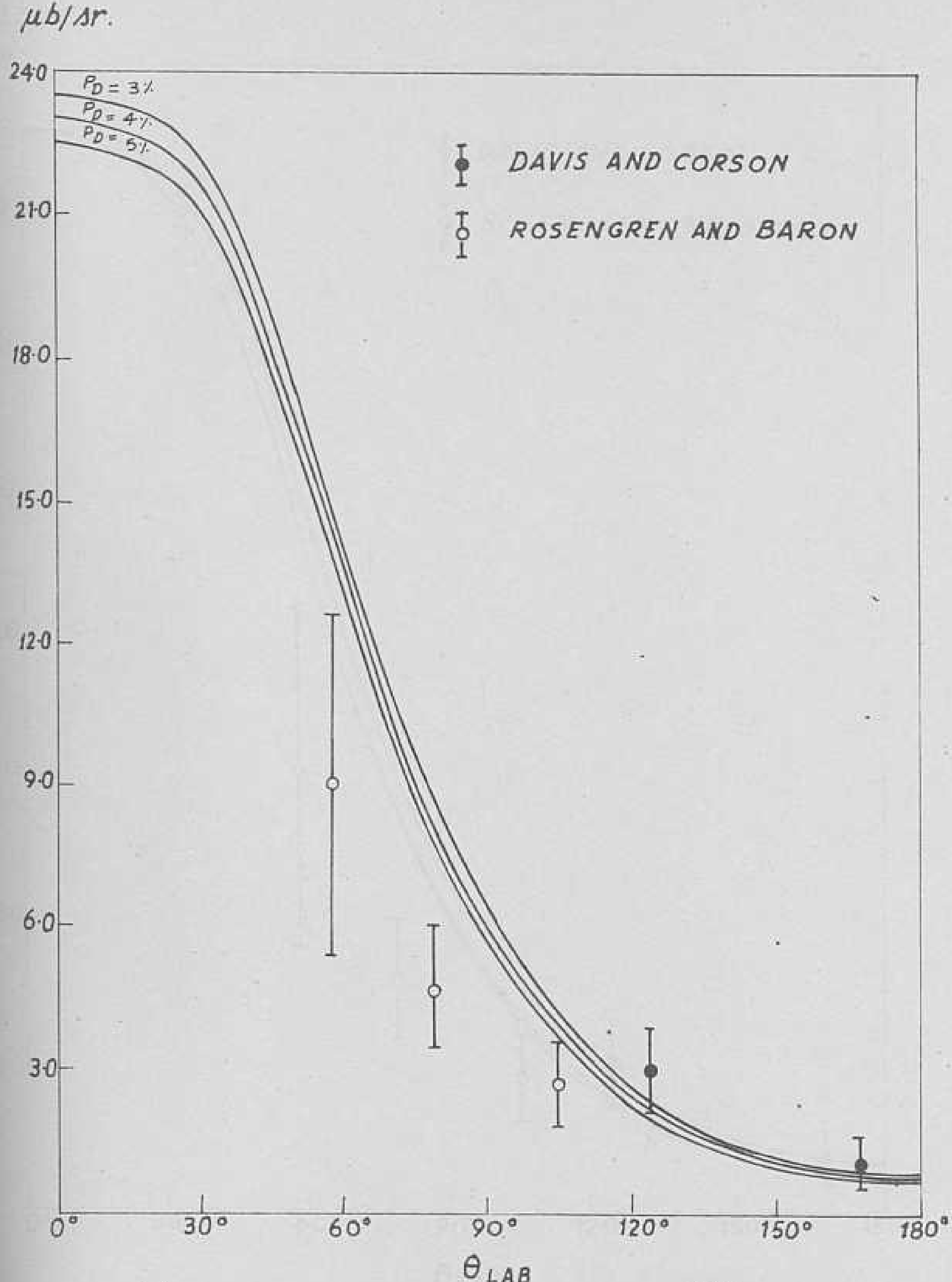
$x_c = 0.10$

FIG.(2·4)



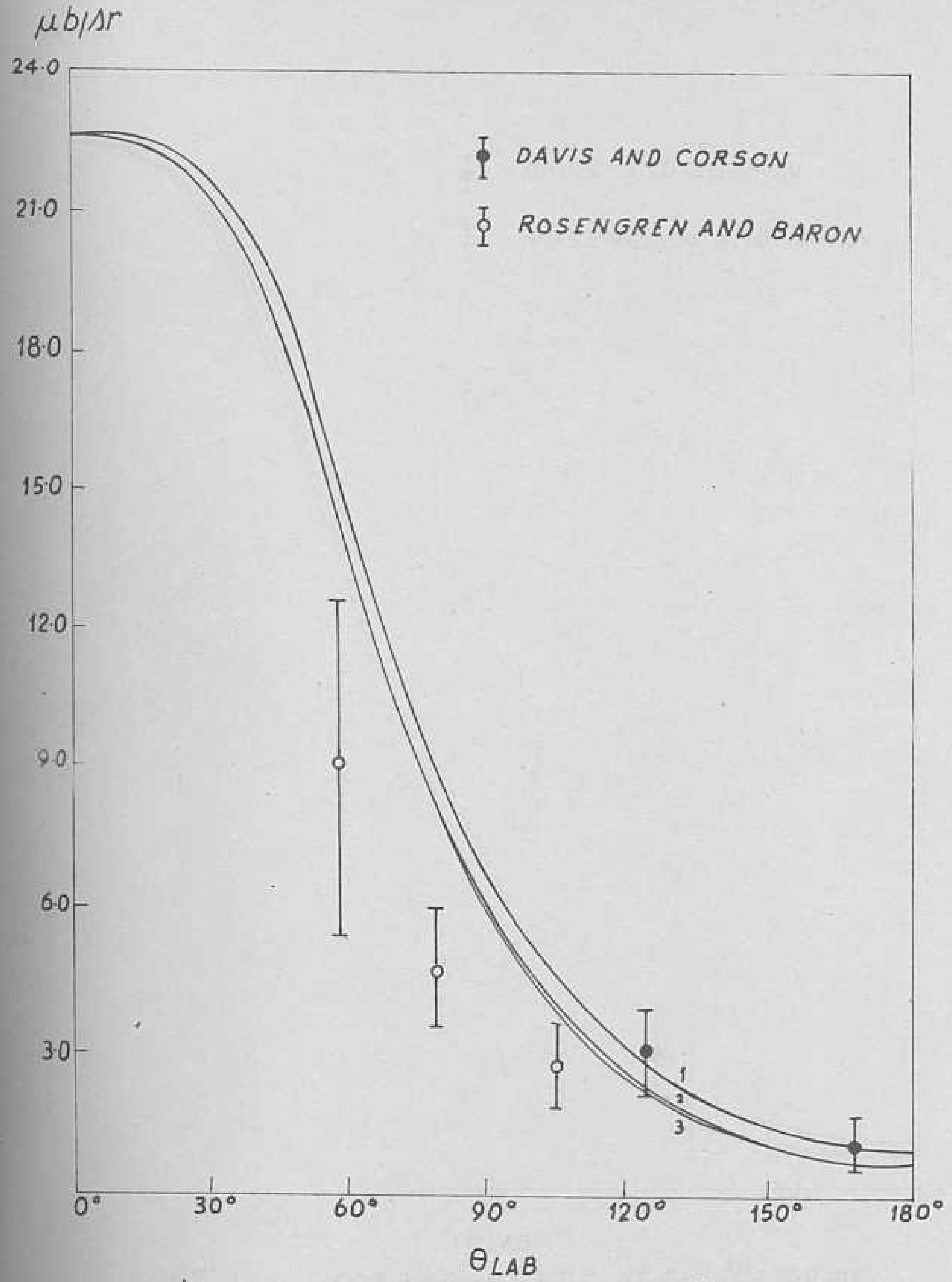
$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev
WITH $\rho_1 = 1.704 \times 10^{-13}$ cm
$x_c = 0.13$

FIG (2.5)



$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev
WITH $\rho_1 = 1{\cdot}734 \times 10^{-13}$ cm.
$x_c = 0{\cdot}13$

FIG.(2·6)



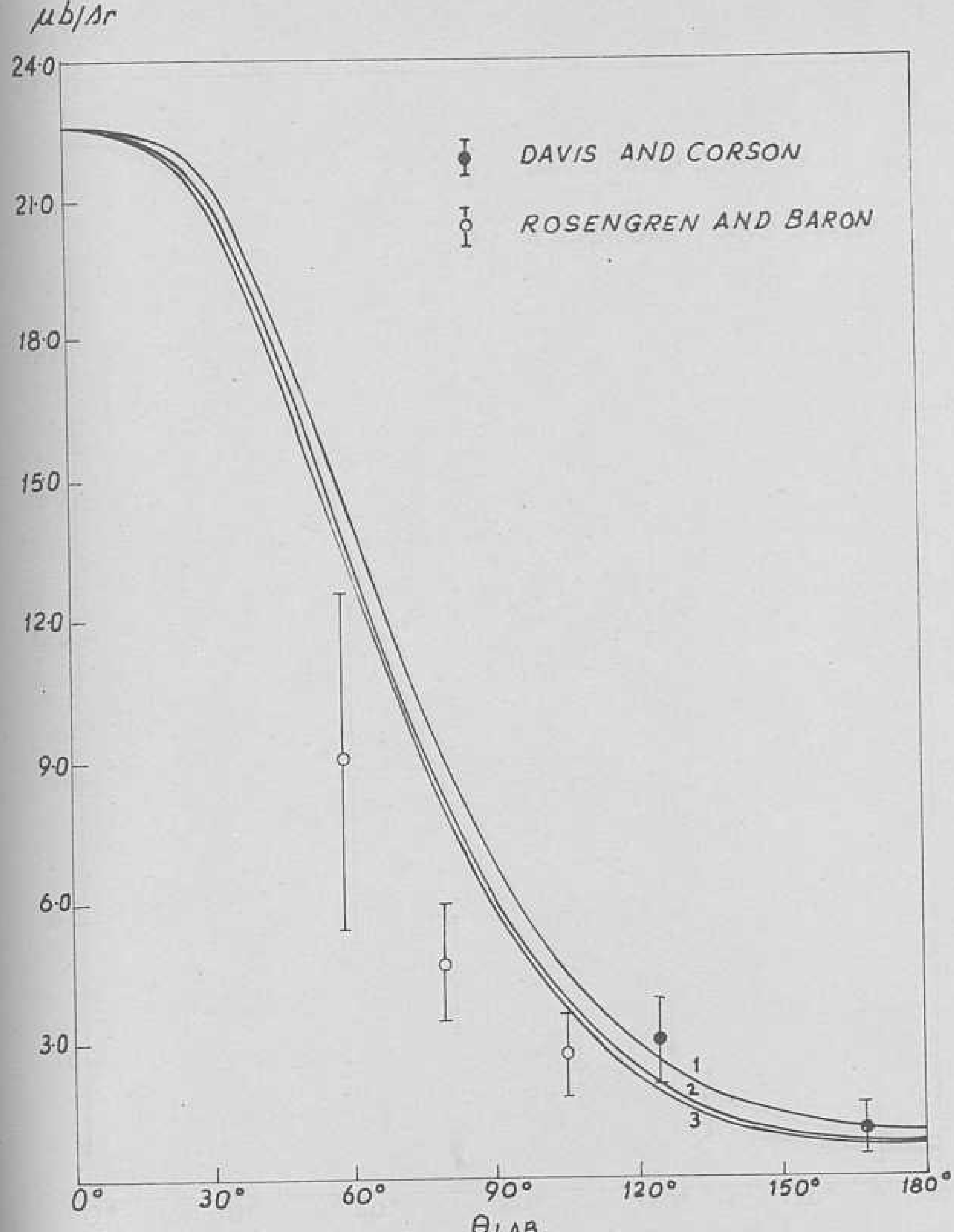
$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^{0}$ AT $E_{\gamma}^{LAB} = 280$ mev

WITH $\rho_1 = 1{\cdot}704 \times 10^{-13}$ cm.

$P_D = 5\%$ (1) $x_c = 0$ (2) $x_c = 0{\cdot}10$

(3) $x_c = 0{\cdot}13$

FIG. (2·7)

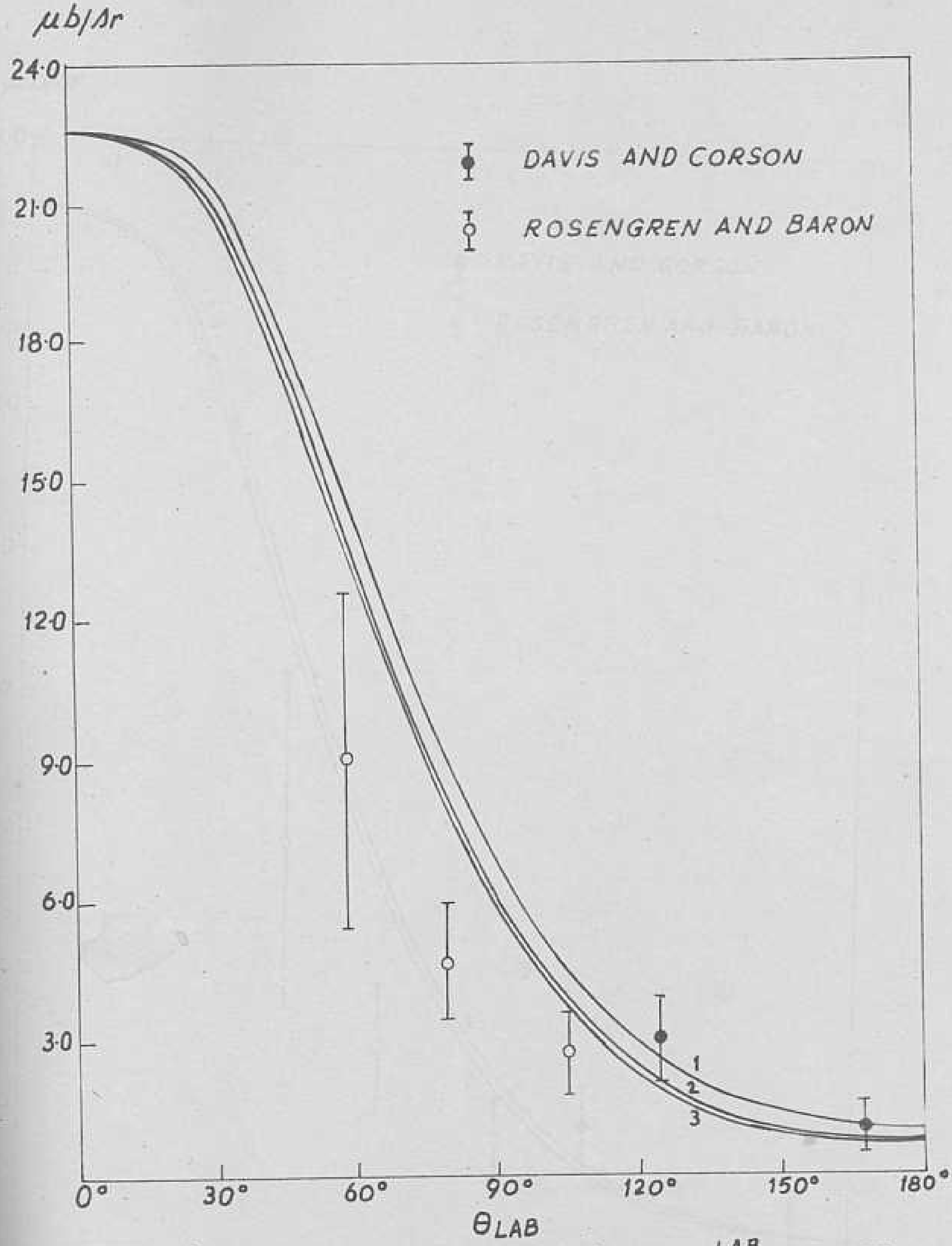


$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^{0}$ AT $E_{\gamma}^{LAB} = 280$ mev

WITH $\rho_{1} = 1{\cdot}734 \times 10^{-13}$ cm. $P_{D} = 5\%$

(1) $x_{c} = 0$ (2) $x_{c} = {\cdot}10$ (3) $x_{c} = {\cdot}13$

FIG. (2·8)

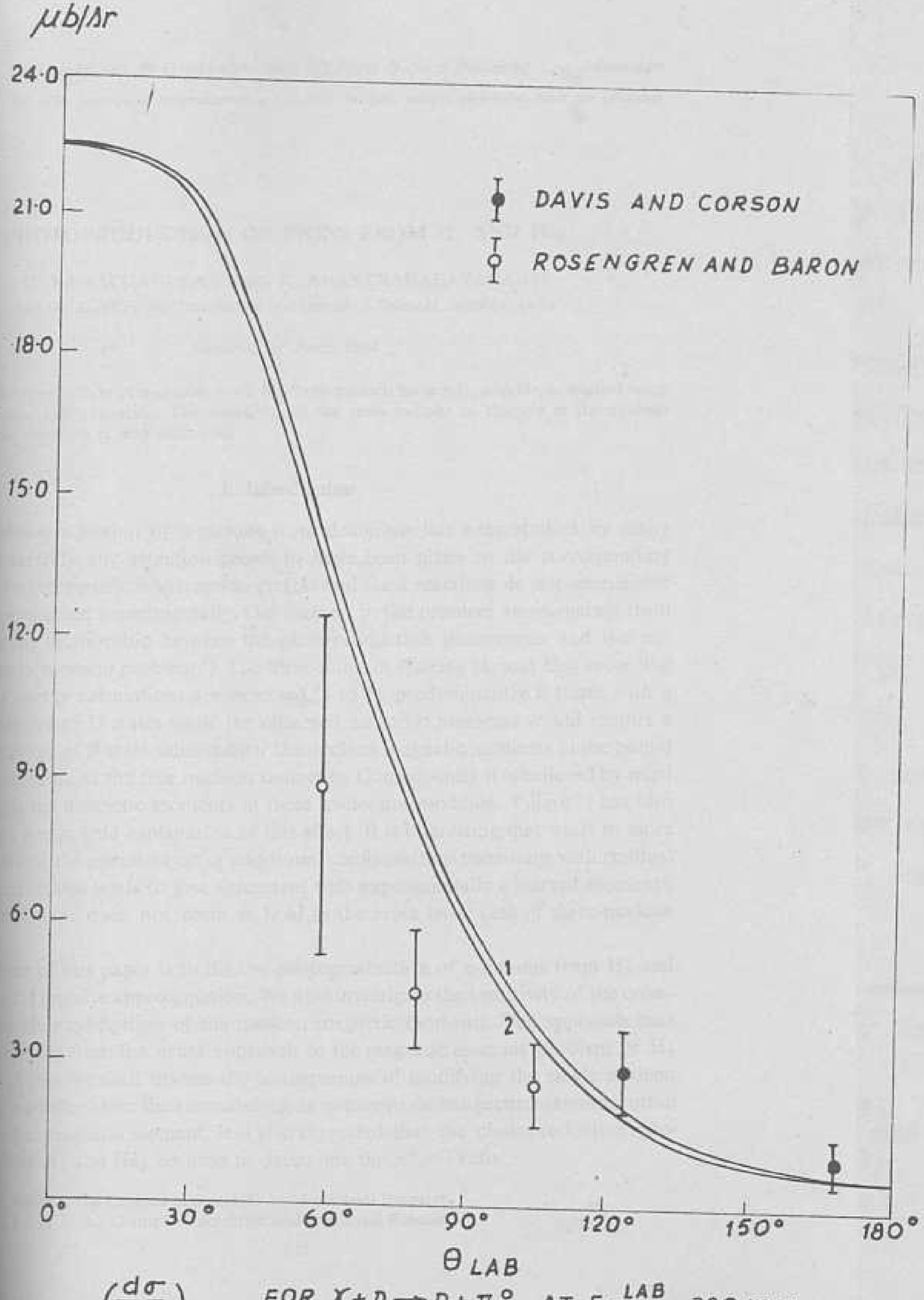


$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev

WITH $\rho_1 = 1{\cdot}734 \times 10^{-13}$ cm. $P_D = 5\%$

(1) $x_C = 0$ (2) $x_C = {\cdot}10$ (3) $x_C = {\cdot}13$

FIG. (2·8)



$\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ FOR $\gamma + D \rightarrow D + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev

WITH $P_D = 5\%$, $x_c = 0.13$

(1) $\rho_1 = 1{\cdot}704$ fm (2) $\rho_1 = 1{\cdot}734$ fm

FIG. (2·9)

Chapter 3 *

1. The study of the mirror nuclei H^3 and He^3 presents a unique opportunity to investigate in detail the three-nucleon problem wherein all the three nucleons interact strongly with each other. The series of nuclides H^1, H^2, H^3, He^3 and He^4 offers 'an unrivalled opportunity to study the relative changes occurring when the number of nucleons changes by units of one and two, thus allowing the charge independent nature of nuclear forces to be scrutinised under carefully controlled conditions.' 'All these informations when available, will certainly throw light on the question of the existence or otherwise of an intrinsic three-nucleon force'. 1)

Extensive experimental studies of electron scattering by three-nucleon systems have been made by Collard et al[1] and their data necessitate a detailed theoretical investigation of the three-nucleon ground states.

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* G. Ramachandran and K. Ananthanarayanan, Nucl. Phys. 52, 633 (1964)

1) H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M.R. Yearian, R.B. Day and R.T. Wagner, Phys. Rev. Letts. 11, 132 (1963).

The study of the photoproduction and scattering of pions from these nuclei will necessarily yield results which together with the electron scattering data might ensure a deeper understanding of the structure of the three nucleon system.

2. While the photoproduction of π mesons from deuteron has been studied by several authors*, scarcely any attention seems to have been bestowed on the corresponding problem of the photoproduction of π mesons from three-nucleon systems. It is indeed surprising why no experiments have been done so far in this case. Our main interest in this problem is due to the interesting relationship between the photoproduction phenomenon and the nuclear magnetic moment problem. The closeness of the magnetic moment of H^3 to that of the proton and the closeness of the magnetic moment of He^3 to that of neutron indicates that the ground states of H^3 and He^3 are predominantly $^2S_{1/2}$. There is nevertheless a descripancy and it is tempting to attribute it to the admixture of other states. However, it turns out[1)] that no reasonable admixture of states can account for the anomalies. Calculations with two body forces suggest that the ground state is primarily $^2S_{1/2}$ with a small

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* see Chapters 1 and 2 .

1) R.G.Sachs, "Nuclear theory" (Addison-Wesley Publishing Company Inc., Cambridge, 1963).
R.Avery and R.G.Sachs, Phys. Rev. 74 , 1320 (1948)
R. Avery and R.H.Adams, Phys. Rev. 75 , 1106 (1949)

admixture (about 4 %) of $^4D_{1/2}$) whereas an admixture of about 40 % of $P_{1/2}$ state and little $D_{1/2}$ state is needed to give the correct magnetic moment.[1] Consequently it is believed by most authors that the magnetic moments in these nuclei are modified. Villars[2] using a more sophisticated meson theoretical approach, has also interpreted these magnetic moment anomalies in terms of exchange effects.

In this chapter we shall study the elastic photo-production of charged and neutral pions from He^3 and H^3 in the impulse approximation. The processes to be considered are :

$$\gamma + He^3 \longrightarrow H^3 + \pi^+ \qquad (3.1)$$

$$\gamma + H^3 \longrightarrow He^3 + \pi^- \qquad (3.2)$$

$$\gamma + He^3 \longrightarrow He^3 + \pi^0 \qquad (3.3)$$

$$\gamma + H^3 \longrightarrow H^3 + \pi^0 \qquad (3.4)$$

We investigate the sensitivity of the cross sections to the modification of the nucleon magnetic moments. Our approach differs from the conventional approach to the magnetic moment problem of He^3 and H^3 in that we shall discuss the consequences of modifying the single nucleon magnetic moments

1) R.J.Blinstoyle, Rev. Mod. Phys. 28 , 75 (1956)

2) F. Villars, Phys. Rev. 72 , 257 (1947);
Helv. Phys. Acta. 20 , 476 (1947).

instead of introducing an unknown exchange current contribution to the nuclear magnetic moment. It is also suggested that the results of the charge pion photoproduction from He^3 and H^3 be used to determine the ratio of the cross-sections for the processes :

$$\gamma + p \rightarrow n + \pi^+ \tag{3.5}$$

and

$$\gamma + n \rightarrow p + \pi^- \tag{3.6}$$

3. The ground state wave functions of H^3 were first studied by Gerjuoy and Schwinger[1)] who obtain D $(L=2)$ state wave functions which are not orthogonal since they have not specified the isospin (T). Furthermore, they describe four (instead of three) ^4D functions which are not linearly independent. Verde[2)] has classified some of the states according to symmetry properties. Later, Sachs[3)] made an extensive study of the ground state wave functions of both He^3 and H^3 using the isospin formalism. He has obtained two S - state wave functions, three P - state wave functions and three D - state wave functions. However Derrick and Blatt[4)]

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1) Gerjuoy and J. Schwinger, Phys. Rev. 61, 138 (1942)
2) M. Verde, Helv. Phys. Acta, 22 , 340 (1949)
ibid. 23 , 453 , (1950)
3) R.G.Sachs: "Nuclear theory" - Addison-Wesley Publishing Company, 1955, p. 182.
4) G. Derrick and J.M.Blatt, Nucl. Phys., 8 , 310 (1958).

have given three S -states, four P- states and three D - states, thus adding one more S- state and P- state which are antisymmetric in the radial coordinates. The classification of Derrick and Blatt is more suitable for theoretical analysis, but the wave function of Sachs are easier to handle. Here we shall indicate the method of constructing the wave functions (as is done by Sachs).

In the three-nucleon system, there are only three partitions (three irreducible representations of the permutation group on three things). These are : the completely symmetric representation, the completely antisymmetric representation and the mixed symmetric representation. The first represents all permutations by 1. In the antisymmetric representation, the even permutations are mapped on to $+1$ and the odd permutations on to -1. The mixed symmetric representation is two dimensional, the matrices for the elements being

$$(1) : \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad ; \qquad (123) : \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \quad ;$$

$$(132) : \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} \quad ; \qquad P_{12} = (12) : \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad ; \tag{3.7}$$

$$P_{23} = (23) : \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \quad ; \qquad P_{31} = (13) : \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$$

Let us denote the doublet spin states for three nucleons by

$$\varphi^m = {}^1\chi_0(1,2)\ \chi^m_{1/2}(3) \tag{3.8}$$

$$\bar{\varphi}^m = \frac{1}{\sqrt{12}}\,(\underset{\sim}{\sigma}_{12}\cdot\underset{\sim}{\sigma}_3)\,\varphi^m \tag{3.9}$$

where m is the projection quantum number, $\underset{\sim}{\sigma}_{12} = \underset{\sim}{\sigma}_1 - \underset{\sim}{\sigma}_2$, $\underset{\sim}{\sigma}_i$ being the Pauli spin vector for the i th nucleon, ${}^1\chi_0(1,2)$ is the singlet spin function of the nucleons 1 and 2, $\chi^m_{1/2}(3)$ is the spin function of nucleon 3. The factor $\frac{1}{\sqrt{12}}$ in (3.9) is inserted in order to normalise $\bar{\varphi}$ to unity.

The φ's transform in the following way under the permutations of the nucleons

$$P_{12}\,\varphi = -\,\bar{\varphi} \tag{3.10}$$

$$P_{12}\,\bar{\varphi} = \bar{\varphi} \tag{3.11}$$

The other relation can be obtained from the matrices given in Eq. (3.7).

The $T=1/2$ isospin doublet wave functions can be formed in the same way as follows :

$$\eta^t = {}^1\xi_0(1,2)\ \xi^t_{1/2}(3) \tag{3.12}$$

$$\bar{\eta}^t = \frac{1}{\sqrt{12}}\,(\underset{\sim}{\tau}_{12}\cdot\underset{\sim}{\tau}_3)\,\eta^t \tag{3.13}$$

where t is the projection quantum number . $t = +\frac{1}{2}$ for He^3 and $t = -\frac{1}{2}$ for H^3 ; $\underset{\sim}{\tau}_{12} = \underset{\sim}{\tau}_1 - \underset{\sim}{\tau}_2$, $\underset{\sim}{\tau}_i$ being the isospin operator for the i th nucleon, ${}^1\xi_0(1,2)$ is the singlet isospin wave function of nucleons 1 and 2 , $\xi^t_{1/2}(3)$ is the isospin function of the nucleon 3 .

From the spin and isospin function one can make the following combinations :

$$\Phi_a = \frac{1}{\sqrt{2}} (\varphi\bar{\eta} - \bar{\varphi}\eta) \tag{3.14}$$

$$\Phi = \frac{1}{\sqrt{2}} (\varphi\bar{\eta} + \bar{\varphi}\eta) \tag{3.15}$$

$$\bar{\Phi} = \frac{1}{\sqrt{2}} (\varphi\eta - \bar{\varphi}\bar{\eta}) \tag{3.16}$$

$$\Phi_s = \frac{1}{\sqrt{2}} (\varphi\eta + \bar{\varphi}\bar{\eta}) \tag{3.17}$$

Φ_a is fully antisymmetric; Φ_s is fully symmetric while Φ and $\bar{\Phi}$ belong to the mixed symmetric representations transforming like φ and $\bar{\varphi}$ respectively.

The internal space coordinates of the three-nucleon systems can be chosen as follows:

$$\underset{\sim}{R} = \frac{1}{3} (\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3) \tag{3.18}$$

$$\underset{\sim}{\rho} = \frac{1}{2} (\underset{\sim}{r}_{23} + \underset{\sim}{r}_{13}) \tag{3.19}$$

$$\underset{\sim}{r} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2 \tag{3.20}$$

where $\underset{\sim}{r}_{ij} = \underset{\sim}{r}_i - \underset{\sim}{r}_j$ and $\underset{\sim}{r}_i$ $(i = 1, 2, 3)$ is the coordinate of the i th nucleon. From the definitions of $\underset{\sim}{r}$ and $\underset{\sim}{\rho}$ it follows that

$$P_{13}^{r} \underset{\sim}{r} = \frac{1}{2}\underset{\sim}{r} - \underset{\sim}{\rho} \qquad (3.21)$$

$$P_{13}^{r} \underset{\sim}{\rho} = -\frac{1}{4}(3\underset{\sim}{r} + 2\underset{\sim}{\rho}) \qquad (3.22)$$

Thus the S functions of the intermediate class are

$$s = \underset{\sim}{r} \cdot \underset{\sim}{\rho} \qquad (3.23)$$

$$\bar{s} = \frac{1}{\sqrt{12}}(4\rho^2 - 3r^2) \qquad (3.24)$$

The symmetric S function is

$$S_{s} = (r_{12}^2 + r_{13}^2 + r_{23}^2) \qquad (3.25)$$

Any function of S_s is also symmetric for the interchange of any pair of nucleon indices.

The quartet spin functions can be found from φ, $\tilde{\varphi}$ by suitable combination of $\underset{\sim}{\sigma}_1$, $\underset{\sim}{\sigma}_2$ and $\underset{\sim}{\sigma}_3$.

4. In forming the S state radial wave functions let us follow the procedure of Schiff[1)] which is more general than the procedures of Sachs as well as Derrick and Blatt.

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1) L.I.Schiff, Phys. Rev. **133**, B 802 (1964)

Consider a function of the radial coordinates r_{12}, r_{23} and r_{31} denoted by $g(12,3)$ which is symmetric in an interchange of nucleons 1 and 2 but neither symmetric nor antisymmetric in an interchange of the nucleons 1 and 3 or 2 and 3 . From this single function $g(12,3)$, we can form the mixed symmetric S functions v and $\bar{v}$ as follows:

$$v = \frac{1}{\sqrt{2}} \left[g(23,1) - g(13,2) \right] \qquad (3.26)$$

$$\bar{v} = \frac{1}{\sqrt{6}} \left[g(23,1) + g(13,2) - 2g(12,3) \right] \qquad (3.27)$$

In addition we have to consider a function $u(123)$ which is fully symmetric and another function which is ~~is~~ fully antisymmetric. The fully antisymmetric radial function is used only by Derrick and Blatt and it can be formed from three functions $v_1(x)$, $v_2(x)$ and $v_3(x)$ as follows:

$$f_a(r_{12}, r_{23}, r_{31}) = \det \begin{vmatrix} v_1(r_{23}) & v_2(r_{23}) & v_3(r_{23}) \\ v_1(r_{13}) & v_2(r_{13}) & v_3(r_{13}) \\ v_1(r_{12}) & v_2(r_{12}) & v_3(r_{12}) \end{vmatrix}$$

(3.28)

The Pauli principle requires that the overall wave function be fully antisymmetric in the interchanges of all of the coordinates (spin, isospin and space) of any pair of nucleons. The function $\Phi_a u$ has this property; this is the dominant $^2S_{1/2}$ state which is denoted by S. One can also form another fully antisymmetric wave function from Φ, $\bar{\Phi}$ and v, $\bar{v}$ which is $(\bar{\Phi} v - \Phi \bar{v})$. This is the $^2S_{1/2}$ state of mixed symmetry which is denoted as S'. One can form yet another S- state wave function (this is not found in the classification by Sachs, but has been added by Derrick and Blatt) using Φ_S and the antisymmetric radial wave function $f_a(r_{12}, r_{23}, r_{31})$ viz. $\Phi_S f_a(r_{12}, r_{23}, r_{31})$

Thus the three $^2S_{1/2}$ wave function are :

$$\psi_1^{m,t} = \Phi_a u \tag{3.29}$$

$$\psi_2^{m,t} = (\bar{\Phi} v - \Phi \bar{v}) \tag{3.30}$$

$$\psi_3^{m,t} = \Phi_S f_a(r_{12}, r_{23}, r_{31}) \tag{3.31}$$

5. A wave function with $J = 1/2$ and $J_z = 1/2$ can be obtained by operating a spherically symmetric operator on φ. For the P state wave function this operator must contain a space vector which should have even parity since all constituents of the ground state have even parity. The one and only such vector is $\dfrac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}$. (This unit vector contains only the angular part and so its spherical components are same as $Y^1_M(\theta, \varphi)$ apart from a constant where θ, φ are the polar angles of the normal to the plane of the three nucleons. $Y^1_M(\theta, \varphi)$ are the spherical hormonics of rank unity). The required spherically symmetric operator is then constructed by taking the scalar product of this unit vector with a spin vector $\underset{\sim}{\sigma}_{12}$, $\underset{\sim}{\sigma}_3$ and $\underset{\sim}{\sigma}_1 \times \underset{\sim}{\sigma}_3$; higher powers can be reduced to these or constants. The expression

$$ {}^2P = i\, \underset{\sim}{\sigma}_3 \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}\, \varphi^m \qquad (3.32) $$

is clearly a ${}^2P_{1/2}$ function which is antisymmetric for the interchange of the spins of nucleons 1 and 2. Since the doublets belong to the intermediate symmetry class, another doublet function can be obtained by applying the operator P^{σ}_{13} given by the expression $\frac{1}{2}(1 + \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_3)$. The function $\frac{1}{2}(1 + \underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_3)$ can be reduced to the following form

$$ {}^2\bar{P} = \frac{1}{\sqrt{12}} \left(i \underset{\sim}{\sigma}_{12} + \underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3 \right) \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|} \varphi^m \qquad (3.33) $$

where the factor $1/\sqrt{12}$ is added for normalization. These two functions of mixed symmetry class should be suitably combined with η^t and $\bar{\eta}^t$ to get one of the fullyx antisymmetric P - state wave functions

$$ \psi_4^{m,t} = \left[i \underset{\sim}{\sigma}_3 \eta^t + \frac{1}{\sqrt{12}} (\underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3 + i \underset{\sim}{\sigma}_{12}) \right] \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|} \varphi^m f_4(S_s) \qquad (3.34) $$

Another P - state wave function can be formed with the use of v and $\bar{v}$. Or, one can use S and $\bar{S}$. For, v and $\bar{v}$ reduce to s and $\bar{s}$ when one takes $g(12,3) = -\left(\frac{8}{3}\right)^{1/2} r_{12}^2$. We need combine v and $\bar{v}$ with η^t and $\bar{\eta}^t$ to form a pair of mixed symmetry functions and then combine these with ${}^2P$ and ${}^2\bar{P}$ to give

$$ \psi_5^{m,t} = \left[(\bar{\eta}^t v + \bar{v} \eta^t) i \underset{\sim}{\sigma}_3 + (v \eta^t - \bar{v} \bar{\eta}^t) \frac{1}{\sqrt{12}} (i \underset{\sim}{\sigma}_{12} + \underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3) \right] \times $$
$$ \times \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|} \varphi^m f_5(S_s) \qquad (3.35) $$

One can absorb $f_5(S_s)$ with v and $\bar{v}$ as Derrick and Blatt have done.

Finally, as pointed out by Derrick and Blatt a third $^2P_{1/2}$ state can be formed by multiplying the fully symmetric wave function formed by the combination

$$({}^2P\eta^t - {}^2\overline{P}\eta^t) \qquad (3.36)$$

by the fully antisymmetric radial wave function $f_a(r_{12}, r_{23}, r_{31})$

$$\psi_6^{m,t} = \left[i\,\underset{\sim}{\sigma}_3 \cdot \overline{\eta}^t - \frac{1}{\sqrt{12}}\left(i\,\underset{\sim}{\sigma}_{12} + \underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3 \right)\eta^t \right] \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}\, \varphi^m \times$$
$$\times\, f_a(r_{12}, r_{23}, r_{31}) \qquad (3.37)$$

The wave functions $\psi_4^{m,t}$, $\psi_5^{m,t}$, $\psi_6^{m,t}$ are the three possible $^2P_{1/2}$ state wave functions. In addition to these we can form one $^4P_{1/2}$ state wave function also. The $^4P_{1/2}$ state wave function is symmetric in the spin variables and therefore can only be a linear combination of

$$\left(\underset{\sim}{\sigma}_{12} \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}\right)\varphi^m \quad \text{and} \quad \left(\underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3\right) \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}\, \varphi^m \qquad (3.38)$$

The appropriate coefficients can be determined by the symmetry under P_{13}^{σ}. Hence

$$^4P = \left(i\,\underset{\sim}{\sigma}_{12} - \tfrac{1}{2}\underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3 \right) \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|}\, \varphi^m \qquad (3.39)$$

Since this function is totally antisymmetric, it must be combined with the symmetric product of η^t, $\bar{\eta}^t$ with $v, \bar{v}$. Hence the ${}^4P_{1/2}$ function is

$$\psi_7^{m,t} = [v\eta^t + \bar{v}\bar{\eta}^t][i\underset{\sim}{\sigma}_{12} - \tfrac{1}{2}\underset{\sim}{\sigma}_{12} \times \underset{\sim}{\sigma}_3] \cdot \frac{\underset{\sim}{r} \times \underset{\sim}{\rho}}{|\underset{\sim}{r} \times \underset{\sim}{\rho}|} \varphi^m f_7(S_3) \quad (3.40)$$

6. The classification of D states by Sachs is simple, and elegant but the three D state wave functions are not orthognal. But these states can be combined to give an orthognal set[1].

For convenience we shall define two vectors $\underset{\sim}{R} = -\underset{\sim}{r}$ and $\underset{\sim}{\bar{R}} = \left(\frac{4}{3}\right)^{1/2} \underset{\sim}{\rho}$. Formation of the D state wave functions requires the use of products such as $(\underset{\sim}{A}\cdot\underset{\sim}{R})(\underset{\sim}{B}\cdot\underset{\sim}{R})\varphi^m$, $(\underset{\sim}{A}\cdot\underset{\sim}{\bar{R}})(\underset{\sim}{B}\cdot\underset{\sim}{\bar{R}})\varphi^m$ and $(\underset{\sim}{A}\cdot\underset{\sim}{R})(\underset{\sim}{B}\cdot\underset{\sim}{\bar{R}})\varphi^m$ where $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are spin operators.

These space-spin functions are combinations of S, P and D states that have even parity, $J = 1/2$ and total angular momentum component m. For a particular choice of $\underset{\sim}{A}$ and $\underset{\sim}{B}$ the four space-spin functions may be grouped into two combinations

$$\Phi = [(\underset{\sim}{A}\cdot\underset{\sim}{R})(\underset{\sim}{B}\cdot\underset{\sim}{\bar{R}}) + (\underset{\sim}{A}\cdot\underset{\sim}{\bar{R}})(\underset{\sim}{B}\cdot\underset{\sim}{R})]\varphi^m \quad (3.41)$$

$$\bar{\Phi} = [(\underset{\sim}{A}\cdot\underset{\sim}{R})(\underset{\sim}{B}\cdot\underset{\sim}{R}) - (\underset{\sim}{A}\cdot\underset{\sim}{\bar{R}})(\underset{\sim}{B}\cdot\underset{\sim}{\bar{R}})]\varphi^m \quad (3.42)$$

which transform like φ^m and $\bar{\varphi}^m$ and so belong to mixed symmetry class and the two combinations

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1) B.F.Gibson and L.I.Schiff, Phys. Rev. 138, B 26 (1965)

$$\Phi_a = \left[(\mathbf{A}\cdot\mathbf{R})(\mathbf{B}\cdot\bar{\mathbf{R}}) - (\mathbf{A}\cdot\bar{\mathbf{R}})(\mathbf{B}\cdot\mathbf{R}) \right] \varphi^m \qquad (3.43)$$

$$\Phi_s = \left[(\mathbf{A}\cdot\mathbf{R})(\mathbf{B}\cdot\mathbf{R}) + (\mathbf{A}\cdot\bar{\mathbf{R}})(\mathbf{B}\cdot\bar{\mathbf{R}}) \right] \varphi^m \qquad (3.44)$$

are respectively antisymmetric and symmetric with respect to permutations of nucleon space coordinates. It is easily seen that $\Phi_a = (\mathbf{A}\times\mathbf{B})\cdot(\mathbf{R}\times\bar{\mathbf{R}})$ and hence yields the P state functions formed earlier when $\mathbf{A}$ and $\mathbf{B}$ are suitably chosen.

The remaining three Φ's are combinations of D and S states. Since higher powers of σ's introduce nothing new, it is sufficient to choose $\mathbf{A} = \boldsymbol{\sigma}_3$ and $\mathbf{B} = \boldsymbol{\sigma}_{12}$. We thus convert the three combinations into pure D states by subtracting their S parts which are the averages over orientations of the space triangle defined by $\mathbf{R}$ and $\bar{\mathbf{R}}$. Consequently

$$D = \left[(\boldsymbol{\sigma}_3\cdot\mathbf{R})(\boldsymbol{\sigma}_{12}\cdot\bar{\mathbf{R}}) + (\boldsymbol{\sigma}_3\cdot\bar{\mathbf{R}})(\boldsymbol{\sigma}_{12}\cdot\mathbf{R}) - \tfrac{2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_{23})(\mathbf{R}\cdot\bar{\mathbf{R}}) \right] \varphi^m \qquad (3.45)$$

$$\bar{D} = \left[(\boldsymbol{\sigma}_3\cdot\mathbf{R})(\boldsymbol{\sigma}_{12}\cdot\mathbf{R}) - (\boldsymbol{\sigma}_3\cdot\bar{\mathbf{R}})(\boldsymbol{\sigma}_{12}\cdot\bar{\mathbf{R}}) - \tfrac{1}{3}(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_{12})(\mathbf{R}^2 - \bar{\mathbf{R}}^2) \right] \varphi^m \qquad (3.46)$$

$$D_s = [(\sigma_3 \cdot R)(\sigma_{12} \cdot R) + (\sigma_3 \cdot \bar{R})(\sigma_{12} \cdot \bar{R}) - \tfrac{1}{3}(\sigma_3 \cdot \sigma_{12})(R^2 + \bar{R}^2)]\,\varphi^m \tag{3.47}$$

These are the $^4D_{1/2}$ states with $J = 1/2$. The quartet character of their spin dependence means that they are fully symmetric with respect to permutations of the nucleon spins. Thus $D's$ have the symmetries indicated by their subscripts with respect to permutations of all (space and spin) coordinates of the three nucleons.

The $D's$ must be combined with or multiplied by isospin functions and perhaps also by spherically symmetric space functions so as to form fully antisymmetric functions that obey Pauli's principle. D_s can be only be multiplied by $(v\bar{\eta}^t - \bar{v}\eta^t)$ which is antisymmetric. As mentioned before v and $\bar{v}$ can be replaced by s and $\bar{s}$.

$$\psi_8^{m,t} = D_s\,(v\bar{\eta}^t - \bar{v}\eta^t)\,f_8(S_s) \tag{3.48}$$

This is equivalent to Sachs' wave function $\psi_6^{m,t}$ with v and $\bar{v}$ replaced by s and $\bar{s}$. D and $\bar{D}$ may be combined with $\eta's$ to give

$$\psi_9^{m,t} = (D\bar{\eta}^t - \bar{D}\eta^t)\,f_9(S_s) \tag{3.49}$$

which is equivalent to Sach's wave function $\psi_7^{m,t}$. Finally mixed symmetry combinations of the $v's$ and $\eta's$ may be combined with the D's to yield a wave function which reduces to Sach's wave function $\psi_{10}^{m,t}$ when v and $\bar{v}$ are replaced by s and $\bar{s}$. The three pairs can be combined in any order and the result is

$$\psi_{10}^{m,t} = \left[(v\eta^t - \bar{v}\bar{\eta}^t) D - (v\bar{\eta}^t + \bar{v}\eta^t)\bar{D} \right] f_8(S_s) \tag{3.50}$$

It has been pointed out by Gibson and Schiff that $\psi_{10}^{m,t}$ is orthognal to both $\psi_8^{m,t}$ and $\psi_9^{m,t}$ but $\psi_6^{m,t}$ and $\psi_7^{m,t}$ are not orthogonal to each other. Replacing v, $\bar{v}$ ~~by~~ by less general S and $\bar{S}$ they redefine $\psi_8^{m,t}$ and $\psi_9^{m,t}$ as follows:

$$\psi_8^{m,t} = \left[(5 D_s S - 2 D S_s)\bar{\eta}^t - (5 D_s \bar{S} - 2\bar{D} S_s)\eta^t \right] f_8(S_s) \tag{3.51}$$

$$\psi_9^{m,t} = (D \bar{S}_s \bar{\eta}^t - \bar{D} S_s \eta^t) f_9(S_s) \tag{3.52}$$

In these three wave functions the angular and radial parts are not factored explicitly as we have ~~asked~~ used $\underset{\sim}{\rho}$ and $\underset{\sim}{r}$ instead of the unit vectors $\hat{\underset{\sim}{\rho}}$ and $\hat{\underset{\sim}{r}}$. So, the wave functions ψ_8, ψ_9, ψ_{10} and the D state wave functions are not the same as those of Derrick and Blatt. Derrick and Blatt have separated the angular and radial parts and further instead of using the $\underset{\sim}{\sigma}_i$ operators for building the D state wave functions they have used the Euler angle wave functions $D^{L}_{\mu\mu_L}(\alpha, \beta, \gamma)$ for the angular parts.

The Derrick and Blatt D state wave functions are:

$$\psi_8' = (\eta\bar{v} - \bar{\eta}v)\, q_3 \left(\frac{(10)^{1/2}}{4\pi} D^2_{0,M}(\alpha, \beta, \gamma) \right) \quad (3.53)$$

$$\psi_9' = (\eta\bar{v} - \bar{\eta}v)\, q_3 \left(\frac{-(5)^{1/2}}{4\pi} \right) \left[D^2_{2M}(\alpha, \beta, \gamma) + D^2_{-2,M}(\alpha, \beta, \gamma) \right] \quad (3.54)$$

$$\psi_{10}' = (\eta v + \bar{\eta}\bar{v})\, q_3 \left(\frac{-i5^{1/2}}{4\pi} \right) \left[D^2_{2,M}(\alpha, \beta, \gamma) + - D^2_{-2,M}(\alpha, \beta, \gamma) \right] \quad (3.55)$$

The ground state wave function of the three nucleon body system is a linear combination of these ten functions :

$$\psi^m = \sum_i a_i \psi_i^m \quad (3.56)$$

The coefficients a_i are shown to be real with the help of a time reversal argument by Sachs. It is of interest however to estimate which of these ten functions are likelty to be more important . It seems reasonable to assume that the predominant term in the ground state wave function will be symmetric under the interchange of space coordinates of any pair of nucleons. Two, rather compelling arguments are available to ~~subt~~antiate this view. First of all the Majorana potential will always favour such a state. Furthermore, the function with the smallest number of nodes is expected to have the lowest kinetic energy and a high degree of symmetry usually implies that the number of nodes is a minimum.

The only space symmetric state is $\psi_1^{m,t}$. So, we are led to the view that not only is the ground state primarily a $^2S_{1/2}$ state but of the three $^2S_{1/2}$ states it is predominantly $\psi_1^{m,t}$. The tensor interaction couples the $^4D_{1/2}$ states directly to the $^2S_{1/2}$ state $\psi_1^{m,t}$. So, some admixture of D functions is to be expected. Though in the Sach's classification the wave function $\psi_9^{m,t}$ seems to be more probable, in the classification of Derrick and Blatt all the three $^4D_{1/2}$ states are equally probable as the internal wave functions of all the three have the same mixed symmetry. The P states occur only in the second order as far as the tensor force is concerned. Finally the states which are likely to be least important are states with antisymmetric internal wave functions, that is, $\psi_3^{m,t}$ and $\psi_7^{m,t}$. The kinetic energy associated with these states is so large that no serious error is made by omitting them altogether.

7. Though the ground state wave functions for three nucleon systems can be a combination of all the possible states $\psi_i^{m,t}$, the binding energy calculation using appropriate two particle interactions show that the ground states of these nuclei are expected to be predominantly ($\approx 96\%$) $\psi_1^{m,t}$. The probability of the D states which occur due to their direct coupling

to the $\psi_1^{m,t}$ is expected to be $\approx 3.3\%$, the admixture of the other states are expected to be negligible. As a first approximation we shall therefore assume that three nucleon systems are sufficiently well described by $\psi_1^{m,t}$ alone.

$$\psi_1^{m,t} = \frac{1}{\sqrt{2}} (\varphi^m \bar{\eta}^t - \bar{\varphi}^m \eta^t) u \qquad (3.57)$$

The amplitudes for photoproduction of π - mesons from He^3 or H^3 in the impulse approximation can now be written as

$$\langle f | T | i \rangle = \langle f | \sum_{i=1,2,3} t_i \, e^{i \underset{\sim}{k} \cdot \underset{\sim}{r}_i} | i \rangle \qquad (3.58)$$

where the initials and final nuclear states have the form (3.57); a factor $e^{i \underset{\sim}{k} \cdot \underset{\sim}{R}}$ is attached to the final state to take into account the nuclear recoil; $\underset{\sim}{k}$ denotes the recoil momentum

$$\underset{\sim}{k} = \underset{\sim}{\nu} - \underset{\sim}{\mu} \qquad (3.59)$$

where $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$ are the momenta of the incident photon and the outgoing pion respectively. Since $|i\rangle$ and $|f\rangle$ are completely antisymmetric in the indices 1 , 2 , 3 we may write

$$\langle f | T | i \rangle = 3 \langle f | t_3 \, e^{i \underset{\sim}{k} \cdot \underset{\sim}{r}_3} | i \rangle \qquad (3.60)$$

$$= \frac{3}{2} F_3 \langle \varphi^{m'} \bar{\eta}^{t'} - \bar{\varphi}^{m'} \eta^{t'} | t_3 | \varphi^m \bar{\eta}^t - \bar{\varphi}^m \eta^t \rangle \tag{3.61}$$

where F_3 is the radial integral

$$F_3 = \int u^2 e^{i \underset{\sim}{k} \cdot (\underset{\sim}{r}_3 - \underset{\sim}{R})} d^3\underset{\sim}{r}_1 \, d^3\underset{\sim}{r}_2 \, d^3\underset{\sim}{r}_3 \tag{3.62}$$

The amplitude t for photoproduction of charged pions has the form as we have seen in chapter 1 of Part 1,

$$t = t_{p,n} \tau^{\mp}$$

where t_p and t_n denote the photoproduction amplitudes for the processes (3.5) and (3.6) respectively. The isospin matrix elements between the various states η^t and $\bar{\eta}^t$ are calculated to give

$$\langle \bar{\eta}^{t'} | \bar{\eta}^t \rangle = \langle \eta^{t'} | \eta^t \rangle = \delta_{t,t'}$$

$$\langle \bar{\eta}^{t'} | \eta^t \rangle = 0$$

$$\langle \bar{\eta}^{t'} | \tau_z(3) | \eta^t \rangle = 0$$

$$\langle \bar{\eta}^{t'} | \tau_z(3) | \bar{\eta}^t \rangle = -\frac{2}{3} t \, \delta_{t,t'}$$

$$\langle \eta^{t'} | \tau_z(3) | \eta^t \rangle = 2t \, \delta_{t,t'}$$

$$\langle \bar{\eta}^{t'} | \tau^{\pm}(3) | \eta^{t} \rangle = 0$$

$$\langle \bar{\eta}^{t'} | \tau^{\pm}(3) | \bar{\eta}^{t} \rangle = -\frac{1}{3}\, \delta_{t', t\pm 1}$$

$$\langle \eta^{t'} | \tau^{\pm}(3) | \eta^{t} \rangle = \delta_{t', t\pm 1} \qquad (3.63)$$

so that we can write (3.60) as

$$\langle f | T | i \rangle = 3 \Big[\langle \bar{\varphi}^{m'} | t(3) | \bar{\varphi}^{m} \rangle + \\ - \frac{1}{3} \langle \bar{\varphi}^{m'} | t(3) | \varphi^{m} \rangle \Big] F_3 \qquad (3.64)$$

where t denotes t_p or t_n respectively for photo production of π^+ from He^3 or π^- photoproduction from H^3.

8. In the case of neutral pion production, the single-nucleon amplitude t has the form

$$t = \frac{1}{2} \left[(t_p^0 + t_n^0) + (t_p^0 - t_n^0) \tau_z \right] \qquad (3.65)$$

where $t^0_{p,n}$ denotes the π^0 photoproduction amplitude from proton and neutron respectively, so that by using the matrix elements (3.63) we have

$$\langle f|T|i\rangle = \frac{3}{2}F_3\Big\{\langle\varphi^{m'}|\tfrac{1}{3}(t_p^0(3) + 2t_n^0(3))|\varphi^m\rangle +$$
$$+\langle\bar{\varphi}^{m'}|\ t_p^0(3)\ |\bar{\varphi}^m\rangle\Big\} \qquad (3.66)$$

for neutral pion production from He^3, and

$$\langle f|T|i\rangle = \frac{3}{2}F_3\Big\{\langle\varphi^{m'}|\tfrac{1}{3}(t_n^0(3) + 2t_p^0(3))|\varphi^m\rangle +$$
$$+\langle\bar{\varphi}^{m'}|\ t_n^0(3)\ |\bar{\varphi}^m\rangle\Big\} \qquad (3.67)$$

for π^0 production from H^3.

9. The spin matrix elements occurring in equations (3.64) to (3.67) can be calculated by writing the operators in the form

$$t(3) = i\underset{\sim}{\sigma}_3\cdot\underset{\sim}{K} + L \qquad (3.68)$$

and again using the following relations :

$$\langle\bar{\varphi}^{m'}|\bar{\varphi}^m\rangle = \langle\varphi^{m'}|\varphi^m\rangle = \delta_{m,m'}$$

$$\langle\bar{\varphi}^{m'}|\varphi^m\rangle = 0$$
$$\langle\bar{\varphi}^{m'}|\sigma_z(3)|\varphi^m\rangle = 0$$
$$\langle\bar{\varphi}^{m'}|\sigma_z(3)|\bar{\varphi}^m\rangle = -\frac{2}{3}m\,\delta_{m,m'}$$

$$\langle\varphi^{m'}|\sigma_z(3)|\varphi^m\rangle = 2m\,\delta_{m,m'}$$

$$\langle \bar{\varphi}^{m'} | \sigma^{\pm}(3) | \varphi^{m} \rangle = 0$$

$$\langle \bar{\varphi}^{m'} | \sigma^{\pm}(3) | \bar{\varphi}^{m} \rangle = -\frac{1}{3} \delta_{m', m\pm 1}$$

$$\langle \varphi^{m'} | \sigma^{\pm}(3) | \varphi^{m} \rangle = \delta_{m', m\pm 1} \tag{3.69}$$

and after summing over the final spin states and averaging over the initial spin states

$$\frac{1}{2} \sum_{f} \overline{\sum_{i}} | \langle f | T | i \rangle |^2 = \frac{1}{2} |F_3|^2 (\underset{\sim}{K} \cdot \underset{\sim}{K}^* + LL^*) \tag{3.70}$$

where $\underset{\sim}{K}$ and L refer to the spin-dependent and spin-independent amplitudes for (3.5) and (3.6) for the charged pion production with He^3 and H^3 targets respectively and

$$\frac{1}{2} \sum_{Spins} \sum | \langle f | T | i \rangle |^2 = \frac{1}{2} |F_3|^2 (\underset{\sim}{K} \cdot \underset{\sim}{K}^* + LL^*) \tag{3.71}$$

for neutral pion photoproduction, where $\underset{\sim}{K}$ refers to the spin-dependent part of the amplitude for

$$\gamma + p \rightarrow p + \pi^{\circ} \tag{3.72}$$

and

$$\gamma + n \rightarrow n + \pi^{\circ} \tag{3.73}$$

with H^3 and He^3 targets respectively while L for He^3 target is given by

$$L = 2L_p + L_n \tag{3.74}$$

and for H^3 target by

$$L = 2L_n + L_p \tag{3.75}$$

10. It is worth observing that the photoproduction cross-sections for the three-nucleon targets take such particularly interesting forms. The combination $\frac{1}{2}(\underset{\sim}{K}\cdot\underset{\sim}{K}^* + LL^*)$ reflects the fact that the targets are spin ½ systems and the structure factor $|F_3|^2$ takes into account the momentum distributions in the initial and final states characteristic of the bound system. Since for the photoproduction of charged pions only the protons in He^3 or the neutron in H^3 can contribute, the respective $\underset{\sim}{K}$ and L for the processes are $\underset{\sim}{K}_p$, L_p and $\underset{\sim}{K}_n$, L_n while all the three nucleons can participate in the π° photoproduction. But we can picture the three-nucleon systems as two particles with the same charge forming a singlet state to which the third particle is coupled. This means that the like-charged particles cannot contribute to the spin-dependent part. Therefore we find $\underset{\sim}{K}$ given by $\underset{\sim}{K}_n$ and $\underset{\sim}{K}_p$ for $\gamma + He^3 \rightarrow He^3 + \pi^\circ$, and $\gamma + H^3 \rightarrow H^3 + \pi^\circ$ respectively, while the spin-independent part L is given by equations (3.74) and

(3.75) respectively, exhibiting the participation of all the three nucleons.

11 9. The differential cross section for the photoproduction process is given by

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \frac{\mu \mu_0}{2} \sum_{\underset{\sim}{\epsilon}} \sum\sum_{spins} |\langle f | T | i \rangle|^2 \qquad (3.76)$$

assuming no energy transfer to the targets. Here $\underset{\sim}{\epsilon}$ denotes the incident photon polarisation and the cross section (3.76) refers to initially unpolarised photons. Using for the different $\underset{\sim}{K}$ and L the appropriate expressions given by Chew et al[1] and averaging over photon polarisations, the following explicit expressions are obtained :

(i) $\gamma + He^3 \longrightarrow H^3 + \pi^+$,

$$\frac{d\sigma}{d\Omega} = \frac{\mu e^2 f^2}{\nu_0} |F_3|^2 |Q_1|^2 \qquad (3.77)$$

where $|Q_1|^2$ is given by the expression (1.49)

(ii) $\gamma + H^3 \longrightarrow He^3 + \pi^-$,

$$\frac{d\sigma}{d\Omega} = \frac{\mu e^2 f^2}{\nu_0} |F_3|^2 |Q_1|^2 \qquad (3.78)$$

where also $|Q_1|^2$ is given by the expression (1.53)

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1) G.F. Chew et al. Phys. Rev. 106, (1957) 1345.

(iii) $\gamma + He^3 \longrightarrow He^3 + \pi^0$,

$$\frac{d\sigma}{d\Omega} = \frac{\mu e^2 f^2}{2\nu_0} |F_3|^2 \left\{ \frac{8\lambda^2\nu^2}{9\mu^4} \sin^2\delta_{33} \left(1 + \frac{35}{2}\sin^2\theta\right) + \right.$$

$$- \frac{4\lambda\alpha}{3\mu^3} \sin\delta_{33}\cos\delta_{33}\left[\nu^2\mu^2(1+\cos^2\theta) - 2\mu_0^2\mu\nu\cos\theta\right] +$$

$$\left. + \alpha^2\left[2\mu_0^2(\mu_0^2 - 2\mu\nu\cos\theta) + \nu^2\mu^2(1+\cos^2\theta)\right] \right\} \quad (3.79)$$

(iv) $\gamma + H^3 \longrightarrow H^3 + \pi^0$,

$$\frac{d\sigma}{d\Omega} = \frac{\mu e^2 f^2}{2\nu_0} |F_3|^2 \left\{ \frac{8\lambda^2\nu^2}{9\mu^4} \sin^2\delta_{33} \left(1 + \frac{35}{2}\sin^2\theta\right) + \right.$$

$$+ \frac{4\lambda\alpha}{3\mu^3} \sin\delta_{33}\cos\delta_{33}\left[\nu^2\mu^2(1+\cos^2\theta) - 2\mu_0^2\mu\nu\cos\theta\right] +$$

$$+ \alpha^2\left[2\mu_0^2(\mu_0^2 - 2\mu\nu\cos\theta) + \nu^2\mu^2(1+\cos^2\theta)\right] +$$

$$\left. + \frac{\mu^4\sin^2\theta}{M^2\mu_0^2} - \frac{2\alpha\mu^2\mu_0\sin^2\theta}{M} \right\} \quad (3.80)$$

where ν_0 and μ_0 refer to the photon and meson energies respectively and θ is the angle between their momenta $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$. Other symbols are explained in the chapter 1 .

11. The form factors have been evaluated by Schiff[1)] for exponential Gaussian and Irving wave functions and by Griffy and Oakes[2)] for Irving-Gunn wave function. The "exponential" wave function

$$u = A(r_{12}\, r_{13}\, r_{23})^{-1/2} \exp\left[-\alpha(r_{12}^2 + r_{23}^2 + r_{31}^2)^{1/2}\right] \qquad (3.81)$$

with $\qquad A^2 = \dfrac{\alpha^3}{2\pi^2}$

would be more plausible physically if it did not contain the reciprocal square root as a factor. It would still be possible to evaluate the needed integrals analytically if this factor were to be omitted, but the labour required would be much greater than with (3.81). The "Gaussian" wave function

$$u = A \exp\left[-\tfrac{1}{2}\alpha^2(r_{12}^2 + r_{13}^2 + r_{23}^2)\right] \qquad (3.82)$$

with $\qquad A^2 = \dfrac{3^{3/2}\alpha^6}{\pi^3}$

is extremely tractable analytically, but its rapid fall-off for large internucleon distances makes it rather implausible physically. The "Irving" wave function[3)]

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1) L.I.Schiff, Phys. Rev. **133** , B 802 (1964)
2) T.A.Griffy and R.J.Oakes, Phys. Rev. **135** , B 1161 (1964)
3) J. Irving, Phil. Mag. **42** , 338 (1951)

$$u = A \exp\left[-\tfrac{1}{2}\alpha(r_{12}^2 + r_{23}^2 + r_{13}^2)^{1/2}\right] \tag{3.83}$$

with
$$A^2 = \frac{3^{1/2}\alpha^6}{120\,\pi^3}$$

is not too difficult to deal ~~xx~~ with analytically and has a high degree of physical plausibility.

The Irving-Gunn[1] wave function is given by

$$u = A(r_{12}^2 + r_{23}^2 + r_{13}^2)^{-1/2} \exp\left[-\frac{\alpha}{2}(r_{12}^2 + r_{23}^2 + r_{13}^2)^{1/2}\right] \tag{3.84}$$

with
$$A^2 = \frac{3^{1/2}\alpha^4}{2\,\pi^3}$$

which behaves as the "Hulthen" function if one of the particle is taken far away.

Analysis of the three body electric charge form factors and magnetic form factors by Schiff show a definite preference for the Gaussian and Irving forms of wave functions over the modified exponential wave function and a slight preference for the Irving over the Gaussian form. Irving-Gunn wave function is found to give a good fit for the analysis of the photo-disintegration of He^3.

Let us first use Gaussian wave function to analyse the photoproduction cross-sections. However the results for the improved wave functions will be given in the later chapters.

13. For the Gaussian form of u the size parameter α has been found by Schiff from the analysis of charge form

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1) J.C.Gunn and J.Irving, Phil. Mag. **42**, (1951) 1353

factors. The value obtained for α, $\alpha = 0.384\ fm^{-1}$, ~~which~~ is in good agreement with the Coulomb energy of He^3. Following Schiff the form factor can be evaluated to give

$$F_3 = \exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (3.84')$$

The numerical estimates have been obtained for incident photon energy of two pion mass units and the results are presented in the figures (3.1), (3.2), (3.3) and (3.4). The curves (a) were obtained using from nucleon magnetic moments : $\mu_p = 2.793$ n.m. and $\mu_n = -1.913$ n.m. while the curves (b) were obtained with the phenomenological values : $\mu_n = -2.127$ n.m in He^3 and $\mu_p = 2.979$ n.m. in H^3. In Fig. (3.1) the curve (a) corresponds to the differential cross section for the process :

$$\gamma + He^3 \rightarrow H^3 + \pi^+ \qquad \text{at} \quad E_\gamma = 280\ \text{MeV}$$

with unquenched magnetic moments whereas curve (b) corresponds to quenched magnetic moments.

In fig. (3.2) the curve (a) corresponds to the differential cross sections for the process $\gamma + H^3 \rightarrow He^3 + \pi^-$ at $E_\gamma = 280$ MeV with unquenched magnetic moments whereas curve (b) corresponds to quenched magnetic moments.

In fig. (3.3) the curve (a) corresponds to the differential cross section for the process $\gamma + He^3 \rightarrow He^3 + \pi^0$ at $E_\gamma = 280$ MeV with unquenched magnetic moments whereas curve (b) corresponds to quenched magnetic moments.

In fig. (3.4) the curve (a) corresponds to the differential cross-section for the process $\gamma + H^3 \rightarrow H^3 + \pi^0$

at $E_\gamma = 280$ MeV whereas curve (b) corresponds to quenched magentic moments. As the form factor is a monotonically decreasing function of θ the cross sections decrease steadily after reaching maximums in the neighbourhood of 45°. In the case of neutral pions the cross-section is dominated by the first term $(1+\frac{35}{2}\sin^2\theta)$ the other terms being of the order $1/M$ or higher. Consequently we find the differential cross sections for $\gamma + He^3 \rightarrow He^3 + \pi^0$ and $\gamma + H^3 \rightarrow H^3 + \pi^0$ show an increase with increasing θ upto $\theta = 45^\circ$. This tendency is however off-set by the steeper decrease due to the form factor which results in the cross-sections falling off at larger angles. Similar results are true for charged pion photoproduction cross-sections also.

14. In the case of charged pion photoproductions the differential cross sections are not much sensitive to the changes in the values of the nucleon magnetic moments. So we can use the free nucleon magnetic moments themselves in the expressions for the differential cross sections as a good approximation. Further, since the probability of admixture of other states is small we can assume that the dominant S state alone represents the three nucleon systems. Under this assumption it is interesting to compare the differential cross-section for $\gamma + He^3 \rightarrow H^3 + \pi^+$ with the differential cross-section for $\gamma + p \rightarrow n + \pi^+$ on the one hand and the differential cross-section for $\gamma + H^3 \rightarrow He^3 + \pi^-$

with the differential cross-section for $\gamma + n \rightarrow p + \pi^-$

It is easy to check to find the following equations to be true:

$$\frac{d\sigma_{He^3}}{d\Omega} = F_3(k^2)\,\frac{d\sigma_p}{d\Omega} \qquad (3.85)$$

$$\frac{d\sigma_{H^3}}{d\Omega} = F_3(k^2)\,\frac{d\sigma_n}{d\Omega} \qquad (3.86)$$

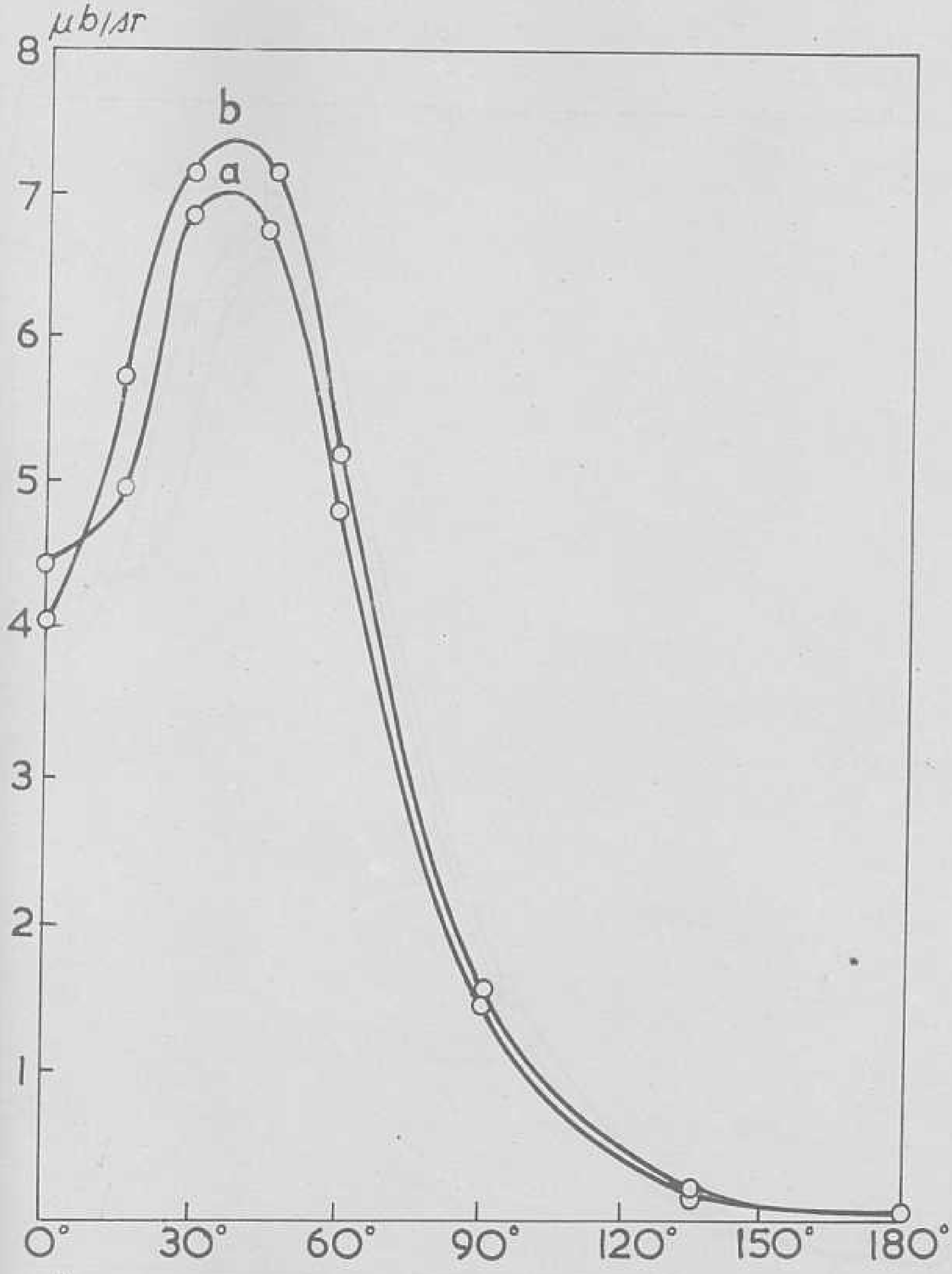
As the form factors can be calculated from the elastic electron scattering experiments equations (3.85) and (3.86) can be used to derive the differential cross-sections for the photoproduction of pions from single nucleons. This is very interesting in view of the nonavailability of free neutron targets. This is an alternative to Chew's extrapolation procedure[1]. Also the ratio of the differential cross-sections $d\sigma(\gamma + He^3 \rightarrow H^3 + \pi^+)$ and $d\sigma(\gamma + H^3 \rightarrow He^3 + \pi^-)$ represents essentially the π^+/π^- ratio for nucleons.

15. In the case of neutral pion production such a simple relation does not exist between the differential cross-sections of the photoproduction of neutral pions from the three ~~body~~ nucleon systems and the single nucleons. But the sensitivity of the differential cross-sections to the changes in the values of the nucleon magnetic moments is appreciable, from $\theta = 30^\circ$ to $\theta = 60^\circ$. The measurement of the cross-section in this region will certainly throw light on the magnetic moment structure of the three nucleon systems.

1) G.F. Chew Phys. Rev. 113 1640 (1959).

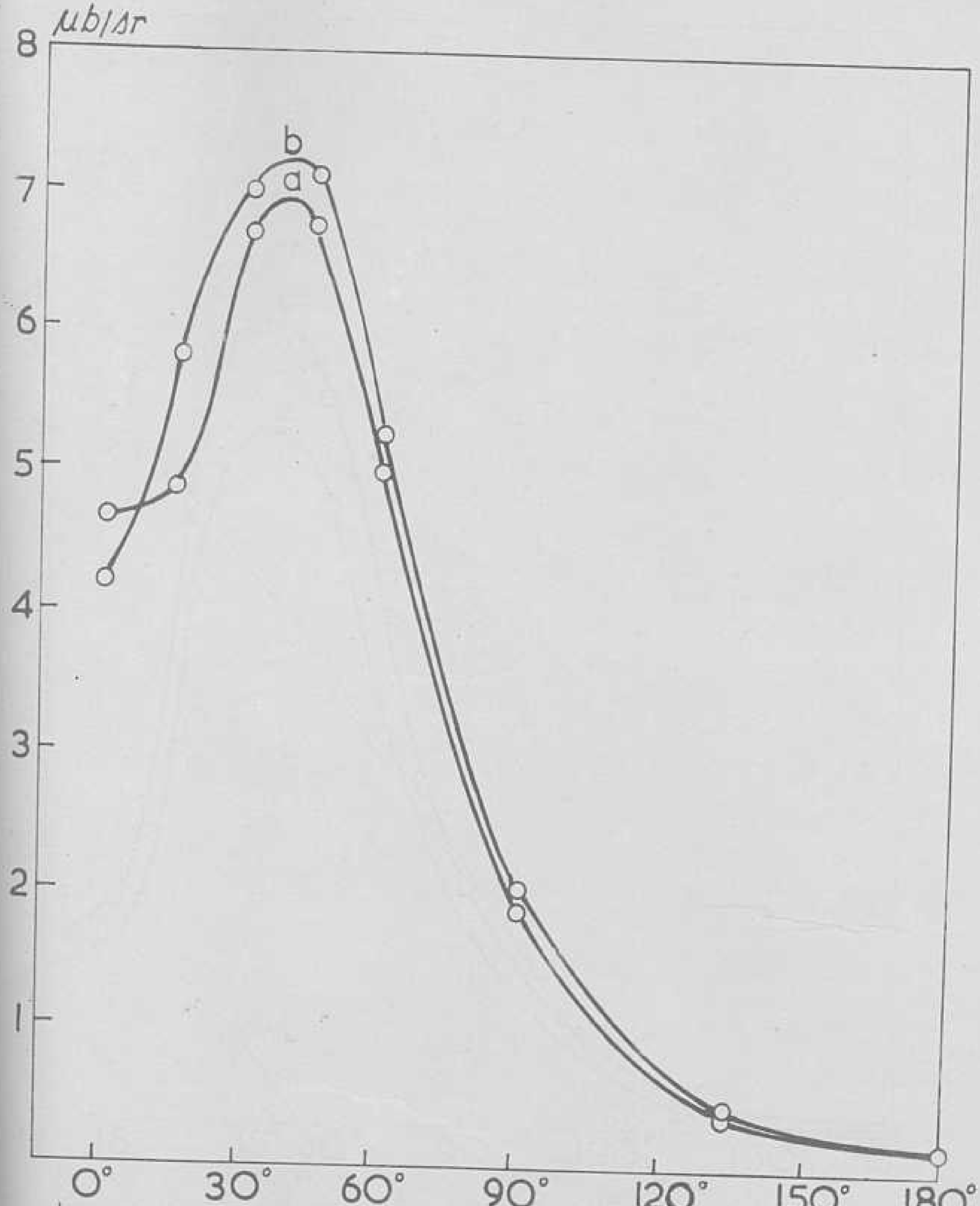
16. ~~21.~~ As the quenching of magnetic moments increases the differential cross-sections for neutral pion photo-production for all angles, it is interesting to estimate the total cross-sections with and without quenching and study whether there is any observable difference. In the following table we give the results for neutral pion photoproduction for He^3 and H^3 for the incoming photon energy $E_\gamma = 2$ pion mass units.

Process	σ_{total} in μb.	
	quenched	unquenched
$\gamma + He^3 \rightarrow He^3 + \pi^0$	109.5	92.92
$\gamma + H^3 \rightarrow H^3 + \pi^0$	109.1	96.18



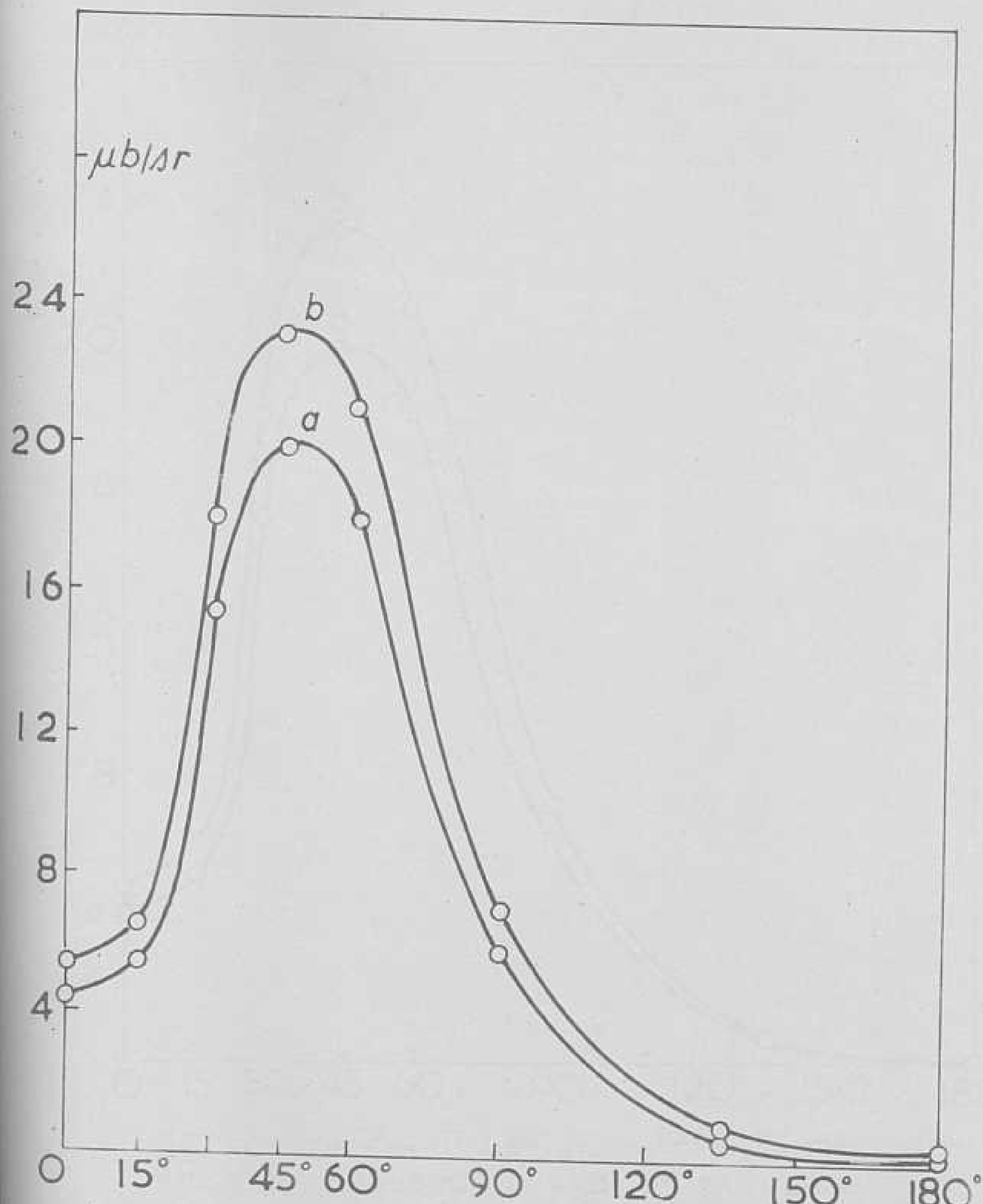
$\frac{d\sigma}{d\Omega}$ FOR $\gamma + H_e^3 \rightarrow H^3 + \pi^+$ AT $E_\gamma^{LAB} = 280$ mev WITH (a) UNQUENCHED AND (b) QUENCHED MAGNETIC MOMENTS

FIG. (3·1)



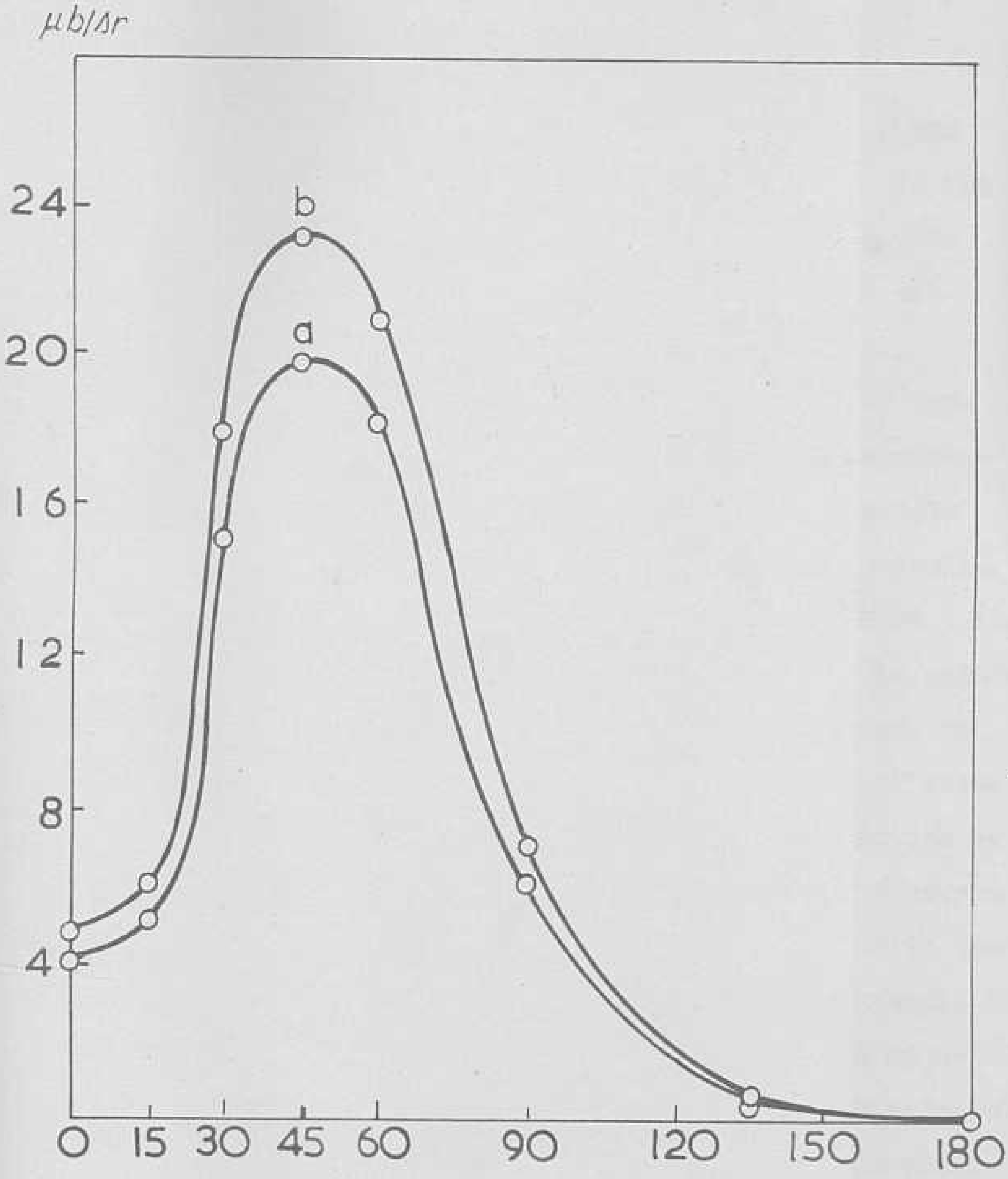
$\frac{d\sigma}{d\Omega}$ FOR $\gamma + H^3 \longrightarrow He^3 + \pi^-$ AT $E_\gamma^{LAB} = 280$ mev WITH (a) UNQUENCHED AND (b) QUENCHED MAGNETIC MOMENTS

FIG. (3·2)



$\frac{d\sigma}{d\Omega}$ FOR $\gamma + He^3 \rightarrow He^3 + \pi^\circ$ AT $E_\gamma^{LAB} = 280$ mev WITH (a) UNQUENCHED AND (b) QUENCHED MAGNETIC MOMENTS

FIG. (3-3)



$\frac{d\sigma}{d\Omega}$ FOR $\gamma + H^3 \rightarrow H^3 + \pi^0$ AT $E_\gamma^{LAB} = 280$ mev WITH
(a) UNQUENCHED AND (b) QUENCHED
MAGNETIC MOMENTS

FIG. (3·4)

CHAPTER. 4 *

1. The calculations for the binding energy of the three nucleon systems have shown that $\psi_1^{m,t}$ is the principal part of the ground state. In addition to this dominant S state, one can expect small admixtures of $\psi_2^{m,t}$, $\psi_8^{m,t}$, $\psi_9^{m,t}$ and $\psi_{10}^{m,t}$. It is important to know what should be the necessary percentages of admixture of these states. Different percentages of admixture are obtained for different potentials when an attempt is made to fit the binding energy calculations with the experimental value. Therefore one is at a loss to know the correct percentages of admixture. This is mainly due to the freedom in the adjustment of the parameters to arrive at a single value. Further binding energy calculations cannot select the form of the radial wave function as all calculations hinge upon a single value. One can have a better insight regarding the percentage of admixture in the ground state of the three nucleon system, if one attempts to make a calculation wherein the parameters should be adjusted to fit a continuous set of values rather than one single value. Such a possibility is wide open in cases such as the cross-sectionof a reaction as a function of the scattering angle

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* G. Ramachandran and K. Anantanarayanan, Nucl. Phys. 64 , 652 (1965)

or nuclear form factors (e.g. electric and magnetic form factors). A group of physicists at Stanford are trying to make a thorough investigation of elastic and inelastic electron scattering to achieve this goal.

The difficulty in dealing with these processes is the inability to tackle the magnetic moment problem of the three nucleon systems. The configuration-mixing could not explain the magnetic moments of H^3 and He^3. The precise way of dealing with the exchange magnetic moment effect - which is expected to explain the deviation of the magnetic moment of He^3 from that of 'n' and the magnetic moment of H^3 from that of 'p' - is not known. In view of this, the choice of a strong interaction process (atleast to the first order) like the elastic pion scattering by three nucleon systems may be a better choice for the analysis. The differential cross-sections for the pions scattered by He^3 and H^3 can be taken to analyse the three nucleon ground states.

2. Schiff[1)] provided a theoretical background for the experiments on the elastic scattering of high energy electrons from H^3 and He^3 conducted at Stanford by Collard et al[2)]. He used for the ground state of He^3 and H^3 a superposition of $\psi_1^{m,t}$ and $\psi_2^{m,t}$. The

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1) L. I. Schiff, Phys. Rev. 133, B 802 (1964)

2) H. Collard, R. Hofstadter, A. Johansson and L. I. Schiff, Phys. Rev. Letters, 11, 132, (1963)

available experimental data was explained with 4 % probability for $\psi_2^{m,t}$. This is against the spirit of the conclusion from the binding energy calculations which set the upper limit of the probability for $\psi_2^{m,t}$ (known as S' state) to be 1 % . ~~For~~ An independent calculation of the S' state probability from the pion scattering experiments will be of course very useful, for reasons explained in the previous section. But no experiment seems to have been performed for this process. Yet anticipating such experiments a calculation has been made which we present in the next section. It is found that the differential cross section for π^+ scattered by He^3 or π^- scattered by H^3 at 90° is very sensitive to the S' state probability. It is suggested that this can be used to measure the S' state probability $P_{S'}$.

3. The amplitude for the scattering of pions from He^3 or H^3 can be written in the impulse approximation as

$$\langle f | T | i \rangle = \langle f | \sum_{i=1}^{3} t_i \, e^{i \underset{\sim}{k} \cdot \underset{\sim}{r}_i} | i \rangle \qquad (4.1)$$

where $\underset{\sim}{r}_i$ denotes the positron coordinates of the nucleons and

$$\underset{\sim}{k} = \underset{\sim}{q}_2 - \underset{\sim}{q}_1 \qquad (4.2)$$

is the momentum transferred to the target, $\underset{\sim}{q}_1$ and $\underset{\sim}{q}_2$ being the initial and final momenta of the pion. Since $|i\rangle$ and $|f\rangle$ are completely antisymmetric in the labels 1 , 2 , 3 we may write

$$\langle f | T | i \rangle = 3 \langle f | t_3 e^{i \underset{\sim}{k} \cdot \underset{\sim}{r}_3} | i \rangle \qquad (4.3)$$

The operators t have the following form in the isospin space

$$t = \frac{1}{2}(t_p + t_n) + \frac{1}{2}(t_p - t_n)\tau_z \qquad (4.4)$$

where t_p and t_n have the following structure (considering, to be precise, π^+ scattering) :

$$t_p = \frac{-2\pi}{\omega q^3}\left[2\underset{\sim}{q}_2 \cdot \underset{\sim}{q}_1 - i\underset{\sim}{\sigma}\cdot(\underset{\sim}{q}_2 \times \underset{\sim}{q}_1)\right] e^{i\delta_{33}} \sin\delta_{33} \quad (4.5)$$

$$t_n = \frac{-2\pi}{3\omega q^3}\left[2\underset{\sim}{q}_2 \cdot \underset{\sim}{q}_1 - i\underset{\sim}{\sigma}\cdot(\underset{\sim}{q}_2 \times \underset{\sim}{q}_1)\right] e^{i\delta_{33}} \sin\delta_{33} \quad (4.6)$$

where $$q = |q_1| = |q_2| ,$$

and ω denotes the pion energy.

Here we have taken into account only the dominant $(3/2, 3/2)$ phase shifts δ_{33}, as the other phase shifts δ_{13} , δ_{11} and δ_3 are negligible compared to δ_{33} in the energy region under consideration. In the case of π^- scattering t_p and t_n interchange their roles we shall further assume that the Coulomb effects are negligible. If we take~~s~~ an admixture of S' state $\psi_2^{m,t}$ alone along with the dominant S state $\psi_1^{m,t}$ the initial state has the following form

$$|i\rangle = \psi_1^{m,t} + \psi_2^{m,t}$$

$$= \frac{1}{\sqrt{2}} \{ (\bar{\varphi}^m \eta^t - \varphi^m \bar{\eta}^t) u \} +$$

$$= \frac{1}{\sqrt{2}} [(\bar{\varphi}^m \eta^t + \varphi^m \bar{\eta}^t) \bar{v} + (\bar{\varphi}^m \bar{\eta}^t - \varphi^m \eta^t) v] \quad (4.7)$$

whereas the final state has the form

$$|f\rangle = \exp(i \underset{\sim}{k} \cdot \underset{\sim}{R}) (\psi_1^{m,t} + \psi_2^{m,t}) \quad (4.8)$$

where $\exp(i\underset{\sim}{k}\cdot\underset{\sim}{R})$ is introduced to take into account the recoil of the nucleus. The wave functions u , v and $\bar{v}$ are normalised as follows:

$$\int u^2 \, d^3 \underset{\sim}{r}_i = 1 \quad (4.9)$$

and

$$P_{S'} = \int (v^2 + \bar{v}^2) \, d^3 \underset{\sim}{r}_i$$

$$= 2 \int [g^2(31,2) - g(31,2) g(23,1)] \, d^3 \underset{\sim}{r}_i \quad (4.10)$$

Evaluating the isospin matrix elements using the equations (3.63) and neglecting $S' \to S'$ transitions as the matrix element is proportional to P and taking into account only $S \to S'$ and $S' \to S$ transition (the $S \to S'$ and $S' \to S$ transition matrix elements are proportional to $\sqrt{P}$) we obtain

$$\langle f | T | i \rangle = \frac{3}{4} \Big[\langle \bar{\varphi}^{m'} | (t_p + t_n) + 2(t_p - t_n) | \bar{\varphi}^{m} \rangle (F_S - F_{S'}) +$$

$$+ \langle \varphi^{m'} | (t_p + t_n) - \tfrac{2}{3}(t_p - t_n) | \varphi^{m} \rangle (F_S + F_{S'}) \Big] \quad (4.11)$$

where $t = \pm 1/2$ for He^3 or H^3 respectively, and F_S and $F_{S'}$ denote

$$F_S = \int u^* u \exp[i \underset{\sim}{k} \cdot (\underset{\sim}{r}_3 - \underset{\sim}{R})] \, d^3 \underset{\sim}{r}_i \quad (4.12)$$

For

$$F_{S'} = -\int (\bar{v}^* u + u^* \bar{v}) \exp[i \underset{\sim}{k} \cdot (\underset{\sim}{r}_3 - \underset{\sim}{R})] \, d^3 \underset{\sim}{r}_i$$

$$= -2 \int v u \exp[i \underset{\sim}{k} \cdot (\underset{\sim}{r}_3 - \underset{\sim}{R})] \, d^3 \underset{\sim}{r}_i \quad (4.13)$$

Here v, $\bar{v}$, u are all assumed to be real. This assumption can be proved from time reversal invariance.[1] Expressing t_p, t_n for convenience in the form

$$t_{p,n} = i \underset{\sim}{\sigma} \cdot \underset{\sim}{K}_{p,n} + L_{p,n} \quad (4.14)$$

1) R.G. Sachs, 'Nuclear theory', (Addison-Wesley Publishing Co., Inc., Cambridge, Massachusetts, 1953). pp (353-357)

and using the results (3.69) of the previous chapter we can evaluate the spin matrix elements in (4.11) to obtain finally to obtain

$$\frac{1}{2}\sum_{\text{Spins}}\sum |\langle f|T|i\rangle|^2 = (\underset{\sim}{K}\cdot\underset{\sim}{K}^* + LL^*) \tag{4.15}$$

where, in the He^3 targets

$$\underset{\sim}{K} = F_s\underset{\sim}{K}_n + F_{s'}(\underset{\sim}{K}_p + \underset{\sim}{K}_n), \tag{4.16}$$

$$L = F_s(2L_p + L_n) + F_{s'}(L_n - L_p), \tag{4.17}$$

and in the case of H^3 targets

$$\underset{\sim}{K} = F_s\underset{\sim}{K}_p + F_{s'}(\underset{\sim}{K}_p + \underset{\sim}{K}_n), \tag{4.18}$$

$$L = F_s(2L_n + L_p) + F_{s'}(L_p - L_n). \tag{4.19}$$

Choosing the Gaussian type of radial wave functions we have

$$u = A\exp\left[-\frac{\alpha^2}{2}(r_{12}^2 + r_{23}^2 + r_{31}^2)\right] \tag{4.20}$$

$$v = \frac{1}{\sqrt{2}}\left[g(23,1) - g(13,2)\right] \tag{4.21}$$

$$\bar{v} = \frac{1}{\sqrt{6}}\left[g(23,1) + g(13,2) - 2g(12,3)\right] \tag{4.22}$$

where $g(12,3)$ are functions of the form

$$g(12,3) = B \exp\left[-\frac{\alpha^2}{2}(r_{13}^2 + r_{23}^2) - \frac{\beta^2}{2} r_{12}^2\right] \qquad (4.23)$$

4. Integrals F_s and $F_{s'}$ can be evaluated by expressimng the internucleon distances in terms of the two vectors $\underset{\sim}{\rho}$ and $\underset{\sim}{r}$ defined in chapter . The integrals F_s and $F_{s'}$ after the coordinate transformation take the forms (without explicit mention of the Jacobians)

$$F_s = |A|^2 \int \exp\left[-\alpha^2\left(2\rho^2 + \frac{3r^2}{2}\right) - \frac{2}{3} i \underset{\sim}{k}\cdot\underset{\sim}{\rho}\right] d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r}_i \qquad (4.24)$$

and

$$F_{P_s'} = 2AB\left[F_1 + F_2 - 2F_3\right] \qquad (4.25)$$

where

$$F_1 = (6)^{-1/2} \int d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r} \exp\left[-\frac{2}{3} i \underset{\sim}{k}\cdot\underset{\sim}{\rho} + \right.$$
$$\left. - \alpha^2\left(\frac{3}{2}\rho^2 + \frac{11}{8} r^2 + \frac{1}{2}\underset{\sim}{\rho}\cdot\underset{\sim}{r}\right) - \frac{\beta^2}{2}\left(\rho^2 + \frac{r^2}{4} - \underset{\sim}{\rho}\cdot\underset{\sim}{r}\right)\right] \quad (4.26)$$

$$F_2 = (6)^{-1/2} \int d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r} \exp\left[-\frac{2}{3} i \underset{\sim}{k}\cdot\underset{\sim}{\rho} + \right.$$
$$\left. - \alpha^2\left(\frac{11}{8} r^2 + \frac{3}{2}\rho^2 - \frac{1}{2}\underset{\sim}{\rho}\cdot\underset{\sim}{r}\right) - \frac{\beta^2}{2}\left(\rho^2 + \frac{r^2}{4} + \underset{\sim}{\rho}\cdot\underset{\sim}{r}\right)\right] \quad (4.27)$$

and $$F_3 = (6)^{-1/2} \int d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r} \exp\left[-\frac{2}{3} i \underset{\sim}{k}\cdot\underset{\sim}{\rho} + \right.$$
$$\left. - \alpha^2(\rho^2 + r^2) - \frac{\beta^2}{2} r^2\right] \qquad (4.28)$$

It is then best to do all the integrations in terms of rectangular components of $\underset{\sim}{\rho}$ and $\underset{\sim}{r}$ 1). The most complicated integral that arises can be reduced to a product of terms of the form

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-ax^2 - by^2 - 2cxy - \tfrac{2}{3}ikx\right] dx\,dy \quad , \; ab > c^2 \qquad (4.29)$$

where x and y are corresponding rectangular components of $\underset{\sim}{\rho}$ and $\underset{\sim}{r}$. It is evaluated by rotating axes in the xy plane with the result :

$$\pi\,(ab-c^2)^{1/2} \exp\left[-\frac{k^2 b}{9(ab-c^2)}\right] \qquad (4.30)$$

The normalisation constants A and B are found from the following relations:

$$1 = A^2 \int \exp\left[-\alpha^2(2\rho^2 + r^2)\right] d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r} \qquad (4.31)$$

and

$$P_{S'} = 2B^2 \int d^3\underset{\sim}{\rho}\, d^3\underset{\sim}{r} \left\{ \exp\left[-\alpha^2(r_{13}^2 + r_{23}^2) - \beta^2 r_{12}^2\right] + \right.$$
$$\left. - \exp\left[-\frac{\alpha^2}{2}(2r_{23}^2 + r_{12}^2 + r_{13}^2) - \frac{\beta^2}{2}(r_{12}^2 + r_{13}^2)\right]\right\} \qquad (4.32)$$

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1) L.I.Schiff, Phys. Rev. 133 B , (1964) 802.

We expect B to be much smaller than A, and β to be very close to α. It turns out that if B and A have the same sign which is chosen positive for definiteness then β must be slightly less than α in order to have F_2 positive as is observed in the electron scattering analysis. Further if we define

$$\epsilon = \alpha - \beta \qquad (4.33)$$

then ~~the~~ to lowest order B and ϵ enter into the expressions for observable quantities only through their product. Thus we could as well choose B and A with opposite signs, and β slightly greater than α.

After integration we have the following results

$$F_s = \exp\left(-\frac{k^2}{18\alpha^2}\right) \quad , \quad A^2 = \frac{3^{3/2}\alpha^6}{\pi^3} \qquad (4.34)$$

and the expressions for $F_{s'}$ and $P_{s'}$ are

$$F_{s'} = +\frac{2}{3}\frac{\sqrt{6}\,\pi^3 AB}{\alpha^3(2\alpha^2+\beta^2)^{3/2}}\left\{\exp\left(-\frac{k^2}{18\alpha^2}\right) + \right.$$
$$\left. - \exp\left[-k^2\left(\frac{1}{72\alpha^2} + \frac{1}{8(2\alpha^2+\beta^2)}\right)\right]\right\} \qquad (4.35)$$

$$P_{s'} = 2\pi^3 B^2\left[\frac{1}{\alpha^3(\alpha^2+2\beta^2)^{3/2}} - \frac{8}{(\alpha^2+\beta^2)^{3/2}(5\alpha^2+\beta^2)^{3/2}}\right] \qquad (4.36)$$

For small ϵ equations (4.36) and (4.33) give

$$P_{s'}^{3} \simeq \frac{\pi^3 B^2 \epsilon^2}{3^{3/2}\alpha^8} \qquad (4.37)$$

Equation (4.35) may be similarly approximated if it is assumed that ϵ is small in comparison with α^3/k^2 as well as with α ; with the help of (4.37) it may be written

$$F_{s'} \simeq + \left(\frac{P_{s'}}{6}\right)^{1/2} \frac{k^2}{9\alpha^2} \exp\left(-\frac{k^2}{18\alpha^2}\right) \tag{4.38}$$

From electron scattering experimental data[1] one can arrive at the following value for α:

$$\alpha = 0.384 \text{ fm}^{-1} \tag{4.39}$$

5. The differential cross-section for scattering of mesons from H^3 and He^3 can now be finally written as

$$\frac{d\sigma_{H^3}}{d\Omega} = \frac{\sin^2\delta_{33}}{q^2}\left[\sin^2\theta\,|F_s + \tfrac{4}{3}F_{s'}|^2 + 4\cos^2\theta\,|\tfrac{5}{3}F_s + \tfrac{2}{3}F_{s'}|^2\right] \tag{4.40}$$

and

$$\frac{d\sigma_{He^3}}{d\Omega} = \frac{\sin^2\delta_{33}}{q^2}\left[\sin^2\theta\,|\tfrac{1}{3}F_s + \tfrac{4}{3}F_{s'}|^2 + 4\cos^2\theta\,|\tfrac{7}{3}F_s - \tfrac{2}{3}F_{s'}|^2\right] \tag{4.41}$$

Expressions (4.40) and (4.41) are evaluated numerically at incident pion energy corresponding to 2 pion mass units where our assumption that the scattering is represented sufficiently accurately by the δ_{33} phase shifts alone holds.[2]

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1) L.I.Schiff, Phys. Rev. **133** B , (1964) 802.
2) J.Hamilton and W.S.Woolcock, Revs. Mod. Phys. **35** , (1963) 737 .

The differential cross sections for π^+ scattered by He^3 and H^3 are given in the tables 1 and 2 below. The $d\sigma/d\Omega$ for π^+ scattered by He^3 is more sensitive to the s'- state admixture at 90^o than $d\sigma/d\Omega$ for π^+ scattered by H^3 at the same angle. Therefore, it is interesting to study the $d\sigma/d\Omega$ for π^+ scattered by He^3 or equivalently π^- scattered by H^3 in the neighbourhood of 90^o for different energies. Hence estimates have been made for $d\sigma_{He^3}/d\Omega$ at incident meson energy equal to two pion mass units and 2.5 pion mass units for the scattering angle $\theta = 80^o$, 85^o , 90^o , 95^o and 100^o , taking for $P_{s'}$ the values 0 % , 1 % , 2 % , 3 % and 4 %.

In fig. (4.1) we have plotted $d\sigma_{He^3}/d\Omega$ against the scattering angle θ for the incident pion energy equal to two pion mass units and for P_s = 0 % , 1 % , 2 % , 3 % , 4 %.

In fig. (4.2) we have plotted $d\sigma_{He^3}/d\Omega$ against the scattering angle θ for the incident from energy equal to 2.5 pion mass units and for P_s = 0 % , 1 % , 2 % , 3 % , 4 %.

By linear extrapolation of the phase shift δ_{33}[1] for incoming pion energy equal to three pion mass units, estimates are made for $d\sigma_{He^3}/d\Omega$ at 90^o . This analysis is not very dependable as the linear extrapolation of δ_{33} is very unlikely[1].

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1) H. A. Bethe and F. De Hoffmann "Mesons and Fields", Vol. II , Row, Peterson and Company, New York, page 124.

<u>Table I</u>

The differential cross-section for the reaction

$H^3 + \pi^+ \longrightarrow H_3 + \pi^+$ in mb/sr.

θ (degrees) \ $P_{S'}$	0 %	1 %	2 %	3 %	4 %
0	34.46	34.46	34.46	34.46	34.46
15	29.94	30.06	30.08	30.10	30.10
30	19.64	19.86	19.97	20.00	20.06
45	9.682	9.934	10.04	10.12	10.19
60	3.534	3.732	3.818	3.882	3.941
90	.3234	0.4078	0.4458	0.4792	.5020
135	.3970	.4504	.4874	.5160	.5276
180	.3752	.4388	.4678	.4774	.4842

Table II

The differential cross-section for the reaction

$He^3 + \pi^+ \rightarrow He^3 + \pi^+$ in mb/sr

θ (degrees) \ $P_{s'}$	0 %	1 %	2 %	3 %	4 %
0	67.56	67.56	67.56	67.56	67.56
15	58.36	58.26	58.24	58.16	58.16
30	37.46	37.22	37.10	37.04	36.98
45	17.51	17.24	17.14	17.07	16.99
60	4.76	4.728	4.722	4.714	4.708
90	.03804	.06736	.08336	.09652	.1086
135	.7154	.6554	.6324	.6148	.6000
180	.7356	.6598	.6296	.6070	.5886

However as the δ_{33} exhibits itself only as a factor $\sin^2\delta_{33}$ an estimate at ω = three pion mass units, will be useful to study the sensitivity of the $S \to S'$ contribution to the variation of energy. The result is given below :

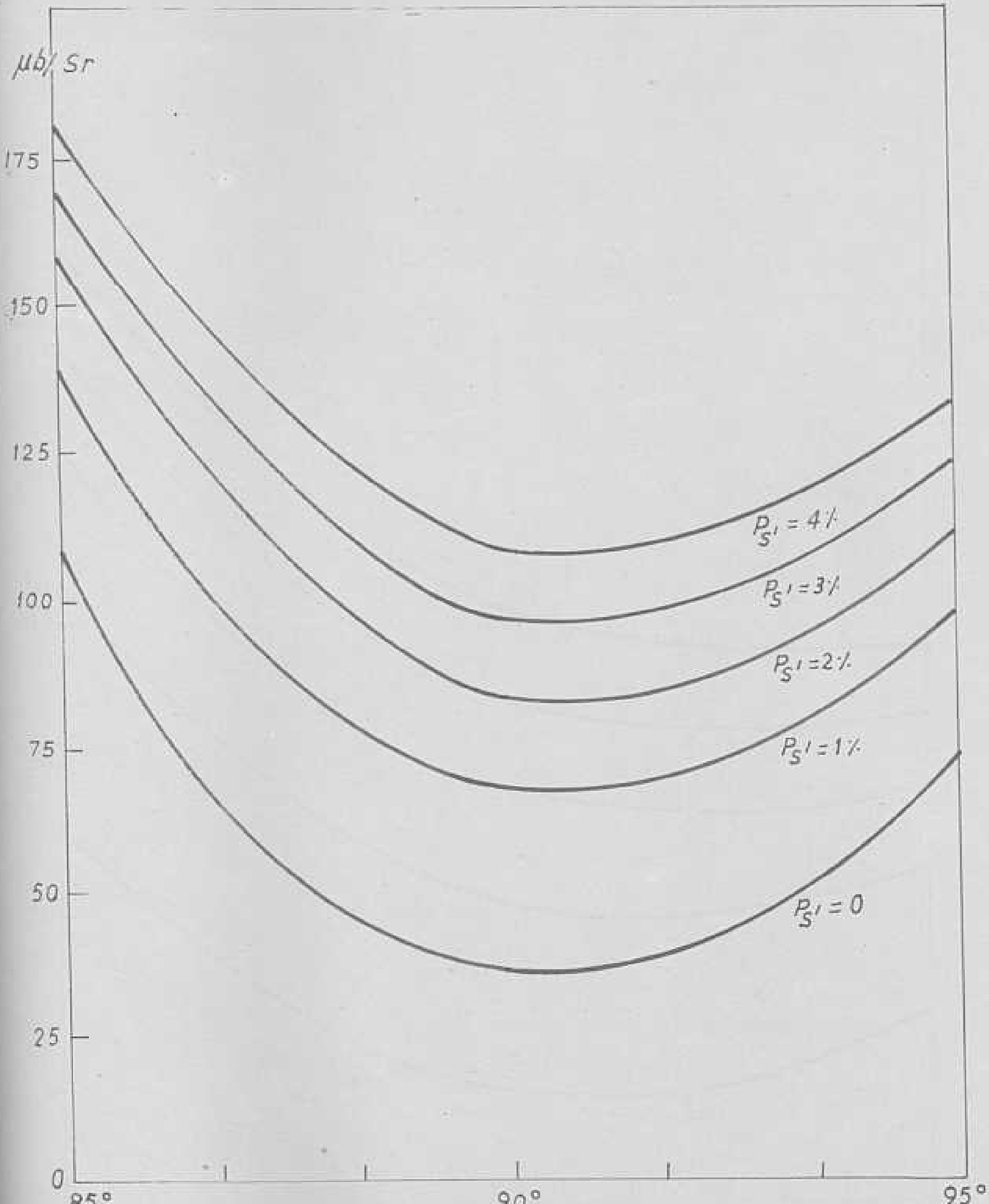
$P_{S'}$	0 %	1 %	2 %	3 %	4 %
$\frac{d\sigma_{He^3}}{d\Omega}$ in $\mu b/sr.$	0.39	1.54	2.24	2.86	3.45

6. We find that the sensitivity of the cross-section to $P_{S'}$ increases with energy. This is because $F_{S'}$ is proportional to ~~\$~~ k^2 (square of the three momentum transferred to the nucleus) . The relative sensitivity of $d\sigma_{He^3}/d\Omega$ to $P_{S'}$ for π^+ scattering compared to $d\sigma_{H^3}/d\Omega$ can be explained as follows: when we assume pure S state, as the like charged particles from a spin singlet , no contribution comes from the protonsin He^3 for the spin flip part of the elastic scattering processes and no contribution comes from the neutrons in H^3 . At 90° the contributions to the cross sections from the non-spin-flip part identically vanishes. As the amplitude for $\pi^+ + n \longrightarrow \pi^+ + n$ is one-third of the amplitude for $\pi^+ + p \longrightarrow \pi^+ + p$,

when only δ_{33} phase shifts are taken into account, at 90^{0} the contribution to ~~the~~ $d\sigma_{He^3}/d\Omega$ is $1/9$ th of ~~the~~ $d\sigma_{H^3}/d\Omega$. The contribution to the spin flip part from the S' state admixture being the same to both $d\sigma_{He^3}/d\Omega$ and $d\sigma_{H^3}/d\Omega$, this contribution compares very well with ($\frac{1}{9}$ th of the $d\sigma_{H^3}/d\Omega$) $d\sigma_{He^3}/d\Omega$ with pure S state than with $d\sigma_{H^3}/d\Omega$ with pure S state.

By charge symmetry π^- scattered by H^3 is equivalent to π^+ scattered by He^3 .

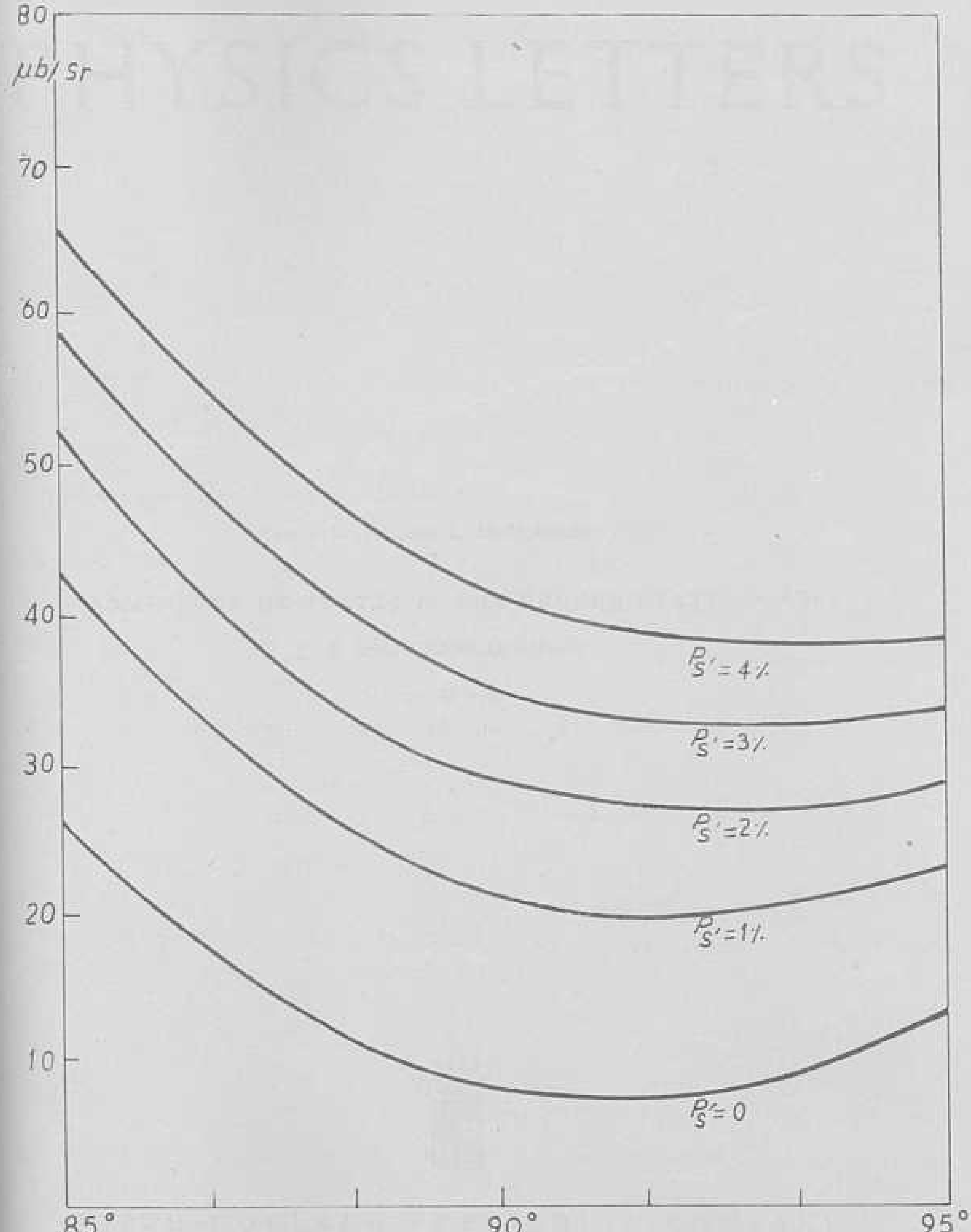
The inclusion of final state interactions may not affect the ratio of the differential cross-sections as much as it affects the differential cross section itself. Also due to the finiteness of the experimental resolution in θ, it may be easier to deal with this ratio. So we suggest measurement of the ratio of the differential cross-sections in the neighbourhood of 90^{0} to the differential cross-section in the neighbourhood of 0^{0} .



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ AT $\omega = 280$ mev

$P_{S'}$ = PERCENTAGE OF THE S'– STATE ADMIXTURE.

FIG.(4·1)



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ AT $\omega = 350$ mev

$P_{S'}$ = PERCENTAGE OF THE S′– STATE ADMIXTURE.

FIG. (4·2)

Chapter 5 *

The percentage of admixture of S' state in the ground state of He^3 and H^3 necessary to explain the electron scattering data is 4 %[1]. But the analysis[2] of the thermal neutron capture by deutron revealed that the value for $P_{S'}$ (S' - state probability) cannot be greater than 2 %. Also the study[3] of inelastic $e - He^3$ and $e - H^3$ scattering assuming that the ejected proton is counted in coincidence with the scattered electron, has revealed that 4 % admixture of S'- state brings in considerable discrepancy with the experimental data whereas a pure S - state gives a good fit. In the absence of S' state Gibson and Schiff[4] have shown that even the admixture of P- states and D - states cannot explain the data and have suggested the possibility of including $T = 3/2$ state admixture to the ground state of He^3 to explain away the discrepancy. An earlier

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* K. Anantanarayanan, Physics Letters, **18** , Number 3, (1966)

1) L.I.Schiff, Phys. Rev. **133** , B 802 (1964)
2) T.K.Radha and N.T.Meister, Phys. Rev., **136** B, 388 (1964)
3) T.A.Griffy and R.J.Oakes, Phys. Rev., **135** B , 1161 (1964)
4) B.F.Gibson and L.I.Schiff, Phys. Rev., **138** B , 26 (1965)

perturbation calculation by Griffy[1] using the Coulomb interaction between the two protons and Irving wave functions for He^3 indicated that the $T=3/2$ state admixture can be as large as 2 %.

We now show that the differential cross-section in a direction perpendicular to the incident beam is considerably decreased by a small admixture of $T=3/2$ state. If either $T=3/2$ state or the S' state alone (see Chapter 4) is present then the admixture can be found from the experimentally measured differential cross-section for π^+ scattered by He^3 at 90^0. On the other hand, if an admixture of both the states exists, we suggest the measurement of the differential cross section at 90^0 of the π^- scattered by H^3 (wherein $T=3/2$ state is assumed to be absent) which along with the measured differential cross-section of π^+ scattered by He^3 at 90^0 will provide the percentage of admixture of both the states.

2. Due to Coulomb effects the ground state of nuclei with more than one proton need not have T (isospin) as a good quantum number. Coulomb effects are important when we think of admixtures of the order of a few per cent, of states other than the dominant S state in He^3. For the Coulomb energy of He^3 is 0.76 Mev which is about 10 % of its total binding energy (7.72 Mev).

% 1) T.A.Griffy, Physics Letters, 11 , 155 (1964)

Thus we have to consider the states with $T = 3/2$ in addition to the states with $T = 1/2$. The iso-spin function with $T = 3/2$, ~~$T_z = 1/2$~~, $T_z = 1/2$ is fully symmetric and it is given by

$$\eta^{1/2}_{3/2}(123) = \frac{1}{\sqrt{3}} \left[\xi^{1/2}_{1/2}(1)\, \xi^{1/2}_{1/2}(2)\, \xi^{-1/2}_{1/2}(3) + \xi^{1/2}_{1/2}(1)\, \xi^{-1/2}_{1/2}(2)\, \xi^{1/2}_{1/2}(3) + \right.$$

$$\left. + \xi^{-1/2}_{1/2}(1)\, \xi^{1/2}_{1/2}(2)\, \xi^{1/2}_{1/2}(3) \right] \qquad (5.1)$$

Now the only S- state with $T = 3/2$, $J_z = 1/2$ and $J = 1/2$ is given by

$$\psi_{T=3/2} = (\bar{\varphi}\bar{v} - \varphi\bar{v})\, \eta^{1/2}_{3/2}(123) \qquad (5.2)$$

3. Let us take for the ground state of He^3, $\psi_{T=3/2}$ in addition to the dominant S state :

$$\psi_{He^3} = \frac{1}{\sqrt{2}}\, u\, [\varphi\bar{\eta} - \bar{\varphi}\eta] + [v\bar{\varphi} - \bar{v}\varphi]\, \eta^{1/2}_{3/2} \qquad (5.3)$$

Following the same procedure as in the chapter 4 we obtain for the differential cross-section of the π^+ elastically scattered by He^3 :

$$\frac{d\sigma_{He^3}}{d\Omega} = \frac{\sin^2\delta_{33}}{9q^2} \left[|F_s - 2F_{T=3/2}|^2 \sin^2\theta + \right.$$

$$\left. + 4\,|7F_s - 2F_{T=3/2}|^2 \cos^2\theta \right] \qquad (5.4)$$

where δ_{33} is the dominant $(3/2, 3/2)$ pion-nucleon phase shifts. (In the energy region chosen the other phase shifts

$\delta_{11}, \delta_{13}, \delta_{31}$ and δ_1 are negligible compared with δ_{33}) $\underset{\sim}{q}$ is the incident pion momentum, θ the scattering angle,

$$F_s = \int u^2 \exp(i \underset{\sim}{k} \cdot \underset{\sim}{r}_3) \, d^3 \underset{\sim}{r}_i \qquad (5.5)$$

$$F_{T=3/2} = 2 \int u v_1 \exp(i \underset{\sim}{k} \cdot \underset{\sim}{r}_3) \, d^3 \underset{\sim}{r}_i \qquad (5.6)$$

The s sign of $F_{T=3/2}$ is so chosen as to yield the correct change in electric change from factors as is known from experiments.

From (5.4) it is obvious that the differential cross-section is sensitive to the admixture of $T = 3/2$ state in the neighbourhood of $\theta = 90^\circ$ irrespective of the form of the radial wave function. Gaussian wave function gives the following expressions for the form factors (see chapter 4)

$$F_s = \exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (5.7)$$

$$F_{T=3/2} = \left(\frac{P_{T=3/2}}{6}\right)^{1/2} \frac{k^2}{9\alpha^2} \exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (5.8)$$

where $P_{T=3/2}$ is the percentage of admixture of $T=3/2$ state, $\alpha = 0.384 \, fm^{-1}$. In fig. (5.1) we present the results for the incoming meson energy $\omega = 2$ pion mass units. In fig. (5.2) we present the results for the incoming meson energy $\omega = 2.5$ pion mass units.

We find that the differential cross sections is very sensitive to $P_{T=3/2}$ at 90° and that the sensitivity increases with energy.

4. It is very interesting to compare the effects of S' state admixture with the effects of the $T=3/2$ state admixture. While the former increases the differential cross section section at 90° the latter decreases the same. In Table I we compare the effects of admixture of these states for the incoming meson energy $\omega = 2$ pion mass units. In Table II, we compare the effect of admixture of these states for the incoming meson energy $\omega = 2{\cdot}5$ pion mass units.

5. If we assume the charge independence of nuclear forces, then the $T=3/2$ state admixture can be present in the ground state of He^3 only due to the Coulomb force between two of its protons, whereas there will be no $T=3/2$ state admixture in H^3. Charge symmetry implies that the mirror nuclei He^3 and H^3 should contain equal amount of S' state admixture. (For, all the properties which are not electromagnetic in character are the same for both nuclei). So, when both the S' state and the $T=3/2$ state are present in He^3 along with the dominant S - state, the two parameters $P_{S'}$ and $P_{T=3/2}$ can be found from two experiments : π^+ scattering by He^3 and π^- scattering by H^3 -- the former involves both $P_{S'}$ and $P_{T=3/2}$ while the latter involves only $P_{S'}$.

The expressions for the differential cross sections for π^+ scattered by He^3 and for π^- scattered by H^3 are given below

$$\frac{d\sigma_{He^3}}{d\Omega} = \frac{\sin^2\delta_{33}}{9q^2}\left[\,|F_S - 2F_{T=3/2} + 4F_{S'}|^2 \sin^2\theta + 4\,|7F_S - 2F_{T=3/2} + 2F_{S'}|^2 \cos^2\theta\,\right] \qquad (5.9)$$

$$\frac{d\sigma_{H^3}}{d\Omega} = \frac{\sin^2\delta_{33}}{9q^2}\left[\,|F_S + 4F_{S'}|^2 \sin^2\theta + 4\,|7F_S + 2F_{S'}|^2 \cos^2\theta\,\right] \qquad (5.10)$$

where

$$F_{s'} = \left(\frac{P_{s'}}{6}\right)^{1/2} \frac{k^2}{9\alpha^2} \exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (5.11)$$

The inclusion of final state interactions may not affect the ratio of the differential cross-sections as much as it affects the differential cross-section itself. Also due to the finiteness of the experimental resolution it may be easier to deal with this ratio. So we suggest the simultaneous measurements of the ratios of the differential cross-section (5.9) and (5.10) in the neighbourhood of 90^0 to the differential cross sections in the neighbourhood of 0^0.

6. It is interesting to study the variation of the differential cross section for the π^+ scattered from He^3 in the neighbourhood of 90^0 as we vary the probability $P_{T=3/2}$.

In Fig. 5.1 the differential cross section for $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ at the incoming pion energy 280 Mev in the laboratory system is plotted for the following values of $P_{T=3/2}$:

$$P_{T=3/2} = 1\%,\ 2\%,\ 3\%,\ \cancel{4\%} \text{ and } 4\%$$

In Fig. 5.2 the differential cross section for $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ at the incoming pion energy 330 Mev in the laboratory system is plotted for the following values of

$$P_{T=3/2} = 1\%,\ 2\%,\ 3\% \text{ and } 4\%$$

($P_{T=3/2}$ is denoted as P_T in figure.)

7. Since the admixtures of both S' state and $T = 3/2$ state give the same change in the electric charge form factors, it is not possible to find out from the required change in the electric charge form factors to explain the experimental data, whether the admixture state is S' state or $T = 3/2$ state. But we can see that whereas S' - state admixture increases the differential cross section for π^+ scattering from He^3 appreciably in the neighbourhood of 90^0, the $T = 3/2$ state admixture decreases the same differential cross section appreciably in the neighbourhood of 90^0. So we can find from the experimentally measured differential cross section for π^+ scattered from He^3 around 90^0 whether the needed admixture is of S' state or of $T = 3/2$ state.

In Fig. (5.3) and (5.4) we plot the differential cross section for π^+ elastically scattered from He^3 at 280 Mev and 330 Mev respectively around $\theta = 90^0$ for three possibilities :

(1) Pure 'S' state i.e. $P_{T=3/2} = P_{S'} = 0$

(2) 'S' state with 1 % $T = 3/2$ state

i.e. $P_{T=3/2} = 1\%$, $P_{S'} = 0$.

(3) 'S'- state with 1 % S' state

i.e. $P_{S'} = 1\%$, $P_{T=3/2} = 0$.

We find that even ~~the~~ an admixture as small as 1 % can be easily detected.

Table I

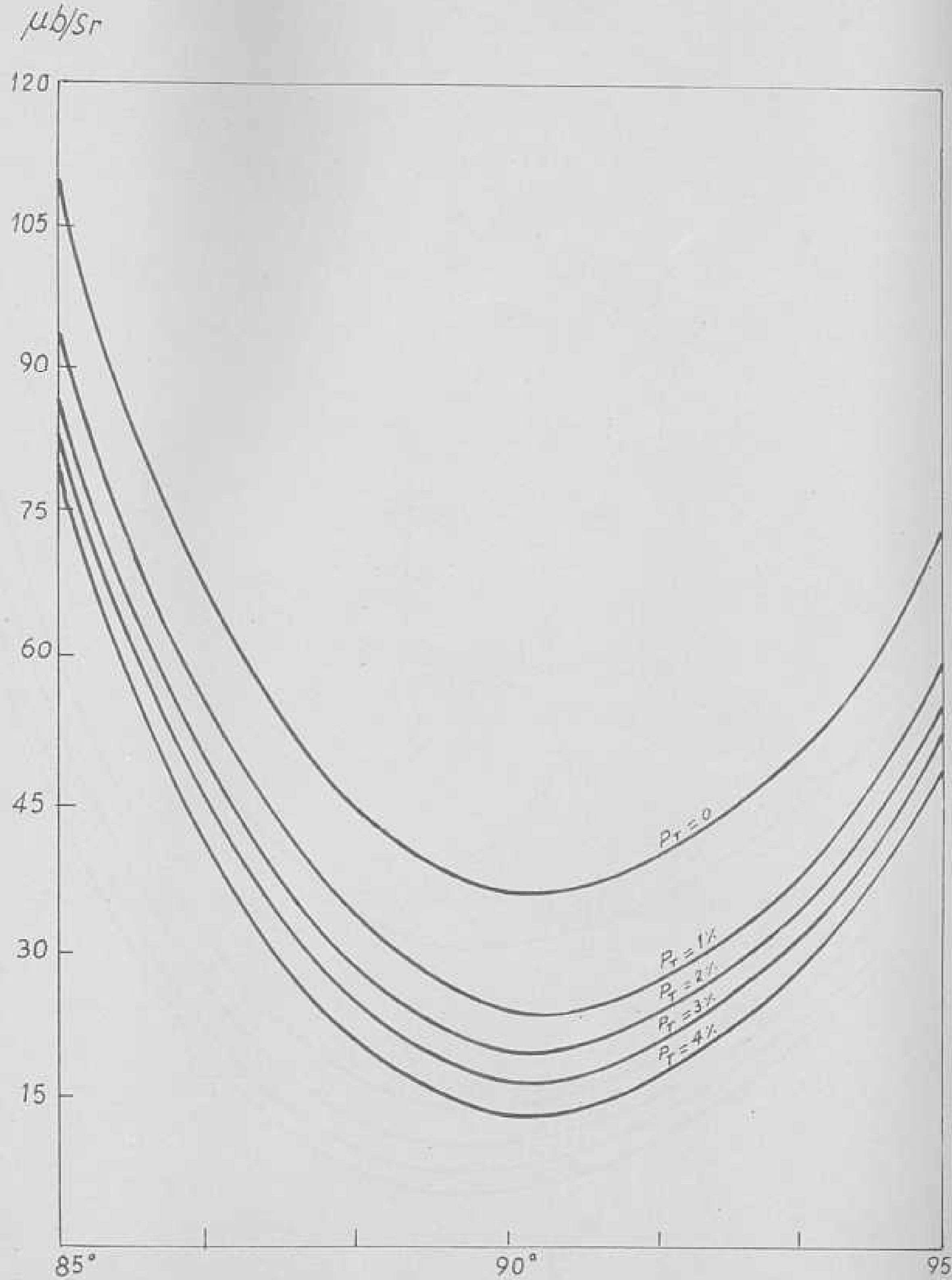
Differential cross section for π^+ scattered by He^3 in $\mu b/sr$ for the incoming pion energy $\omega = 2$ pion mass units ($\approx$ 280 Mev)

Admixture state	Percentage of admixture				
	0	1	2	3	4
S' state	35.93	77.36	83.26	96.52	108.06
$T=3/2$ state	35.93	23.90	19.63	16.63	14.30

<u>Table II</u>

Differential cross-section for π^+ scattered by He^3 in $\mu b/sr$ for the incoming pion energy $\omega = 2.5$ pion mass units ($\approx$ 350 Mev)

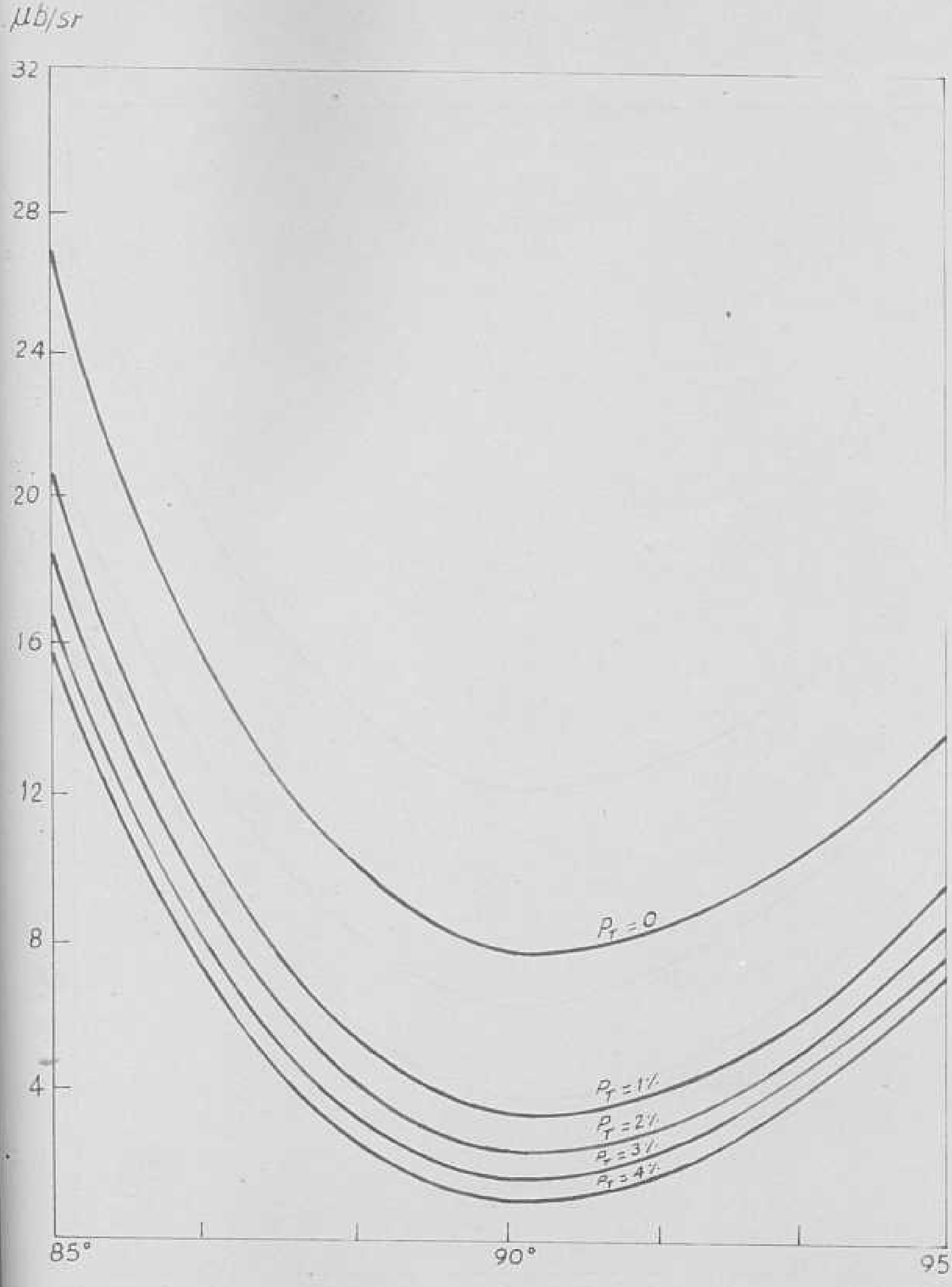
Admixture state	Percentage of admixture				
	0	1	2	3	4
S' state	7.805	21.14	28.59	35.04	41.01
$T = 3/2$ state	7.805	3.580	2.303	1.515	0.9786



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ AT $\omega = 280$ mev

P_T = PERCENTAGE OF $T = 3/2$ – STATE ADMIXTURE.

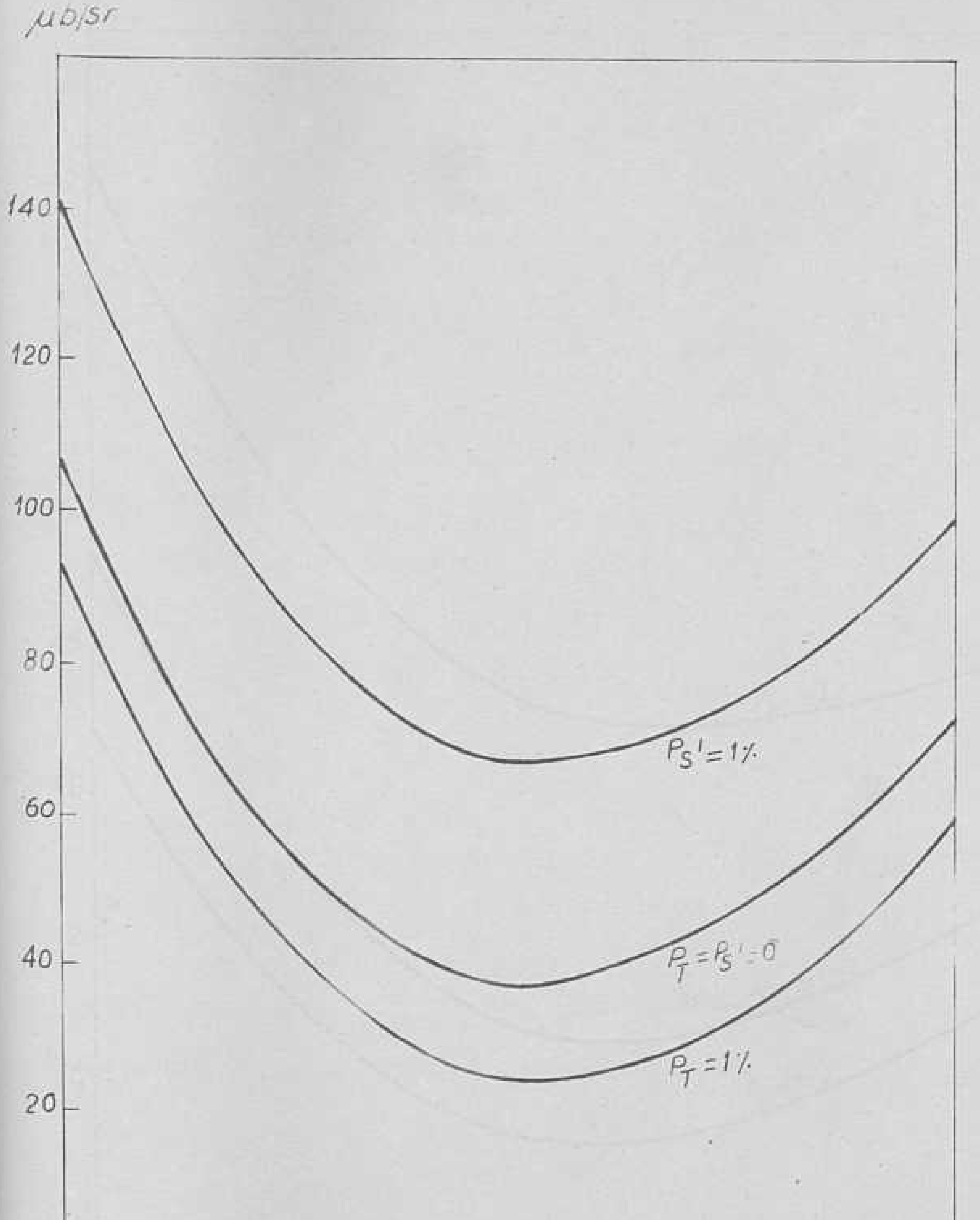
FIG. (5·1)



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ AT $\omega = 350$ mev

P_T = PERCENTAGE OF $T = 3/2$ – STATE ADMIXTURE

FIG. (5·2)

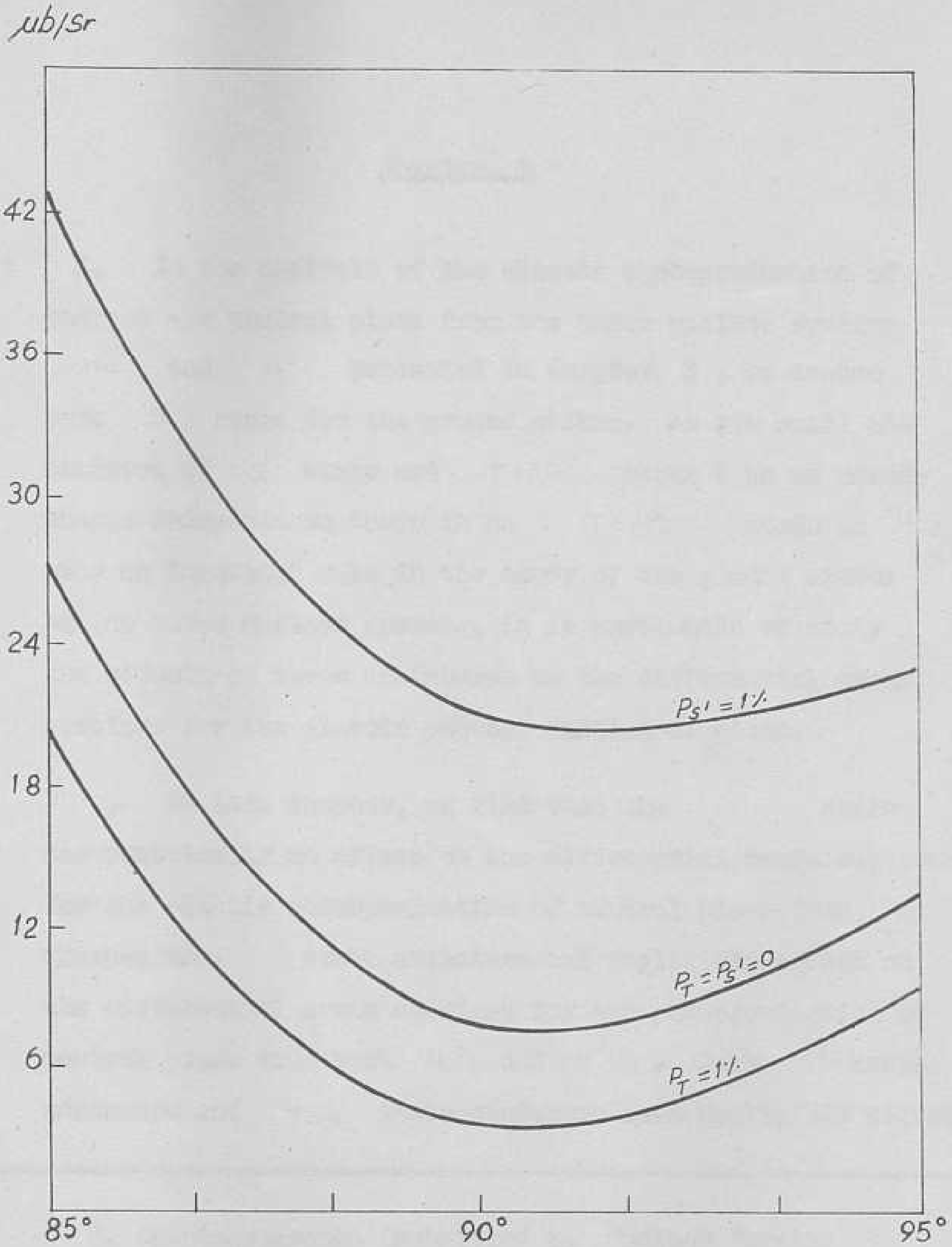


$\frac{d\sigma}{d\Omega}$ FOR $He^3 + \pi^+ \rightarrow He^3 + \pi^+$ AT $\omega = 280$ mev;

$P_S{}'$ = PERCENTAGE OF STATE ADMIXTURE

P_T = PERCENTAGE OF T=3/2 STATE ADMIXTURE

FIG. (5·3)



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \rightarrow \pi^+ + He^3$ AT $\omega = 350$ mev;

P_S' = PERCENTAGE OF S'- STATE ADMIXTURE

P_{+1} = PERCENTAGE OF T=3/2 STATE ADMIXTURE

FIG. (5.4)

Chapter 6 *

1. In the analysis of the elastic photoproduction of charged and neutral pions from the three nucleon systems He^3 and H^3 presented in Chapter 3 , we assume pure S state for the ground states. As the small admixtures of S' state and $T=3/2$ state (as we assume charge independence there is no $T=3/2$ state in ^3H) play an important role in the study of the ground states of the three nucleon systems, it is worthwhile to study the effects of these admixtures on the differential cross sections for the elastic photoproduction of pions.

In this chapter, we find that the state has practically no effect on the differential cross sections for the elastic photoproduction of neutral pions from whereas the state admixture has negligible effect on the differential cross sections for the photoproduction of neutral pions from both He^3 and H^3 . Both S' state admixture and $T=3/2$ state admixture have negligible effect

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on the forward angle differential cross section of the photoproduction of charged pions and their contributions to the backward angle cross sections are very small. Especially there is no contribution from the state at 180°.

The results indicate that it is a reasonable approximation to neglect these states in the analysis of the photoproduction of pions from H^3 and He^3.

2. While the S' state may be present both in the ground state of H^3 and He^3, $T=3/2$ state will be present only in He^3 if charge independence of the nuclear forces holds. Of course there are some indications for the violation of charge independence and charge symmetry. We shall consider this possibility in a latter chapter. Here there is no meaning in taking $T=3/2$ state in ^3H to account for the charge ~~ind~~ dependence and charge asymmetry, while using a charge independent Chew-Low theory for the pion-nucleon interactions. When an admixture of both S' state and $T=3/2$ state is present, the ground state of H^3 and He^3 can be written as

$$\psi^{m,t} = \psi_1^{m,t} + \psi_2^{m,t} + \psi_{T=3/2}^{m,t} \qquad (6.1)$$

$$= \frac{1}{\sqrt{2}} \left(\varphi^m \bar{\eta}^t - \bar{\varphi}^m \eta^t \right) u +$$

$$- \frac{1}{\sqrt{2}} \left[\left(\bar{\varphi}^m \eta^t + \varphi^m \bar{\eta}^t \right) \bar{v}_{s'} + \right.$$

$$\left. + \left(\bar{\varphi}^m \bar{\eta}^t - \varphi^m \eta^t \right) v_{s'} \right] +$$

$$+ \epsilon \left(v_T \bar{\varphi}^m - \bar{v}_T \varphi^m \right) \eta^t_{3/2} \qquad (6.2)$$

where $t = +1/2$ and $\epsilon = 1$ for He^3 and $t = -1/2$ and $\epsilon = 0$ for H^3 and $v_{s'}$ or v_T and $\bar{v}_{s'}$ or $\bar{v}_T$ differ from v and $\bar{v}$ only by constant factors. $v_{s'}$; v_T ; $\bar{v}_{s'}$; $\bar{v}_T$ are normalized as follows :

$$\int \left(v_{s'}^2 + \bar{v}_{s'}^2 \right) d^3 \underset{\sim}{r}_i = P_{s'} \qquad (6.3)$$

$$\int \left(v_T^2 + \bar{v}_T^2 \right) d^3 \underset{\sim}{r}_i = P_{T=3/2} \qquad (6.4)$$

where $P_{s'}$ and $P_{T=3/2}$ are the probabilities of

s' state admixture and the probability of $T=3/2$ state admixture respectively.

3. In the case of neutral pion photoproduction the single-nucleon amplitude can be written (as in the chapter 3) as

$$t = \frac{1}{2}\left[(t_p^0 + t_n^0) + (t_p^0 - t_n^0)\,\tau_z \right] \qquad (6.5)$$

The additional isobaric matrix elements between the various states η^t, $\bar{\eta}^t$ and $\eta^t_{3/2}$ can be calculated to give

$$\begin{aligned}
\langle \eta^{t'} \,|\, \tau_z(3) \,|\, \eta^t_{3/2} \rangle &= 0 \\
\langle \eta^{t'} \,|\, \tau^\pm(3) \,|\, \eta^t_{3/2} \rangle &= 0. \\
\langle \bar{\eta}^{t'} \,|\, \tau_z(3) \,|\, \eta^t_{3/2} \rangle &= 2t\,\delta_{t',t}\left(\frac{2\sqrt{2}}{3}\right) \\
\langle \bar{\eta}^{t'} \,|\, \tau^\pm(3) \,|\, \eta^t_{3/2} \rangle &= \delta_{t',t\pm1}\left(\frac{\sqrt{2}}{3}\right)
\end{aligned} \qquad (6.6)$$

Now the matrix element $\langle f \,|\, T \,|\, i \rangle$ of the transition amplitude T between the initial state $|i\rangle$ and the final state $|f\rangle$ can be written using (6.6) and (3.63) in the impulse approximation, neglecting the terms proportional to $P_{S'}$, $P_{T=3/2}$ and $(P_{S'} \cdot P_{T=3/2})^{1/2}$ as follows :

$$\langle f|T|i\rangle = \frac{3}{8}\Big[\langle\bar{\varphi}^{m'}|\,(t_p^0(3)+t_n^0(3)) + 2t\,(t_p^0(3)-t_n^0(3))|\bar{\varphi}^m\rangle \times$$
$$\times\, 2(F_S - F_{S'}) +$$

$$+ \langle\varphi^{m'}|(t_p^0(3)+t_n^0(3)) - \frac{2t}{3}(t_p^0(3)-t_n^0(3))|\varphi^m\rangle \times$$
$$\times\, 2(F_S + F_{S'}) +$$

$$- \epsilon\, F_{T=3/2}\langle\varphi^{m'}|(t_p^0(3)-t_n^0(3))|\varphi^m\rangle\Big] \qquad (6.7)$$

Here $t(3)$ acts only upon the wave functions of the nucleon with index 3 . The $F_{S'}$ and $F_{T=3/2}$ are defined by the equation

$$-2\int u\bar{v}_{S'}\, d^3 r_i = F_{S'} \qquad (6.8)$$

$$2\int u\bar{v}_{T=3/2}\, d^3 r_i = F_{T=3/2} \qquad (6.9)$$

The spin matrix element in the equation (6.7) can be calculated by writing $t(3)$ in the form

$$t(3) = i\,\underset{\sim}{\sigma}_3 \cdot \underset{\sim}{K} + L \qquad (6.10)$$

and using results (3. 69) , we thus have, after summing and averaging over the nuclear spin states,

$$\frac{1}{2}\sum_{\text{spins}}\sum \left|\langle f|T|i\rangle\right|^2 = \underset{\sim}{K}\cdot\underset{\sim}{K}^* + LL^* \qquad (6.11)$$

where in the case of He^3 targets

$$\underset{\sim}{K} = F_S \underset{\sim}{K}_n + F_{S'}(\underset{\sim}{K}_p + \underset{\sim}{K}_n) - \frac{1}{2}F_{T=3/2}(\underset{\sim}{K}_p - \underset{\sim}{K}_n) \qquad (6.12)$$

$$L = F_S(2L_p + L_n) + (L_n - L_p)(F_{S'} + \frac{1}{2}F_{T=3/2}) \qquad (6.13)$$

and in the case of H^3 targets

$$\underset{\sim}{K} = F_S \underset{\sim}{K}_p + F_{S'}(\underset{\sim}{K}_p + \underset{\sim}{K}_n) \qquad (6.14)$$

$$L = F_S(2L_n + L_p) + F_{S'}(L_p - L_n) \qquad (6.15)$$

Choosing the Gaussian forms for u and v's we have as in the chapter 5 :

$$F_S = \exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (6.16)$$

$$F_{S'} = \left(\frac{P_{S'}}{6}\right)^{1/2}\frac{k^2}{9\alpha^2}\exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (6.17)$$

$$F_{T=3/2} = \left(\frac{P_{T=3/2}}{6}\right)^{1/2}\frac{k^2}{9\alpha^2}\exp\left(-\frac{k^2}{18\alpha^2}\right) \qquad (6.18)$$

4. The amplitude for the photoproduction of charged pions has the form

$$t = t_{p,n} \tau^{\mp} \qquad (6.19)$$

where t_p and t_n denote the photoproduction amplitudes from protons and neutrons respectively. Using the matrix elements (3.63) and (6.6) the matrix elements $\langle f|T|i\rangle$ can be written in the impulse approximation neglecting terms proportional to $P_{S'}$, $P_{T=3/2}$ and $(P_{S'} \cdot P_{T=3/2})^{1/2}$:

$$\langle f|T|i\rangle = \frac{3}{2}\Big[\langle \bar{\varphi}^{m'} |\, t(3) \,| \bar{\varphi}^{m}\rangle (F_S + 2F_{S'})$$

$$- \frac{1}{3}\langle \varphi^{m'} |\, t(3) \,| \varphi^{m}\rangle (F_S - 2F_{S'} + F_{T=3/2})\Big] \qquad (6.20)$$

where for the process $\gamma + He^3 \rightarrow H^3 + \pi^+$

$$t(3) = t_p(3) \qquad (6.21)$$

and in the case of $\gamma + H^3 \rightarrow He^3 + \pi^-$

$$t(3) = t_n(3) \qquad (6.22)$$

Evaluating the spin matrix elements as in the previous section, averaging over initial nuclear spins and summing over final nuclear spins we get

$$\frac{1}{2} \sum_{spins} \sum |\langle f | T | i \rangle|^2 = \underset{\sim}{K} \cdot \underset{\sim}{K}^* + LL^* \tag{6.23}$$

where for the processes $\gamma + He^3 \rightarrow H^3 + \pi^+$

$$\begin{aligned} \underset{\sim}{K} &= \underset{\sim}{K}_p \left(F_S - \tfrac{1}{2} F_{T=3/2} \right) \\ L &= L_p \left(F_S + 2F_{S'} - \tfrac{1}{2} F_{T=3/2} \right) \end{aligned} \tag{6.24}$$

and in the case of $\gamma + H^3 \longrightarrow He^3 + \pi^-$

$$\begin{aligned} \underset{\sim}{K} &= \underset{\sim}{K}_n \left(F_S - \tfrac{1}{2} F_{T=3/2} \right) \\ L &= L_n \left(F_S + 2F_{S'} - \tfrac{1}{2} F_{T=3/2} \right) \end{aligned} \tag{6.25}$$

5. The amplitudes of Chew et al for the photoproduction of charged and neutral pions from single nucleons can be written as

$$t_p^0 = f^{(+)} + f^{(0)} \tag{6.26}$$

$$t_n^0 = f^{(+)} - f^{(0)} \tag{6.27}$$

$$t_p = \sqrt{2}\left(f^{(-)} + f^{(0)} \right) \qquad (6.28)$$

$$t_n = \sqrt{2}\left(-f^{(-)} + f^{(0)} \right) \qquad (6.29)$$

The expression for $f^{(+)}$, $f^{(-)}$ and $f^{(0)}$ were given in chapter 1. The terms in $f^{(0)}$ are proportional to $1/M$, M being the nucleon mass and so $f^{(0)}$ is small compared with $f^{(+)}$ and $f^{(-)}$. Thus we can neglect $F_{S'} f^{(0)}$ and $F_{T=3/2}\, f^{(0)}$ as $F_{S'}$ and $F_{T=3/2}$ are small compared with F_S.

In case of neutral pion production we can thus neglect the term $(\underset{\sim}{K}_p - \underset{\sim}{K}_n)\, F_{T=3/2}$ and $(L_p - L_n) F_{T=3/2}$ is identically zero. We find that there are no other terms containing $F_{T=3/2}$ in the expression (6.12) and (6.13). Thus there is practically no contribution from the $T = 3/2$ state to the neutral pion photoproduction from He^3.

With the use of the expressions givent in chapter 1 for $f^{(+)}$ and $f^{(0)}$ we can write the expression for the differential cross sections for the neutral pion photoproduction, neglecting terms proportional to $P_{S'}$ as follows:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{s-s} + \left(\frac{d\sigma}{d\Omega}\right)_{s-s'} \qquad (6.30)$$

where $(d\sigma/d\Omega)_{s-s}$ is the contribution from the $s-s$ transition and $(d\sigma/d\Omega)_{s-s'}$ is the contribution from $s-s'$ and $s'-s$ transitions.

While $\left(\frac{d\sigma}{d\Omega}\right)_{S-S}$ is given by (3.79) for He^3 and by (3.80) for H^3, $\left(\frac{d\sigma}{d\Omega}\right)_{S-S'}$ is the same for both He^3 and H^3 and is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{SS'} = F_S F_{S'} \left(\frac{8 \lambda^2 \nu^2 e^2 f^2}{9 \mu^3} \right) \sin^2 \delta_{33} \; (1 + \cos^2\theta) \quad (6.31)$$

In Table 1 we have given the $\left(\frac{d\sigma}{d\Omega}\right)_{S-S'}$ contributions $\left(\frac{d\sigma}{d\Omega}\right)_{S'-S'}$ for He^3 and H^3 taking for $P_{S'}$ the values 1 %, 2 %, 3 % and 4 %, against the S-S contributions. We have assumed free nucleon magnetic moments as the quenching effects are negligible (being small themselves) while calculating the small S' S and S-S' contributions.

Table 1

0	4.169	4.503	.019	.027	.033	.038
15	4.984	5.880	.073	.103	.126	.146
30	15	~~[illegible]~~ 15.39	.164	.233	.285	.329
45	19.63	19.70	.196	.277	.339	.391
60	17.98	17.93	.160	.226	.277	.320
90	5.939	5.850	.069	.097	.119	.137
135	.516	.476	.028	.039	.048	.055
180	~~.024~~ .039	~~.020~~ .014	~~.022~~ .020	.028	.035	.040

Contributions from $\left(\frac{d\sigma}{d\Omega}\right)_{S-S'}$ is negligible except for very backward angles ($\sim 180^\circ$).

6. The differential cross sections for the elastic process $\gamma + He^3 \rightarrow H^3 + \pi^+$ is given by the expression

$$\frac{d\sigma}{d\Omega} = \left(1 - \frac{1}{2}\frac{F_{T=3/2}}{F_S}\right)^2 \left(\frac{d\sigma}{d\Omega}\right)_0 + \left(\frac{d\sigma}{d\Omega}\right)_{S-S'} \qquad (6.32)$$

where $(d\sigma/d\Omega)_0$ is given the right hand side of the equation (3.77) and

$$\left(\frac{d\sigma}{d\Omega}\right)_{S-S'} = F_S F_{S'} \left(\frac{8\lambda^2 \nu e^2 f^2}{9\mu^3}\right) \sin^2 \delta_{33} \cdot 2\sin^2\theta \qquad (6.33)$$

The differential cross section for the elastic process $\gamma + H^3 \rightarrow He^3 + \pi^-$ is given by the expression

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(1 - \frac{1}{2}\frac{F_{T=3/2}}{F_{S^\circ}}\right)^2 \left(\frac{d\sigma}{d\Omega}\right)_0 + \left(\frac{d\sigma}{d\Omega}\right)_{S-S'} \qquad (6.34)$$

where $(d\sigma/d\Omega)_0$ is given by the right hand side of the expression (3.78) and $(d\sigma/d\Omega)_{S-S'}$ is given by the expression (6.33).

In Table 2 the contributions from S' state admixture to the differential cross section for both $\gamma + He^3 \rightarrow H^3 + \pi^+$ and $\gamma + H^3 \rightarrow He^3 + \pi^-$

are presented for various values of $P_{s'}$ along with the differential cross sections for $\gamma + He^3 \to H^3 + \pi^+$ and $\gamma + H^3 \to He^3 + \pi^-$ with pure S states neglecting quenching effects.

Table 2

Contributions from S' state admixture to the differential cross sections for $\gamma + He^3 \rightarrow H^3 + \pi^+$ and $\gamma + H^3 \rightarrow He^3 + \pi^-$

0	4.383	4.766	0	0	0	0
15	4.900	4.970	.005	.007	.009	.010
30	6.829	6.693	.047	.066	.081	.093
45	6.784	6.708	.130	.180	.230	.260
60	4.784	5.021	.191	.269	.330	.381
90	1.433	1.858	.137	.194	.238	.274
135	0.179	.334	.018	.026	.032	.037
180	.066	.107	0	0	0	0

7. Now let us discuss the data when we take pure S state for H^3 and an admixture of $T = 3/2$ state along with S-state for He^3.

In Table 3, we have presented the differential cross sections for the process $\gamma + He^3 \to H^3 + \pi^+$ taking for $P_{T=3/2}$ (the probability of $T = 3/2$ state admixture in He^3) the values 0 %, 1 %, 2 %, 3 % and 4 %.

In Table 4, we have presented the differential cross sections for the process $\gamma + H^3 \to He^3 + \pi^-$ taking for $P_{T=3/2}$, the values 0 %, 1 %, 2 %, 3 %, 4 %.

We find that the differential cross sections in the backward angles are not very sensitive to $P_{T=3/2}$.

8. When we take an admixture of S' state along with the dominant S state for H^3 and ~~ad~~ an admixture of both S' state and $T = 3/2$ state along with the dominant S state for He^3 we can see from the equations (6.32) and (6.34) that to get the differential cross sections for both the processes, we have to add the values of $(d\sigma/d\Omega)_{S-S'}$ presented in the Table 2 to the values of $\left(1 - \frac{1}{2}\frac{F_{T=3/2}}{F_S}\right)^2 \left(\frac{d\sigma}{d\Omega}\right)_0$. Presented in the Tables 3 and 4.

Table 3

Differential cross-section for $\gamma + He^3 \rightarrow H^3 + \pi^+$ when $T = 3/2$ state admixture is taken for the ground state in

$P_{T=3/2}$	0 %	1 %	2 %	3 %	4 %
0°	4.383	4.379	4.377	4.375	4.373
15°	4.900	4.877	4.868	4.860	4.853
30°	6.829	6.723	6.680	6.649	6.602
45°	6.784	6.512	6.419	6.354	6.296
60°	4.784	4.529	4.424	4.343	4.278
90°	1.433	1.283	1.223	1.178	1.141
135°	.1790	.1478	.1357	.1268	.1115
180°	.0660	.0529	.0475	.0438	.0408

Table 4

Differential cross-section for $\gamma + H^3 \rightarrow He^3 + \pi^-$ when $T = 3/2$ state admixture is taken for the ground state.

$P_{T=3/2}$	0	1 %	2 %	3 %	4 %
0	4.775	4.771	4.768	4.766	4.764
15°	4.971	4.948	4.939	4.929	4.922
30°	6.692	6.589	6.546	6.516	6.489
45°	6.708	6.495	6.403	6.339	6.281
60°	5.021	4.753	4.643	4.558	4.489
90°	1.858	1.663	1.584	1.527	1.480
135°	.3840	.3169	.2910	.2719	.2563
180°	.1970	.1579	.1418	.1306	.1218

CHAPTER. 7

1. Whereas the theoretical analysis[1] of the elastic scattering data[2] shows a definite preference for the Gaussian and Irving wave functions over exponential wave function and a slight preference for Irving over the Gaussian type wave function for the radial part of the ground state wave functions of the three nucleon systems He^3 and H^3, the analysis of the inelastic electron scattering[3] $e - He^3$ and $e - H^3$ assuming an ejected proton is counted in coincidence with the scattered electron, prefers the Irving-Gunn wave function to the Irving wave function. Also the analysis of the thermal neutron capture[4] prefers the Irving-Gunn wave function to the Irving type wave function. But the calculation for the muon capture rate in He^3 [5] for the process $\mu^- + He^3 \rightarrow H^3 + \nu$ reports good fit with Irving-wave function while the Irving-Gunn wave function gives too small a capture rate for the process.

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1) L. I. Schiff, Phys. Rev. **133** B, 802 (1964)
2) H. Ellard, R. Hofstadter, A. Johansson, R. Parks, A. Waller, M. R. Yearean, R. B. Day and R. T. Wagner, Phys. Rev. Letters, **11**, 132 (1963)
3) T. A. Griffy and R. J. Oakes, Phys. Rev. **135** B, 1161 (1964)
4) T. K. Radha and N. T. Meister, Phys. Rev., **136** B, 388 (1964)

5) R. J. Oakes, Physical Review,

Thus it remains an open question whether Irving or Irving-Gunn wave function should be used. All the calculations in the earlier chapters were done for the Gaussian wave functions due to its simplicity. Inthis chapter, we estimate the differential cross sections for the photoproduction of charged and neutral pions and scattering of pions from He^3 and H^3 using the Irving and Irving-Gunn wave functions. As the small admixtures of S' state and $T=3/2$ state do not give considerable contributions to the differential cross-sections for the photoproduction of pions we take pure S states for the ground state wave functions of He^3 and H^3. In the case of scattering of pions the sensitivity to S' state or the $T=3/2$ state admixture is found only when the scattering angle is in the neighbourhood of 90^0. The calculations with Gaussian-Irving and Irving-Gunn wave functions for the dominant x S state shows that the cross sections are sensitive to the form of the wave function only in the forward angles especially between 30^0 and 60^0. So it is reasonable to neglect these states for the pion-scattering analysis also.

It is to be noted that the sensitivity of the differential cross sections for the photoproduction of pions was observed to be in the forward angles. Thus just from the photoproduction cross sections we cannot conclude which wave functions have to be preferred. But from the differential cross sections for the pion-scattering one can make a choice for the correct wave functions. This wave function can be used for the calculation of photoproduction cross sections to look for quenching effects.

2. Assuming a pure S - state, the ground states of He^3 and H^3 can be written as

$$\psi^{m,t} = (\bar{\varrho}^m \eta^t - \varrho^m \bar{\eta}^t) u \qquad (7.1)$$

The Irving form of u can be written as

$$u = A \exp\left[- \frac{\alpha}{2} (r_{12}^2 + r_{23}^2 + r_{31}^2)^{1/2} \right]$$

$$A = \frac{3^{3/4} \alpha^3}{(120 \pi^3)^{1/2}} \qquad (7.2)$$

whereas the Irving-Gunn form is given by

$$u = \frac{A \exp\left[- \frac{\alpha}{2} (r_{12}^2 + r_{23}^2 + r_{31}^2)^{1/2} \right]}{(r_{12}^2 + r_{23}^2 + r_{31}^2)^{1/2}}$$

$$A = \frac{3^{1/4} \alpha^2}{(2 \pi^3)^{1/2}} \qquad (7.3)$$

3. The square of the matrix element $\langle f | T | i \rangle$ for the photoproduction of pions after summing and averaging over nuclear spins can be written as

$$\frac{1}{2} \sum_{spins} \sum |\langle f | T | i \rangle|^2 = |F_s|^2 (\underset{\sim}{K} \cdot \underset{\sim}{K}^* + L L^*) \qquad (7.4)$$

where $\underset{\sim}{K}$ and L refer to the spin dependent and spin independent amplitude for $\gamma + p \rightarrow n + \pi^+$ and $\gamma + n \rightarrow p + \pi^-$ for the π^+ production from He^3 and π^- production from H^3 respectively while for neutral pion production $\underset{\sim}{K}$ refers to the spin-dependent part of the amplitude for $\gamma + p \rightarrow p + \pi^0$ and $\gamma + n \rightarrow n + \pi^0$ with H^3 and He^3 ~~that~~ targets respectively whereas L for He^3 targets is given by

$$L = 2L_p + L_n \qquad (7.5)$$

and for H^3 targets by

$$L = 2L_n + L_p \qquad (7.6)$$

The nuclear form factor F_s is defined by the equation

$$F_s = \int u^2 \exp[i \underset{\sim}{k} \cdot (\underset{\sim}{r}_3 - \underset{\sim}{R})] \, d^3 \underset{\sim}{r}_i \qquad (7.7)$$

where $\underset{\sim}{k}$ is equal to $\underset{\sim}{\kappa} - \underset{\sim}{\mu}$, $\underset{\sim}{\kappa}$, $\underset{\sim}{\mu}$ being the momenta of the incident photon and the outgoing meson.

Using for different $\underset{\sim}{k}$ and L the appropriate expressions by Chew et al[1] and averaging over photon polarizations explicit expressions for the different processes are given by the equations (3.77), (3.78), (3.79) and (3.80). The form factor F_s coming in all these integrals can be evaluated by the method of Schiff[2] for the different type of wave functions. For the Irving form (7.2) of u, F_s is given by

$$F_s = \left(1 + \frac{2k^2}{9\alpha^2}\right)^{-7/2} \tag{7.9}$$

and for the Irving-Gunn form (7.3), F_s is given by

$$F_s = \frac{4/3 \cdot \left[1 + 2\left(1 + \frac{2k^2}{9\alpha^2}\right)^{1/2}\right]}{\left(1 + \frac{2k^2}{9\alpha^2}\right)\left[1 + \left(1 + \frac{2k^2}{9\alpha^2}\right)^{1/2}\right]^2} \tag{7.10}$$

The parameter α in Irving wave function can be found from the electron scattering data or from the measurement of the Coulomb energy of He^3. It was found by Schiff

$$\alpha = 1.27\ fm^{-1} \tag{7.11}$$

For the Irving-Gunn wave function the parameter

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1) G.F.Chew et al, Phys. Rev. **106**, 1345 (1957)
2) L.I.Schiff, Phys. Rev. **133 B**, 802 (1964)

was found by Berman[1] et al to have the value

$$\alpha = 0.769 \quad fm^{-1} \tag{7.12}$$

7. In fig. (7.1) the differential cross section $d\sigma/d\Omega$ in $\mu \cdot b$ for the process $\gamma + He^3 \rightarrow H^3 + \pi^+$ is plotted against the scattering angle θ in degrees, using (1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

In fig (7.2) the differential cross-section $d\sigma/d\Omega$ in $\mu \cdot b$ for $\gamma + H^3 \rightarrow He^3 + \pi^-$ plotted against the scattering angle θ in degrees, Using

(1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

In fig (7.3) the differential cross-section $d\sigma/d\Omega$ in $\mu \cdot b.$, for $\gamma + He^3 \rightarrow He^3 + \pi^0$ is plotted against the scattering angle θ in degrees, using

(1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

In fig. (7.4) the differential cross-section $d\sigma/d\Omega$ in $\mu \cdot b.$ for $\gamma + H^3 \rightarrow H^3 + \pi^0$

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1) B.L.Berman, L.J.Koester and J.H.Smith, Phys. Rev. 133 B , 117 , (1964)

is plotted against the scattering anglex θ in degrees using

(1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

The differential cross section is sensitive to the form of the radial wave function in the forward angle especially between 30° and 60°. But the sensitivity of the differential cross-section to the quenching also is in the forwarding angles. So we have to analyse a process wherein there will be no quenching effects. The pion sqattering by the three nucleon systems offers itself for this analysis.

6. When pure S state is assumed for the ground state differential cross section for the elastic scattering of π^+ by He^3 is given by the expression

$$\frac{d\sigma_{He^3}}{d\Omega} = F_S^2 \frac{\sin^2 \delta_{33}}{9q^2} \left(1 + 195 \cos^2 \theta\right) \qquad (7.13)$$

For the elastic scattering of π^+ by H^3 the differential cross section xi is given by

$$\frac{d\sigma_{H^3}}{d\Omega} = F_S^2 \frac{\sin^2 \delta_{33}}{9q^2} \left(9 + 91 \cos^2 \theta\right) \qquad (7.14)$$

In Fig. (7.5) the differential crosssection $d\sigma/d\Omega$ in m.b. for $\pi^+ + He^3 \rightarrow He^3 + \pi^+$ is plotted against the scattering angle θ in degrees between $\theta = 30^\circ$ and $\theta = 90^\circ$ using

(1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

In Fig. (7.6) the differential cross section $d\sigma/d\Omega$ in m.b for $\pi^+ + H^3 \rightarrow H^3 + \pi^+$ is plotted against the scattering angle in degrees between $\theta = 30^\circ$ and $\theta = 90^\circ$ using

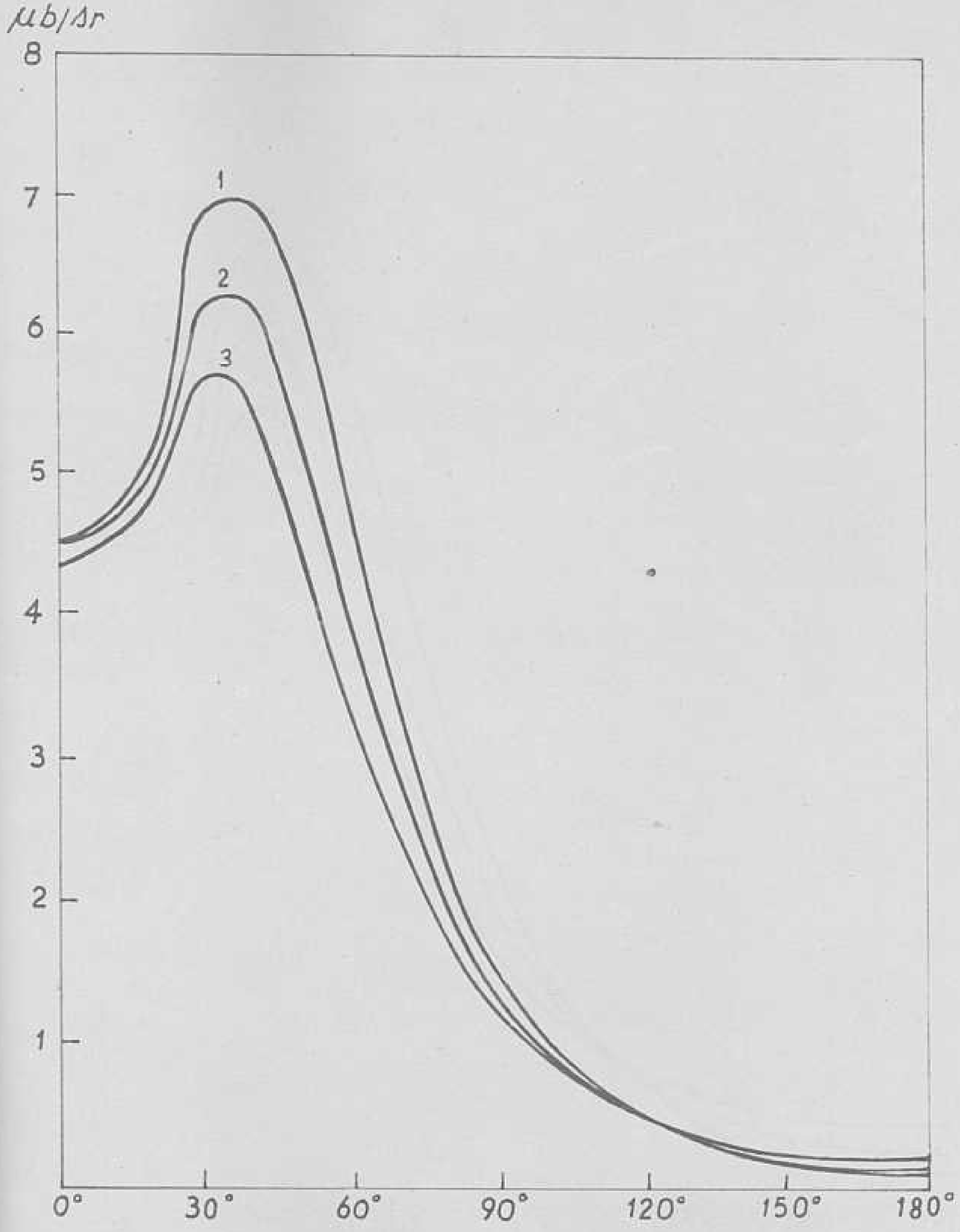
(1) Gaussian function for u

(2) Irving function for u

(3) Irving-Gunn function for u

The differential cross section is sensitive to the form of the radial function in the forward angles. As the admixture of S' state and $T = 3/2$ state have negligible effects in the forward angles, we can choose the correct form of the wave function from experimental data for π^+ scattered by He^3 and H^3.

The chosen form of the radial wave function can be used in the analysis of the photoproduction of pions and the effect of quenching can be measured from the experimental data for the photoproduction of pions.



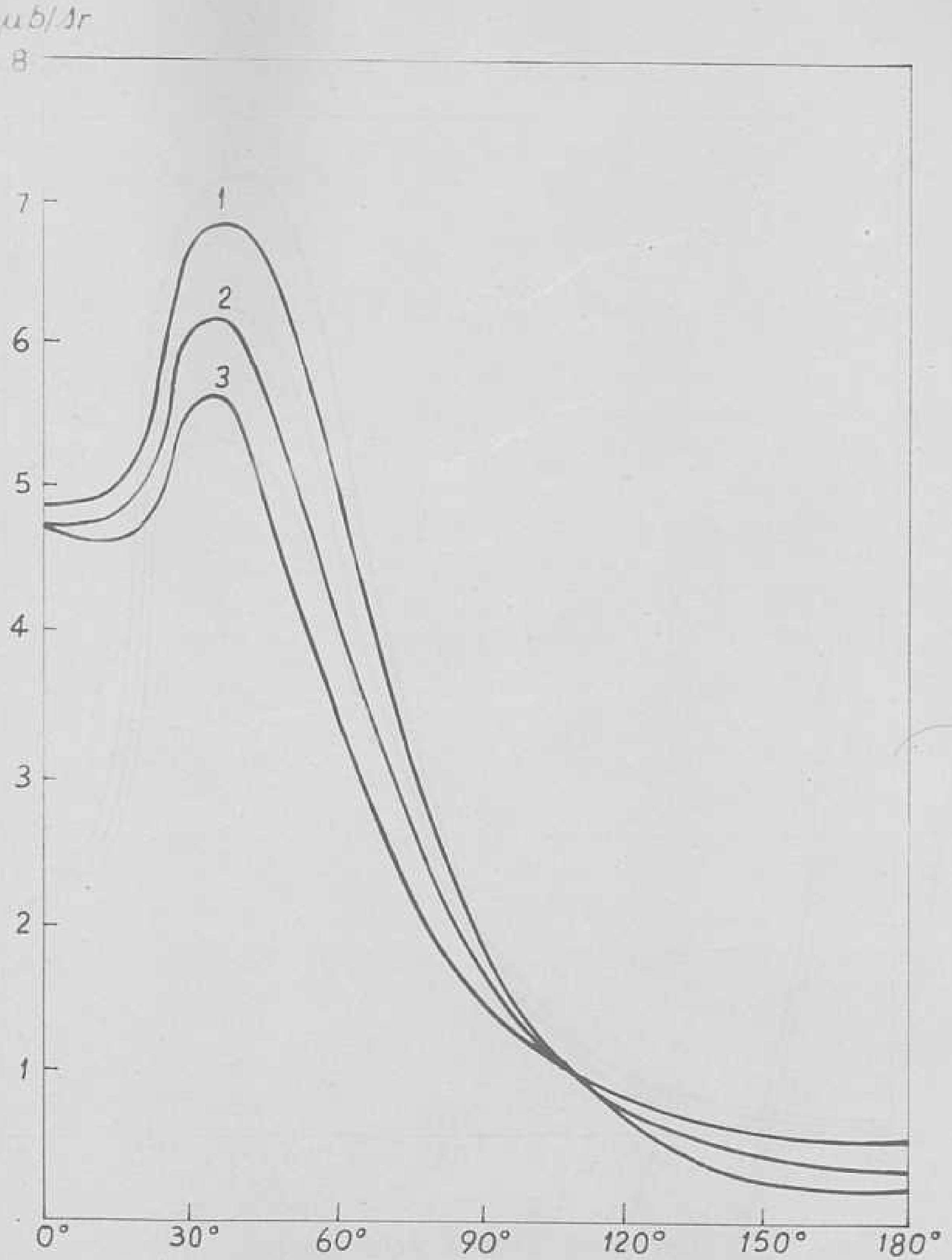
$\frac{d\sigma}{d\Omega}$ FOR $\gamma + He^3 \rightarrow H^3 + \pi^+$ USING

1) GAUSSIAN RADIAL FUNCTION

2) IRVING RADIAL FUNCTION

3) IRVING-GUNN RADIAL FUNCTION

FIG. (7·1)



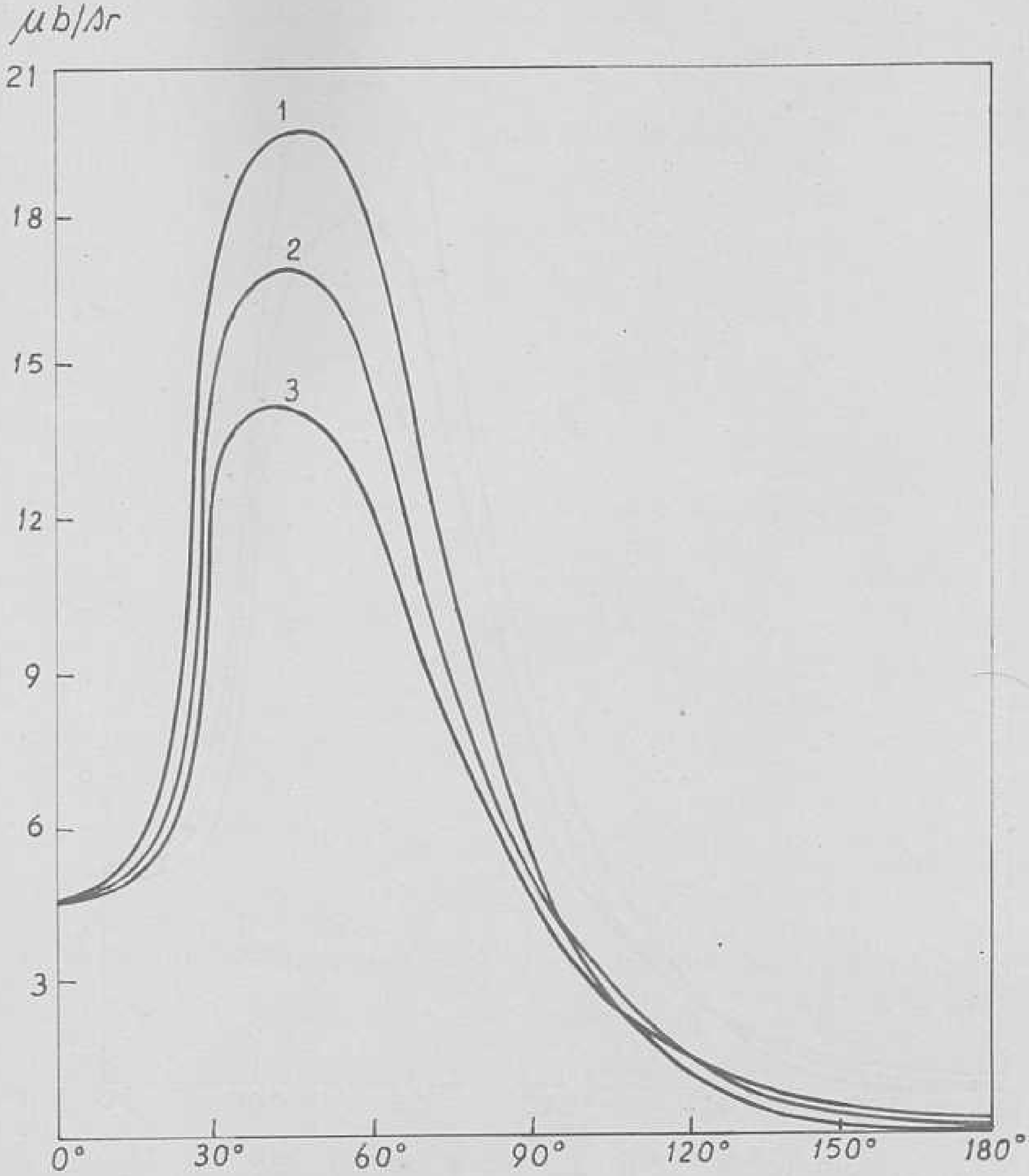
$\frac{d\sigma}{d\Omega}$ FOR $\gamma + H^3 \longrightarrow He^3 + \pi^-$ USING

1) GAUSSIAN RADIAL FUNCTION

2) IRVING RADIAL FUNCTION

3) IRVING-GUNN RADIAL FUNCTION

FIG. (7-2)



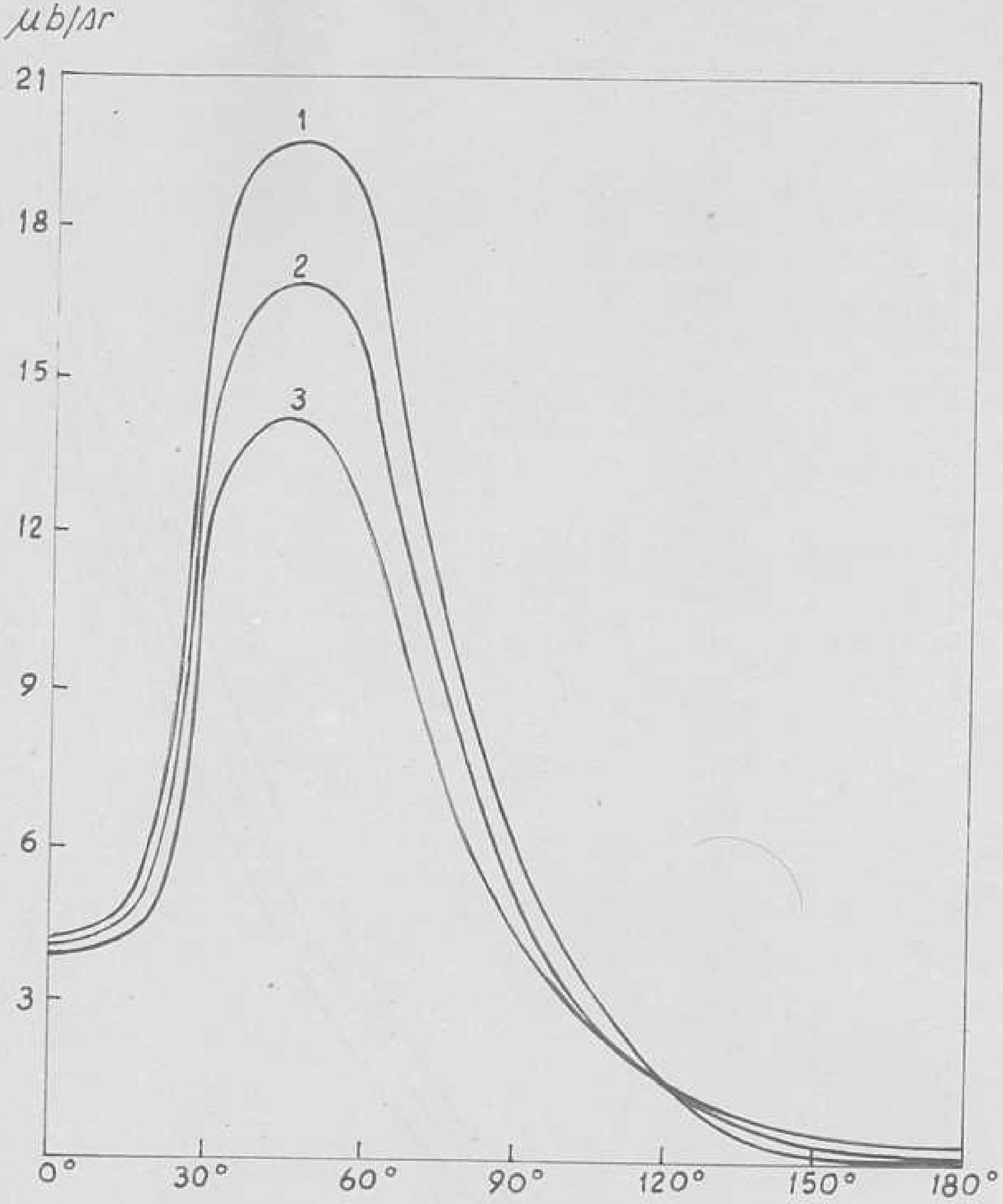
$\frac{d\sigma}{d\Omega}$ FOR $\gamma + He^3 \rightarrow He^3 + \pi^0$ USING

1) GAUSSIAN RADIAL FUNCTION

2) IRVING RADIAL FUNCTION

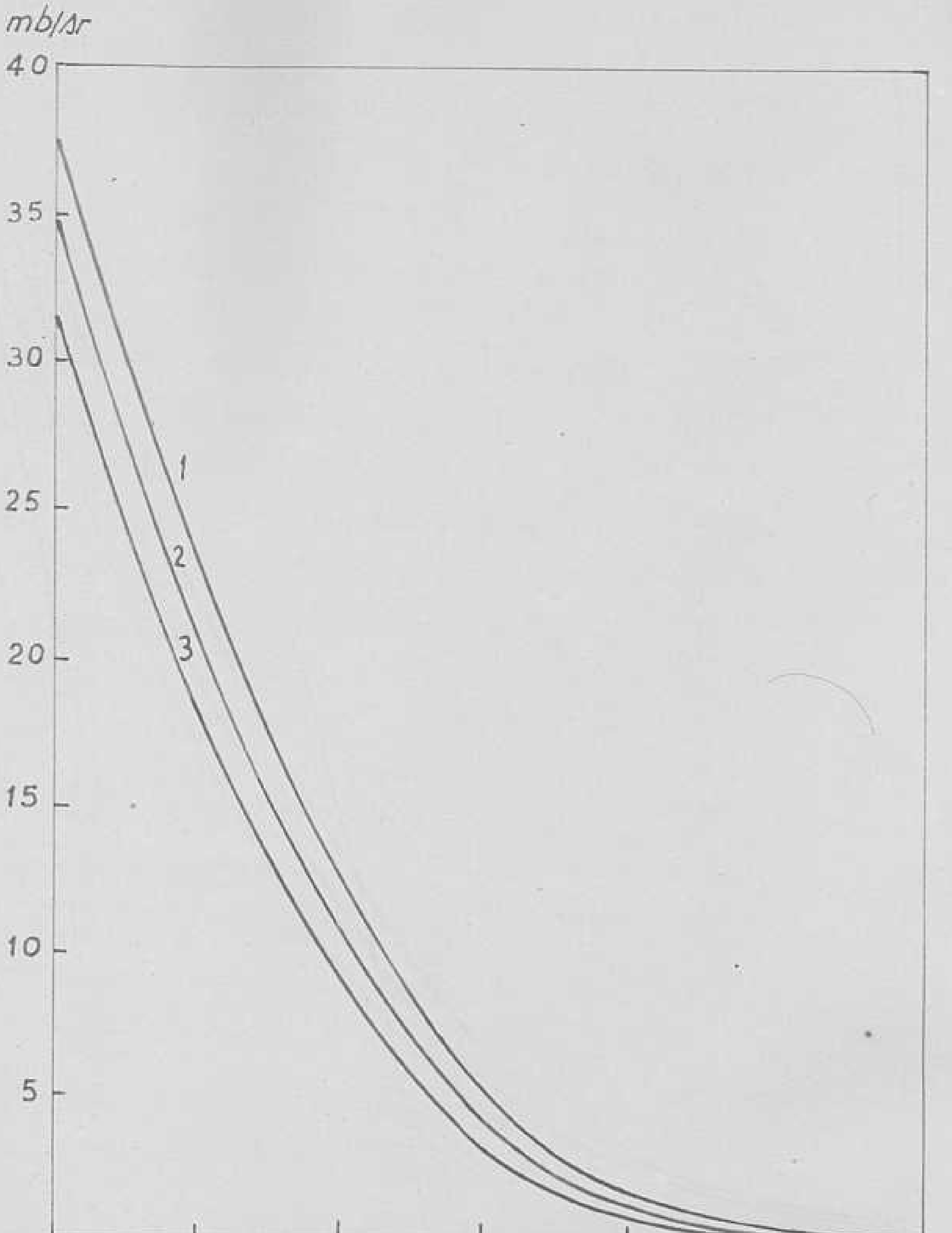
3) IRVING-GUNN RADIAL FUNCTION

FIG. (7.3)



$\frac{d\sigma}{d\Omega}$ FOR $\gamma + H^3 \rightarrow H^3 + \pi^0$ USING
1) GAUSSIAN RADIAL FUNCTION
2) IRVING RADIAL FUNCTION
3) IRVING-GUNN RADIAL FUNCTION

FIG. (7·4)



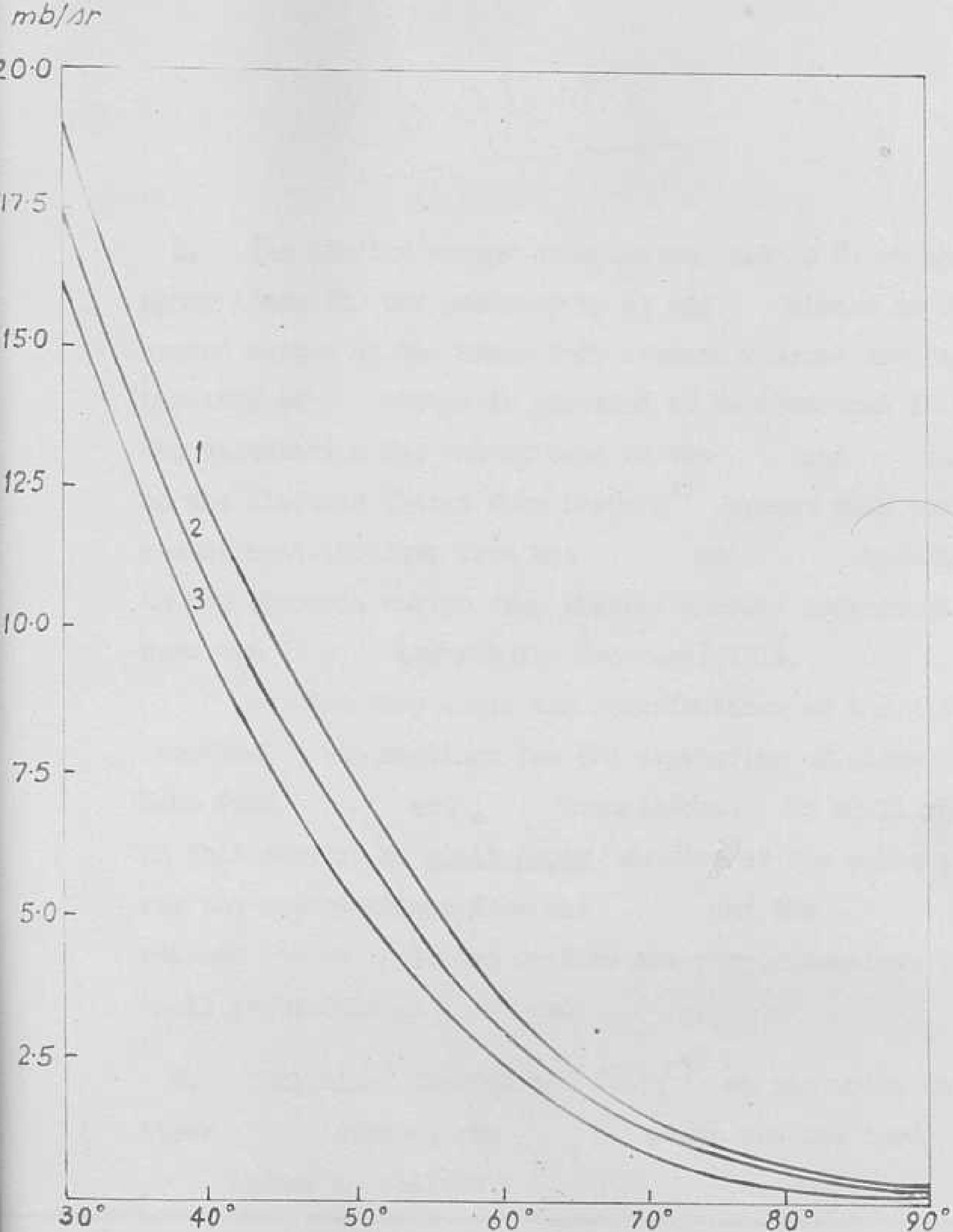
$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + He^3 \longrightarrow He^3 + \pi^+$ USING

1) GAUSSIAN RADIAL FUNCTION

2) IRVING RADIAL FUNCTION

3) IRVING-GUNN RADIAL FUNCTION

FIG. (7·5)



$\frac{d\sigma}{d\Omega}$ FOR $\pi^+ + H^3 \rightarrow \pi^+ + H^3$ USING

1) GAUSSIAN RADIAL FUNCTION

2) IRVING RADIAL FUNCTION

3) IRVING-GUNN RADIAL FUNCTION

FIG. (7·6)

Chapter 8

1. The binding energy calculations set 4 % as the upper limit for the probability of the states in the ground states of the three body systems whereas the probability of states is expected to be less than 1 %. The calculation for the effects of the and states on the electric charge form factors[1] proved that there are no contributions from the and transitions to the electric charge from factors whereas contributions from the trabsutuibs are negligible.

We find that there are contributions to the differential cross sections for the scattering of pions both from and transitions. We shall present in this chapter a preliminary version of the calculations for the contributions from the and the contributions can be neglected as they are proportional to the small probabilities and .

2. Following Derrick and Blatt[2] we can write the three states, one state and the three states as follows :

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1) B.F.Gibson and L.I.Schiff, Phys. Rev. 138 B 26 (1965)
2) G.Derrick and J.M.Blatt, Nuclear Physics, 8, 310 (1958)

Here are the Eulirean angle wave functions. The radial wave functions and are normalized as follows :

where refers to the probability of the corresponding state. We shall neglect for the reasons stated in chapter .

The ground of the three nucleon systems can now be written as

where is the dominant state and it is given by

and and can be written down explicitly from the expressions and .

3. The amplitude for the scattering of pions from and can be written down in the impulse approximation as follows :

where and are the momenta of the incoming and the outgoing pions.

Using a for the expression and neglecting the and transitions can be written as

The amplitude can be written in the iso-spin space as

Using the relations for the matrix and appealing to the form for and the expression can be rewritten as follows :

for the elastic scattering pions from and

for the elastic scattering of pions from .

The spin matrix elements occurring in and can be calculated by writing the operatfors in the form

Using the results and the following additional matrix elements :

4. We can see from the relations and that

vanish identically. The contributions from the remaining state can be written in the case of targets as where the nuclear form factors and are defined by

and

Using the results and summing over the matric elements for the different values and are given in the Table 1 .

The contributions to from the state can be written as

Using the matric elements and summing over the expression is given in the Table 2 for different values of and .

6. The contributions to from is given by

whereas the contributions from and are respectively given by the equations and given below .

Define

After using and summing over ,
we have given in the Table 3 the contributions to
from the three states in the case of
targets.

7. The contributions to from the and states in the case of targets can be got from the Tables 1 , 2 and 3 after interchanging and i.e. after interchanging , , , and , , , .

The differential cross sections for the pions scattered by and can be written as

where the cross section refers to unpolarised targets.

The integrals are to be worked out knowing the correct phase factors to be attached to or corresponding to the different states.

Table 1 *

Contribution to from in the case of

Contribution to	from

* Use has been made of the relations

a)

b)

Table

Contribution from state to

for the process

Table 3

Contributions to from the states for the process

CHAPTER 9

1. We can find an upper bound to the $T = 3/2$ state admixture in He^3 from a perturbation calculation assuming that the charge independence is violated only due to Coulomb forces between the two protons. An estimate of this upperbound was made by Werntz and Valk[1)] usingg Gaussian radial wave function for the dominant S state and they give 0.17 % for the upperbound. Here we find the estimate using Irving and Irving-Gunn wave functions for the dominant S -state. An estimate with Irving function gives 0.19 % for the upperbound whereas the estimate with Irving-Gunn wave function gives 0.23 % for the same.

2. The Coulomb potential between any two nucleons with indices i and j can be writtenaas

$$V(r_{ij}) = \frac{e^2}{r_{ij}} \left[\frac{1}{4} (1+\tau_{iz})(1+\tau_{jz}) \right] \tag{9.1}$$

where e^2 is the fine structure constant and is equal to $1/137$, $r_{ij} = |\underset{\sim}{r}_i - \underset{\sim}{r}_j|$ is the internucleon distance, τ_{iz} is the projection of the iso-spin operator for the i^{th} nucleon.

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* K. Ananthanarayanan, Proceedings of the Second Matscience Summer School, Bangalore, Plenum Press, New York.

1) C. Werntz and H. S. Valk, Phys. Rev. Letts. 14, 910 (1965).

Following the method of Wertz and Valk we can write the following inequality for $|A_{3/2}|^2$, the probability of admixture of the $T = 3/2$ state :

$$|A_{3/2}|^2 < \sum_{i \leq 3/2} \frac{\langle \psi_g | V | \psi_i \rangle \langle \psi_i | V | \psi_g \rangle}{E_g^2} \qquad (9.2)$$

where ψ_g is the solution of the Hamiltonian H_0 without Coulomb force, ψ_i is the solution of the total Hamiltonian $H = H_0 + V, (V = \sum_{i<j} V_{ij})$. The suffix i in ψ_i refers to the iso-spin. E_g is the binding energy of the He^3 ground state and it is equal to 7.72 Mev.

Using the projection operator $|3/2, 1/2\rangle\langle 3/2, 1/2|$ for the state with $T = 3/2$, $T_z = 1/2$ we can rewrite the inequality as

$$|A_{3/2}|^2 < \frac{1}{E_g^2} \left\{ \langle \psi_g | V | 3/2, 1/2 \rangle \langle 3/2, 1/2 | V | \psi_g \rangle \right\} \qquad (9.3)$$

We use pure S-state for ψ_g. Substituting for V and using the matrix elements for τ_{iz} we can arrive at the inequality

$$|A_{3/2}|^2 < \frac{1}{3E_g^2} \left\{ \langle u([123]) | \frac{e^4}{r_{23}^2} - \frac{e^4}{r_{23} r_{13}} | u([123]) \rangle \right\} \qquad (9.4)$$

where u is the radial wave function for the dominant S-state. The inequality (9.4) can be re-written in the form

$$|A_{3/2}|^2 < \frac{e^4}{3E_g^2}(F_1 - F_2) \qquad (9.5)$$

where

$$F_1 = \int \frac{u^2}{r_{23}^2}\, d^3\underset{\sim}{r}_i \qquad (9.6)$$

and

$$F_2 = \int \frac{u^2}{r_{23}\, r_{31}}\, d^3\underset{\sim}{r}_i \qquad (9.7)$$

3. Assuming Irving form for u and applying the transformation

$$\underset{\sim}{\rho} = \underset{\sim}{r}_{23}, \quad \underset{\sim}{\rho} + \tfrac{1}{2}\underset{\sim}{r} = \underset{\sim}{r}_{12}$$

$$\underset{\sim}{\rho} - \frac{\underset{\sim}{r}}{2} = \underset{\sim}{r}_{13} \qquad (9.8)$$

we can rewrite F_1 as follows :

$$F_1 = (4\pi)^2 A^2 \int \exp\left[-\alpha\left(2\rho^2 + \tfrac{3}{2}r^2\right)^{1/2}\right] \rho^2\, d\rho\, dr \qquad (9.9)$$

with

$$A^2 = \frac{3^{3/2}\alpha^6}{120\,\pi^3} \quad \text{and} \quad \alpha = 250 \text{ MeV}.$$

Considering $\sqrt{2}\,\rho$ and $\left(\frac{3}{2}\right)^{1/2} r$ as components of a two dimensional vector and using polar coordinates the integration F_1 can be evaluated easily and we get the expression

$$F_1 = \frac{3\alpha^2}{10} \qquad (9.10)$$

F_2 can be written as

$$F_2 = 8\pi^2 A^2 \int_0^\infty \int_0^\infty \int_{-1}^{+1} \frac{\exp\left[-\alpha\left(2\rho^2 + \frac{3}{2}r^2\right)\right]\rho^2\, d\rho\, dr\, dz}{\left[\rho^2 - \rho r z + \frac{r^2}{2}\right]^{1/2}} \quad (9.11)$$

where $z = \cos\theta$, θ being the angle between $\underset{\sim}{\rho}$ and $\underset{\sim}{r}$.
The integration over z can be done first to give

$$F_2 = 16\pi^2 A^2 \int_0^\infty d\rho \int_0^\infty dr \cdot \exp\left[-\alpha\left(2\rho^2 + \frac{3}{2}r^2\right)^{1/2}\right] \cdot$$

$$\cdot\, \rho \left\{ \left|\rho + \frac{r}{2}\right| - \left|\rho - \frac{r}{2}\right| \right\} \quad (9.12)$$

By the substitution $\sqrt{2}\rho = R\cos\theta$ and $\sqrt{\frac{3}{2}}\, r = R\sin\theta$ the integral F_2 can be evaluated giving the following expression

$$F_2 = \frac{\alpha^2}{5} \quad (9.13)$$

Now the upperbound for $|A_{3/2}|^2$ is given by

$$|A_{3/2}|^2 < \frac{e^4}{3E_g^2} \cdot \frac{3\alpha^2}{10} \quad (9.14)$$

we get the value 0.10 % for the upperbound.

4. When Irving-Gunn function is used for u the integration can be performed as in the case of Irving function. Now we get for the upperbound of $|A_{3/2}|^2$ the inequality

$$|A_{3/2}|^2 < \frac{e^4}{9E_g^2} \cdot \alpha^2 \quad (9.15)$$

with $\alpha = 152$ MeV . We get 0.23 % for the upperbound.

5. There are some evidences for the violation of charge independence of nuclear forces. The difference in np force and pp force or nn force can be understood as follows : The proton and neutron exchange charged pions whereas proton and proton or neutron and neutron exchange neutral pions. The difference in the masses of charged and neutral pions may cause difference in the np and the pp or the nn forces.

Thus in addition to the Coulomb forces the violation of charge independence may result in the admixture of $T=3/2$ state in the ground state of He^3. But now even the H^3 ground state will contain $T=3/2$ state.

An upperbound to $|A_{3/2}|^2$ has been found by Werntz and Valk[1] including the charge dependent nuclear potential in V. Their V is given by

$$V = \sum_{i<j} \frac{e^2}{r_{ij}} T_c + \sum_{i<j} \frac{1}{6} V_0(r_{ij}) \sigma_i \cdot \sigma_j \tau_{iz} \tau_{jz} \quad (9.16)$$

where

$$T_c = \frac{1}{4} (1+\tau_{iz})(1+\tau_{jz})$$

The extra term included in the potential V is charge dependent but charge symmetric. Using a Gaussian form for u they have obtained a value 0.66 %, for the upperbound of $|A_{3/2}|^2$. It is worth examining their result by taking Irving and Irving-Gunn forms for u. The

1) C. Mantz Werntz and H.S.Valk, Phys. Rev. Letters, **14**, 910 (1965).

integrals now cannot be solved analytically and calculations are being done using a computor.

6. The measurement of the mass radii[1] of He^3 and H^3 from the d-d interactions show that

$$R(He^3) > R(H^3) \;;\quad R(H^3) = 1.6 \text{ fermi}$$

$$\text{and} \quad R(He^3) = 1.75 \text{ fermi.}$$

Such a wide difference may demand charge asymmetric nuclear forces. Also the calculation by Okomoto[2] for the difference of the $He^3 - H^3$ binding energies shows the necessity of charge asymmetric nuclear forces, as the Coulomb forces are not sufficient to explain the difference in the binding energies.

Hence a modification of the potential V used by Wertz and Valk becomes necessary. We have to replace (9.16) by

$\frac{1}{6}(\vec{\sigma_i}\cdot\vec{\sigma_j})$

$$V = V_c + \frac{1}{4}\sum_{i<j}\Big[V_{pp}(r_{ij})(1+\tau_z(i))(1+\tau_z(j)) + V_{nn}(r_{ij})(1-\tau_z(i))(1-\tau_z(j))\Big] + \frac{1}{4}\sum_{i,j} V_{np}(r_{ij})(1+\tau_z(i))(1-\tau_z(j)) \qquad (9.17)$$

1) R.B. Theus, W.I. McGarry and L.A. Beach, Phys. Rev. Letters, 14, 282 (1965).

2) K. Okomoto, Phys. Letters, 11, 152, (1964)

Each of the potentials V_{pp}, V_{nn} and V_{np} contains two parameters one giving the strength of the force and the other, the range of the force. The six parameters have to be calculated from a careful analysis of nucleon-nucleon scattering experiments. Once these parameters are known (they are not known yet) a correct upperbound to $|A_{3/2}|^2$ can be found. *

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* The author would like to acknowledge the useful discussion with Dr. R.J.Oakes in the evaluation of the integrals.

CHAPTER 10 *

1. Just as the photoproduction and scattering of pions from the three nucleon systems He^3 and H^3 can be used to study the structure of these nuclei, the same processes can be used to analyse the grand state of He^4. One interesting quantity for analysis is the root mean square radius of He^4.

In the analysis of electron scattering by He^4 Hofstadter[1)] using Gaussian wave function found a good fit taking the value 1.61 fermi for the root mean square radius of He^4. But when Dalitz and Ravenhall[2)] computed the rootmean square radius from the wave function of Clark[3)] the resulting radius was only about 1.07 fermi. Squires et al[4)] while calculating the contributions due to exchange effects in $H^3 - He^4$ scattering with the use of Gaussian wave function obtained good agreement using a value (1.42 ± 0.14) fermi for the root mean square radius.

When we consider the elastic photoproduction of neutral pions from He^4, the differential cross section is necessarily a function of the root mean square radius.

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* K. Ananthanarayanan ~~(submitted to~~ Nuclear Physics) (in print)
1) R. Hofstadter, Revs. Mod. Phys. 28, 214 (1956)
2) R.H. Dalitz and Ravenhall (unpublished)
3) A.C. Clark, Proc. Phys. Soc. (London) A 67, 323 (1954)
4) E.J. Squires, A.B. Forest and P.E. Hodgson, Nucl. Phys. 42, 490, (1962)

The root mean square radius enters in the calculation of the cross-section through the nuclear wave function. Thus the measurements of differential cross section will give the correct root mean square radius of He^4.

Both He^4 and π° has spin zero while the photon has spin unity. So, in the elastic photoproduction of π° from He^4, $(\gamma + He^4 \longrightarrow He^4 + \pi^\circ)$ if the mesons are produced in the S -(angular momentum zero) - state, then the initial total angular momentum will be unity whereas in the final state the total angular momentum will be zero so that the law of addition of angular momentum is violated. Hence the elastic photoproduction in the S state is excluded and at fairly low energies we must ascribe all elastic production to P - states. Thus the theory of Chew and Low, where only P wave phase shifts are taken, holds perfectly well for this nucleus.

The calculation gives a very low value (~ 1 fermi) for the root mean square radius of He^4. Though due to the large binding energy the meson clouds in the He^4 nucleus are contracted to give a rather low root mean square radius a value as low as one fermi cannot be understood this way. Perhaps inclusion of other states increases the value of the rootmean square radius.

2. Cohen[1] has classified the ground state wave functions of the He^4 nucleus. He has given three S-state functions, eight P-state functions and six D-state wave functions. Of these seventeen functions there is only one S-state, one P-state and one D-state in which the internal part (radial part) is completely symmetric. If only central, spin-independent forces are present, only the symmetric S-state occurs. The introduction of a tensor force couples the D states to this S-state. The P states occur only in second order, coupled to D states and are usually neglected. One might expect the principal D-state to be the most important one (among the D-states) but calculations indicate that other D-states also make a significant contribution to the binding energy[2]. In general, a convenient guide to constructing trial wave function would be to commence with the fully symmetric S and D states.

3. As a first approximation let us assume that the ground state of He^4 is represented completely by the fully symmetric S-state. This is reasonable as the percentages of D state admixtures is known to be small from binding energy calculations. of Abraham et al[2]. Now the ground state of He^4, without using isospin formalism, can be written as

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1) L. Cohen, Nucl. Phys. 20, 690, (1960); Nucl. Phys. 22, 492, (1961).

2) G. Abraham L. Cohen and A.S. Roberts, Proc. Phys. Soc. A 68, 265 (1955).

$$|He^4\rangle = \frac{1}{2}\left(\alpha(1)\beta(2) - \alpha(2)\beta(1)\right)\left(\alpha(3)\beta(4) - \alpha(4)\beta(3)\right) \cdot$$

$$\cdot \exp(i\underset{\sim}{k}\cdot\underset{\sim}{R})\, u(\underset{\sim}{r}_1, \underset{\sim}{r}_2, \underset{\sim}{r}_3, \underset{\sim}{r}_4)$$

where $\underset{\sim}{r}_i$ is the positron coordinate of the i th nucleon, nucleons 1 and 2 are like charged (say proton) and hence nucleons 3 and 4 are like charge (neutrons), α represents the spin up nucleon state and β the spin down nucleon state. $\underset{\sim}{R}$ is the centre of mass of He^4 and it is given by the expression

$$\underset{\sim}{R} = \frac{1}{4}(\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3 + \underset{\sim}{r}_4)$$

$\underset{\sim}{k}$ is the momentum of the centre of mass of He^4 .

$u(\underset{\sim}{r}_1, \underset{\sim}{r}_2, \underset{\sim}{r}_3, \underset{\sim}{r}_4)$ is the normalised radial wave function and we assume the following Gaussian form for u :

$$u = A \exp\left[-\lambda \sum_{i>j=1}^{4} r_{ij}^2\right]$$

where

$$r_{ij} = |\underset{\sim}{r}_i - \underset{\sim}{r}_j|$$

and the parameter λ is related to the root mean square radius $\langle r\rangle$ by the equation

$$\langle r\rangle = \left(\frac{9}{64\lambda}\right)^{1/2}$$

The matrix element of the transition operator T for the elastic photoproductions of neutral pions from τ can be written as

$$\langle f|T|i\rangle = \langle f| \sum_j t^{(j)} e^{i(\underset{\sim}{\nu}-\underset{\sim}{\mu})\cdot \underset{\sim}{r}_j} |i\rangle$$

where $\underset{\sim}{\nu}$ and $\underset{\sim}{\mu}$ are the momenta of the incident photon and the emitted pion respectively. $t^{(j)}$ has the form

$$t^{(j)} = \frac{1}{2} t_p^{(j)} (1+\tau_3^{(j)}) + t_n^{(j)} (1-\tau_3^{(j)})$$

where t_p and t_n can be written as usual

$$t_p = i\underset{\sim}{\sigma}\cdot \underset{\sim}{K}_p + L_p$$

$$t_n = i\underset{\sim}{\sigma}\cdot \underset{\sim}{K}_n + L_n$$

The differential cross section

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \frac{\mu\mu_0}{2} \sum_{\underset{\sim}{\epsilon}} |\langle f|T|i\rangle|^2$$

$$= (2\pi)^{-2} \mu\mu_0 \sum_{\underset{\sim}{\epsilon}} 2|L|^2$$

where μ_0 denotes the energy of the emitted pion and $\underset{\sim}{\epsilon}$ the polarization of the photon. L is given by

$$L = \frac{1}{2}(L_p + L_n)$$

Using the expression of Chew et al[1] for L and retaining only the dominant δ_{33} phase shift, (in the energy region under consideration, the other phase shifts δ_{13}, δ_{31} and δ_{11} are negligible in comparison with ~~δ_{33})~~ δ_{33}) we get

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2}\mu_0 \frac{128}{9\mu^3} c^2 \lambda^2 \nu_0^2 \sin^2\theta \cdot \sin^2\delta_{33} \, |F_0|^2$$

where

$$\lambda = \frac{\mu_p - \mu_n}{4Mf^2} \quad ; \qquad c = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}}$$

and

$$F_0 = \int u^2 \exp[i(\underset{\sim}{\kappa} - \underset{\sim}{\mu}) \cdot (\underset{\sim}{r}_4 - \underset{\sim}{R})] \, d^3\underset{\sim}{r}_i$$

4. In order to evaluate

$$F_0 = |A|^2 \int \exp[-2\lambda \sum_{i>j=1}^{4} r_{ij}^2 + i\underset{\sim}{k} \cdot (\underset{\sim}{r}_4 - \underset{\sim}{R})] \, d^3\underset{\sim}{r}_i$$

we change the coordinates to the following

$$\frac{1}{4}(\underset{\sim}{r}_1 + \underset{\sim}{r}_2 + \underset{\sim}{r}_3 + \underset{\sim}{r}_4) = \underset{\sim}{R}\,, \qquad \underset{\sim}{r}_4 - \underset{\sim}{r}_2 + \underset{\sim}{r}_3 - \underset{\sim}{r}_1 = 2\underset{\sim}{r}\,,$$

$$\underset{\sim}{r}_2 - \underset{\sim}{r}_1 = \sqrt{2}\,\underset{\sim}{\rho}_1 \,, \qquad \underset{\sim}{r}_4 - \underset{\sim}{r}_3 = \sqrt{2}\,\underset{\sim}{\rho}_2 \,.$$

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1) G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, Phys. Rev., 106, 1345, (1957)

so that (without mentioning the Jacobian explicitly)

$$F_0 = |A|^2 \int \exp\left[-8\alpha\left(\rho_1^2+\rho_2^2+r^2\right) + i\,\underset{\sim}{k}\cdot\underset{\sim}{r} + \frac{1}{\sqrt{2}}\, i\,\underset{\sim}{k}\cdot\rho_2\right] d^3\underset{\sim}{\rho_1}\, d^3\underset{\sim}{\rho_2}\, d^3\underset{\sim}{r} .$$

As in the three body case, we take the cartesian system of coordinates for $\underset{\sim}{\rho_1}$, $\underset{\sim}{\rho_2}$ and $\underset{\sim}{r}$. The integral can be expressed then as a product of the integrals of the following type:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-a\left(x^2+y^2+z^2+2gx+2by\right)\right] dx\,dy\,dz$$

which when subjected to a translation in the x-y-z space can be proved to be equal to

$$\left(\frac{\pi}{3}\right)^{3/2} \exp\left[a\left(g^2+b^2\right)\right] .$$

Thus it is found

$$F_0 = \exp\left[\frac{-3k^2}{128\alpha}\right]$$

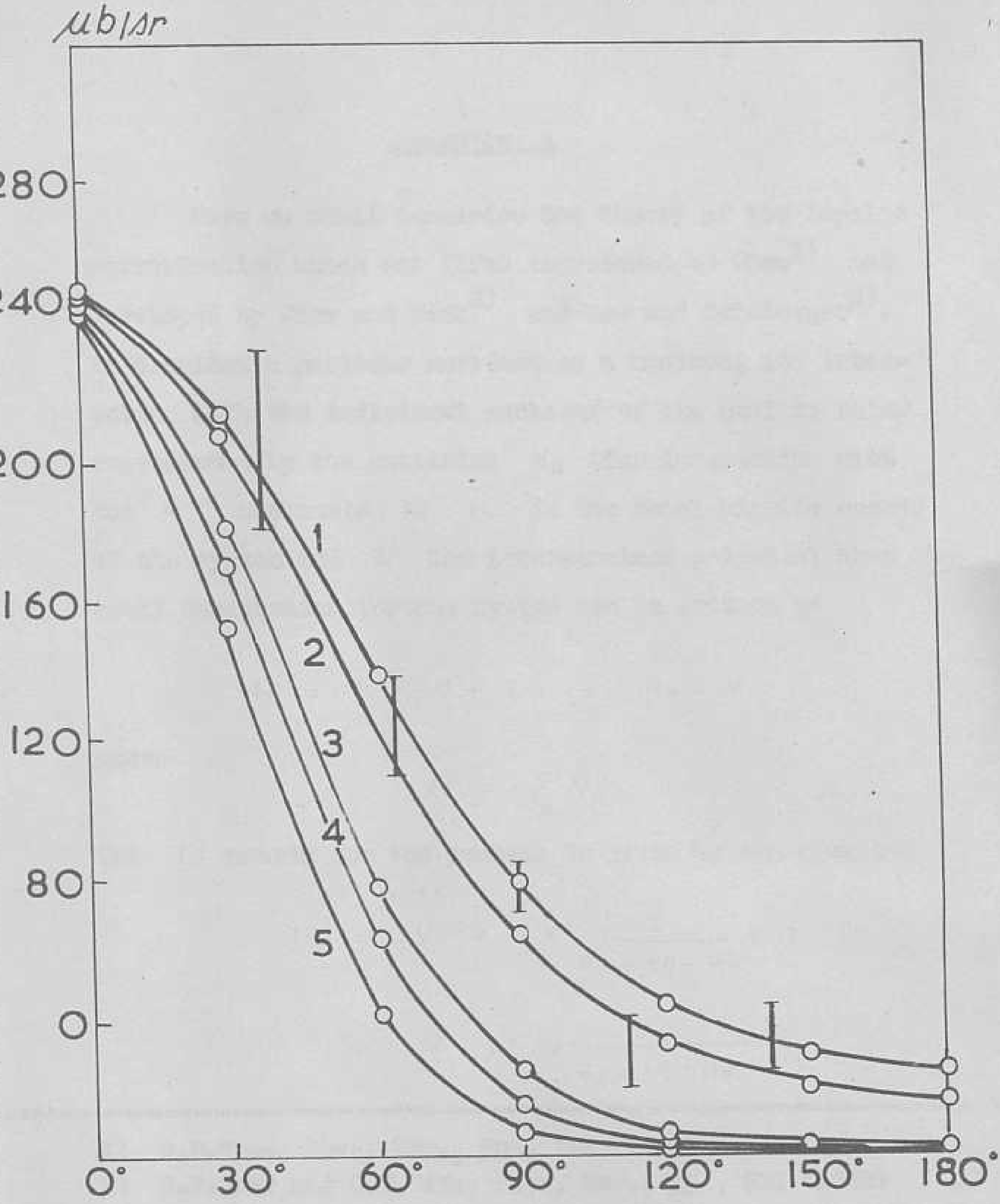
5. Numerical estimates of $\frac{d\sigma}{d\Omega} / \sin^2\theta$ are made for the incident photon energy of 300 Mev in the centre of momentum system for various values of θ .

In fig. (10.1) $\frac{d\sigma}{d\Omega} / \sin^2\theta$ is plotted against θ and the theoretical values represented by curves 1, 2, 3, 4 and 5 correspond to .9191 fermi, 1.006 fermi , 1.3 fermi , 1.42 fermi and 1.61 fermi respectively.

Comparison has been made with the experimental data of de Saussure and Osborne[1] and curves 1 and 2 give

1) G. de Saussure and L.S. Osborne, Phys. Rev. 22 , 843 (1955)

reasonably good fits. The corresponding values of the root mean square radius are smaller than the values used by Hofstadter and Squires et al but very near to the value computed by Dalitz and Ravenhall.



$\frac{d\sigma}{d\Omega}$ FOR $\gamma + He^4 \rightarrow He^4 + \pi^0$ AS A FUNCTION OF THE ROOT MEAN SQUARE RADIUS AT $E_\gamma^{CM} = 300$ mev.

FIG.10·1

Appendix A

Here we shall summarise the theory of the impulse approximation which was first introduced by Chew[1] and developed by Chew and Wick[2] and Chew and Goldberger[3]. We consider a particle incident on a nucleus, its interaction with the individual nucleons of the nucleus being represented by the potential V_k (for interaction with the k^{th} nucleon). If K is the total kinetic energy of the system and U the inter-nucleon potential then the total Hamiltonian for the system can be written as

$$H = K + U + V = H_0 + V \tag{A.1}$$

where

$$V = \sum_k V_k$$

The T- matrix for the process is given by the equation

$$T = V + V \frac{1}{E_a + i\eta - H_0} V T$$

$$= V + V \frac{1}{E_a + i\eta - H_0 - V} V \tag{A.2}$$

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1) G.F.Chew, Phys. Rev., 80, 196 (1950)
2) G.F.Chew and G.C.Wick, Phys. Rev., 85, 636 (1952)
3) G.F.Chew and M.L.Goldberger, Phys. Rev., 87, 778 (1952)

The first form for T in (A.2) is obtained from the S-matrix expansion of Dyson by doing the space and (time-ordered) time integrations separately. The term $+i\eta$ in the denominator represents the outgoing wave boundary condition. A limit of $\eta \to 0$ is implied in (A.2)

We define the two-particle scattering matrix

$$t_k = V_k \, \omega_k \tag{A.3}$$

where

$$\omega_k = 1 + \frac{1}{E_\ell + i\eta - K - V_k} V_k \tag{A.4}$$

Now if B and b are two operators defined by

$$B = \frac{1}{E_a + i\eta - H_o - V} A$$

$$b_k = \frac{1}{E_\ell + i\eta - K - V_k} A$$

where A is any operator, then

$$B = b_k + \frac{1}{E_a + i\eta - H_o - V} \{ [U, b_k] + (V - V_k) b_k \} \tag{A.5}$$

This result follows on using the operator identity

$$\frac{1}{x-y} - \frac{1}{x} = \frac{1}{x-y} \, y \, \frac{1}{x}$$

We use (A.5) in (A.2) which we rewrite as

$$T = \sum_{k=1}^{N} \left\{ V_k + V \frac{1}{E_a + i\eta - H_o - V} V_k \right\} \tag{A.6}$$

From (A.5) and (A.4) we have

$$\frac{1}{E_a + i\eta - H_0 - V} V_k = (\omega_k - 1) +$$

$$+ \frac{1}{E_a + i\eta - H_0 - V} \{ [U, \omega_k] + (V - V_k)(\omega_k - 1) \} \qquad (A.7)$$

Substituting (A.7) in (A.6) we finally obtain

$$T = \sum_{k=1}^{N} \{ t_k + V \frac{1}{E_a + i\eta - H_0 - V} [U, \omega_k] +$$

$$+ [1 + V \frac{1}{E_a + i\eta - H_0 - V}] (V - V_k)(\omega_k - 1) \} \qquad (A.8)$$

which may be called the impulse series. The impulse approximation consists in retaining only the first term in (A.8) which represents a sum of two-body matrix elements. The second term will be negligible if the inter-nucleon potential U is negligible. The third term of (A.8) gives the effect of multiple scattering.

Appendix B

The precursor of the dispersion theoretic approach of Chew et al[1] to the pion-nucleon scattering and the photo-pion-production was the Chew-Low theory[2] which had the following main features :

(1) Pion-nucleon interaction is linear in the pion field variables (Yukawa type),

(2) Charge independence of pion-nucleon interaction,

(3) Predominance of P- wave scattering due to psuedo-scalar nature of pion,

(4) Neglect of antinucleon states which are important for the relativistic γ_5 theory, necessitating a static nucleon and $\underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla}$ type interactions which requires a cut-off in the momentum space (equivalently evaluation of the meson nucleon fields at slightly different points of space and multiplication by a weighting factor which is function of the distance between the two points) to avoid divergences.

With these assumptions and the two parameters $f^2 = 0.08$ the renormalized unrationalized coupling constant and an energy cut-off ω_{max} ($\sim 6\, m_\pi$) they were able to explain the most striking feature of the low energy pion-nucleon scattering, namely, the existence of the resonance

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1) G.F.Chew, M.L.Goldberger, F.E.Low and Y. Nambu, Phys. Rev., 106 , 1345, (1957)

2) G.F.Chew and F.E.Low, Phys. Rev. 101, 1570, 1579 (1956)

in the $T = 3/2$, $J = 3/2$ state at a mass 1238 mev. With the same parameters, they were able to explain the photo-pion-production also.

The essential feature of the photoproduction theory was the decomposition of the current operator into : (1) nucleon part, (2) meson part, and (3) pion-nucleon interaction part. The first part depends on the pion nucleon scattering phase shifts, the second part, the pion propagator term gives rise to all partial waves and the third term, namely, the catastropic term, gives rise to the S - wave photoproduction.

The work of Chew, Goldberger, Low and Nambu was to extend this static model to a full relativistic version by the use of single variable dispersion relation. The solution of the equation for the low partial waves gave rise to matrix ~~as~~ elements which contains $1/M$ corrections to the Chew-Low theory and also does not depend on a cut-off factor. These are the amplitudes which have been used for the photoproduction processes in this thesis.

It is interesting to note that the application of double (Mandelstam) dispersion relations leads to essentially the same features.[1]

However for the pion nucleon scattering processes we use only the amplitudes of Chew and Low.