STUDIES

IN

ELEMENTARY PARTICLE INTERACTIONS IN IMPULSE OR SINGLE PARTICLE EXCHANGE APPROXIMATION

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PREFACE

This thesis comprises work done by the author during the period 1961-1964 under the supervision of Professor Alladi Ramakrishnan, formerly Professor of Theoretical Physics, at the University of Madras and now Director of the Institute of Mathematical Sciences, Madras.

The thesis consists of work done in the field of Elementary Particle Interactions in the framework of Impulse or Single-Particle exchange approximation. It is divided into three parts, Part I dealing with a study of Deuteron Polarization in certain reactions, Part II with the scattering of pions from nuclei in the framework of the Shell Model and Part III with Strong Interactions. Part III is a detailed study of isobar production in nucleon-nucleon collisions, where a generalization of the one pion exchange model has been outlined.

Six papers which form the subject matter of this thesis have been published or are in the process of publication as research papers in established journals. Collaboration in some papers either with my guide Professor Alladi Ramakrishnan or with my colleagues was necessitated by the nature and range of the problems dealt with and due acknowledgment is made in the chapters.

The author is particularly grateful to Professor Alladi Ramakrishnan for his guidance and encouragement throughout the course of this work. He is also grateful to the University of Madras and the Institute of Mathematical Sciences, Madras for offering the facilities that enabled this work to be carried out. The author acknowledges with thanks the fellowship received from the Council of Scientific and Industrial Research (India), since February 1962.

It is a pleasure to acknowledge the benefit of various discussions with my colleagues at the Institute of Mathematical Sciences on the work presented here.

R.K. Umerjee.

The Institute of
Mathematical Sciences, Madras.
October, 1964.

(R.K. UMERJEE)

CONTENTS

PREFACE Page

CHAPTER I . GENERAL INTRODUCTION 1

PART I

STUDY OF DEUTERON POLARIZATION

CHAPTER II . DEUTERON POLARIZATION FOLLOWING ELASTIC SCATTERING OF POSITIVE PIONS. 15

1. Introduction

2. Method

3. Deuteron polarization in $\pi^+ + D \longrightarrow \pi^+ + D$.

4. Numerical Results

5. Conclusion

CHAPTER III . POLARIZATION PHENOMENA IN ELASTIC ELECTRON-DEUTERON AND ELECTRON-PROTON SCATTERING. 42

1. Introduction

2. Elastic e-D scattering

3. Deuteron Polarization

4. Helicity of the electron in the final state of elastic e-D scattering

5. Electron Helicity with linearly polarized target deuterons

6. Electron Helicity with linearly polarized target protons.

Appendix

CHAPTER IV . DEUTERON POLARIZATION FOLLOWING ELASTIC NEUTRAL PION PHOTO-PRODUCTION. 66

1. Introduction

2. Calculation using CGLN amplitudes

3. Numerical results

4. Conclusions

PART II

SCATTERING OF PIONS FROM NUCLEI

CHAPTER V . SCATTERING OF PIONS FROM NUCLEI. 88

1. Introduction

2. Method

3. Numerical results and conclusions

PART III

GENERALIZATION OF THE ONE-PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS.

CHAPTER VI. GENERALIZATION OF THE ONE PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS. 116

1. Introduction

2. Matrix element and pole contributions

3. Isospin

4. Coupling constants and form factors

5. Total cross-sections

6. C. m. Energy spectra

7. C. m. Angular Distributions

8. Conclusions and Tests of the Model

CHAPTER VII. ISO-SPIN ANALYSIS OF ISOBAR PRODUCTION IN NUCLEON-NUCLEON COLLISIONS. 195

CHAPTER VIII. POLARIZATION EFFECTS IN THE GENERALIZED SINGLE PARTICLE EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS. 227.

1. Introduction

2. Polarization formalism

3. Incident Nucleon and Target nucleon both polarized.

4. Polarization correlations with one initial nucleon polarized: Pion exchange.

5. Polarization correlations with one initial nucleon polarized: ρ exchange.

6. Conclusions.

Appendix.

CHAPTER I

GENERAL INTRODUCTION

This thesis comprising studies on some elementary particle interactions has a two-fold objective, namely

1) to study polarization and scattering phenomena in collisions involving composite systems of particles using impulse approximation; and
2) to analyse isobar production in nucleon-nucleon collisions in the framework of the single particle exchange model and study some related polarization effects.

The inadequacy of perturbation methods to tackle the problem of strong interactions is too well-known and the only approach which has obtained a certain measure of success in this context has been that of dispersion theory based on the analytic continuation of the S-matrix[1] and the use of symmetry laws. Even in this approach it is fortuitous that the discovery of a host of resonances has introduced considerable simplification, in the sense that many-particle

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1) G.F.Chew, "S-matrix Theory of Strong Interactions", W.A.Benjamin, Inc., New York (1961)

intermediate states can be assumed to be dominated by the corresponding resonances and can be approximated by these single particle states. Hence an attempt has been made to study isobar production in nucleon-nucleon collisions in the framework of the single particle exchange model and investigate some related polarization effects. This constitutes one half of the thesis (Part III).

Another useful approximation which is expected to play a fundamental role in the S-matrix theory of strong interactions[2] is the impulse approximation, dealing with collisions involving composite systems of particles. The principle of superposition of potentials is the usual starting point for the derivation of impulse approximation. But if S-matrix theory is treated as more fundamental than potential theory, the principle of superposition of potentials must in fact follow as a consequence of the impulse approximation. Hence this approximation can be regarded as a fundamental law at least in the case of strong interactions where the Born approximation is not valid, while this (impulse approximation) seems to hold.[3] Hence it has

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2) R.E.Cutkosky, Phys. Rev. 125 (1962), 745
J.G.Belinfante, Phys. Rev. 128 (1962), 2403

3) H.A.Bethe, Ann. Phys. 3 (1958), 190.

been considered worthwhile to apply the method of impulse approximation to the study of 1) interactions involving the deuteron with particular emphasis on its polarization properties; and 2) scattering of pions from nuclei. This constitutes another half of the thesis (Parts I and II).

In the first part, we are concerned with a study of the residual deuteron polarization in certain reactions where we consider interactions of the deuteron with spin zero, half and one (pion, electron and the photon) particles in impulse approximation.

It is expected that new techniques in the development of low temperatures and high magnetic fields can lead to the availability of polarized targets for use in the study of nuclear reactions and indeed Chamberlain et al[4] have produced a target containing polarized Hydrogen. Hence it is hoped that polarized deuterons will also be available in the near future and with this in view it is considered worthwhile to study polarization of the deuteron and proton in certain reactions as well as electron helicity with polarized deuteron and proton targets.

The ease of calculational technique afforded by our trace rules for spin one matrices is the characteristic feature of our approach to discuss deuteron polarization.

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4) E. Segre, Lectures given at the Institute of Mathematical Sciences, Madras (1962) : (MATSCIENCE Report 18).

The deuteron being a spin one particle, its state of polarization is not completely specified by the orientation of the spin axis. There are also orientational features which may be described in terms of tensors, as opposed to the vectors which specify the spin orientation. In our approach, the calculation of the orientation parameters is facilitated by a method slightly different from the one[5] usually used to discuss deuteron polarization. The set of operators given by ~~the~~ Lakin[5] includes the unit matrix, two linear combinations and three second-rank tensor products of spin operators, as well as the Hermitian adjoint of the latter three. We extend the usual trace rules for Pauli spin matrices to the 3 x 3 matrices for spin one and choose a set of base matrices which are not necessarily the representations of certain spherical tensor operators, but correspond to linear combinations of them. The evaluation of the average expectation values of these is accomplished elegantly with the help of the trace rules and given these values the estimates of the conventional parameters are easily obtained. In addition we have given certain trace rules which are of great utility in helicity calculations, as amply borne out by the examples shown in this thesis.

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5) W. Lakin, Phys. Rev. 98 , (1955), 139

In the next part, scattering of pions from nuclei is studied in view of the increasing importance[6] given to the pion in its role as a tool to probe the structure of nuclei. The scattering of pions from nuclei has been studied experimentally for several years and thus a considerable body of experimental fact is already established. Studies of the elastic scattering of pions by atomic nuclei have frequently been made within the framework of the optical model. But the usual optical model approach does not make use of the complete angular distribution of the elementary pion-nucleon scattering process, but uses only the forward scattering amplitude. We give an alternative point of view (based on the impulse approximation) which uses the complete angular dependence of the $\pi-N$ scattering process modified by a form factor of the nucleus. The agreement obtained with experimental results is quite good.

In the last part of the thesis, we study isobar production in nucleon-nucleon collisions in the framework of the single particle exchange model.

Two of the important steps in achieving an understanding of pion productions in nucleon-nucleon collisions were (i) the isobar model and (ii) the one-pion-exchange (OPE) model. Recently Ferrari and Selleri have put forward a theory

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6) T.Ericson: 1963 International Conference on High Energy Physics and Nuclear Structure, Geneva, CERN 63-28, p. 47.

of single pion production between 1 and 3 Gev in which the OPE model is modified by including a form factor[7]. In our work, we attempt to go beyond the OPE model by taking other mechanisms into account. Although the extended OPE model of Ferrari and Selleri agrees well with the observed total cross-sections and laboratory energy spectra, one is not justified in concluding that the basic mechanism is OPE ~~nucleus~~ unless one can also show that other contributions are negligible in comparison. Detailed calculations have led us to believe that other contributions are not negligible.

We have also analysed some of the polarization effects in these processes of isobar production. The main object of studying the polarization effects is to gain some insight into the mechanisms involved in isobar production. Apart from this, such a study is also of intrinsic interest as an example of polarization effects in a single particle exchange model.

The thesis is thus divided into three parts.

In chapter II (Part I), the first two sections are devoted to the motivation and the method for our study of deuteron polarization and in the rest of the chapter, the spin state of the deuteron following elastic scattering of positive pions[8] is studied in impulse approximation with

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7) E. Ferrari and F. Selleri, Nuovo Cim. 27 (1963) 1450

8) R.K.Umerjee, Nuclear Physics (in print)

unpolarized target. We use the basic amplitudes of Chew and Low[9] for pion-nucleon scattering. It is observed that taking into consideration only the dominant δ_{33} phase-shifts, the differential alignment paramter is independent of the incident pion energy. Also the vanishing of the spin expectation value seems to indicate the presence of "aligned" deuterons in the final state. The estimates of these parameters taking into account all the p-wave phase-shifts exhibit a small energy dependent correction in the values of $\langle T_0^2 \rangle$ and a small percentage polarization. In both cases, it is found that the density matrix in the final state is diagonalised in a natural manner thus obviating the necessity for constructing a new set of basic states as is sometimes necessary. It is also shown that the ellipsoid associated with the "tensor polarization" reduces to an oblate spheroid.

Chapter III is concerned with a study of the following polarization phenomena[10] : 1) The final spin state of the deuteron in the elastic e-D scattering with unpolarized initial particles and 2) the helicity of the electron in the final state of the elastic e-p and e-D scattering with the target particles linearly polarized in the direction

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9) G.F.Chew and F.E.Low, Phys. Rev. 101 (1956), 1570.

10) G.Ramachandran and R.K.Umerjee, Physical Review (to be published)

of the incoming electron beam. The polarization and alignment parameters of the deuteron are expected to be independent of the radial wave function if we neglect the d-state admixture and by studying the spin-state of the deuteron in the final state at two energies and angles corresponding to a given momentum transfer one can determine the electric and magnetic form factors of the neutron (with a knowledge of the proton form factors). This is in contrast to the study of the differential cross-section where such a paired observation is primarily used to eliminate the body form factor and consequently leads only to a determination of the ratio of the two iso-scalar form factors. Also in view of the availability of polarized proton targets, we have studied the electron helicity in elastic e-p scattering with the protons linearly polarized in the direction of the incident electron beam of energy 900 Mev and found that the back scattered electrons are completely polarized, a result which is quite striking. Similar calculation has been done for elastic e-D scattering under identical conditions and the percentage polarization for back scattered electrons is 75 % . In addition, k other modes of target polarization for the deuteron are discussed.

In chapter IV, the spin state of the deuteron following elastic photo-production of neutral pions is

investigated[11] in impulse approximation using the amplitudes of Chew et al[12] . It is found that the deuterons in the final state are 'polarized' perpendicular to the reaction plane and one of the axes of alignment coincided with this direction. Now, one can easily determine the co-ordinate system defined by the principal axes and estimate the values of the parameters in that system. It is found that corresponding to pion production at an angle θ with respect to the beam direction, the residual deuteron is aligned with one of the principal axes making nearly an angle $-\theta$ and the other axis ($90 + \theta$) in the plane of production. In addition, some interesting comparisons have been made with the results of chapter II.

Chapter V (Part II) is concerned with a study of scattering of pions from nuclei in the framework of the shell model and the impulse approximation.[13] Cross-sections for scattering from closed shell nuclei like carbon and oxygen are presented and these are compared with the experimental values of Fujii[14] , Edelstein et al[15] and Baker et al.[16]

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11) G.Ramachandran and R.K.Umerjee, Nuclear Physics 54 (1964), 665
12) Chew et al , Phys. Rev. 106 (1957), 1345.
13) R.K.Umerjee, Nuclear Physics (In print)
14) T.A.Fujii, Phys. Rev. 113 (1959), 695
15) R.M.Edelstein, W.F.Baker and J.Rainwater, Phys. Rev. 122 (1961), 252.
16) W.F.Baker, J. Rainwater and R.E.Willians, Phys. Rev. 112 (1958), 1763.

The agreement obtained is quite good. Alternatively the scattering of pions from nuclei can be used as a tool to probe the structure of nuclei.

Chapter VI (Part III) of this thesis comprises a detailed study[17] of the production of $P_{3/2}$ and $D_{3/2}$ nucleon isobars in nucleon-nucleon collisions. These processes are of interest experimentally in connection with single pion production in nucleon-nucleon collisions. Lindenbaum and Sternheimer's work[18] showed that at not too low values of the incident nucleon energy, isobar formation could be assumed to be the dominant mechanism. Later work using the OPE model[19] showed the importance of the latter mechanism. The object of the present work is to examine whether mechanisms other than OPE are also important and how one can detect them. In our model, we approximate the production amplitude by the exchange of scalar, pseudo-scalar, vector and axial vector states with definite isospin, which may in some of the cases be identified with known multi-pion resonances. In this chapter we calculate the total cross-sections, c.m. energy spectra

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17) Alladi Ramakrishnan, K. Raman and R.K.Umerjee, Nuclear Physics (In print)
18) S.J.Lindenbaum and R.Sternheimer, Phys. Rev. 105 (1957), 1874; 123 (1961), 333.
19) G. da Prato, Nuovo Cim. 22 (1961), 123; J.Iizuka and A. Klein, Phys. Rev. 123 (1961), 669; F. Selleri, Phys. Rev. Lett. 6 (1961), 64.

and c.m. angular distributions (for different assumptions about the form factors) and compare the results of the calculation with the available experimental data. We have analyzed isobar production under assumptions, which were made in order tokeep the calculations manageable. We believe that most of the gross features are predicted correctly by this simplified model.

The main conclusion is that although OPE is probably the most important single contribution to isobar production between 1 and 3 Gev, other contributions when summed are by no means negligible. Measurements on pp reactions alone will tend to give a wrong indication of the importance of the N_2^* production and of the exchange of states of zero isospin, as in these reactions, N_2^* production is suppressed by the isospin factors alone. To understand the relative importance of $T = 1/2$ isobars in single pion production in nucleon-nucleon collisions in general, extensive measurements on np reactions are necessary. Experiments that would test the presence of the various possible mechanisms are outlined.

In chapter VII[20] we have derived the expressions for the differential and total cross-sections for the different charge states in terms of the exchange of states with definite isospin. We note that although the isospin analysis of

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20) Alladi Ramakrishnan, K. Raman and R.K.Umerjee (to be published)

production reactions has often been made, no systematic analysis has been made in terms of the exchange of states with definite isospins. Such an analysis is of interest in view of the importance of peripheral-isobar models.

In the concluding chapter (Chapter VIII), we analyze some of the polarization effects in these processes of isobar production.[21] In order to obtain more information about the types of exchange that dominate, it is necessary to have some knowledge of the angular momenta of the exchanged states. Knowledge of this may be obtained by measuring the various polarization correlations. We note that there are a comparatively simple tests which can detect whether the exchange of states with non-zero spin is important. For instance, if one performs an experiment with a polarized nucleon beam incident on a polarized nucleon target, then if the angular distribution or the total cross-section shows a dependence on the relative orientation of the polarization vectors of the two nucleons in the initial state, then this is evidence that there is an exchange of a state of non-zero spin. The type of correlation expected for ρ exchange is given in this chapter and we also discuss correlations between the polarizations of an initial nucleon and a final nucleon.

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21) Alladi Ramakrishnan, K. Raman and R.K.Umerjee, Nuclear Physics (In print)

We conclude by saying that the polarization correlations considered here when combined with a measurement of correlations of the type discussed by Treiman and Yang[22] and by Stodolsky and Sakurai[23] should give reliable information on the angular momentum of the exchanged states.

22) S.B.Treiman and C.N.Yang, Phys. Rev. Lett. 8 (1962), 140.

23) L. Stodolsky and J.J.Sakurai, Phys. Rev. Lett. 11 (1963), 90.

PART I

STUDY OF DEUTERON POLARIZATION

CHAPTER II

DEUTERON POLARIZATION FOLLOWING ELASTIC SCATTERING OF POSITIVE PIONS

Abstract

(1) The calculation of the orientation parameters of the deuteron is facilitated by a method slightly different from the one usually used to discuss deuteron polarization. We extend the usual trace rules for Pauli spin matrices to the 3 x 3 matrices for spin one and we also choose a set of base matrices which are not necessarily the representations of certain spherical tensor operators, but correspond to linear combinations of them. The evaluation of the average expectation values of these is accomplished elegantly with the trace rules and given these values the estimates of the conventional orientation parameters of the deuteron are easily obtained.

(2) The spin state of the deuteron following elastic scattering of positive pions is studied in impulse approximation using the Chew-Low amplitude for pion-nucleon scattering. Taking into consideration only the dominant δ_{33} phase shifts it is found that the deuteron "polarization" vanishes and that the differential alignment parameter is independent of the incident pion energy. It is also shown that the ellipsoid associated with the "tensor polarization" reduces to an oblate spheroid.

DEUTERON POLARIZATION FOLLOWING ELASTIC SCATTERING OF POSITIVE PIONS[+]

1. Introduction

The successful experiments that were carried out with polarized beams of protons and neutrons gave an impetus to the theoretical possibilities of extending such experiments to spin 1 particles, in particular, the deuteron. While in the case of particles of spin 1/2 the most general spin state can be described as a mixture of a completely polarized state with statistical weight P and an unpolarized state with weight (1 - P), P being the percentage polarization, no such simple picture exists for particles with spin greater than 1/2. In the case of the deuteron, as its spin is one, its state of polarization is not completely specified by the orientation of the spin axis. There are also orientational features which may be described in terms of tensors, as opposed to the vectors which specify the spin orientation.

The Von Neumann density matrix ρ is a convenient starting point to discuss polarization.[1, 2] The theoretical

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[+] R.K.Umerjee, Nuclear Physics (In print).

1) L.Wolfenstein and J.Ashkin, Phys. Rev. 85, 947 (1952)

2) R.H.Dalitz, Proc. Phys. Soc. (London), A 65, 175 (1952)

aspects of oriented beams with particle spins higher than 1/2 have been formulated by Dalitz[2] and applied to the case of the deuteron by Lakin and Wolfenstein[3, 4]. General treatment of the higher order tensor moments has been given by Simon and Welton[5, 6] and Goldfarb[7]. Numerous other theoretical treatments[8, 9, 10] are of value in understanding the tensor orientation problems and in particular the review articles of Wolfenstein[11], Devons and Goldfarb[12] and Blin-Stoyle and Grace[13] are very comprehensive.

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2) R.H.Dalitz, Proc. Phys. Soc. (London), A 65 , 175 (1952)
3) W. Lakin and L. Wolfenstein, Phys. Rev., 90 , 365 A (1953)
4) W. Lakin, Phys. Rev., 98 , 139 (1955)
5) A. Simon and T.A.Welton, Phys. Rev., 90 , 1036 (1953)
6) A. Simon, Phys. Rev., 92, 1050 (1953)
7) L.J.B.Goldfarb, Nuclear Physics, 7 , 622 (1958)
8) H.P.Stapp, University of California, Radiation Laboratory Report UCRL - 3098, August 1955 (unpublished)
9) H.P.Stapp, Phys. Rev., 107 , 607 (1957)
10) S. Watanabe, Nuclear Phys., 8 , 484 (1958)
11) L. Wolfenstein, Ann. Rev. Nuclear Sci., 6 , 43 (1956)
12) S. Devons and L.J.B.Goldfarb, Angular Correlations, Handbuch der Physik, XLII , 443 (Springer, 1957)
13) R.J.Blin-Stoyle and M.A.Grace, Oriental Nuclei, Handbuch der Physik, XLII , 555 (Springer, 1957)

Polarized deuteron beams have been studied[14] using double scattering experiments. Oriented deuterons can be produced from ionized atomic beams. Also, scattering and reactions are possible sources of oriented deuterons. Elastic deuteron scattering in He^4 [15, 16, 17, 18] and C^{12}, (p , d) and (n , d) reactions whether proceeding by compound nucleus formation or pick up reactions are all possible sources of highly oriented deuterons. Thermal neutron capture in hydrogen has also been suggested[19] as an aligned deuteron source. Now it is expected that new techniques in the development of low temperatures and high magnetic fields[20] can lead in the near future to the availability of polarized targets for use in the study of nuclear reactions. Recently Chamberlain et al[21] have produced a target containing polarized protons. By scattering a polarized proton beam off a polarized target of protons, spin correlation parameters of p-p scattering have been measured.[22] It is believed[23] that polarized targets may provide simple tests to reaction mechanism.

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14) J.Baldwin et al, Phys. Rev., 103, 1502 (1956)
J.Button and R.Mermod, Phys. Rev., 118, 1333 (1960)
J. Zamick, Annals of Physics, 21, 550 (1963)
15) L.J.B.Goldfarb, Nuclear Phys., 7, 622 (1958)
16) L.J.B.Goldfarb and J.R.Rook, Nuclear Phys., 12, 494 (1959)
17) L.G.Pondrom, Phys. Rev. Letters, 2, 346 (1959)
18) R.J.N.Phillips, Phys. Rev. Letters, 3, 101 (1959)
19) M.E.Rose, Phys. Rev. Letters, 3, 387 (1959)
20) L.D.Roberts and J.W.T.Dabbs, Ann. Rev. Nuclear Sci. 11, 175 (1961).
21) E.Segre, Lecture delivered at the Institute of Mathematical Sciences, Madras (November 1962)
22) A. Abragam et al, 1962, Proc.of the Conference on Direct Interactions and Nuclear Reaction mechanism, Padua, 617.
23) L.J.B.Goldfarb and D.A.Bromley, 1962,Proc. of the Conference on Direct Interactions and nuclear Reaction Mechanism, Padua, 609.

In this and the two succeeding chapters we will be concerned with a study regarding the final spin-state of the deuteron in certain reactions.† The calculation of the orientation parameters is facilitated by a method slightly different from the one[4, 24] usually used to discuss deuteron polarization. The set of operators given by Lakin[4] includes the unit matrix, two linear combinations of spin operators and three second-rank tensor products of spin operators, as well as the Hermitian adjoint of three of these. We extend the usual trace rules[11, 25] for Pauli spin matrices to the 3 x 3 matrices for spin one and we also choose a set of base matrices which are not necessarily the representations of certain spherical tensor operators, but correspond to linear combinations of them. The evaluation of the average expectation values of these is accomplished elegantly with the help of the trace rules and given these values the estimates of the conventional parameters are easily obtained. We will outline the method in the next section.

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24) J.Hamilton, The theory of Elementary Particles, (Oxford, 1969)

25) G. Ramachandran, MATSCIENCE Report 2.

† R. K. Umerjee, Nuclear Physics (in Print.)
G. Ramachandran and R. K. Umerjee, Physical Review (To be published).
G. Ramachandran and R. K. Umerjee, Nucl. Phys. 54, 665 (1964).

2. Method

In general, the density matrix ρ [11] is of order n, where n is the number of basic states and consequently could be expressed as a linear combination of n^2 independent matrices S_μ of order n which may be chosen suitably.

$$\rho = \sum_\mu C^\mu S_\mu \quad . \qquad (1)$$

The linear ~~dependence~~ independence of the base matrices is expressed by the orthogonality relation

$$Tr\,(S_\mu S_\nu) = n\,\delta_{\mu\nu} \quad , \qquad (2)$$

suitably normalizing the base matrices. The average expectation value of S_μ is given by

$$\langle S_\mu \rangle = \frac{Tr(\rho S_\mu)}{Tr\,\rho} = \frac{n C^\mu}{Tr\,\rho} \quad . \qquad (3)$$

or

$$C^\mu = \frac{Tr\,\rho}{n} \langle S_\mu \rangle \quad . \qquad (4)$$

Hence

$$\rho = \frac{Tr\,\rho}{n} \sum_{\mu=1}^{n^2} \langle S_\mu \rangle S_\mu \quad . \qquad (5)$$

One of the S_μ's can always be chosen as the unit matrix I, so that

$$\rho = \frac{Tr\,\rho}{n}\left[I + \sum_{\mu=1}^{n^2-1} \langle S_\mu \rangle S_\mu\right] \qquad (6)$$

which provides a representation for the density matrix in terms of a set of (n^2-1) matrices which may be chosen to correspond to the matrices of (n^2-1) linearly independent operators on the system and (n^2-1) parameters, $\langle S_\mu \rangle$.

In the case of the deuteron as its spin is one, its state of polarization is completely specified[4] by a set of 8 parameters $\langle S_\mu \rangle$ where S_μ are a set of linearly independent 3 x 3 base matrices usually chosen[4] to be certain spherical tensor operators constructed from the angular momentum operators $\vec{J}$ for the system and normalized to satisfy

$$Tr\,(S_\mu S_\nu^\dagger) = 3\,\delta_{\mu\nu} \quad , \qquad (7)$$

where $S_\nu^\dagger$ denotes the hermitian conjugate of S_ν .

For convenience we shall however choose a set of base matrices comprising of J_x , J_y , J_z and certain linear combinations of the five independent components $Q_{\alpha\beta}$ of the quadrupole tensor in angular momentum operators

$$Q_{\alpha\beta} = \frac{1}{2}\left[\frac{3}{2}\left(J_\alpha J_\beta + J_\beta J_\alpha\right) - 2\,\delta_{\alpha\beta}\right] \qquad (8)$$

Our base matrices shall be

$$S_1 = \sqrt{\frac{3}{2}}\, J_x = \frac{\sqrt{3}}{2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{9}$$

$$S_2 = \sqrt{\frac{3}{2}}\, J_y = \frac{i\sqrt{3}}{2} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{10}$$

$$S_3 = \sqrt{\frac{3}{2}}\, J_z = \sqrt{\frac{3}{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{11}$$

$$S_4 = \sqrt{\frac{8}{3}}\, Q_{xy} = i\sqrt{\frac{3}{2}} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{12}$$

$$S_5 = \sqrt{\frac{8}{3}}\, Q_{xz} = \frac{\sqrt{3}}{2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{13}$$

$$S_6 = \sqrt{\frac{8}{3}}\, Q_{yz} = \frac{i\sqrt{3}}{2} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \tag{14}$$

$$S_7 = \sqrt{2}\, Q_{zz} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}$$

$$S_8 = \sqrt{\frac{2}{3}}\,(Q_{xx} - Q_{yy}) = \sqrt{\frac{3}{2}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{16}$$

together with the unit matrix. These matrices are linearly independent and satisfy the normalization condition (7). In fact to suit particular physical situations, many alternate choices for the 3 x 3 base matrices could be made.[26)]

We shall now generalise the well-known Trace rules[11,25)] for Pauli spin operators viz.,

$$Tr\,(\vec{\sigma}) = Tr\,(\vec{\sigma}.\vec{A}) = 0 \qquad (17)$$

$$Tr\,[\vec{\sigma}(\vec{\sigma}.\vec{A})] = 2\vec{A} \qquad (18)$$

$$Tr\,(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = 2(\vec{A}.\vec{B}) \qquad (19)$$

$$Tr\;\vec{\sigma}(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = 2i(\vec{A}\times\vec{B}) \qquad (20)$$

to the spin 1 case. In the above expressions $\vec{A}$ and $\vec{B}$ denote two arbitrary vectors and ($\vec{\sigma} \cdot \vec{A}$) is given by

$$\vec{\sigma}.\vec{A} = \sigma_x A_x + \sigma_y A_y + \sigma_z A_z \; . \qquad (21)$$

We find in the spin 1 case

$$Tr\,(\vec{J}) = Tr\,(\vec{J}.\vec{A}) = 0 \; , \qquad (22)$$

$$Tr\;\vec{J}(\vec{J}.\vec{A}) = 2\vec{A} \qquad (23)$$

26) P. Roman, Proc. Phys. Soc., A 74 , 649 (1959)

$$Tr\,(\vec{J}.\vec{A})(\vec{J}.\vec{B}) = 2(\vec{A}.\vec{B}) \qquad (24)$$

$$Tr\,\vec{J}(\vec{J}.\vec{A})(\vec{J}.\vec{B}) = i(\vec{A}\times\vec{B}) \qquad (25)$$

which can be arrived at by a direct calculation using the 3 x 3 spin matrices J_x , J_y , and J_z given by (9), (10) and (11) respectively.

We further calculate

$$Tr\,(J_\alpha J_\beta) = 2\,\delta_{\alpha\beta} \qquad (26)$$

$$Tr\left[J_\alpha J_\beta(\vec{J}.\vec{A})\right] = i\,\epsilon_{\alpha\beta r}\,A_r \qquad (27)$$

where $\epsilon_{\alpha\beta r}$ denotes the Levi-Civita symbol in three dimensions. (27) implies

$$Tr\left\{\left(\frac{J_\alpha J_\beta + J_\beta J_\alpha}{2}\right)(\vec{J}.\vec{A})\right\} = 0 \qquad (28)$$

Finally,

$$Tr\left[J_\alpha J_\beta(\vec{J}.\vec{A})(\vec{J}.\vec{B})\right] = A_\beta B_\alpha + (\vec{A}.\vec{B})\delta_{\alpha\beta} \qquad (29)$$

For the different processes involving the deuteron, the quantities $\langle S_\mu \rangle$ can now be calculated elegantly with the help of the above trace rules and with a knowledge of the amplitude for the

process. In terms of these expectation values, the conventional orientation parameters are given by

$$\langle T_0^1 \rangle = \langle S_3 \rangle \tag{30}$$

$$\langle T_{\pm 1}^1 \rangle = \mp \frac{1}{\sqrt{2}} \left[\langle S_1 \rangle \pm i \langle S_2 \rangle \right] \tag{31}$$

$$\langle T_0^2 \rangle = \langle S_7 \rangle \tag{32}$$

$$2\langle T_{\pm 1}^2 \rangle = \mp \sqrt{2} \left[\langle S_5 \rangle \pm i \langle S_6 \rangle \right] \tag{33}$$

and

$$\langle T_{\pm 2}^2 \rangle = \frac{1}{\sqrt{2}} \left[\langle S_8 \rangle \pm i \langle S_4 \rangle \right] \tag{34}$$

The quantity $\langle i T_1^1 \rangle$ is referred to as "vector polarization" while the $\langle T_M^2 \rangle$ are components of "tensor polarization" and are associated with a spin alignment rather than an orientation. The $\langle T_M^2 \rangle$ tensor can be represented by an ellipsoidal surface and this will be discussed at the end of the chapter. In the next section, we will study the deuteron polarization in the final state in the case of elastic scattering of pions.

3. Deuteron polarization in $\pi^+ + D \longrightarrow \pi^+ + D$.

The scattering of pions by deuterons has been discussed in impulse approximation[27] by many authors[28] and in particular the elastic scattering of positive pions has been studied[29] theoretically using the basic amplitudes of Chew and Low.[30] The impulse approximation assumes that the matrix element for the process is the sum of the matrix elements for interaction with each of the two nucleons in the deuteron assuming that during this interaction, the other nucleon plays the role of a spectator. The early experiments of Ashkin et al[31] have shown that there is no great difference between the scattering of π^+ and π^- indicating that charge symmetry holds. It is meaningful to assume that the amplitudes for scattering of π^+ and π^- from deuterons should be the same, in which case, the present calculation can be extended to the case of π^- as well. An interesting observable in this scattering process is the spin state of the deuteron in the final state though no systematic calculation seems to have been done so far and it is our object to study this final spin state using impulse approximation and the Chew-Low amplitudes.[30]

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27) G.F.Chew, Phys. Rev., 80, 196 (1950)
G.F.Chew, and G.C.Wick, Phys. Rev., 85, 636 (1952)
G.F.Chew and M.L.Goldberger, Phys. Rev., 87, 778 (1952)

28) S.Fernbach, T.A.Green and K.M.Watson, Phys. Rev., 80, 1084 (1951).
P.M.Rockmore, Phys. Rev., 105, 256 (1957).

29) Alladi Ramakrishnan, V.Devanathan and K. Venkatesan, Nuclear Physics, 29 , 680 (1962)

30) G.F.Chew and F.E.Low, Phys. Rev., 101 , 1570 (1956)

31) J.Ashkin et al , Phys. Rev., 96 , 1104 (1954)

The impulse approximation requires that we should know the matrix elements for the scattering of a π^+ from a proton and a neutron. The amplitude for scattering of a pion by a nucleon is given by

$$t = \left[\frac{t_p + t_n}{2} + \frac{t_p - t_n}{2}\,\tau_3\right] \qquad (35)$$

where τ_3 is the third component of the isotopic spin vector of the nucleon and t_p , t_n denote the scattering amplitudes[30] from the proton and neutron respectively and these are given by

$$t_p = t\,(p\pi^+ \longrightarrow p\pi^+)$$

$$= -\frac{2\pi}{\omega}\left[2\,\vec{q}_2\cdot\vec{q}_1 - i\,\vec{\sigma}\cdot(\vec{q}_2\times\vec{q}_1)\right]\cdot\frac{e^{i\delta_{33}}\sin\delta_{33}}{q^3} \qquad (36)$$

$$t_n = t\,(n\pi^+ \longrightarrow n\pi^+)$$

$$= -\frac{2\pi}{\omega q^3}\left[\frac{1}{3}\left\{2\,\vec{q}_2\cdot\vec{q}_1 - i\,\vec{\sigma}\cdot(\vec{q}_2\times\vec{q}_1)\right\}e^{i\delta_{33}}\sin\delta_{33}\right.$$

$$\left. + \frac{1}{3}\left\{5\,\vec{q}_2\cdot\vec{q}_1 - i\,\vec{\sigma}\cdot(\vec{q}_2\times\vec{q}_1)\right\}\right.$$

$$e^{i\delta_{31}}\sin\delta_{31}$$

$$+ \frac{2}{3} \left\{ \vec{q}_2 \vec{q}_1 + i\vec{\sigma}.(\vec{q}_2 \times \vec{q}_1) \right\} .$$

$$e^{i\delta_{11}} \sin\delta_{11} \Big\} \qquad (37)^{\dagger}$$

where $\vec{q}_1$ and $\vec{q}_2$ are the momenta of the initial and final pions and $\omega_1 (=\omega)$ and $\omega_2 (=\omega)$ are their energies.

If T denotes the transition matrix for the process $\pi^+ + D \rightarrow \pi^+ + D$, thedensity matrix for the final state deuteron is given by

$$\rho = T \cdot \frac{1}{3} T^{\dagger} \qquad (38)$$

corresponding to unpolarized target and where the transition matrix T in impulse approximation is given by

$$\langle {}^3\chi_m | T | {}^3\chi_m \rangle$$

$$= \left\langle {}^3\chi_m \left| \frac{1}{2} (t_p^{(1)} + t_n^{(1)}) + \frac{1}{2} (t_p^{(2)} + t_n^{(2)}) \right| {}^3\chi_m \right\rangle . E \qquad (39)$$

following the notation in reference[29] and noting that τ_3 in (35) does not c onnect the charge singlet initial and final states. χ denotes the spin wave function and E , the overlap integral which can be calculated using the Hulthen wave function for the deuteron[32]

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32) G.F.Chew and H.W.Lewis, Phys. Rev., 84, 779 (1951)

† Here, δ_{33} etc. denote the p-wave phase-shifts for pion-nucleon scattering in Fermi's notation.

We can express $(t_p + t_n)$ in terms of the spin-dependent and spin-independent parts as

$$t_p + t_n = i \vec{\sigma} . \vec{K} + L \qquad (40)$$

where $\vec{K}$ and L can be readily obtained from the expressions (36) and (37). Hence the amplitude (39) has the form

$$T_{m,m} = \langle {}^3\chi_m \mid i \vec{J} . \vec{K} + L \mid {}^3\chi_m \rangle \cdot E \qquad (41)$$

With the help of (38) and (41) , the quantities $\langle S_\mu \rangle$ are now given by

$$\langle S_\mu \rangle = \frac{Tr \left\{ S_\mu \left(i \vec{J} . \vec{K} + L \right) \left(-i \vec{J} . \vec{K}^* + L^* \right) \right\}}{Tr \left\{ \left(i \vec{J} . \vec{K} + L \right) \left(-i \vec{J} . \vec{K}^* + L^* \right) \right\}} \qquad (42)$$

These quantities are independent of the overlap integral E and also the denominator is proportional to the differential cross-section. In the above $\vec{K}^*$ and L^* denote the complex conjugates of $\vec{K}$ and L respectively.

Using the trace rules (22 to 29) , the above quantities can be evaluated as follows

$$\begin{aligned} D &= Tr \left[\left(i \vec{J} . \vec{K} + L \right) \left(-i \vec{J} . \vec{K}^* + L^* \right) \right] \\ &= Tr \left[\left(\vec{J} . \vec{K} \right) \left(\vec{J} . \vec{K}^* \right) + i L^* \vec{J} . \vec{K} - i L \vec{J} . \vec{K}^* + L L^* \right] \\ &= 2 \vec{K} . \vec{K}^* + 3 L L^* \end{aligned} \qquad (43)$$

$$D.\langle\vec{J}\rangle = Tr\left[\vec{J}\,(i\vec{J}.\vec{K}+L)(-i\vec{J}.\vec{K}^{*}+L^{*})\right]$$

$$= Tr\left[\vec{J}\left\{(\vec{J}.\vec{K})(\vec{J}.\vec{K}^{*})+LL^{*}\right.\right.$$

$$\left.\left.+iL^{*}(\vec{J}.\vec{K})-iL(\vec{J}.\vec{K}^{*})\right\}\right]$$

$$= i\left[\vec{K}\times\vec{K}^{*}+2L^{*}\vec{K}-2L\vec{K}^{*}\right] \qquad (44)$$

Also

$$D\langle Q_{\alpha\beta}\rangle = \frac{3}{4}\left[K_{\alpha}K_{\beta}^{*}+K_{\beta}K_{\alpha}^{*}\right]-\frac{1}{2}(\vec{K}.\vec{K}^{*})\delta_{\alpha\beta} \qquad (45)$$

Let us first take into consideration only the dominant δ_{33} phase shifts. Under this assumption with the help of (36) and (37), $\vec{K}$ and L are given by

$$\vec{K} = \frac{8\pi}{3\omega q^{3}}\, e^{i\delta_{33}} \sin\delta_{33}\,(\vec{q}_{2}\times\vec{q}_{1}), \qquad (46)$$

$$L = -\frac{16\pi}{3\omega q^{3}}\, e^{i\delta_{33}} \sin\delta_{33}\,(\vec{q}_{2}.\vec{q}_{1}) \qquad (47)$$

Let θ be the angle between $\vec{q}_1$ and $\vec{q}_2$ where the x-axis is taken along the incident pion direction and the scattering is assumed to take place in the z-plane. Substituting the above values of $\vec{K}$ and L in the expressions (43), (44) and (45), the conventional orientation parameters are given by

$$\langle T_0^1 \rangle = 0 \tag{48}$$

$$\langle T_{\pm 1}^1 \rangle = 0 \tag{49}$$

$$\langle T_{\pm 1}^2 \rangle = 0 \tag{50}$$

$$\langle T_{\pm 2}^2 \rangle = 0 \tag{51}$$

and

$$\langle T_0^2 \rangle = \frac{1}{\sqrt{2}} \frac{\sin^2\theta}{1+5\cos^2\theta} \tag{52}$$

The above equations imply that the density matrix for the final deuteron is diagonal in the frame of reference defined by the principal axes of alignment and using (6) it can be explicitly written as

$$\rho' = \frac{1}{3} \times \begin{bmatrix} 1+\alpha & 0 & 0 \\ 0 & 1-2\alpha & 0 \\ 0 & 0 & 1+\alpha \end{bmatrix} \tag{53}$$

where

$$\alpha = \frac{1}{2} \frac{\sin^2\theta}{1+5\cos^2\theta} \tag{54}$$

Numerical estimates of the parameter $\langle T_0^2 \rangle$ given by (52) are given in Table 1 .

We will now include all the p-wave phase shifts in the calculation of the orientation parameters. With the help of (36) and (37), $\vec{K}$ and L are now given by

$$\vec{K} = \frac{2\pi}{3\omega q^3} \left[4a + b - 2c\right] (\vec{q}_2 \times \vec{q}_1) \qquad (55)$$

$$L = \frac{-2\pi}{3\omega q^3} \left[8a + 5b + 2c\right] (\vec{q}_2 \cdot \vec{q}_1) \qquad (56)$$

where

$$a = e^{i\delta_{33}} \sin\delta_{33}$$

$$b = e^{i\delta_{31}} \sin\delta_{31}$$

$$c = e^{i\delta_{11}} \sin\delta_{11}$$

and the axes are defined as before. As before, the conventional parameters can be calculated and are given by

$$\langle T_{\pm 1}^1 \rangle = 0 \qquad (57)$$

$$\langle T_{\pm 1}^2 \rangle = 0 \qquad (58)$$

$$\langle T_{\pm 2}^2 \rangle = 0 \qquad (59)$$

$$\langle T_0^1 \rangle = \frac{\sqrt{6} \times 24 \sin\theta \cos\theta}{D} \Big[\sin\delta_{33} \sin\delta_{31} \sin(\delta_{33}-\delta_{31})$$

$$+ 2 \sin\delta_{33} \sin\delta_{11} \sin(\delta_{33}-\delta_{11}) + \sin\delta_{31} \sin\delta_{11} \sin(\delta_{33}-\delta_{11}) \Big] \quad (60)$$

$$\langle T_0^2 \rangle = \frac{\sqrt{2}}{D} \sin^2\theta \Big[16 \sin^2\delta_{33} + \sin^2\delta_{31} + 4 \sin^2\delta_{11}$$

$$+ 8 \sin\delta_{33} \cdot \sin\delta_{31} \cos(\delta_{33}-\delta_{31})$$

$$-16 \sin\delta_{33} \sin\delta_{11} \cos(\delta_{33}-\delta_{11})$$

$$-4 \sin\delta_{31} \sin\delta_{11} \cos(\delta_{31}-\delta_{11}) \Big] \quad (61)$$

where

$$D = 2 \sin^2\theta \Big[16 \sin^2\delta_{33} + \sin^2\delta_{31} + 4 \sin^2\delta_{11}$$

$$+ 8 \sin\delta_{33} \sin\delta_{31} \cos(\delta_{33}-\delta_{31})$$

$$- 16 \sin\delta_{33} \sin\delta_{11} \cos(\delta_{33}-\delta_{11})$$

$$- 4 \sin\delta_{31} \sin\delta_{11} \cos(\delta_{31}-\delta_{11}) \Big]$$

$$+ 3 \cos^2\theta \Big[64 \sin^2\delta_{33} + 25 \sin^2\delta_{31} + 4 \sin^2\delta_{11}$$

$$+ 80 \sin\delta_{33} \sin\delta_{31} \cos(\delta_{33}-\delta_{31})$$

$$+ 32 \sin\delta_{33} \sin\delta_{11} \cos(\delta_{33}-\delta_{11})$$

$$+ 20 \sin\delta_{31} \sin\delta_{11} \cos(\delta_{31}-\delta_{11}) \Big]$$

(62)

In this case too, the density matrix for the final deuteron is diagonal and it can be explicitly written as

$$\rho' = \frac{1}{3} \times \begin{bmatrix} 1+\beta+\gamma & 0 & 0 \\ 0 & 1-2\beta & 0 \\ 0 & 0 & 1+\beta-\gamma \end{bmatrix} \qquad (63)$$

where

$$\beta = \frac{1}{\sqrt{2}} \langle T_0^2 \rangle \qquad (64)$$

and

$$\gamma = \sqrt{\frac{3}{2}} \langle T_0' \rangle \qquad (65)$$

Numerical estimates of $\langle T_0^2 \rangle$ and $\langle T_0' \rangle$ in the present case are given in Tables 2 and 3 .

A useful method of describing the state of polarization is to consider the representation in which ρ is diagonal.[4] Any state may then be described as a statistical mixture of the three pure mutually orthogonal states ψ_n which form the basis of this representation, and the corresponding eigenvalues λ_n of ρ are the weights. Expressions (53) and (63) exhibit the density matrix of the final state and they are diagonal. Hence the corresponding weights can be calculated from the diagonal elements. We will now show how the eigenvalues or the weights are

related to the semi-axes of the ellipsoid associated with the "tensor polarization"[33)]

The ellipsoid associated with "tensor polarization" is similar to the moment-of-inertia ellipsoid. It can be represented by the equation

$$\langle J_x^2 \rangle x^2 + \langle J_y^2 \rangle y^2 + \langle J_z^2 \rangle z^2 + \langle J_x J_y + J_y J_x \rangle xy + \langle J_y J_z + J_z J_y \rangle yz + \langle J_z J_x + J_x J_z \rangle xz = 1 \qquad (66)$$

From equations (50) and (51) it is clear that the product terms xy, yz and zx in (66) drop out and hence the ellipsoid reduces to one referred to its principal axes and the semi-axes of the ellipsoid are the reciprocals of $\left(\langle J_x^2 \rangle\right)^{1/2}$, $\left(\langle J_y^2 \rangle\right)^{1/2}$ and $\left(\langle J_z^2 \rangle\right)^{1/2}$. Using expressions (45) and (52), these quantities can be evaluated as follows:

$$\left(\langle J_x^2 \rangle\right)^{1/2} = \left(\frac{3}{2}\,\frac{1}{1-\lambda}\right)^{1/2} \qquad (67)$$

$$\left(\langle J_y^2 \rangle\right)^{1/2} = \left(\frac{3}{2}\,\frac{1}{1-\lambda}\right)^{1/2} \qquad (68)$$

$$\left(\langle J_z^2 \rangle\right)^{1/2} = \left(\frac{3}{2}\,\frac{1}{1+2\lambda}\right)^{1/2} \qquad (69)$$

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33) J. Button and R. Mermod, Phys. Rev., 118, 1333 (1960)

where $\lambda = \frac{\alpha}{2}$ given by (54). From (67), (68) and (69) it is clear that the ellipsoid reduces to an "oblate spheroid" with the semi-axes related to the eigenvalues of the density matrix as seen by the relation $\lambda = \alpha/2$. Also this spheroid can be fixed in position and magnitude as λ is the same as $\left(\frac{1}{8}\right)^{1/2} \cdot \langle T_0^2 \rangle$ which in turn can be calculated from Table 2.

4. Numerical Results

An examination of expression (52) shows that taking into consideration only the dominant δ_{33} contributions, the differential alignment parameter $\langle T_0^2 \rangle$ is independent of the pion energy. This result, though surprising at first sight can be accounted for by considering expressions (46) and (47); They are homogenous in the kinematical factors and also in the phase shifts. As $\langle T_0^2 \rangle$ can be expressed as

$$\langle T_0^2 \rangle = \sqrt{2}\, \frac{K_z K_z^*}{2 K_z K_z^* + 3 L L^*} \qquad (70)$$

it is clear that the kinematical factors and phase shifts cancel out leaving the term as a pure function of θ. Numerical estimates of expression (52) are given in Table 1.

Inclusion of all p-wave phase shifts is exhibited in expressions (55) and (56) and this brings a small energy dependent correction to the above values in Table 1 through the interference terms. The relevant estimates aregiven in Table 2 for incident pion K-E 's 120 , 144 and 169 Mev.

Equations (48) and (49) indicate the presence of 'aligned' deuterons since the polarization vanishes. But this is the case when only the dominant δ_{33} phase shifts are taken into consideration. Inclusion of all the p-wave phase shifts introduces 1 to 3 percent polarization for incident pion kinetic energies k_o = 120, 144 Mev and 4 to 9 percent in the case of k_o = 169 Mev . The relevant estimates are given in Table 3 .

5. Conclusions

It is observed that taking into consideration only the dominant δ_{33} terms, the differential alignment parameter is independent of the incident pion energy. Also the vanishing of the spin expectation value seems to indicate the presence of "aligned" deuterons in the final state. The estimates of these parameters taking into account all the p-wave phase shifts exhibit a small energy dependent correction in the values of $\langle T_o^2 \rangle$ and a small percentage polarization. In both cases, it is found that the density matrix in the final state seems to be diagonalised

in a natural manner thus obviating the necessity for constructing a new set of basic states as is sometimes necessary. Also, the ellipsoid associated with the tensor polarization reduces to an oblate spheroid in the present case.

Table 1

Numerical estimates of the parameter $\langle T_0^2 \rangle$ as a function of the angle of meson emission (for the case of the dominant δ_{33} phase-shift)

0°	30°	60°	90°	120°	150°	180°
0	0.0372	0.2358	0.707	0.2358	0.0372	0

Table 2

Numerical Estimates of the parameter $\langle T_o^2 \rangle$ as a function of the angle of meson emission for incident pion kinematic energies k_o = 120 , 144 and 169 Mev.

(All p-wave phase shifts being included)

k_o \ θ	0°	30°	60°	90°	120°	150°	180°
120	0	0.03823	0.2402	0.707	0.2402	0.03823	0
140	0	0.03892	0.2431	0.707	0.2431	0.03893	0
169	0	0.03284	0.2154	0.707	0.2154	0.03284	0

Table 3

Numerical estimates of the parameter $\langle T_0' \rangle$ as a function of the angle of meson emission for incident pion kinetic energies k_0 = 120, 144 and 169 Mev.

(All p-wave phase-shifts being included)

k_0 \ θ	0°	30°	60°	90°	120°	150°	180°
120	0	0.01073	0.02247	0	-0.02247	-0.01073	0
144	0	0.01724	0.0359	0	-0.0359	-0.01724	0
169	0	0.04362	0.0956	0	-0.0956	-0.04362	0

CHAPTER III

POLARIZATION PHENOMENA IN ELASTIC ELECTRON-DEUTERON AND ELECTRON-PROTON SCATTERING

ABSTRACT

(1) The final spin state of the deuteron in the elastic electron-deuteron scattering is discussed with unpolarized initial particles. It is found that a measurement of the deuteron polarization in the final state can be used to evaluate the form factors of the neutron with some advantage.

(2) Helicity of the final electron in elastic e-p scattering is studied at 900 Mev. It is found that with the target linearly polarized in the direction of the incident electron beam, the electron helicity reaches a value 100 % for back-scattering. This is of high experimental interest in view of the availability of polarized proton targets.

(3) Similar calculation for electron helicity in elastic e-D scattering under identical conditions, has yielded a value 75 % .

Percentages of the electron helicity for other angles of scattering have been obtained at 900 Mev in both cases and it is noted that the values obtained for back scattering are independent of the incident energy.

POLARIZATION PHENOMENA IN ELASTIC ELECTRON-DEUTERON AND ELECTRON-PROTON SCATTERING[+*]

1. Introduction

Elastic as well as inelastic scattering of electrons from deuterons has been discussed extensively in the context of determining the electro-magnetic form factors of the neutron[1]. This is essentially due to the fact that since neutron targets are not easily available, the main source of information about

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+ G.Ramachandran and R.K.Umerjee, Physical Review (to be published).

* This work was done by the author in collaboration with G.Ramachandran. Part of the results of this chapter has been sent for publication to PHYSICAL REVIEW and it has been accepted for publication. Also, the work was presented by the author at the first MATSCIENCE Summer School held at Bangalore in August 1964, under the sponsorship of the Council of Scientific and Industrial Research (INDIA).

1) J.A.McIntyre, Phys. Rev. 103 (1956), 1464.
L.I.Schiff, Rev. Mod. Phys. 30 (1958), 462.
J.I.Friedman, H.W.Kendall and P.A.M.Gram, Phys. Rev. 120 (1960), 992.
N.K.Glendenning and G.Kramer, Phys. Rev. Letters, 7 (1961), 471.
V.Z.Jankus, Phys. Rev. 102 (1956), 1586.
A.Goldberg, Phys. Rev. 112 (1958), 618.
S.Sobottka, Phys. Rev. 118 (1960), 831.
Loyal Durand III, Phys. Rev. Letters, 6 (1961), 631.

the neutron structure comes from the use of deuteron targets which due to the small Binding Energy and the corresponding large spread of the wave function is a very good source of quasi-free neutrons. The elastic e-D scattering could be mentioned as a possible source of information on the coherent combination of proton and neutron amplitudes. But the main interest in elastic deuteron scattering lies in the fact that only the isoscalar form factors appear.

The elastic electron-proton scattering[2] has been the subject of investigation for eliciting information on the proton structure. The extensive experiments on the scattering of High Energy electrons by Hydrogen of Hofstadter et al[3] have given much detailed information about the structure functions of the proton. The analysis uses the formula first derived by Rosenbluth[4] for the cross-section for the scattering of a relativistic electron by a proton.

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2) E. Clemental and C.Villi, Nuovo Cimento, 4 , (1956), 1207.
D. Yennie, M.M.Levy and D.G.Ravenhall, Rev. Mod. Phys. 29, (1957), 144.
R. Hofstadter and R.R.Wilson: Ninth Ann. Int. Conf. on High Energy Physics at Kiev, 1959.
M. Croissanx, E. Dally and R.Hofstadter, Phys. Rev., 124, (1961), 1623.

3) R.Hofstadter and R.W.McAllister, Phys. Rev., 102 , (1956), 851; E.E. CHAMBERS & R. HOFSTADTER, Phys. Rev. 103 (1956), 1454.

4) M.N.Rosenbluth, Phys. Rev., 79 , (1950), 615.

In this chapter, we concern ourselves with the following polarization phenomena:

(1) The final spin state of the deuteron in the elastic case with unpolarized initial particles;

(2) The helicity of the electron in the final state of the elastic e-D scattering with the target deuterons linearly polarized in the direction of the incoming electron beam direction;

(3) The helicity of the electron in the final state of the elastic electron proton scattering with the target protons linearly polarized along the incident beam direction.

The polarization and alignment parameters of the deuteron are expected to be independent of the radial wave function if we neglect the d-state admixture and by studying the spin-state of the deuteron in the final state at two energies and angles corresponding to a given momentum transfer, one can determine the electric and magnetic form factors of the neutron (sect. 3). This is in contrast to the study of the differential cross-section where such a paired observation is primarily used to eliminate the body form factor and consequently leads only to a determination of the ratio of the iso-scalar form factors; one would in principle need four different observations of the cross-section corresponding to same momentum-transfer in order to be able to determine absolutely the two form factors.

Another interesting polarization phenomenon associated with the elastic e-D scattering with which we are concerned in this

chapter is the helicity of the electron in the final state. However with unpolarized targets, the electron helicity in the final state is found to be zero[5] (sect.4). In view of the increasing interest in polarized targets[6] it was considered worthwhile to investigate the effect of polarizing the deuterons in the initial state. Different types of initial deuteron polarization are considered (sects. 4 and 5) and we find that at an energy of 900 Mev and with targets linearly polarized along the beam, the electron helicity in the final state can be as high as 75 per cent. (sect. 5). This is compared with the percentage helicity expected in the elastic e-p scattering with polarized protons where we find that the back scattered electrons are completely polarized, if the target polarization is unity along the direction of the incident beam (sect. 6). In the Appendix, we derive some simple trace rules which will be used in the helicity calculations.

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5) This might be anticipated since with unpolarized targets and the final target spin state unobserved, the polarization of a spin 1/2 projectile in the final state is expected to be normal to the scattering plane. For instance, the transverse proton polarization in elastic p-D scattering is of interest in understanding the nucleon-nucleon interaction and experimental data on 22 Mev protons is reported recently by H.E.Conzett, G. Igo and W.J.Knox, Phys. Rev. Letters, 12, 222 (1964).

6) E. Segre, Lectures given at the Institute of Mathematical Sciences, Madras (1962) (Matscience Report 18); L.J.B.Goldfarb and D.A.Bromley; A. Abragam et al, Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962 (Gordon and Breach, New York, .1964).

NOTE :- After the completion of our work, we found that somewhat similar considerations have been studied by Schildknecht and Gourdin et al.

a) D. Schildknecht : Phys. Lett. 10, 254 (1964).

b) M. Gourdin and C.A. Piketty : Nuovo Cimento 32, 1137(1964).

2. Elastic e-D scattering

The amplitude for scattering of electrons from deuterons can be written using impulse approximation, in the form

$$T = \langle {}^3\chi_{m'} | i(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{K} + 2L | {}^3\chi_m \rangle \cdot E \qquad (1)$$

where $\vec{K}, L$ denote the spin-dependent and spin independent isoscalar amplitudes for the scattering of an electron from a nucleon and E denotes the body form factor of the deuteron taking into account only the s -state wave function. We shall write as usual, the electron-nucleon interaction in the form

$$\tilde{u}_{p'}\left[\gamma_\mu F_E^{p,n} + \frac{K^{p,n}}{2M}\cdot F_M^{p,n}\sigma_{\mu\nu}Q_\nu\right]u_p A_\mu \qquad (2)$$

where p, p' indicate the four-momenta of the nucleon before and after the interaction, K the anamolous magnetic moment, M the nucleon mass and F_E , F_M the electric and magnetic form factors. A_μ denotes

$$A_\mu = \frac{4\pi e^2}{Q^2}\cdot\left(\tilde{u}_{p_2}\gamma_\mu u_{p_1}\right) \qquad (3)$$

where p_1, p_2 denote the initial and final four-momenta of the electron. Q , the four-momentum transfer is given by

$$Q = P - P' = p_2 - p_1 \qquad (4)$$

and

$$Q^2 = (Q\cdot Q) = Q_t^2 - \vec{Q}\cdot\vec{Q} \qquad (5)$$

The isoscalar part of the amplitude (2) for electron-nucleon interaction is given by

$$\tilde{u}_{p'} \left[F_1^{\circ} \gamma_\mu + \frac{F_2^{\circ}}{2M} \sigma_{\mu\nu} Q_\nu \right] u_p \cdot A_\mu \qquad (6)$$

where the isoscalar form factors F_1°, F_2° are given by

$$2 F_1^{\circ} = F_E^{p}(Q^2) + F_E^{n}(Q^2) \qquad (7)^{\dagger}$$

$$2 F_2^{\circ} = K^{p} F_M^{p}(Q^2) + K^{n} F_M^{n}(Q^2) \qquad (8)^{\dagger}$$

Treating the nucleons non-relativistically, the above interaction can be written in the form

$$i \vec{\sigma} \cdot \vec{K} + L \qquad (9)$$

where $\vec{K}$, L are given by

$$\vec{K} = \frac{F_1^{\circ} + F_2^{\circ}}{2M} \cdot (\vec{Q} \times \vec{A}) \qquad (10)$$

$$L = \left[\frac{F_1^{\circ} + F_2^{\circ}}{2M} \cdot Q_t + F_1^{\circ} \right] \cdot A_t \qquad (11)$$

Having thus identified the quantities $\vec{K}$ and L in (1) we now essentially follow the ~~WXYZ~~ trace method[7] described in the previous chapter to calculate the parameters characterizing the final spin state of the deuteron in $e + D \rightarrow e + D$ with initially unpolarized deuterons.

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7) G. Ramachandran and R.K.Umerjee, Nuclear Physics, 54 , (1964), 665.
R.K.Umerjee, Nuclear Physics (In Press)

† Constants like 2 which occur here will cancel out in our calculation as our polarization parameters are homogenous expressions in $\vec{K}$ and L of the same degree in both numerator and denominator.

3. Deuteron Polarization

In the previous section, we have expressed the amplitude for e-D scattering in the form $i\vec{\sigma}\cdot\vec{K}+L$ where $\vec{K}$ and L have the prescribed expressions. Using the trace rules defined in chapter II for spin one matrices, the orientation parameters of the deuteron in the final state with initially unpolarized deuterons are givenx by the following (equations 30 to 34 of chapter II).

$$\langle T_0^1 \rangle = \sqrt{\tfrac{3}{2}}\cdot i\cdot\frac{1}{D}\sum_{Spins}\left[(\vec{K}\times\vec{K}^*)_z + 2K_z L^* - 2LK_z^*\right] \tag{12}$$

$$\langle T_{\pm 1}^1 \rangle = \mp\frac{\sqrt{3}}{2}\cdot\frac{1}{D}\cdot\sum_{Spins}\left\{i\left[(\vec{K}\times\vec{K}^*)_x + 2K_x L^* - 2LK_x^*\right] \mp \right.$$
$$\left. \mp\left[(\vec{K}\times\vec{K}^*)_y + 2K_y L^* - 2LK_y^*\right]\right\} \tag{13}$$

$$\langle T_0^2 \rangle = \frac{1}{\sqrt{2}}\cdot\frac{1}{D}\cdot\sum_{Spins}\left[3K_z K_z^* - \vec{K}\cdot\vec{K}^*\right] \tag{14}$$

$$\langle T_{\pm 1}^2 \rangle = \mp\frac{\sqrt{3}}{2D}\cdot\sum_{Spins}\left[(K_x K_z^* + K_z K_x^*) \pm i\left(K_y K_z^* + K_z K_y^*\right)\right] \tag{15}$$

$$\langle T_{\pm 2}^2 \rangle = \frac{\sqrt{3}}{2D}\cdot\sum_{Spins}\left[(K_x K_x^* - K_y K_y^*) \pm i\left(K_x K_y^* + K_y K_x^*\right)\right] \tag{16}$$

where

$$D = \sum_{Spins}\left[2\vec{K}\cdot\vec{K}^* + 3LL^*\right] \tag{17}$$

In the above expressions, the summation over spins denotes the summing over the initial and final electron spin states. We note that the parameters (12) to (16) are independent of the body form factor , E . We further observe that each of the expressions (12) - (16) is bilinear in A and the summation over electron spins can be elegantly carried out using the trace rules for the γ -matrices[8].

As an example, let us consider the LL^* term. The contribution of this term can be written as

$$\left[(F_1^\circ+F_2^\circ)(E_2-E_1)+2MF_1^\circ\right]^2 \sum_{spins}(\tilde{u}_{p_2}\gamma_t u_{p_1})(\tilde{u}_{p_2}\gamma_t u_{p_1})^* \qquad (18)$$

$$=\left[(F_1^\circ+F_2^\circ)(E_2-E_1)+2MF_1^\circ\right]^2 Tr\cdot\left[(\not{p}_2+m)\gamma_t(\not{p}_1+m)\gamma_t\right]$$

$$=\left[(F_1^\circ+F_2^\circ)(E_2-E_1)+2MF_1^\circ\right]^2 Tr\cdot\left[\not{p}_2\gamma_t\not{p}_1\gamma_t+m^2\gamma_t\gamma_t\right]$$

$$=\left[(F_1^\circ+F_2^\circ)(E_2-E_1)+2MF_1^\circ\right]^2\cdot 4\cdot\left[E_1E_2+\vec{p}_1\cdot\vec{p}_2+m^2\right]$$

$$=4\left[(F_1^\circ+F_2^\circ)(E_2-E_1)+2MF_1^\circ\right]^2\cdot\left[E_1E_2+\vec{p}_1\cdot\vec{p}_2+m^2\right] \qquad (19)$$

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8) J.M.Jauch and F.Rohrlich: The theory of Photons and Electrons , (Addison-Wesley, 1959), p. 436.

In a similar way, terms like $(\vec{K}\cdot\vec{K}^*)$ can be calculated. Also, it is simple to see that terms like $(\vec{K}\times\vec{K}^*)$ and $(\vec{K}L^*-LK^*)$ do not contribute. In addition, we can choose the axes in such a way that the incident electron beam is along the x -direction and that the scattering takes place in the xy plane. Then with the help of trace calculations outlined above, the orientation parameters can be calculated as follows.

$$\langle T_0^1 \rangle = 0 \tag{20}$$

$$\langle T_{\pm 1}^1 \rangle = 0 \tag{21}$$

$$\langle T_{\pm 1}^2 \rangle = 0 \tag{22}$$

$$\langle T_0^2 \rangle = \frac{2\sqrt{2}}{D}\cdot\left(F_1^0+F_2^0\right)^2 \times$$

$$\times\left\{\left(E_1E_2-\vec{p}_1\cdot\vec{p}_2-m^2\right)\left(|\vec{p}_2-\vec{p}_1|^2\right)+4p_1^2p_2^2\sin^2\theta\right\} \tag{23}$$

$$\langle T_{\pm 2}^2 \rangle = \frac{2\sqrt{3}}{D}\cdot\left(F_1^0+F_2^0\right)^2 \times$$

$$\times\left\{\left(E_1E_2-\vec{p}_1\cdot\vec{p}_2-m^2\right)\left(c\mp id\right)\right\} \tag{24}$$

where

$$C = p_2^2 \left(\sin^2\theta - \cos^2\theta\right) - p_1^2 + 2 p_1 p_2 \cos\theta . \qquad (25)$$

$$d = 2\left(p_2 \cos\theta - p_1\right) p_2 \sin\theta . \qquad (26)$$

and

$$D = 16 \left(F_1^\circ + F_2^\circ\right)^2 \times$$

$$\times \left\{ (E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2 - m^2)\left(|\vec{p}_2 - \vec{p}_1|^2\right) + p_1^2 p_2^2 \sin^2\theta \right\} +$$

$$+ 12 \left[\left(F_1^\circ + F_2^\circ\right)\left(E_2 - E_1\right) + 2MF_1^\circ \right]^2 \times$$

$$\times \left\{ E_1 E_2 + \vec{p}_1 \cdot \vec{p}_2 + m^2 \right\} \qquad (27)$$

Hence as mentioned at the outset, by studying the final spin state of the deuteron at two energies and angles corresponding to a given momentum transfer, one can determine absolutely the two form factors. This paired observation at a given momentum transfer enables the calculation of F_1°, F_2° and together with a knowledge of the proton form factors, the neutron form factors can be evaluated.

4. Helicity of the electron in the final state of elastic $e-D$ scattering

In the first place, let us consider the helicity of the electron with unpolarized targets. The transition probability for the elastic scattering with incident deuterons unpolarized can now be written in impulse approximation as

$$|E|^2 \sum_{m',m''} \langle {}^3\gamma_m | \sum_{n=1,2} (i\vec{\sigma}_n \cdot \vec{K} + L) | {}^3\gamma_{m'} \rangle \cdot \frac{1}{3} \cdot$$

$$\cdot \langle {}^3\gamma_m | \sum_{n=1,2} (i\vec{\sigma}_n \cdot \vec{K} + L) | {}^3\gamma_{m''} \rangle^* \qquad (28)$$

where E is the deuteron body form factor neglecting the d-state contribution.

To evaluate the cross-section between definite spin states of the electron we now sum over the final deuteron spin states making use of the trace rules obtained earlier. We have

$$\frac{|E|^2}{3} \cdot \left[2\vec{K}\cdot\vec{K}^* + 3LL^* \right] \qquad (29)$$

The sum over the electron spin states is carried out using standard techniques[8], introducing however the projection operator $\frac{1}{2}\left(1 \pm \frac{\vec{\sigma}\cdot\vec{p}_2}{|p_2|}\right)$ to project the required final state with projection[9] along or opposite the direction of motion.

Taking the LL^* term in (29), we have to evaluate quantities like

$$Tr.\left\{(\vec{\sigma}\cdot\vec{p}_2)\left[\not{p}_2 p_t \not{p}_1 p_t + m^2 p_t p_t \right]\right\} \qquad (30)$$

and these vanish as seen from the Appendix, and also the $\vec{K}\cdot\vec{K}^*$ term.

Hence the percentage helicity is zero with incident deuterons unpolarized.

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9) Alladi Ramakrishnan: Elementary Particles and Cosmic Rays, (Pergamon, 1962) p.26.

Now let us discuss the helicity of the electron with the target deuterons polarized. If the initial deuteron spin direction is not random, then the density matrix ρ_f describing the final state can be written in the form

$$\rho_f = T \rho_i T^{\dagger} \tag{31}$$

where ρ_i denotes the density matrix characterizing the initial state of the deuteron and T the scattering matrix with elements defined by equation (1) . Hence our knowledge of the initial deuteron polarization must be fed ~~with~~ into matrix ρ_i .

The spin state of the deuteron is specified by the vector polarization $\langle T^1_\nu \rangle$ and the tensor polarization $\langle T^2_\nu \rangle$. If however, in analogy to the photon (for which the state $m = 0$ is unphysical) one defines linear combination corresponding to elliptic or circular polarization (these types of polarization are obtained[7] for deuterons in the final state of the reactions $\gamma + D \rightarrow \pi^0 + D$, $\pi + D \rightarrow \pi + D$) and considers all the particles to be in one of the 'circular polarized' states, the situation will be described in terms of $\langle T^\mu_\nu \rangle$ by saying that

$$\langle T^2_{\pm 2} \rangle = \pm \frac{i\sqrt{3}}{2} \; ; \; \langle T^2_0 \rangle = \frac{1}{\sqrt{2}} \; ; \tag{32}$$

the rest being zero. The normalization for the operators T^μ_ν is the same as in the previous chapter and the circular polarization is with reference to the Z -axis. Also the situation of a "cylindrically symmetric" target polarization

is described by saying that $\langle T_0^2 \rangle$ is the only non-vanishing parameter. If the deuterons are linearly polarized, then

$$\langle T_0^1 \rangle = \sqrt{\frac{3}{2}} \langle J_z \rangle = \sqrt{\frac{3}{2}} . \qquad (33)$$

and the rest of the parameters are zero, the linear polarization being with reference to the Z-axis . Thus the different situations can be taken into account by feeding the appropriate values of the non-vanishing parameters into the density matrix ρ_i of equation (31) .

The transition probability for the elastic scattering with incident deuterons polarized can now be written in impulse approximation as

$$|E|^2 \sum_{m',m''} \langle {}^3\chi_m \Big| \sum_{n=1,2} \left(i\vec{\sigma}_n \cdot \vec{K} + L \right) \Big| {}^3\chi_{m'} \rangle \times$$

$$\times \frac{1}{3} \left[1 + \sum_{\mu,\nu} (-1)^\nu \langle T_\nu^\mu \rangle T_{-\nu}^\mu \right]_{m',m''} \times$$

$$\times \langle {}^3\chi_m \Big| \sum_{n=1,2} \left(i\vec{\sigma}_n \cdot \vec{K} + L \right) \Big| {}^3\chi_{m''} \rangle^* \qquad (34)$$

where we take into account all possible types of deuteron polarization in the initial state. To evaluate the cross-section

between definite spin states of the electron, we now sum over the final deuteron spin states making use of the trace rules of chapter II . We have

$$|E|^2 \cdot \Big[2\vec{K}\cdot\vec{K}^* + 3LL^* + $$

$$+\sqrt{\tfrac{3}{2}}\cdot C\langle\vec{T}'\rangle\cdot\Big\{(\vec{K}^*\times\vec{K})+2(\vec{K}L^*-L\vec{K}^*)\Big\} +$$

$$+\sqrt{3}\sum_{\nu}(-1)^{\nu}(K\times K)^2_{\nu}\langle T^2_{-\nu}\rangle\Big] \qquad (35)$$

where $(K\times K)^2_{\nu}$ denotes a spherical tensor of rank 2 obtained on combining $\vec{K}$ with $\vec{K}$ through the usual Clebsch-Gordan addition. The above expression is bilinear in A and the sum over the electron spin states is carried out using the standard techniques, introducing as before the projection operator $\frac{1}{2}\left(1 \pm \frac{\vec{\sigma}\cdot\vec{p}_2}{|p_2|}\right)$ to project the required final spin state. In the process of the actual calculation, one can use the trace rules outlined in the Appendix.

We define the percentage helicity in the usual manner as

$$P = 100.\frac{\uparrow-\downarrow}{\uparrow+\downarrow} \qquad (36)$$

$\uparrow$, $\downarrow$ referring to the numbers of the electrons in the forward, backward spin projection states. Clearly P is independent of the deuteron body form factor. This simplification arises due to the neglect of the d -state admixture.

The following results are obtained with different modes of polarization for the initial spin state of the deuterium.

The percentage helicity is zero with the incident deuterons either unpolarized or 'circularly polarized' with respect to the incident beam direction which is chosen as Z -axis. The percentage helicity is again zero with 'cylindrically symmetric' target polarization, when $\langle T_0^2 \rangle$ is the only non-vanishing parameter. In the next section we discuss the case of linearly polarized targets.

5. Electron Helicity with linearly polarized target deuterons

In the previous section, we referred to the linearly polarized deuterons, where the situation is described by $\langle T_0^1 \rangle = \sqrt{\frac{3}{2}} \langle J_z \rangle = \sqrt{\frac{3}{2}}$ and the rest of the parameters are zero, the linear polarization being with reference to the Z -axis (incident beam direction).

Using expression (35) and the trace rules given in the Appendix, the percentage helicity in this case can be written in the form

$$P = \frac{\frac{3}{2} \cdot \frac{\langle \vec{J} \rangle}{|\vec{p}_2|} \cdot \left(A\vec{p}_1 + B\vec{p}_2 \right)}{C \langle \vec{J} \rangle \cdot \left(\vec{p}_1 \times \vec{p}_2 \right) + D} \qquad (37)$$

where A, B, C and D are given by

$$A = 8\left(F_1^\circ + F_2^\circ\right)^2 \left[\left(m^2 - E_1E_2 + \vec{p}_1\cdot\vec{p}_2\right)\left\{\vec{p}_2\cdot\left(\vec{p}_2 - \vec{p}_1\right)\right\} + \left|\vec{p}_1\times\vec{p}_2\right|^2\right] +$$

$$+ 16\left(F_1^\circ + F_2^\circ\right)\left[\left(F_1^\circ + F_2^\circ\right)\left(E_2 - E_1\right) + 2MF_1^\circ\right] E_2\left[\left(\vec{p}_1\cdot\vec{p}_2\right) - \left(\vec{p}_2\cdot\vec{p}_2\right)\right]; \quad (38)$$

$$B = -8\left(F_1^\circ + F_2^\circ\right)^2 \left[\left(m^2 - E_1E_2 + \vec{p}_1\cdot\vec{p}_2\right)\left\{\vec{p}_2\cdot\left(\vec{p}_2 - \vec{p}_1\right)\right\} + \left|\vec{p}_1\times\vec{p}_2\right|^2\right] +$$

$$+ 16\left(F_1^\circ + F_2^\circ\right)\left[\left(F_1^\circ + F_2^\circ\right)\left(E_2 - E_1\right) + 2MF_1^\circ\right] E_2\left[\left(\vec{p}_1\cdot\vec{p}_2\right) - \left(\vec{p}_1\cdot\vec{p}_1\right)\right]; \quad (39)$$

$$C = 0 \; ;$$

$$D = 16\left(F_1^\circ + F_2^\circ\right)^2 \times$$

$$\times\left[\left(E_1E_2 - \vec{p}_1\cdot\vec{p}_2 - m^2\right)\left(\left|\vec{p}_2 - \vec{p}_1\right|^2\right) + \left|\vec{p}_1\times\vec{p}_2\right|^2\right] +$$

$$+ 12\left[\left(F_1^\circ + F_2^\circ\right)\left(E_2 - E_1\right) + 2MF_1^\circ\right]^2 \times$$

$$\times\left[E_1E_2 + \vec{p}_1\cdot\vec{p}_2 + m^2\right]. \quad (41)$$

The percentage helicity is zero with target polarization perpendicular to thescattering plane. Numerical estimates of P with the target polarization along the direction of the incident electron beam of energy 900 Mev are given in Table 1 .

Here we make use of the results given in ref.[10] for the isoscalar form factors F_1° and F_2° .

$$F_1^\circ = F_{1S} \; ; \; F_2^\circ = -0.12\, F_{2S} \qquad (42)$$

The percentage helicity is expected to be small with the target polarization perpendicular to the beam and in the scattering plane. The $P_{\parallel}$ and $P_{\perp}$ can be coupled like vectors to give the percentage helicity for any arbitrary direction of target polarization.

6. Electron Helicity with linearly polarized target protons

The electron helicity in the elastic e-p scattering is easily computed in a similar way and is given by an expression similar to equation (37) , with $\langle \vec{J} \rangle$ replaced by the target polarization vector $\vec{P}_T$ but without the factor 3/2 and where A , B , C , D are given by

$$A = 16\left(F_1^p + K^p F_2^p\right)^2 \times$$

$$\times \left[\left(m^2 - E_1 E_2 + \vec{p}_1 \cdot \vec{p}_2\right) \left\{ \vec{p}_2 \cdot (\vec{p}_2 - \vec{p}_1) \right\} + \left| \vec{p}_1 \times \vec{p}_2 \right|^2 \right] +$$

$$+ 16\left(F_1^p + K^p F_2^p\right) \times$$

$$\times \left[\left(F_1^p + K^p F_2^p\right)(E_2 - E_1) + 2M F_1^p \right] E_2 \left[\vec{p}_1 \cdot \vec{p}_2 - \vec{p}_2 \cdot \vec{p}_2 \right] \; ; \qquad (43)$$

10) R.Hofstadter, C. de Vries and R.Herman, Phys. Rev. Letters, 6 , (1961) 290.
R. Hofstadter and R.Herman, Phys. Rev. Letters, 6 , (1961), 293.

$$B = -16\left(F_1^p + K^p F_2^p\right)^2 \times$$

$$\times\left[\left(m^2 - E_1 E_2 + \vec{p}_1\cdot\vec{p}_2\right)\left\{\vec{p}_2\cdot\left(\vec{p}_2 - \vec{p}_1\right)\right\} + \left|\vec{p}_1 \times \vec{p}_2\right|^2\right] +$$

$$+16\left(F_1^p + K^p F_2^p\right)\times$$

$$\times\left[\left(F_1^p + K^p F_2^p\right)\left(E_2 - E_1\right) + 2MF_1^p\right] E_2\left[\vec{p}_1\cdot\vec{p}_2 - \vec{p}_1\cdot\vec{p}_1\right] \; ; \qquad (44)$$

$$C = 0 \; ; \qquad (45)$$

$$D = 16\left(F_1^p + K^p F_2^p\right)^2 \times$$

$$\times\left[\left(E_1 E_2 - \vec{p}_1\cdot\vec{p}_2 - m^2\right)\left(\left|\vec{p}_2 - \vec{p}_1\right|^2\right) + \left|\vec{p}_1 \times \vec{p}_2\right|^2\right] +$$

$$+ 8\left[\left(F_1^p + K^p F_2^p\right)\left(E_2 - E_1\right) + 2MF_1^p\right]^2 \times$$

$$\times\left[E_1 E_2 + \vec{p}_1\cdot\vec{p}_2 + m^2\right] . \qquad (46)$$

where $K^p = 1.79$ and F_1^p, F_2^p denote the proton form factors.

Taking the target proton polarization to be parallel to the incident beam of energy 900 Mev , the estimates of the percentage helicity of the final electron are also given in Table 1 . It isvery interesting to observe that with polarized proton targets, the back scattering electrons are completely polarized opposite their direction of motion, independent of the particulár energy chosen. This is of high experimental interest in view of the availability of polarized proton targets.

It is interesting to note that the expression D in equation (46) is just the Rosenbluth cross-section without the kinematical factors. In fact attaching the density of states and the other kinematical factors and simplifying, with the neglect of higher order terms, we can put the expression (46) in the Rosenbluth form. This affirms that the quantities $\vec{K}$ and L of equations (10) and (11) are in good shape as the expression D in (46) can reproduce the Rosenbluth formula when all the kinematical factors are taken into account.

Appendix

In the evaluation of Helicity ~~amplitudes~~ states, one comes across expressions of the following type :

$$Tr.\left[(\vec{\sigma}.\vec{p})\,\not{A}\not{B}\right] \quad ; \tag{A.1}$$

$$Tr.\left[(\vec{\sigma}.\vec{p})\,\not{A}\not{B}\not{C}\not{D}\right] ; \tag{A.2}$$

and

$$Tr.\left[(\vec{\sigma}.\vec{p})\,\not{A}\not{B}\not{C}\not{D}\not{E}\not{F}\right] . \tag{A.3}$$

(A.1) and (A.2) are the general form of the expressions occurring in our chapter (from the projection operator $\frac{1}{2}\left(1 \pm \frac{\vec{\sigma}.\vec{p}_2}{|p_2|}\right)$) . We shall evaluate these and put them in an elegant form so that they can be used in sections 4 and 5 .

$$Tr.\left[(\vec{\sigma}.\vec{p})\not{A}\not{B}\right] = Tr.\left[(i\gamma_y\gamma_z p_x + i\gamma_z\gamma_x p_y + i\gamma_x\gamma_y p_z)\not{A}\not{B}\right] \quad (A.4)$$

Taking the ip_x term, its co-efficient can be written as

$$Tr.\left[\gamma_y\gamma_z\not{A}\not{B}\right] = -4(A_y B_z - A_z B_y) \quad (A.5)$$

Hence (A.4) can be written as

$$-4ip_x(A_y B_z - A_z B_y) + \text{Cyclic terms.}$$

$$= -4i\vec{p}.(\vec{A}\times\vec{B})$$

$$\therefore Tr.\left[(\vec{\sigma}.\vec{p})\not{A}\not{B}\right] = -4i\vec{p}.(\vec{A}\times\vec{B}) \quad (A.6)$$

Similar calculation yields

$$Tr.\left[(\vec{\sigma}.\vec{p})\not{A}\not{B}\not{C}\not{D}\right] = -4i\vec{p}.\left[(\vec{A}\times\vec{B})(C.D) - (\vec{A}\times\vec{C})(B.D) + (\vec{A}\times\vec{D})(B.C) + (\vec{B}\times\vec{C})(A.D) - (\vec{B}\times\vec{D})(A.C) + (\vec{C}\times\vec{D})(A.B)\right] \quad (A.7)$$

and

$$Tr.\left[(\vec{\sigma}.\vec{p})\not{A}\not{B}\not{C}\not{D}\not{E}\not{F}\right] = -4i\vec{p}.\vec{\Lambda} \quad (A.8)$$

where

$$\vec{\Lambda} = (\vec{A}\times\vec{B})\left[(C\cdot D)(E\cdot F)-(C\cdot E)(D\cdot F)+(C\cdot F)(D\cdot E)\right]$$

$$-(\vec{A}\times\vec{C})\left[(B\cdot D)(E\cdot F)-(B\cdot E)(D\cdot F)+(B\cdot F)(D\cdot E)\right]$$

$$+(\vec{A}\times\vec{D})\left[(B\cdot C)(E\cdot F)-(B\cdot E)(C\cdot F)+(B\cdot F)(C\cdot E)\right]$$

$$-(\vec{A}\times\vec{E})\left[(B\cdot C)(D\cdot F)-(B\cdot D)(C\cdot F)+(B\cdot F)(C\cdot D)\right]$$

$$+(\vec{A}\times\vec{F})\left[(B\cdot C)(D\cdot E)-(B\cdot D)(C\cdot E)+(B\cdot E)(C\cdot D)\right]$$

$$+(\vec{B}\times\vec{C})\left[(D\cdot E)(F\cdot A)-(D\cdot F)(E\cdot A)+(D\cdot A)(E\cdot F)\right]$$

$$-(\vec{B}\times\vec{D})\left[(C\cdot E)(F\cdot A)-(C\cdot F)(E\cdot A)+(C\cdot A)(E\cdot F)\right]$$

$$+(\vec{B}\times\vec{E})\left[(A\cdot C)(D\cdot F)-(A\cdot D)(C\cdot F)+(A\cdot F)(C\cdot D)\right]$$

$$-(\vec{B}\times\vec{F})\left[(A\cdot C)(D\cdot E)-(A\cdot D)(C\cdot E)+(A\cdot E)(C\cdot D)\right]$$

$$+(\vec{C}\times\vec{D})\left[(A\cdot B)(E\cdot F)-(A\cdot E)(B\cdot F)+(A\cdot F)(B\cdot E)\right]$$

$$-(\vec{C}\times\vec{E})\left[(A\cdot B)(D\cdot F)-(A\cdot D)(B\cdot F)+(A\cdot F)(B\cdot D)\right]$$

$$+(\vec{C}\times\vec{F})\left[(A\cdot B)(D\cdot E)-(A\cdot D)(B\cdot E)+(A\cdot E)(B\cdot D)\right]+$$

$$+(\vec{D}\times\vec{E})\left[(A\cdot B)(C\cdot F)-(A\cdot C)(B\cdot F)+(A\cdot F)(B\cdot C)\right]$$

$$-(\vec{D}\times\vec{F})\left[(A\cdot B)(C\cdot E)-(A\cdot C)(B\cdot E)+(A\cdot E)(B\,C)\right]$$

$$+(\vec{E}\times\vec{F})\left[(A\cdot B)(C\cdot D)-(A\cdot C)(B\cdot D)+(A\cdot D)(B\cdot C)\right]\ . \qquad (A.9)$$

(A.6) and (A.7) are repeatedly used in sections 4 and 5 to evaluate the traces.

Table 1

Percentage helicity of the electrons in the final state of $e+D \rightarrow e+D$ and $e+p \rightarrow e+p$ at an energy 900 Mev. and with target polarization along the direction of the beam.

θ	0°	30°	60°	90°	120°	150°	180°
$-P(e+D \rightarrow e+D)$	0	6.4	10.2	11.1	12.8	22.4	75
$-P(e+p \rightarrow e+p)$	0	13.8	5.99	1.36	36.67	80.09	100

CHAPTER IV

DEUTERON POLARIZATION FOLLOWING ELASTIC NEUTRAL PION PHOTO-PRODUCTION

Abstract

The spin state of the deuteron following elastic photo-production of neutral pions is investigated in impulse approximation using the photo-production amplitudes of CHEW et al. It is found that the deuterons in the final state are 'polarized' perpendicular to the reaction plane and one of the axes of alignment coincides with this direction. One can easily determine now the coordinate system defined by the principal axes and estimate the values of the parameters in that system. It is found that corresponding to pion production at an angle θ with respect to the beam direction, the residual deuteron is aligned with one of the principal axes making nearly an angle $-\theta$ and the other axis $90^\circ + \theta$ in the plane of production.

DEUTERON POLARIZATION FOLLOWING ELASTIC NEUTRAL PION PHOTO-PRODUCTION *

1. Introduction

The photo-production of pions from deuterons has been discussed in impulse approximation by many authors[1] and in particular the elastic photo-production of pions has been studied[2] theoretically using the basic amplitudes of Chew, Goldberger, Low and Nambu[3] . This process is of interest since it involves no ambiguities regarding the final two-nucleon state and is one where the corrections[4] due to final state nuclear interactions are taken fully into consideration through the bound state. A reliable calculation is possible in this case since the spatial wave function of the deuteron is known

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* G. Ramachandran and R.K.Umerjee, Nuclear Phys. 54 , 665 (1964)

1) G.F.Chew and H.W.Lewis, Phys. Rev. 84 , 779 (1951)
M. Lax and H. Feshbach, Phys. Rev. 88 , 509 (1952)
Y. Saito, Y. Watanabe and Y. Yamaguchi, Prog. Th. Phys. 7 , 103 (1952)
J. Chapplear, Phys. Rev. 99, 254 (1955)
V. Devanathan and G. Ramachandran, Nuclear Phys. 23, 312 (1961)
2) A Alladi Ramakrishnan, V. Devanathan and G. Ramachandran, Nuclear Phys., 24 , 163 (1961)
3) G.F.Chew, M.L.Goldberger, F.E.Low and Y. Nambu, Phys. Rev. 106 , 1345 (1957)
4) N. Mac Donald, Il Nuovo Cimento, 15 , 301 (1950).

and the complete amplitude for the photo-production of π^0 from nucleons is given by Chew et al.[3] Such a calculation has already been made[2] as well as a detailed comparison with the experimental results.[5] An interesting observable in this reaction is the spin state of the deuteron in the final state. This has been mentioned by Chew and Lewis[6] though no systematic calculation seems to have been made so far.

We concern ourselves in this chapter with this problem and study theoretically the final spin state of the deuteron in the reaction $\gamma + D \rightarrow \pi^0 + D$ using impulse approximation and CGLN amplitudes. The calculation of the orientation parameters is facilitated by the method outlined in Chapter II. Numerical estimates of the parameters are presented for incident photon energies 1.5, 1.75 and 2.00 pion mass units taking also into consideration the kinematics.

2. Calculation using CGLN amplitudes

The amplitude T for photo-production of π^0 mesons from deuterons in impulse approximation can be written as[2]

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5) B. Wolfe, A.Silverman and J.W. DeWire, Phys. Rev., 99, 268 (1955)
H.L.Davis and D.R.Corson, Phys. Rev., 99, 273 (1955)
J.W.Rosengren and N.Baron, Phys. Rev., 101, 410 (1956)

6) G.F.Chew and H.W.Lewis, Phys. Rev., 84, 779 (1951)

$$T = \langle f | t_1 e^{i\vec{k}\cdot\vec{r}_1} + t_2 e^{i\vec{k}\cdot\vec{r}_2} | i \rangle \qquad (1)$$

where the t's which are the basic photo-production amplitudes from free nucleons (position coordinates $\vec{r}_1$ and $\vec{r}_2$) now contain also the isotopic spin operator and can be expressed as

$$t = t_p \left(\frac{1+\tau_z}{2}\right) + t_n \left(\frac{1-\tau_z}{2}\right) \qquad (2)$$

where t_p , t_n are the amplitudes for neutral π meson production from the proton and the neutron respectively and $\frac{1+\tau_z}{2}$ and $\frac{1-\tau_z}{2}$ are the projection operators respectively for the proton and the neutron . Using the CGLN amplitudes t_p , t_n are given by

$$t_p = t(\gamma + p \rightarrow p + \pi^\circ) = f^+ + f^\circ \qquad (3)$$

$$t_n = t(\gamma + n \rightarrow n + \pi^\circ) = f^+ - f^\circ \qquad (4)$$

where f^+ and f° are given with the neglect of secondary scattering corrections by

$$f^+ = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \left[i\vec{\sigma}\cdot\vec{\mu} \times (\vec{\nu}\times\vec{\epsilon})\, \lambda b^{+-} + \frac{i(\vec{\sigma}\cdot\vec{\mu})(\vec{\mu}\cdot\vec{\epsilon})}{2M\mu_0} + \vec{\mu}\cdot(\vec{\nu}\times\vec{\epsilon})\, \lambda b^{++} \right] \qquad (5)$$

and $$f^{\circ} = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \left[-i(\vec{\sigma}.\vec{\epsilon})\alpha\mu_0^2 - i\vec{\sigma}.\vec{\mu}\times(\vec{\nu}\times\vec{\epsilon})\alpha + \frac{i(\vec{\sigma}.\vec{\mu})(\vec{\mu}.\vec{\epsilon})}{2M\mu_0} \right] \quad (6)$$

where

$$h^{++} = \frac{4}{3\mu^3} \cdot e^{i\delta_{33}} \sin\delta_{33} \quad (7)$$

and

$$h^{+-} = -\frac{2}{3\mu^3} \cdot e^{i\delta_{33}} \sin\delta_{33} \quad (8)$$

Here, $\vec{\mu}, \vec{\nu}$ denote the meson and photon momenta, μ_0, ν_0 their energies, M the nucleon mass, μ_p, μ_n the magnetic dipole moments of the proton and neutron, e^2 the fine structure constant, $\vec{\epsilon}$ the photon polarization and the pion nucleon coupling constant f is taken to be $\sqrt{0.08}$.

Also

$$\vec{k} = \vec{\nu} - \vec{\mu}$$

$$\lambda = \frac{\mu_p - \mu_n}{4Mf^2} \quad (9)$$

$$\alpha = \frac{\mu_p + \mu_n}{2M\mu_0}$$

The initial state $|i\rangle$ representing the deuteron (at rest) can be written neglecting the small D-state admixture as

$$|i\rangle = \frac{1}{\sqrt{2}} \left[p(1)n(2) - p(2)n(1) \right] \chi_m \frac{u(r)}{r} (2\pi)^{-3/2} \quad (10)$$

where p and n denote the two cha rge states proton and neutron, ${}^3\chi_m$ the triplet spin state with projection quantum number m , $\frac{u(r)}{r}$ the s-state radial wave function, $\vec{r}$ being the relative position co-ordinate

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \tag{11}$$

Denoting by

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

the position co-ordinate corresponding to the centre of mass of the two nucleon system the final state $|f\rangle$ can be written as

$$|f\rangle = \frac{1}{\sqrt{2}} \left[p(1)\, n(2) - p(2)\, n(1) \right] {}^3\chi_{m'} \frac{u(r)}{r} (2\pi)^{-3/2} e^{i\vec{k}\cdot\vec{R}} \tag{12}$$

Re-writing (2) as

$$t = \frac{t_p + t_n}{2} + \frac{t_p - t_n}{2} \tau_z \tag{13}$$

and observing that the matrix elements of τ_z (1) and τ_z (2) vanish taken between the isotopic spin singlet states of the deuteron we have after eliminating $\vec{R}$ and using (3) and (4) for t_p , t_n

$$T = \langle f | \frac{t_p(1) + t_n(1)}{2} e^{\frac{i\vec{k}\cdot\vec{r}}{2}} + \frac{t_p(2) + t_n(2)}{2} e^{\frac{-i\vec{k}\cdot\vec{r}}{2}} | i \rangle \tag{14}$$

$$= \langle {}^3\chi_{m'} | f^+(1) + f^+(2) | {}^3\chi_m \rangle \, Ed \tag{15}$$

where E_d the overlap integral can be evaluated as in Chew and Lewis[6].

Before evaluating the orientation pa_rameters we shall write f^+ in terms of spin deppendent and spin independent parts as

$$f^+ = C\vec{\sigma}.\vec{K} + L \tag{16}$$

so that we can write the spin matrix element as

$$\langle {}^3\chi_{m'} | f^+(1) + f^+(2) | {}^3\chi_m \rangle$$

$$= \langle {}^3\chi_{m'} | 2C\vec{J}.\vec{K} + 2L | {}^3\chi_m \rangle \tag{17}$$ †

$\vec{K}$ and L are given by

$$\vec{K} = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \left[\vec{u} \times (\vec{\nu} \times \vec{\epsilon}) \lambda h^{+-} + \frac{(\vec{u}.\vec{\epsilon})}{2M\mu_0} \vec{u} \right] \tag{18}$$

$$L = \frac{2\pi e f}{\sqrt{\mu_0 \nu_0}} \vec{u}.(\vec{\nu} \times \vec{\epsilon}) \lambda h^{++} \tag{19}$$

If T denotes the transition matrix element for the process $p + D \rightarrow \pi^0 + D$, the density matrix for the final deuteron is given by

$$\rho = \frac{1}{2} \sum_{\vec{\epsilon}} T \cdot \frac{1}{3} \cdot T^\dagger \tag{20}$$

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† The factor 2 appears in the spin-dependent part since the $\vec{\sigma}$ denote twice the angular momentum matrices for a nucleon. This is a feature in all our deuteron calculations.

corresponding to unpolarized targets, $\vec{\epsilon}$ being the polarization of the photon . Using (43) to (45) of Chapter II and substituting for $\vec{K}$ and L from (18) and (19) and summing over photon polarizations, we have

$$D = \frac{8\lambda^2\nu^2}{9\mu^4} \sin^2\delta_{33}(2 + 5\sin^2\theta) + \frac{\mu^4}{2M^2\mu_0^2}\sin^2\theta \qquad (21)$$

where θ is the angle between $\vec{\mu}$ and $\vec{\nu}$. Also

$$\langle \vec{J} \rangle = \frac{1}{D} \times \frac{2\lambda \sin^2\delta_{33}}{3M\mu\mu_0} (\vec{\mu} \times \vec{\nu}) \qquad (22)$$

If we now choose following Lakin[7] the z-axis of our co-ordinate system to be along $\langle \vec{J} \rangle$, i.e perpendicular to the reaction plane, we find

$$\langle T_0^2 \rangle = \sqrt{2} \left\{ -\frac{1}{4} + \frac{3}{4D} \cdot \frac{8\lambda^2\nu^2}{9\mu^4} \sin^2\delta_{33}(1 + \sin^2\theta) \right\} \qquad (23)$$

$$\langle Q_{xz} \rangle = 0 \qquad (24)$$

and

$$\langle Q_{yz} \rangle = 0. \qquad (25)$$

In other words,

$$\langle T_{\pm 1}^2 \rangle = 0. \qquad (26)$$

which means that the deuteron in the final state is 'polarized' perpendicular to the reaction plane and one of the principal axes of alignment is along the same direction. To fix the other principal

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7) W.Lakin, Phys. Rev., 98 , 139 (1955)

axes, let α be the angle which $\vec{\nu}$ makes with one of the principal axes in the plane of the reaction chosen to be along the x -axis. Then $\langle Q_{xy} \rangle$ is given by

$$\frac{4D}{3} \langle Q_{xy} \rangle = a \sin 2(\alpha+\theta) + b \cos 2(\alpha+\theta) \qquad (27)$$

where

$$a = \frac{-4\lambda^2 \nu^2}{9\mu^4} \sin^2 \delta_{33} + \frac{\mu^4}{4M^2\mu_0^2} \sin^2\theta \qquad (28)$$

and

$$b = \frac{2\nu}{3M\mu_0} \sin\theta \sin 2\delta_{33} \qquad (29)$$

$\langle Q_{xy} \rangle$ should vanish if (x y) are to be the principal axes. We can therefore solve for α which thus fixes the alignment of the deuteron spin in the final state. The degree of 'polarization' as a function of the meson angle of emission is given by

$$|\langle \vec{J} \rangle| = \frac{1}{D} \cdot \frac{2\lambda\nu \sin^2\delta_{33} \sin\theta}{3M\mu_0} \qquad (30)$$

Also

$\langle (Q_{xx} - Q_{yy}) \rangle$ is given in terms of α and θ by

$$\frac{2D}{3} \langle (Q_{xx} - Q_{yy}) \rangle = a \cos 2(\alpha+\theta) - b \sin 2(\alpha+\theta) \qquad (31)$$

Therefore in the frame of reference defined by the principal axes (i.e. where $\langle Q_{xy} \rangle = 0$) the parameter $\langle T^2_{+2} \rangle = \langle T^2_{-2} \rangle$ and is given by

$$\langle T^2_2 \rangle = \frac{\sqrt{3}}{2D} (a^2 + b^2)^{1/2} \qquad (32)$$

which is not necessarily zero. Consequently the density matrix for the final deuteron is not diagonal in the frame of reference defined by the principal axes of alignment. If $'\chi_{+1}$, $'\chi_{-1}$ and $'\chi_0$ denote the basic spin states the density matrix ρ could be diagonalised by choosing a new set of basic states as follows.[7]

$$\Psi_1 = A|'\chi_{+1}\rangle + B|'\chi_{-1}\rangle \tag{33}$$

$$\Psi_2 = |'\chi_0\rangle \tag{34}$$

$$\Psi_3 = B^*|'\chi_{+1}\rangle - A^*|'\chi_{-1}\rangle \tag{35}$$

with

$$|A|^2 + |B|^2 = 1 \tag{36}$$

The basic states (33) and (35) correspond to the states of elliptic polarization in the case of photons.

As in Chapter III, the density matrix for the deuteron can be written as

$$\rho = \frac{1}{3} \mathrm{Tr}\, \rho \left[I + \sum_{\mu=1}^{8} \langle S_\mu \rangle S_\mu \right] \tag{37}$$

In the new representation, the density matrix for the deuteron is given by

$$\rho' = U \rho U^\dagger \tag{38}$$

where U is given by

$$\begin{bmatrix} A & 0 & B \\ 0 & 1 & 0 \\ B^* & 0 & -A^* \end{bmatrix} \tag{39}$$

U is hermitian, as can be seen by the relation $UU^+ = 1$ which can be verified with the help of (36). From (37) and using the notation of Chapter II , we have

$$\rho = \frac{1}{3} \begin{bmatrix} 1+\alpha_1+\beta_1 & 0 & 3\gamma_1 \\ 0 & 1-2\beta_1 & 0 \\ 3\gamma_1 & 0 & 1-\alpha_1+\beta_1 \end{bmatrix} \tag{40}$$

where $\alpha_1, \beta_1, \gamma_1$ denote the parameters

$$\alpha_1 = \frac{3}{2} \langle J_z \rangle \tag{41}$$

$$\beta_1 = \langle Q_{zz} \rangle \tag{42}$$

$$\gamma_1 = \frac{1}{3} \langle Q_{xx} - Q_{yy} \rangle \tag{43}$$

The density matrix written explicitly in terms of the new basic set of states is given by

$$\rho' = \frac{1}{3} \begin{bmatrix} 1+\beta_1+\alpha_1(|A|^2-|B|^2) & 0 & 2AB\alpha_1 \\ +3\gamma_1(AB^*+A^*B) & & -3\gamma_1(A^2-B^2) \\ 0 & 1-2\beta_1 & 0 \\ 2A^*B^*\alpha_1 & & 1+\beta_1-\alpha_1(|A|^2-|B|^2) \\ -3\gamma_1(A^{*2}-B^{*2}) & 0 & -3\gamma_1(AB^*+A^*B) \end{bmatrix} \tag{44}$$

For (44) to be diagonal, A and B have to be real and given as solutions of

$$2AB\alpha_1 = 3\gamma_1 (A^2 - B^2) \tag{45}$$

$$A^2 + B^2 = 1 \tag{46}$$

The eigen values λ_1 , λ_2 and λ_3 of (44) are obtained in terms of α_1 , β_1 and γ_1 as

$$\lambda_1 = \frac{1}{3} \left\{ (1+\beta_1) \pm \sqrt{\alpha_1^2 + 9\gamma_1^2} \right\} \tag{47}$$

$$\lambda_2 = \frac{1}{3} (1 - 2\beta_1) \tag{48}$$

$$\lambda_3 = \frac{1}{3} \left\{ (1+\beta_1) \mp \sqrt{\alpha_1^2 + 9\gamma_1^2} \right\} \tag{49}$$

Thus the spin state of the deuteron in the final state of $\gamma + D \rightarrow \pi^0 + D$ consists of a statistical mixture of the elliptic polarized states Ψ_1 , Ψ_2 and Ψ_3 with corresponding statistical weights λ_1 , λ_2 and λ_3 in the frame of reference defined by the principal axes of alignment.

3. Numerical Results

An examination of the expressions in (22), (23), (28), (29) and (32) shows that the deuteron polarization in the final state is small of order M^{-1} compared to the other parameters. Numerical estimates of the parameters in (22), (23) and (32) for photon energies. ν_0 = 1.5 , 1.75 and 2.00 pion mass units are obtained taking exactly into consideration the kinematics of the reaction.[8] (Fig.1) . Taking into account the recoil of the target particle (in our case, the deuteron) the energy E_1 of the outgoing meson can be written as[8]

$$E_1 = \frac{A E_T + p_I \cos\theta \left[A^2 - 4 m_1^2 (E_T^2 - p_I^2 \cos^2\theta)\right]^{1/2}}{2 (E_T^2 - p_I^2 \cos^2\theta)} \qquad (50)$$

where

$$E_T = m_{II} + E_I \qquad (51)$$

and

$$A = E_T^2 + m_1^2 - m_2^2 - p_I^2 \qquad (52)$$

From (50) it is clear that the pion energy μ_0 is a function of the angle θ of meson emission and fig. 2 exhibits the variation of μ_0 with θ for the three incident photon energies ν_0 = 1.5 , 1.75 and 2.00 in pion mass units. Taking into consideration the kinematics, the numerical results are presented

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8) A.M.Baldin, V.I.Gol'danski and I.L.Rozenthal, Kinematics of Nuclear Reactions (Pergamon) 1961,

in Tables 1 , 2 and 3 and refer to the parameters in the frame of reference defined by the principal axes.

Table 4 gives α as a function of ν_0 and the angle of meson emission , θ . We observe that approximately $\alpha \approx -\theta$ and this could be expected since

$$Tan\ 2(\alpha+\theta) = \frac{-b}{a} \qquad (53)$$

where 'b' given by (29) in an expression of order M^{-1} compared to 'a' which is given by (28) . Hence corresponding to pion production at an angle θ with respect to beam direction, the residual deuteron is aligned with one of the principal axes making nearly an angle $-\theta$ and the other axis $90+\theta$ in the plane of production. Thus all the three principal axes can be fixed in position at any angle of pion emission.

4. Conclusions

Some interesting comparisons can be made with the results of Chapter III where the polarization of the deuteron in the final state of the pion-deuteron scattering reaction was studied. In that chapter, the differential alignment parameter $\langle T_0^2 \rangle$ was found to be independent of the pion energy and the phase-shifts and this was accounted for by the homogeneity of $\vec{K}$ and L in the kinematical factors and the phase shifts. In the present chatper, the values of $\langle T_0^2 \rangle$ for three incident

photon energies are given in Table 2 and they are found to be almost the same. The very small variations involved can be traced to the M^{-1} term occurring in the expression for $\vec{K}$ in (18) and to the kinematical considerations outlined above. Also in Chapter II, the density matrix in the final state was diagonalised in a natural manner, thus obviating the necessity for constructing a new set of basic states (as is the case in this chapter). This can be attributed to the non-vanishing of $\langle T^2_{\pm 2} \rangle$ as can be seen from (40) and (43). Also, it is found that corresponding to pion production at an angle θ with respect to the beam direction, the residual deuteron is aligned with one of the principal axes making nearly an angle $-\theta$ and the other axis (90 + θ) in the plane of production thus enabling the fixing of all the principal axes in position at any angle of pion emission.

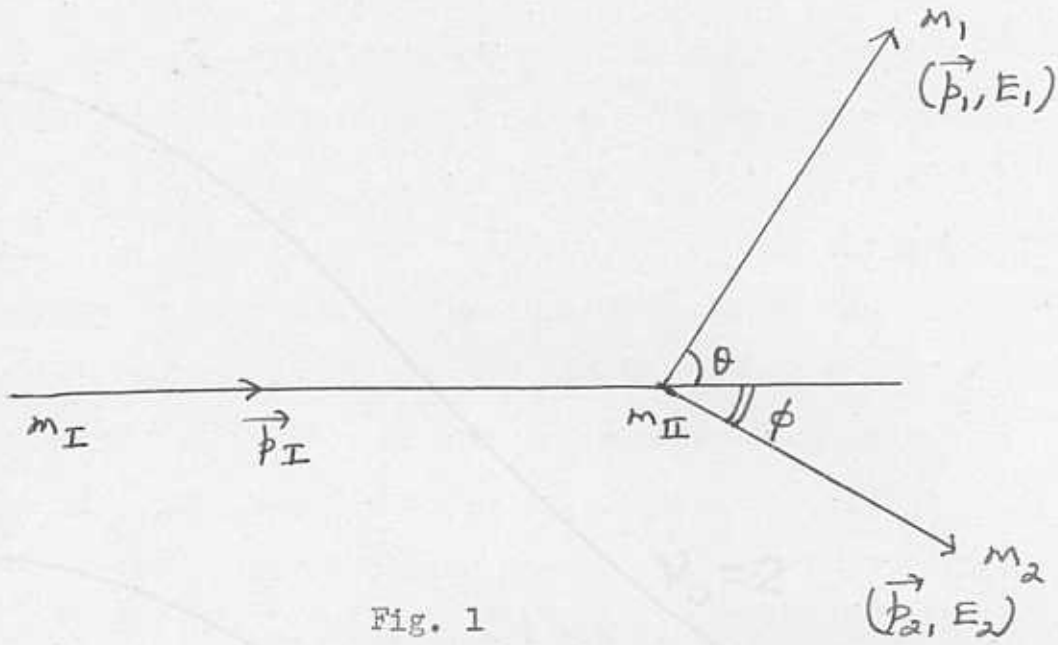

Fig. 1

m_I : Incoming particle of momentum $\vec{p}_I$

m_{II} : Target particle

m_1 and m_2 : Outgoing particles of momenta $\vec{p}_1$ and $\vec{p}_2$.

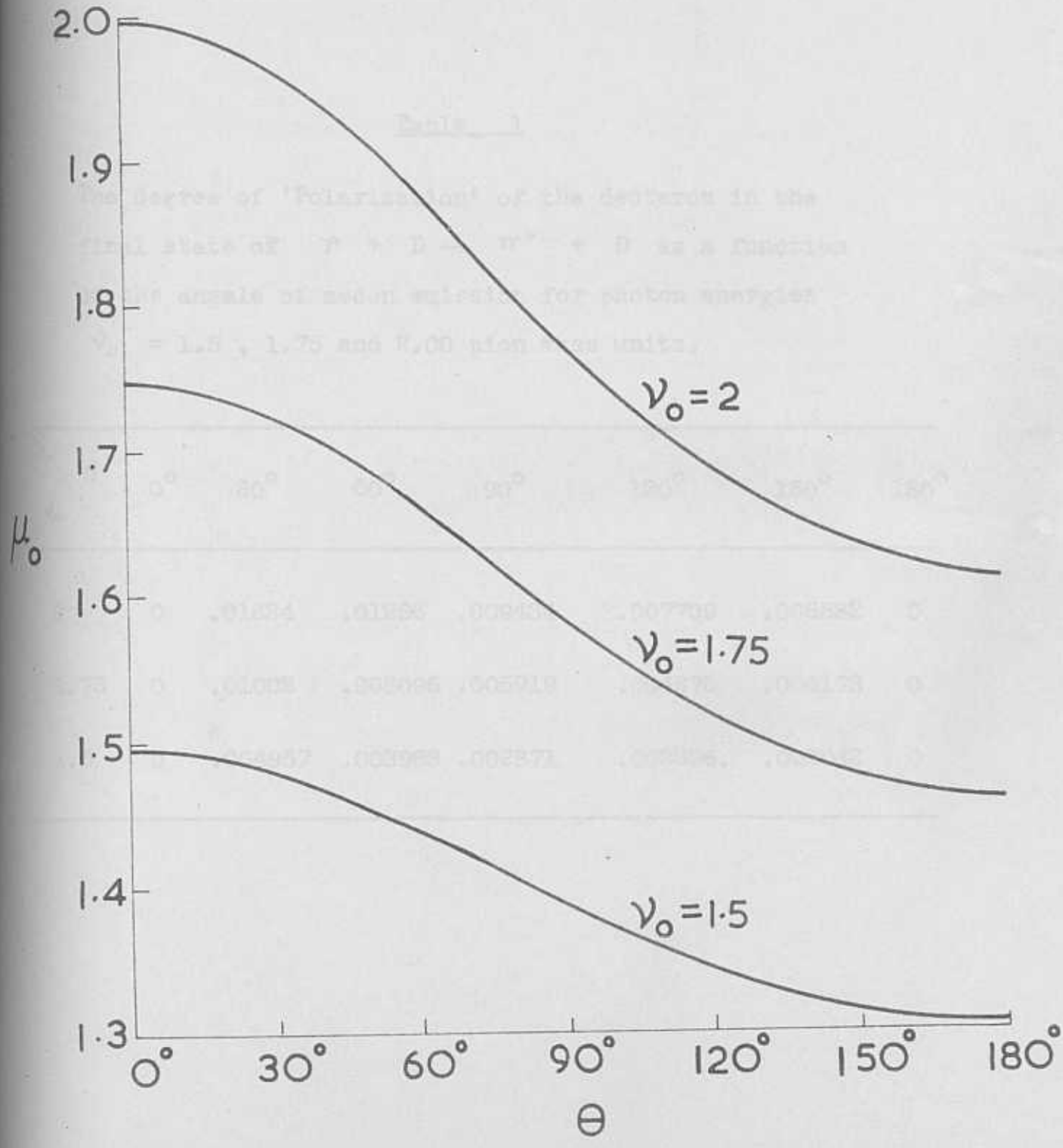
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
μ_0
$\nu_0 = 2$
$\nu_0 = 1.75$
$\nu_0 = 1.5$
0°
30°
60°
90°
120°
150°
180°
θ

Fig. 2

Table 1

The degree of 'Polarization' of the deuteron in the final state of $\gamma + D \rightarrow \pi^{\circ} + D$ as a function of the angle of meson emission for photon energies ν_o = 1.5 , 1.75 and 2.00 pion mass units.

ν_o \ θ	0°	30°	60°	90°	120°	150°	180°
2	0	.01624	.01296	.009434	.007709	.006582	0
1.75	0	.01002	.008096	.005919	.004876	.004178	0
1.5	0	.004957	.003983	.002871	.002396	.002043	0

Table 2.

Numerical estimates of the parameter $\langle T^2 o \rangle$ as a function of the angle of meson emission.

ν_o \ θ	0°	30°	60°	90°	120°	150°	180°
2	0.1770	0.05373	0.03196	0.05175	0.03181	0.05378	0.1768
1.75	0.1768	0.05316	0.03196	0.05175	0.03196	0.05358	0.1768
1.5	0.1768	0.05330	0.03223	0.05217	0.03181	0.05373	0.1768

Table 3

Numerical estimates of the parameter $\langle T_2^2 \rangle$ as a function of the angle of meson emission

γ_0 \ θ	0°	30°	60°	90°	120°	150°	180°
2	.2166	.1331	.07575	.06272	.07615	.1338	.2163
1.75	.2165	.1337	.07642	.06301	.07638	.1338	.2166
1.5	.2165	.1340	.07683	.06380	.07645	.1338	.2165

Table 4

Numerical estimates of the angle α which determines the principal axis in the plane of the reaction, as a function of the angle of meson emission.

ϑ_0 \ θ	0°	30°	60°	90°	120°	150°	180°
2.0	0	-26°.19'	-52°.55'	-81°.29'	-112°.37'	-145°.44'	-180°
1.75	0	-25°	-51°.48'	-80°.53'	-112°.7'	-145°.28'	-180°
1.50	0	-24°-45'	-50°.69'	-79°.7'	-112°.11'	-145°.52'	-180°

PART II

SCATTERING OF PIONS FROM NUCLEI

CHAPTER V

SCATTERING OF PIONS FROM NUCLEI.

Abstract

Elastic scattering of pions from nuclei is studied using the shell-model and the impulse approximation. Numerical results are presented in the energy range 65 to 150 Mev for targets carbon and oxygen and comparison has been made with the available experimental results. The agreement obtained is quite good.

SCATTERING OF PIONS FROM NUCLEI[+]

1. Introduction

The scattering of pions from nuclei has been studied experimentally for several years and the precision of the measurements has strongly improved through this period. Thus a considerable body of experimental fact, relevant to the interaction of pions in complex nuclei is already established. Nuclear emulsion experiments have demonstrated the principal mechanism of the interaction and given information regarding their energy dependence. More recent experiments with counters[1], clould chambers[2] and emulsions[3] have continued these studies and have more clearly shown the dependence of these processes on nuclear size.

[+] R.K.Umerjee, Nuclear Physics. (In print)

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Studies of the elastic scattering of pions by atomic nuclei have frequently been made within the frame-work of the optical model, according to which one replaces the interaction between the incident pion and the nucleus by a complex potential well having the form

$$V_c(x) = V(\omega) f(x)$$

where $f(x)$ is the nuclear density and $V(\omega)$ is a complex quantity called the "well-depth" and is a function of ω, the energy of the pion. The procedure then is to obtain $V(\omega)$ for some $f(x)$ which best fits the scattering data at the energy ω. Alternatively from a knowledge of the pion-nucleon scattering, the complex potential representing the interaction of the incident pion and the nucleus can be evaluated[4]. However it is expected that such a purely phenomenological treatment cannot be used as a tool to probe the structure of nuclei. Also the usual optical model approach does not make use of the angular distribution of the elementary pion-nucleon scattering process, but uses only the forward scattering amplitude $f(o)$. However it should be mentioned that a modified optical model analysis has been developed[5] which takes better account of $f(\theta)$ and that the experimental data of Baker et al[6] seem to support this type of modification

4) R.M.Frank, J.L.Gammel and K.M.Watson, Phys.Rev.101 (1956) 891.
5) L.S.Kisslinger, Phys.Rev. 98, (1955) 761.
6) W.F.Baker, J.Rainwater and R.E.Williams, Phys.Rev.112 (1958) 1763.

to some extent.

In this chapter we give an alternative point of view (based on the Impulse approximation) which uses the complete angular dependence of the π-N scattering modified by a form factor of the nucleus. We study the scattering process using the shell model and the impulse approximation. Our treatment is similar to that for photo-production of charged pions, where certain impulse approximation procedures have been outlined[7]†. It should be noted that while for charge exchange or inelastic scattering only the nucleons outside the closed shells contribute, the amplitude for elastic scattering contains contributions from all the nucleons in the nucleus. The method of calculation for scattering is outlined in Section 2. In Section 3, numerical results are presented for closed shell nuclei like carbon and oxygen and these are compared with the experimental values of Fujii[8], Edelstein et al[9] and Baker et al[6]. The agreement is found to be quite good.

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7) V. Devanathan and G. Ramachandran, Nuclear Physics, 42 (1963) 254.

8) T.A. Fujii, Phys. Rev. 113 (1959) 695.

9) R.M. Edelstein, W.F. Baker and J. Rainwater, Phys. Rev. 122 (1961) 252.

† Please see the Note at the end of this Chapter. (p.101)

2. Method

The amplitude for scattering of a pion from a nucleus (consisting of A nucleons) can be written in impulse approximation using shell model wave functions for the initial and final nuclear states as

$$\langle f | T | i \rangle = \langle f | \sum_{i=1}^{A} t_i | i \rangle$$

$$= \delta_{fi} \Big[n_1 \langle \psi_{00}(\cdots n_1) | t | \psi_{00}(\cdots n_1) \rangle +$$

$$+ n_2 \langle \psi_{00}(\cdots n_2) | t | \psi_{00}(\cdots n_2) \rangle +$$

$$+ \text{----------} \Big] +$$

$$+ \langle \psi_f(\cdots n) | \sum_{i=1}^{n} t_i | \psi_i(\cdots n) \rangle \qquad (1)$$

if we assume that the n nucleons outside closed shells are equivalent and closed shells remain closed shells during the process. The fact that the 'n' particles are equivalent is important to be able to evaluate their contribution since in this case we can make use of the well-known fractional parentage expansions.[+] The remaining $(A-n)$ nucleons

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[+] Such considerations will play an important part when we consider nuclei with nucleons outside closed shells. Studies on the scattering of pions from nuclei like Lithium are in progress and results will be published shortly: G. Ramachandran and R. K. Umerjee (to be published). Kindly see the Note at the end of this Chapter. (p. 101)

are distributed as $n_1, n_2, \cdots\cdots$ in the various closed shells. The matrix elements $\langle \psi_{00}(\cdots n_1)|t|\psi_{00}(\cdots n_1)\rangle$ (the suffixes 0,0 are used to denote that the wave functions refer to closed shells characterized by zero angular momentum and zero isotopic spin) can be easily calculated as we shall see below. The expansion (1) could be obtained[7] using the laplace expansion for the determinantal A-nucleon wave functions.

The single nucleon amplitudes t have the explicit form

$$t = e^{i\vec{k}\cdot\vec{r}}\left[\frac{t_p+t_n}{2} + \frac{t_p-t_n}{2}\,\tau_3\right] \qquad (2)$$

where t_p, t_n denote the scattering amplitudes[10] from the proton and neutron respectively, τ_3 the third component of isotopic spin of the nucleon, $\exp(i\vec{k}\cdot\vec{r})$ the momentum transfer to the nucleon whose position coordinate is denoted by $\vec{r}$.

Let us for the present (in this chapter) confine our attention to closed shell nuclei. As total isotopic spin $T = 0$ and since τ_3 is a spherical tensor operator of rank 1 in that space, the τ_3 term in (2) contributes zero to the

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10) G.F.Chew and F.E.Low, Phys. Rev. 101 (1956) 1570.

amplitude; i.e. the isovector part of the amplitude contributes zero to the closed shell nucleon amplitudes. Hence the non-vanishing contributions come to $\langle \psi_{00}(\cdots n_1) | t | \psi_{00}(\cdots n_1) \rangle$ only through the first term $\frac{t_p + t_n}{2}$.

$t_{p,n}$ have again the general form

$$t_{p,n} = i \vec{\sigma} . \vec{K}_{p,n} + L_{p,n} . \tag{3}$$

in terms of the spin-dependent and non-spin-dependent parts $\vec{K}$ and L .

If we write

$$t_p + t_n = i \vec{\sigma} . \vec{K} + L \tag{4}$$

then (from chapter II)

$$\vec{K} = \frac{8\pi}{3\omega q^3} \cdot e^{i\delta_{33}} \sin\delta_{33} (\vec{q}_2 \times \vec{q}_1) \tag{5}$$

$$L = -\frac{16\pi}{3\omega q^3} \cdot e^{i\delta_{33}} \sin\delta_{33} (\vec{q}_2 \cdot \vec{q}_1) \tag{6}$$

where $\vec{q}_1 . \vec{q}_2$ are the momenta of the initial and final pions and $\omega_1 (= \omega)$ and $\omega_2 (= \omega)$ are their energies.

We saw that the contribution to the closed ~~matrix~~ shell matrix elements came from the part $\frac{t_p + t_n}{2}$. Even here we see that the spin-dependent part contributes zero[7] by

seeking the scalar part of the operator $e^{i\vec{k}.\vec{r}}\; i\vec{\sigma}.\vec{K}$

viz.,

$$\frac{4\pi i j_1(kr)}{2kr} \cdot \left\{ \vec{k} \cdot \vec{K} \right\} (\vec{\sigma} . \vec{r}) \qquad (7)$$

and this again contributes zero since the matrix element of $(\vec{\sigma}.\vec{r})$ between two angular momentum states with same ℓ gives $C(\ell 1 \ell ; 00)$ which is zero. Or, in other words, the contribution of $(\vec{\sigma}.\vec{r})$ which has odd parity vanishes when taken between same states of definite parity. The non-vanishing contribution comes from the spin independent part, viz.,

$$\frac{L_p + L_n}{2} \cdot j_0(kr) \qquad (8)$$

Hence a typical matrix element can be written as

$$n_1 \langle \psi_{00}(\cdots n_1) | t | \psi_{00}(\cdots n_1) \rangle = n_1 \frac{L_p + L_n}{2} \cdot F_{0, n_1} \qquad (9)$$

where F_{ℓ, n_1} are the radial integrals

$$F_{\ell, n_1} = \int_0^\infty u^*_{n_1}(r)\, j_\ell(kr)\, u_{n_1}(r)\, r^2 dr . \qquad (10)$$

the u referring to the single nucleon radial wave functions in the particular shell. (This is indicated by the suffix n_1).

Hence in the case of elastic scattering from closed shell nuclei the square of the matrix element for scattering is given by

$$|Q|^2 = |\langle f|T|i\rangle|^2 \qquad (11)$$

$$= \left| (n_1 F_{o,n_1} + n_2 F_{o,n_2} + \cdots\cdots) \frac{L_p + L_n}{2} \right|^2$$

Now we can specialize the above expression for the case of C^{12} and O^{16}.

In the case of C^{12},

$$|Q|^2 = \left| (4 F_{o,s} + 8 F_{o,p}) \frac{L_p + L_n}{2} \right|^2 \qquad (12)$$

In the case of O^{16},

$$|Q|^2 = \left| (4 F_{o,s} + 12 F_{o,p}) \frac{L_p + L_n}{2} \right|^2 \qquad (13)$$

The radial integrals can be calculated using oscillator wave functions[11] and we have

$$F_{o,s} = \exp.(-k^2/4\beta) \qquad (14)$$

and

$$F_{o,p} = \left(1 - \frac{k^2}{6\beta}\right) \exp.(-k^2/4\beta) \qquad (15)$$

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11) J.P. Elliott and A.M. Lane, Handbuch der Physik, Vol. XXXIX (Springer-Verlag).

where value of β can be obtained from the relation between β and the mean square radius of the nucleus, viz.,

$$\frac{5}{2\beta} = \langle r^2 \rangle = A^{2/3} \tag{16}$$

A being the mass number of the nucleus in question.

$\vec{k}$ is given by the relation

$$\vec{k} = (\vec{q}_2 - \vec{q}_1) \tag{17}$$

The differential cross-section[+] is given by

$$\frac{d\sigma}{d\Omega} = (2\pi)^{-2} \mu_0^2 . |Q|^2 . \tag{18}$$

where μ, μ_0 are the momentum and energy respectively of the pion. The relevant numerical results are given in the next section.

Alternatively, we can also discuss how the pion can be used as a nuclear probe. Taking the case of scattering of pions from carbon, we have from equations (12), (14) and (15),

$$|Q|^2 = 16 \left(\frac{L_p + L_n}{2} \right)^2 . e^{-k^2/2\beta} . \left(3 - \frac{k^2}{3\beta} \right)^2 \tag{19}$$

[+]We use natural units in which $\hbar = c$ = pion mass = 1.

We just now saw that the value of the parameter β, which depends on the oscillation potential well depth can be determined by finding the expectation value of $\langle r^2 \rangle$ and equating it with the experimentally determined value. Alternatively, the experimental data of the scattering of pions from nuclei can be used to determine the value of β which gives the best fit. Thus the scattering of pions from nuclei.can be used as a tool to probe the structure of nuclei. This view gains support from Prof.T.Ericson's[+] remarks, namely, "With the availability of pion beams of high intensity it is no longer appropriate to consider the pion an exotic tool for nuclear investigations. We should rather see it as a nuclear probe on the same footing as the nucleons, the electron or the alpha particle. Its radically different properties, as compared to electrons and nucleons make it a potentially powerful addition to the arsenal we dispose of for nuclear study".

In the next section we present the numerical results and our conclusions.

[+] T.Ericson, 1963 International Conference on High Energy Physics and Nuclear structure, Geneva. CERN 63-28, p.47

3. Numerical Results and Conclusions

The differential cross-section for elastic scattering by carbon at 150 MeV have been tabulated in Table 1 along with the experimental values of Fujii[8]. Both the results have also been shown in Fig.1. There seems to be good agreement between the theoretical and experimental values.

Numerical results are also presented for carbon at 87.5, 80 and 69.5 Mev and for Oxygen at 87.5 Mev in tables 2 - 5 (and also figs.2–5) and comparison has been made with the experimental values of Edelstein et al[9] and Baker et al[6]. Again there is good agreement with the experimental results.

In both the cases of carbon and oxygen, the agreement is good at low angles, but the theoretical values obtained are much smaller at large angle scattering. In this connection, attention may be drawn to the experimental results of Baker Rainwater and Williams[6]. Their results agree well with the previously reported results at small angles but smaller at large angles by a factor varying between two and ten. It may be pointed out that at large angles, the inelastic events predominate due to large momentum transfers; and it is very likely that some of these events with low loss of energy may be counted as elastic since in almost all experiments, the elastic and inelastic events are distinguished only by measuring the energy of the scattered pions and the reliability of the experimental result thus largely depends on the energy resolution of the apparatus.

We are led to a feeling that as a result of the lowering of the values of the cross-section at large angles (because of improvement in experimental technique) we can expect that with further improvement, the large angle cross-sections may further decrease giving an agreement with our theoretical findings. On the other hand, in order to obtain a *fit* with the existing experimental results for the elastic scattering at large angles, we have to take into account some of the inelastic events with low energy losses.

NOTE

In this note a few remarks are made about the problem of scattering of pions as compared to the problem of photo-production of charged pions from nuclei.

The scattering of pions from nuclei differs essentially in two respects from the corresponding problem of charged pion photo-production. 1) The pion scattering amplitude ' t ' contains an isoscalar part in addition to the isovector part met with in charged pions; and 2) the nucleons in closed shells also contribute to the nuclear amplitude. It can be shown[1] that these two effects could be taken into account with ease by a simple extension of the exact impulse approximation procedure outlined earlier[2,3] for photo-production of charged pions.

If the nucleus considered has n-nucleons outside closed shells, then the contribution to the amplitude from these nucleons (assuming them to be equivalent) can be evaluated

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1) G. Ramachandran and R.K. Umerjee. (to be published)

2) V. Devanathan and G. Ramachandran, Nuclear Physics 42 (1963) 254

3) G. Ramachandran and V. Devanathan, Nuclear Physics (to be published)

using the appropriate fractional parentage expansions[3], thereby reducing the n-nucleon problem to that of a single nucleon. Thus with $j-j$ coupling,

$$\langle \Psi_f(\cdots n) | \sum_{i=1}^{n} t_i | \Psi_i(\cdots n) \rangle$$

$$= n \sum_{\alpha' J' T'} \langle (j)^{n-1} (\alpha' J' T') j' ; \alpha_f J_f T_f | \} (j)^{n-1} j' \alpha_f J_f T_f \rangle^* \times$$

$$\cdot \langle (j)^{n-1} (\alpha' J' T') j ; \alpha_i J_i T_i | \} (j)^{n} \alpha_i J_i T_i \rangle \times$$

$$\sum_{\lambda} U(J' j J_f \lambda ; J_i j') \sum_{\tau} U(T' \tfrac{1}{2} T_f \tau ; T_i \tfrac{1}{2})$$

$$\sum_{m, m_t} \sum C(j \lambda j' ; m m_\lambda m') C(\tfrac{1}{2} \tau \tfrac{1}{2} ; m_t m_\tau m_{t'}) \cdot$$

$$\langle j' m' ; \tfrac{1}{2} m_{t'} | t | j m ; \tfrac{1}{2} m_t \rangle$$

where the first 2 terms on the R.H.S. denote the coefficients of fractional parentage and $C(j \lambda j' ; m m_\lambda m')$ and $U(J' j J_f \lambda ; J_i j')$ etc denote the corresponding Clebsch-Gordan and recoupling coefficients respectively. For the case of elastic scattering we have to replace j' by j since the above general form includes the singly excited state of the final nucleus.

The above expression differs essentially from the corresponding expression obtained in reference 3) since in the case of charged pion production, τ could take only one value viz. $\tau = 1$, and the isotopic spin dependence could be factored out while in the scattering problem τ can take two values $\tau = 0, 1$.

This is the essential difference in the calculational technique in the two problems and this could be taken into account by just writing

$$t = \sum_{\tau=0,1} t_\tau$$

and following the procedure adopted in reference 3). An L-S coupling calculation for scattering of pions from Lithium is in progress and the results will be published shortly[1].

Captions to Figures

Fig.1. Differential cross-section in the laboratory system for the scattering of π^- by carbon at 150 Mev. The experimental values are those of Fujii[8)]

Fig.2. Differential cross-section in the laboratory system for the scattering of π^- by carbon at 87.5 Mev. The experimental values are those of Edelstein et al[9)]

Fig.3. Differential cross-section in the laboratory system for the scattering of π^- by carbon at 80 Mev. The experimental values are those of Baker et al[6)].

Fig.4. Differential cross-section in the laboratory system for the scattering of π^- by carbon at 69.5 MeV. The experimental values are those of Edelstein et al[9)]

Fig.5 Differential cross-section in the laboratory system for the scattering of π^- by oxygen at 87.5 MeV. The experimental values are those of Edelstein et al[9)]

Table 1.

The differential cross-section $d\sigma/d\Omega$ for the elastic scattering of π^- by carbon at 150 Mev.(In units of mb/sr).

Values of FUJII[8)]

θ in degrees	18,5	24.3	30.8	36.5	43.6	44.8	61.0	74.8	90.1	115.0	138.6
$\frac{d\sigma}{d\Omega}$	295 ± 17	178 ± 10	104 ± 6	50.6 ± 3.3	16 ± 1.3	15.1 ± 1.5	2.4 ± 0.3	1.2 ± 0.2	0.52 ± 0.07	0.78 ± 0.07	0.38 ± 0.07

Present calculation:

θ in degrees	20	25	30	35	40	45	60	75	90	115	140
$\frac{d\sigma}{d\Omega}$	323.4	201.7	113.6	60.25	21.69	17.14	4.823	.7654	.2547	.1711	.0252

Fig.1

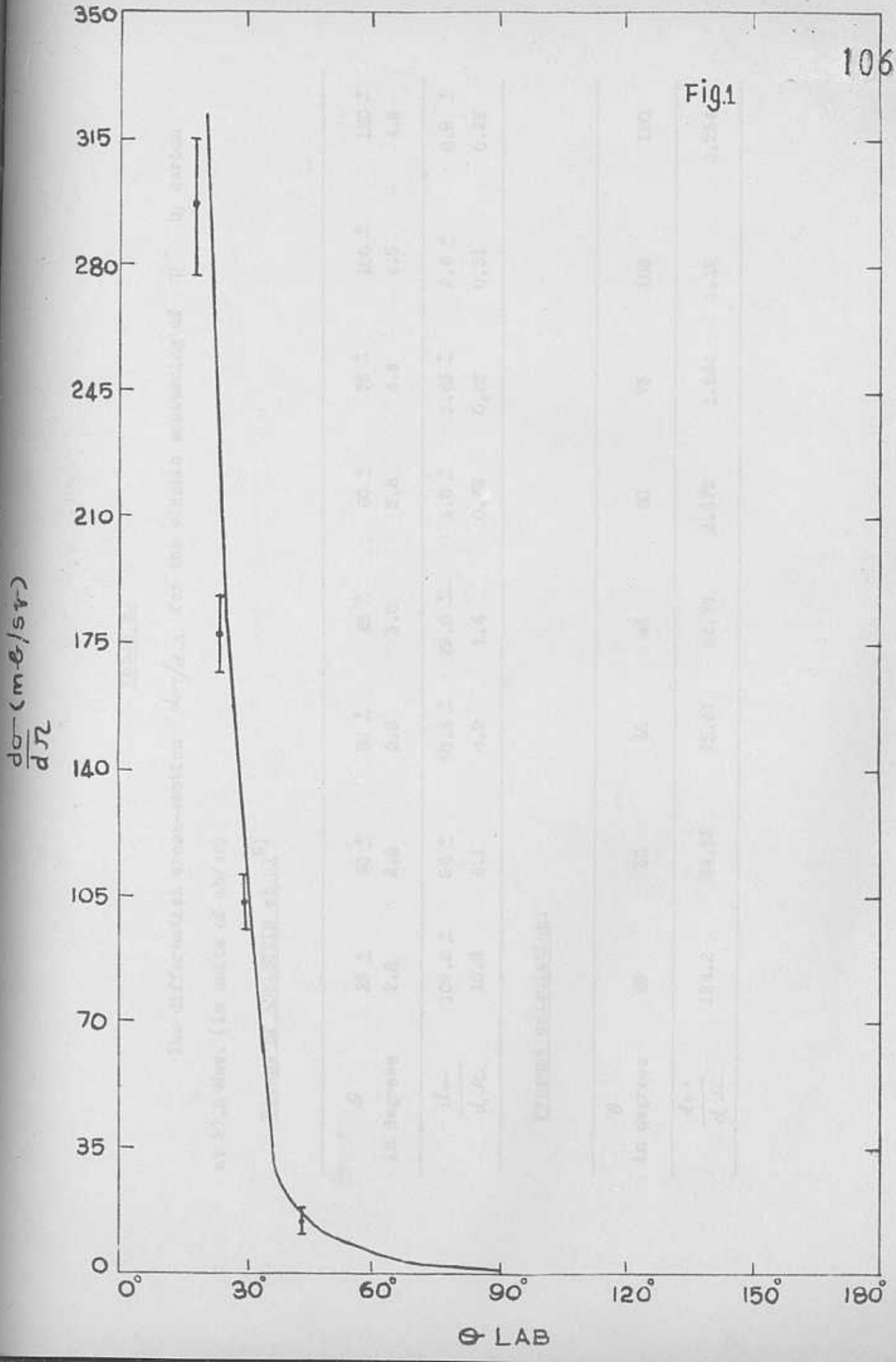

350
315
280
245
210
175
140
105
70
35
0
dσ/dΩ (mb/sr)
0°
30°
60°
90°
120°
150°
180°
Θ LAB

Table 2.

The differential cross-section $d\sigma/d\Omega$ for the elastic scattering of π^- by carbon at 87.5 Mev. (In units of mb/sr)

Values of EDELSTEIN et al[9)]

θ in degrees	25 ± 2.8	30 ± 2.8	35 ± 2.8	45 ± 3.8	60 ± 3.8	75 ± 4.5	100 ± 4.5	120 ± 4.5
$\frac{d\sigma}{d\Omega}$	109.2 ± 10.6	86 ± 6.1	68.4 ± 4.9	29.0 ± 1.6	5.8 ± 0.49	3.63 ± 0,26	2.6 ± 0.31	0.9 ± 0.28

Present calculation.

θ in degrees	25	30	35	45	60	75	100	120
$\frac{d\sigma}{d\Omega}$	124.2	94.62	75.31	32.75	3.872	1.831	1.12	0.234

Fig.2

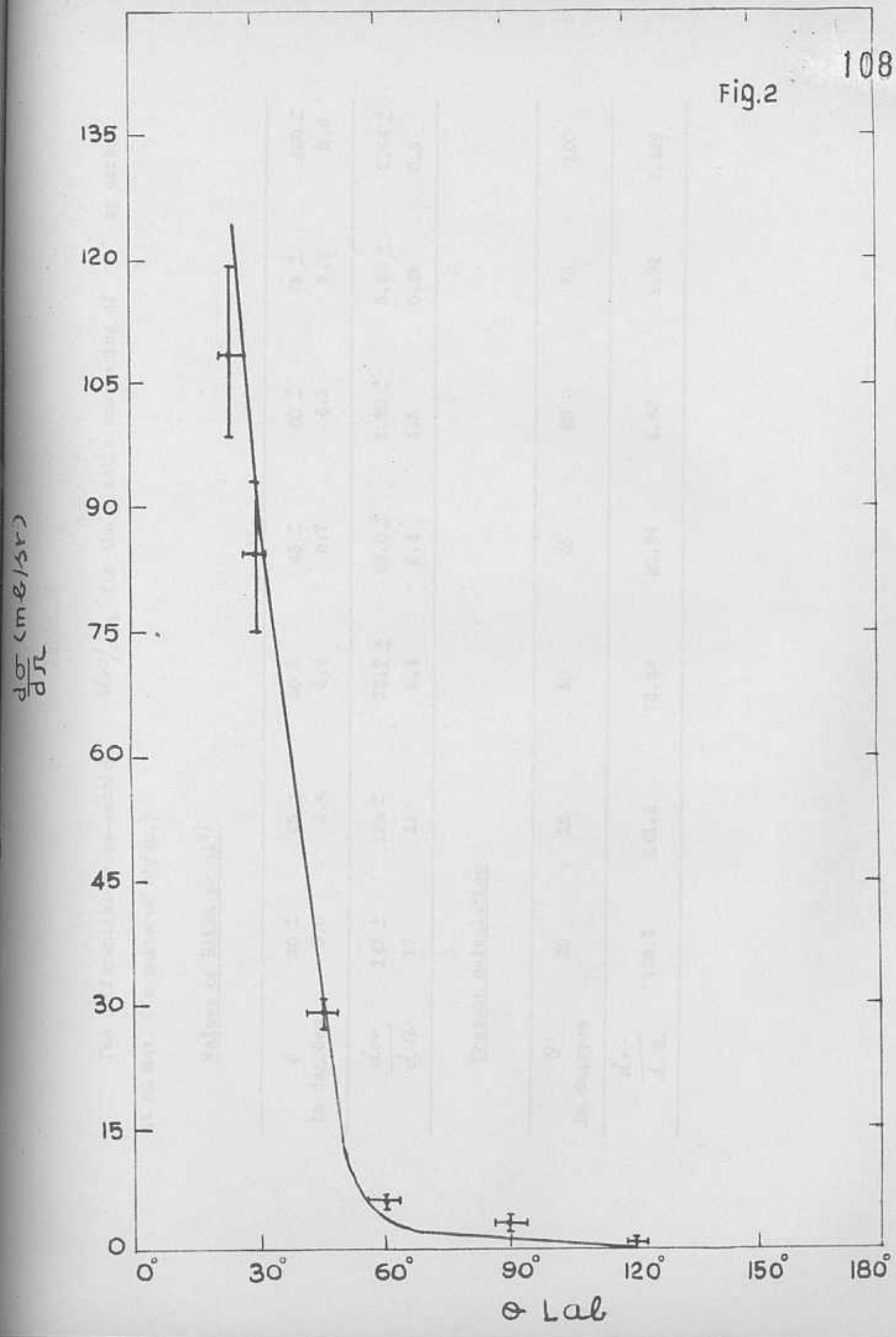

135
120
105
90
75
60
45
30
15
0
dσ/dΩ (mb/sr)
0°
30°
60°
90°
120°
150°
180°
θ Lab

Table 3

The differential cross-section $d\sigma/d\Omega$ for the elastic scattering of π^- by carbon at 80 Mev. (In units of mb/sr.)

Values of BAKER et al[6)]

θ in degrees	20 ± 3.0	25 ± 3.4	30 ± 3.4	45 ± 3.7	60 ± 4.1	75 ± 5.6	100 ± 5.6
$\frac{d\sigma}{d\Omega}$	147 ± 17	123 ± 11	73.7 ± 5.4	29.0 ± 2.4	5.28 ± 1.5	3.33 ± 0.65	2.84 ± 0.5

Present calculation

θ in degrees	20	25	30	45	60	75	100
$\frac{d\sigma}{d\Omega}$	168.3	141.2	78.62	20.24	4.62	1.92	0.426

Fig 3

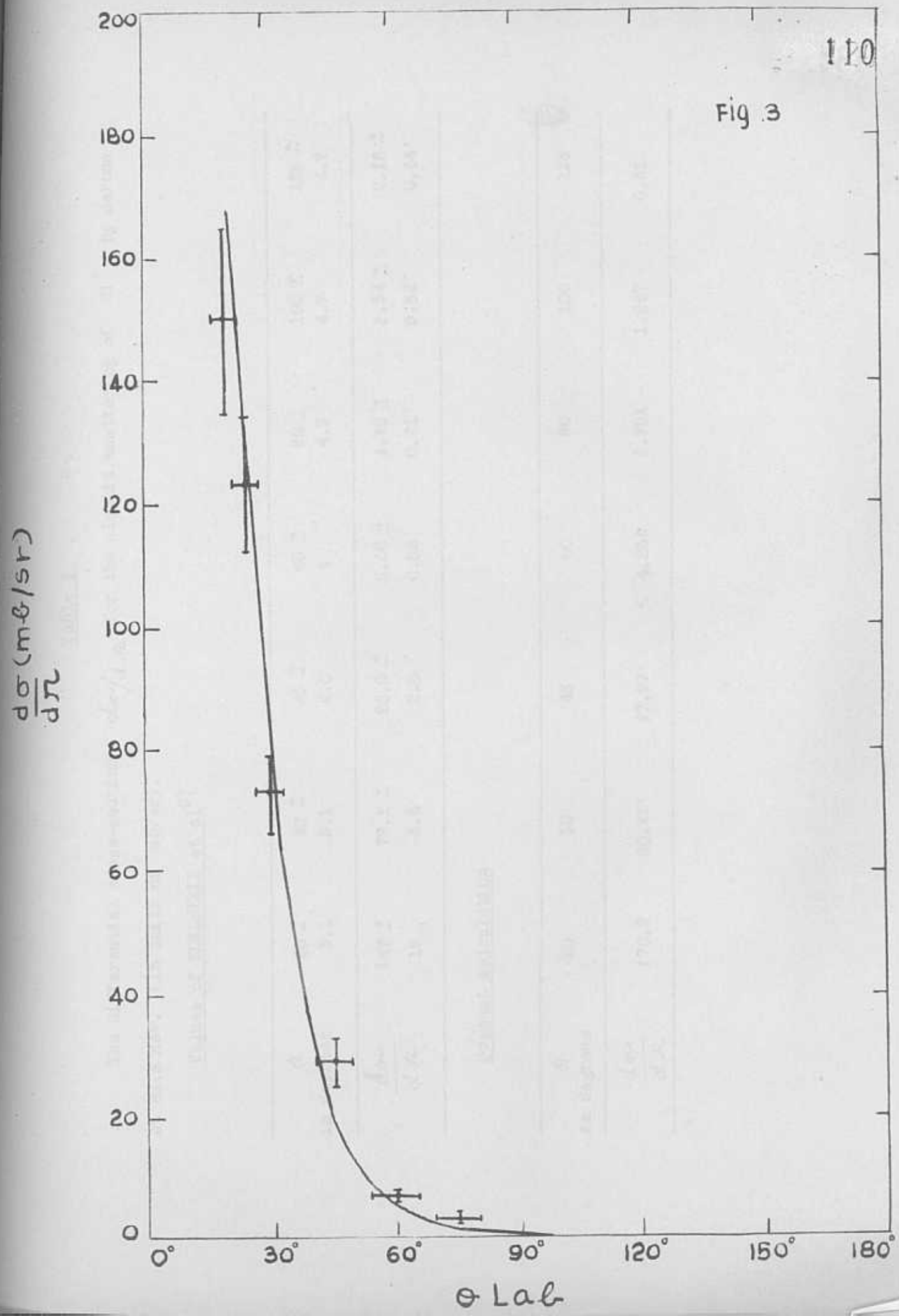

200
180
160
140
120
100
80
60
40
20
0
0°
30°
60°
90°
120°
150°
180°
dσ/dΩ (mb/sr)
θ Lab

Table 4

The differential cross-section $d\sigma/d\Omega$ for the elastic scattering of π^- by carbon at 69.5 Mev. (In units of mb/sr).

Values of EDELSTEIN et al[9)]

θ in degrees	20 ± 3.1	30 ± 3.1	45 ± 4.0	60 ± 4	80 ± 4.7	100 ± 4.7	125 ± 4.7
$\frac{d\sigma}{d\Omega}$	147 ± 16	79.1 ± 6.6	26.9 ± 2.0	5.08 ± 0.56	4.48 ± 0.71	5.34 ± 0.62	2.16 ± 0,44

Present calculation

θ in degrees	20	30	45	60	80	100	125
$\frac{d\sigma}{d\Omega}$	170.2	80.23	17.97	4.233	2.781	1.867	0.31

Fig.4

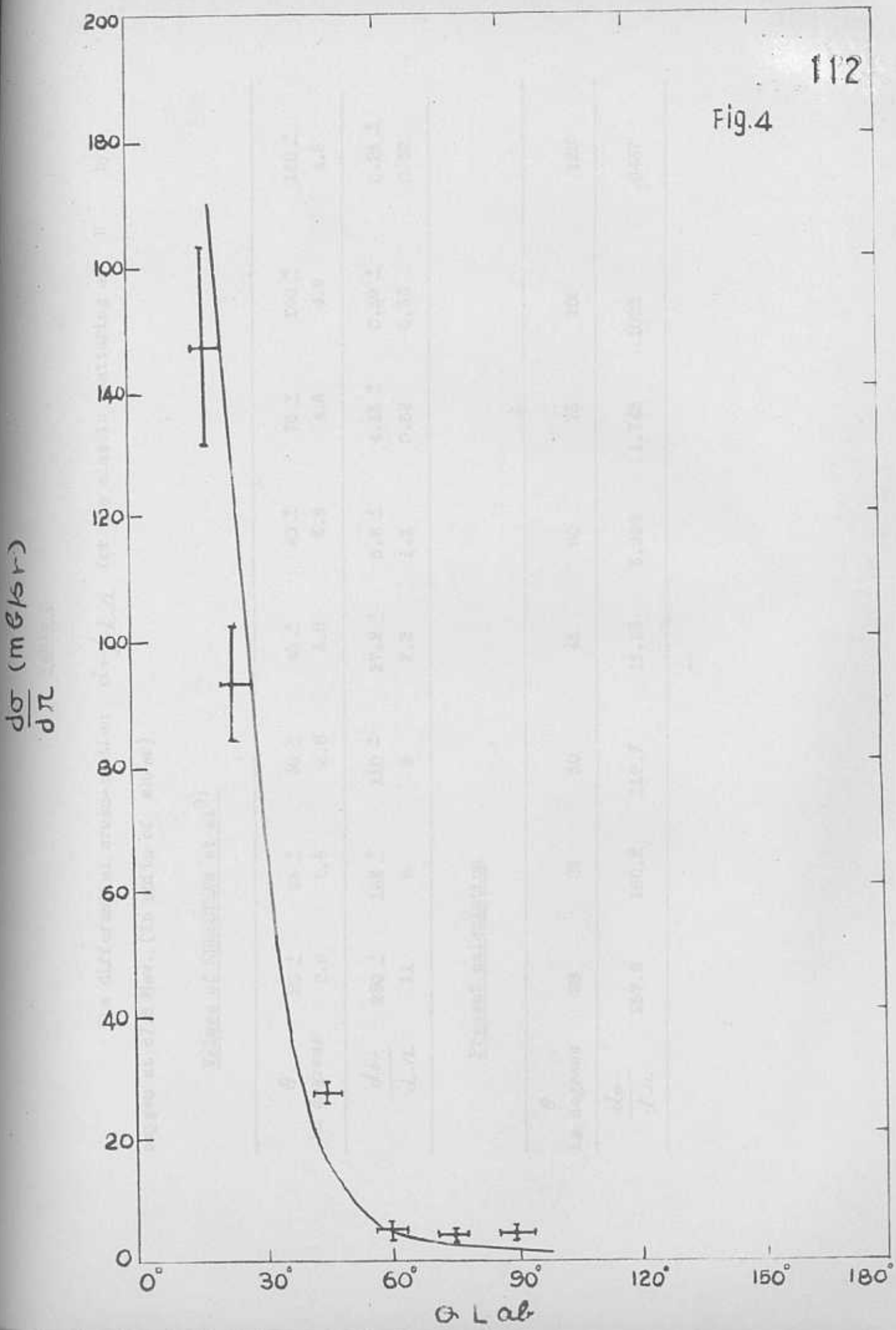

200
180
100
140
120
100
80
60
40
20
0
0°
30°
60°
90°
120°
150°
180°
θ Lab
dσ/dΩ (mb/sr)

Table 5

The differential cross-section $d\sigma/d\Omega$ for the elastic scattering of π^- by oxygen at 87.5 Mev. (In units of mb/sr)

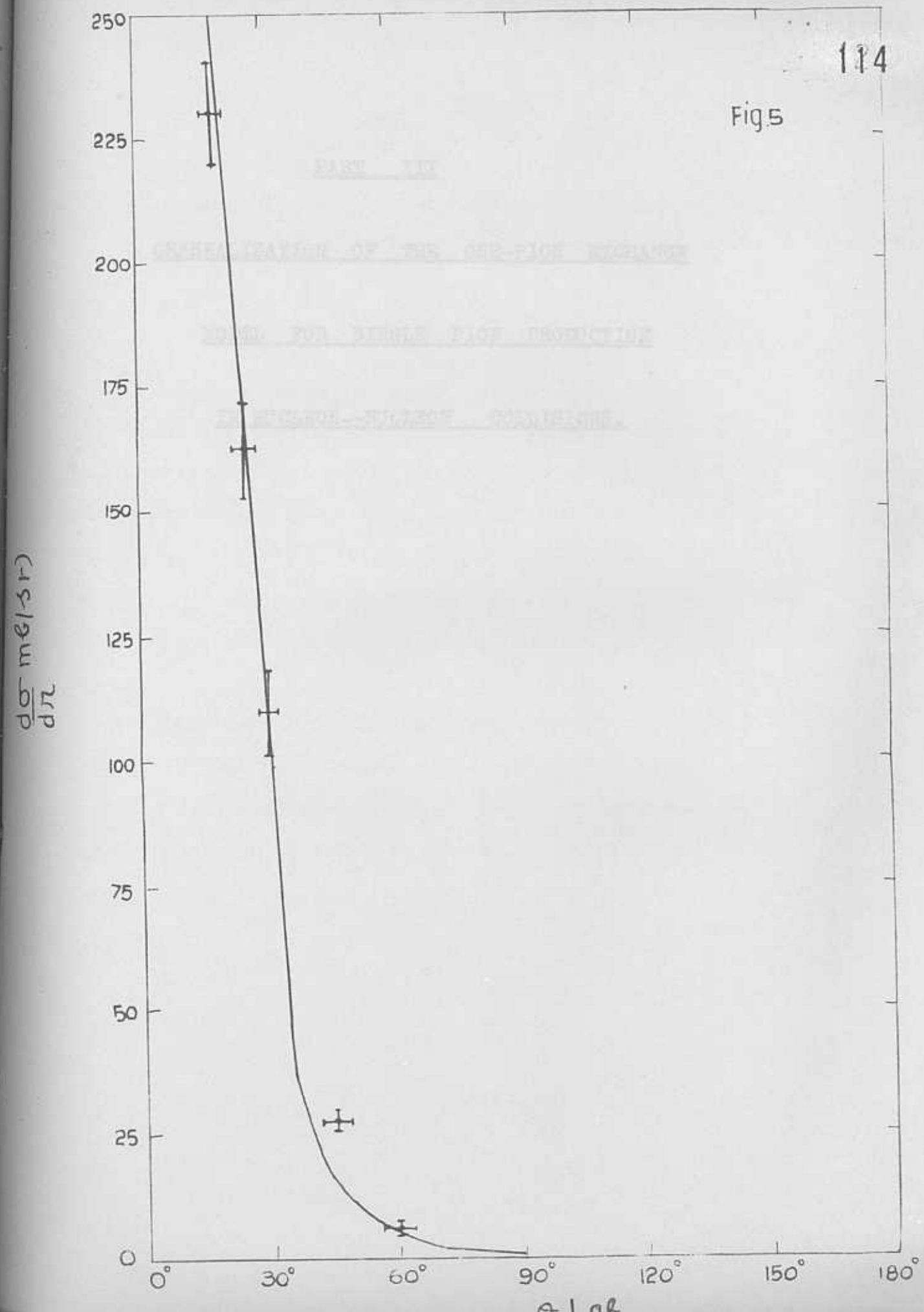
Values of EDELSTEIN et al[9)]

θ in degrees	20 ± 2.8	25 ± 2.8	30 ± 2.8	45 ± 3.8	60 ± 3.8	75 ± 4.5	100 ± 4.5	120 ± 4.5
$\frac{d\sigma}{d\Omega}$	230 ± 11	162 ± 9	110 ± 8	27.2 ± 2.2	5.8 ± 1.1	4.15 ± 0.62	0.59 ± 0.33	0.25 ± 0.32

Present calculation

θ in degrees	20	25	30	45	60	75	100	120
$\frac{d\sigma}{d\Omega}$	257.6	180.2	112.7	15.23	3.926	1.742	.1023	.0467

Fig 5


250
225
200
175
150
125
100
75
50
25
0
dσ/dΩ mB/sr
0°
30°
60°
90°
120°
150°
180°
θ Lab

PART III

GENERALIZATION OF THE ONE-PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON--NUCLEON COLLISIONS.

CHAPTER VI

GENERALIZATION OF THE ONE-PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS.

Abstract

Single-isobar production in NN collisions is considered in a model in which the forces are approximated by the exchange of the different multi-pion resonances. It is shown that contributions other than single-pion exchange are important in understanding the isobar production process. Total cross-sections, c.m. energy spectra and c.m. angular distributions are calculated for different assumptions about the form factors and the type of information that can be derived from each is discussed. The results of the calculation are compared with experimental data and experiments are outlined which will enable one to achieve a better understanding of the mechanism of isobar production.

GENERALIZATION OF THE ONE-PION EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS+

1. Introduction

Pion production in nucleon-nucleon collisions is one of the most familiar phenomena in the Bev region. In fact such processes are among the oldest examples of inelastic elementary particle interactions, especially because they are used for the production of pion beams.

After the failure of the first perturbative approaches, a complete theoretical explanation of such phenomena has not yet been found. Certain models have been proposed to study processes of this type, the earliest version being the statistical model which consisted essentially in saying that cross-sections for any particular process are mainly given by the corresponding phase-space available. Experiments above the Gev region already show a backward-forward peaking of secondaries that is in disagreement with the statistical model. Besides it would be difficult to conciliate the energy dependence of phase-space with certain properties of high energy interactions now available. Also in the energy range from threshold upto 600 Mev a phenomenological theory has been developed by Mandelstam which

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+ Alladi Ramakrishnan, K.Raman and R.K.Umerjee, Nuclear Physics (In print).

reproduces the experimental data rather well with certain assumptions about the matrix elements for pion production and their variations.

At higher energies (> 1GEv) the deuteron production becomes very small. This fact can be interpreted as an indication that final state interactions between nucleons are no longer important. This conclusion is indirectly confirmed by the success of the isobar model, the assumptions and range of validity of which have been discussed by Lindenbaum and Sternheimen[1] and which reproduces the experimental behaviour of the c.m. energy spectra for the final particles.

Another important step in achieving an understanding of single pion production processes is the one-pion exchange (OPE) model[2] which in most of its form is also essentially an isobar model as the final pion is assumed to interact strongly with only one of the final nucleons. This model could quantitatively reproduce some rather marked experimental features such as the forward-backward c.m. peaking of the nucleons and their corresponding high and low energy laboratory peaks. Recently Ferrari and Selleri have put forward a theory of single pion production between 1 and 3 Gev in which the OPE model is modified by including a form factor.[3]

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1) S.J.Lindenbaum and R.Sternheimer: Phys. Rev., 105 (1957), 1874; Phys. Rev., 123, (1961), 333.
2) e.g. see G. da Prato, Nuovo Cimento, 22 (1961), 123. J.Izuka and A.Klein, Phys. Rev. 123 (1961), 669 and F.Selleri, Phys. Rev. Lett., 6 (1961), 64.
3) E.Ferrari and F.Selleri, Nuovo Cimento, 27 (1963) 1450 (This will be referred to as FS)

In this chapter, we shall attempt to go beyond the OPE model by taking other mechanisms into account. Although the extended OPE model of FS agrees well with the observed total cross-sections and laboratory energy spectra in pp collisions one is not justified in concluding that the basic mechanism is OPE unless one can also show that other contributions are negligible in comparison. Detailed calculations have led us to believe that other contributions are not negligible and this is the motivation for this work.

The following considerations suggest that the dominance of OPE in certain regions of phase space is largely a kinematical accident arising from the small mass of the pion and that more complex mechanisms may be important.

(a) The allowed exchanges in single-pion production are the same as those in elastic NN scattering, and since short-range forces are known to be important in the latter, it would be surprising if they are negligible in the inelastic process. The main difference between the elastic and inelastic processes would be expected to arise from the difference in the kinematics and the greater restrictive effect of the Pauli principle in the elastic process.

(b) The suppression of N_1^* production at high energies and the increase in the production of the $I = 1/2$ isobars has been interpreted as caused by the increasing importance of

I = 0 exchange[4, 5]. Even at an energy of 2.9 Bev, N^*_2 and N^*_3 production dominate over N^*_1 production at larger angles where the momentum transfer is much larger than the pion mass[6]. This suggests that I = 0 exchange is important even at such energies.

Of the various questions one is faced with in attempting to go beyond the isobar model with OPE, we shall, in this chapter consider only one viz., the effect of forces other than OPE. We assume that the process we are considering is essentially the production of a single nucleon isobar, either the N^*_1 at 1238 Mev or the N^*_2 at 1520 Mev.

Our model is defined by the assumption that the important part of multi-pion exchange may be approximated by a few of the lowest partial waves in the crossed channel; we shall further approximate each partial wave (with a definite spin, parity and isospin) by a pole. (In our work we have kept only the $J = 0^{\pm}$ and $J = 1^{+}$ partial waves for simplicity, although the J = 2 partial wave may be important). Such an approximation has been shown to be very useful in an analysis of NN scattering by Scotti and Wong[7] and by Ramsay.[8]

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4) G.Cocconi, A.N.Diddens, E.Lillethun, G.Manning, A.E.Taylor, T.G.Walker and A.M.Wetherell, Phys. Rev.Lett., 7, (1961) 450.
5) A.P.Contogouris, S.C.Frautschi and How-sen Wong, Phys. Rev., 129 (1963), 974.
6) A.M.Wetherell, Proc. Phys. Soc., 80 (1962), 63.
7) A.Scotti and D.Y.Wong, Phys. Rev., Lett., 10 (1963), 142.
8) W.Ramsay, Phys. Rev., 130 (1963), 1553.

While this work was in progress it came to our notice that a similar description of single-pion production has been considered earlier by T.Ueda[9]. Our work differs from Ueda's in several respects: (i) we go much further than Ueda in considering not only N_1^* production as he has done, but also N_2^* production and the effects of $I \neq 0$ exchange, (ii) our calculations are more extensive, and (iii) our arguments and conclusions are quite different.

To keep the complications manageable, we shall treat each isobar as a real 'particle' with a Breit-Wigner mass distribution. In calculating the energy spectra we take the mass distribution into account, while in obtaining the cross-sections, the isobar is treated as a particle of fixed mass. With these approximations we can claim to make statements only about the gross features of the process ; we hope that our results will serve as a starting point for more refined calculations.

In section 2 are discussed briefly the general form of the matrix element and the pole contributions to the differential cross-sections. In section 3 the cross-sections are analysed in terms of the exchange of systems with definite isospin. The methods of separating the contribution of $I = 0$ exchange are discussed. In section 4 we state our assumptions about the coupling constants and form factors. In sections 5, 6 and 7 we discuss the results for the total cross-sections, c.m. energy

9) T.Ueda, Prog.Theor. Phys., 29, (1963) 829.

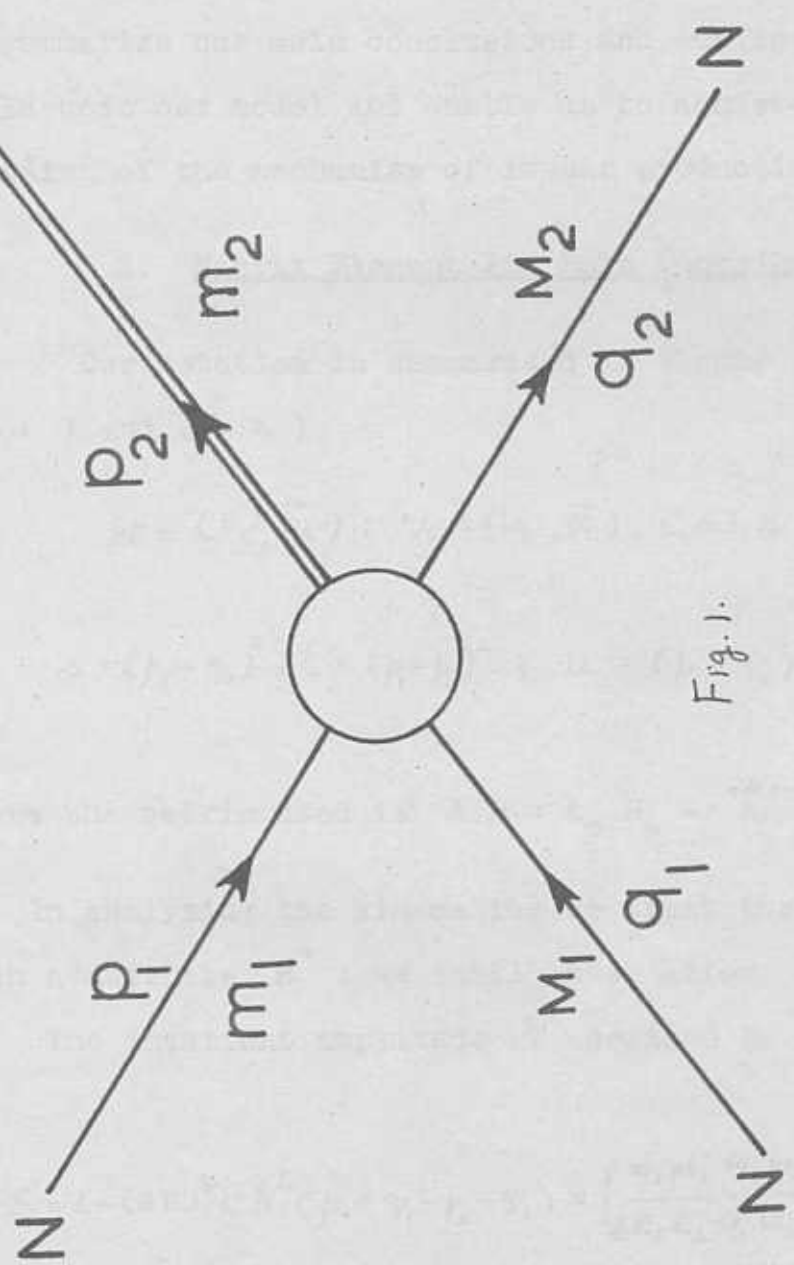

Fig. 1.

spectra, and c.m. angular distributions and examine what type of information may be derived from each. In the last section we summarize our main conclusions and outline experiments that would test our model and enable us to achieve a deeper understanding of the mechanism of isobar production.

2. Matrix Element and Pole Contributions

Our notation is summarized in figure 1 and in equations (2.1) and (2.2)

$$p_i = (E_i, \vec{p}_i) \; ; \; q_i = (\omega_i, \vec{q}_i) \, , \; i = 1, 2 \qquad (2.1)$$

$$s = (p_1 + q_1)^2 \; ; \; t = (p_1 - p_2)^2 \; ; \; u = (p_1 - q_2)^2 \qquad (2.2)$$

where the metric used is $A.B = A_0 B_0 - \vec{A}.\vec{B} = A_\alpha \delta_{\alpha\beta} B_\beta$.

In analyzing the kinematics we treat the isobar as a particle with a definite M^* ; we shall later allow M^* to vary.

The invariant amplitude T defined by

$$S = 1 - (2\pi)^4 i \, \delta^4(p_1 + q_1 - p_2 - q_2) \times \left(\frac{m_1 m_2 M_1 M_2}{4 E_1 E_2 \omega_1 \omega_2} \right)^{1/2} \times T \qquad (2.3)$$

has the form

$$T = \left[\bar{\psi}_\alpha(p_2) \mathcal{M}^A u(p_1) \right] \left[\bar{u}(q_2) \mathcal{M}^B u(q_1) \right] \qquad (2.4)$$

where $\Psi_\alpha(p)$ describes a $P_{3/2}$ or $D_{3/2}$ N* and $u(p)$ describes a nucleon. Spin operators between the incident nucleon spinor $u(p_1)$ and the isobar spinor $\overline{\Psi}_\alpha(p_2)$ are distinguished by a superscript A, and the operators between $u(q_1)$ and $u(q_2)$ by a superscript B.

Defining

$$P = \tfrac{1}{2}(p_1 + p_2) \quad , \quad Q = \tfrac{1}{2}(q_1 + q_2) \tag{2.5}$$

the general form of the matrix element may be written as

$$\mathcal{M} = \mathcal{M}^A . \mathcal{M}^B = \sum_{r=1}^{8} i \left[P_\alpha F_r(s,t,u) + Q_\alpha G_r(s,t,u) \right] I_r \tag{2.6}$$

The functions $F_r(s,t,u)$ and $G_r(s,t,u)$ are the 16 invariant amplitudes for the process, while the I_r are a set of linearly independent products of spin operators, which may be conveniently as chosen follows:

$$\begin{aligned}
&I_1 = 1^A . T^B \quad ; \quad I_2 = \gamma_5^A . \gamma_5^B T^B \\
&I_3 = \gamma_\mu^A . \gamma_\mu^B T^B \quad ; \quad I_4 = i(\gamma_\mu \gamma_5)^A . i(\gamma_\mu \gamma_5)^B T^B \\
&I_5 = \sigma_{\mu\nu}^A . \sigma_{\mu\nu}^B T^B \quad ; \quad I_6 = (\gamma . Q)^A . T^B \\
&I_7 = 1^A . (\gamma . P)^B T^B \quad ; \quad I_8 = (\gamma . Q)^A . (\gamma . P)^B T^B
\end{aligned} \tag{2.7}$$

where

$$T^B = \gamma_5^B \quad \text{for} \quad a\ P_{3/2}\ N^* \qquad (2.8)$$
$$= 1^B \quad \text{for} \quad a\ D_{3/2}\ N^*$$

A detailed analysis of the kinematics and the polarization is givenin the next chapter.

States that can be exchanged in the process $N + N \rightarrow N^* + N$ must have quantum numbers that are common to $(N\bar{N})$ and $(\bar{N}N^*)$ systems, which are the same as the allowed quantum numbers for the $(N\bar{N})$ system. These are well-known[10] and are given in Table 1 at the end of the chapter.

Of these, the η, π, ω, ρ and f° may be considered as well-established; the other states have been denoted by symbols mainly for notational convenience.

For elastic NN scattering there is evidence for the presence of pion exchange, for ρ exchange (if we can interpret the recently observed forward peak in np charge-exchange scattering at 2.04 and 2.85 Gev as arising from ρ exchange[11]) and for ω exchange (for instance, from the analysis of NN and $N\bar{N}$ scattering with a few Regge poles.[12]) There is also evidence for the exchange of a vacuum trajectory; however this has not been included in our calculations.

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10) e.g. see D.Lichtenberg and G.C.Summerfield, Phys. Rev., 127, (1962), 1806.
11) M.M.Islam and T.W.Preist, Phys. Rev. Lett., 11, (1963), 444.
H.Palevsky et al, Phys. Rev. Lett., 9 (1962), 509.
12) e.g. see S.D.Drell, Proc. International Conference on High Energy Physics, CERN, 1962, 897.

Table 1

The allowed quantum numbers for the $J = 0$ and $J = 1$ partial waves in $N + \bar{N} \rightarrow N^* + \bar{N}$. (The $J = 2^{++}$, $I = 0$ state f^0 is also included)[a)]

Isospin (I)	Spin-parity and G-parity (J^{PG})	Charge conjugation parity C	Symbol	Assumed mass (in Mev)
0	0^{++}	+	σ	395
			S_0	550
1	0^{+-}	+	ε, S_1	550
0	0^{-+}	+	η	550
1	0^{--}	+	π	139.6
0	1^{--}	-	ω	785
1	1^{-+}	-	ρ	750
0	1^{++}	++	a_0	
0	1^{+-}	-	A_0	750
1	1^{++}	-	A_1	750
1	1^{+-}	+	a_1	
0	2^{++}	+	f^0	

a) The first 3 columns of this table are taken from reference 10.

The analysis of NN scattering by Scotti and Wong[7] and by Ramsay[8] suggests that the exchange of an I = 0 scalar system with mass between $3\mu_\pi$ and $4\mu_\pi$ may also be important. More recently, the analysis of η decay suggests the existence of an I = 0 ($\pi\pi$) resonance (σ) at about 380 Mev, with a width of about 50 Mev[13]. These different pieces of evidence as well as the experiment by Samios et al[14] suggest that an I = 0 ($\pi\pi$) resonance with a mass $m_\sigma \approx$ 400 Mev does exist; we shall assume a mass $m_\sigma^2 \approx 8\, m_\pi^2$.

If an I = 1 scalar state (S_1) exists, it is expected to have a higher mass than the I = 0 state. We shall study the effect of such a resonance at $m_s \approx$ 550 Mev, as suggested by the observation of the φ peak[15]. We have calculated the contributions of these mesons to isobar production and, in addition to these, also the effect of an axial vector state (A). Such a state has not been observed with any certainty, although a 1^+ state at 625-650 Mev was suggested by certain experiments[16]. It is also possible that the $\omega\pi$ resonance recently observed (at 1250 Mev)[17] has $J = 1^+$. In our calculations we have

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13) Crawford et al. Phys. Rev.Lett., 11, (1963), 564 and L.M.Brown and P.Singer, Phys. Rev. to be published.
14) N.P.Samios et al, Phys. Rev. Lett., 9 (1962), 139.
15) R.Barloutaud et al, Nuovo Cimento, 27, (1963), 238; B.Sechi Zorn, Phys. Rev. Lett., 8, (1962), 282.
16) E.Pickup, D.K.Robinson and E.O.Salant, Phys. Rev. Lett., 8, (1962), 329 and B.Sechi Zorn, Phys. Rev. Lett., 8, (1962), 282.
17) Abolins et al, Phys. Rev.Lett., 11, (1963).

taken m_A = 750 Mev, in order to obtain the general effect of such a resonance. The effect of a higher or lower mass is easily obtained.

We shall now write down the contribution of scalar, pseudo sclar, vector and axial vector poles to the differential cross-sections.

We assume the following expressions for the couplings at the different vertices, (S, P, V and A denote scalar, pseudo-sclar, vector and axial vector mesons).

$$
\begin{aligned}
&\text{NNS} \quad : \quad G_S\, \bar{u}(q_2)\, u(q_1) \\
&\text{NN}_1^*\text{S} \quad : \quad i f_S k_\mu \bar{\psi}_\mu(p_2)\, \gamma_5\, u(p_1) \\
&\text{NNP} \quad : \quad G_P\, \bar{u}(q_2)\, \gamma_5\, u(q_1) \\
&\text{NN}_1^*\text{P} \quad : \quad i f_P\, \bar{\psi}_\mu(p_2)\, k_\mu u(p_1) \qquad (2.9) \\
&\text{NNV} \quad : \quad \bar{u}(q_2)\left[G_V \gamma_\alpha + i F_V \sigma_{\alpha\beta} k_\beta\right] u(q_1) \\
&\text{NN}_1^*\text{V} \quad : \quad \bar{\psi}_\mu(p_2)\left[g_V \gamma_5\, \delta_{\mu\alpha} - f_V \gamma_5 k_\mu \gamma_\alpha - h_V \gamma_5 k_\mu k_\alpha\right] u(p_1) \\
&\text{NNA} \quad : \quad \bar{u}(q_2)\left[G_A \gamma_5 \gamma_\alpha + i F_A \gamma_5 \sigma_{\alpha\beta} k_\beta\right] u(q_1) \\
&\text{NN}_1^*\text{A} \quad : \quad \bar{\psi}_\mu(p_2)\left[g_A \delta_{\mu\alpha} - f_A k_\mu \gamma_\alpha - h_A k_\mu k_\alpha\right] u(p_1)
\end{aligned}
$$

These couplings are for a $P_{3/2}$ N^* †. For a $D_{3/2}$ N^* the corresponding vertices differ by a factor γ_5 .

† For a few remarks relating to NN^*V and NN^*A couplings, please refer to the Note I at the end of this Chapter. (p.189)

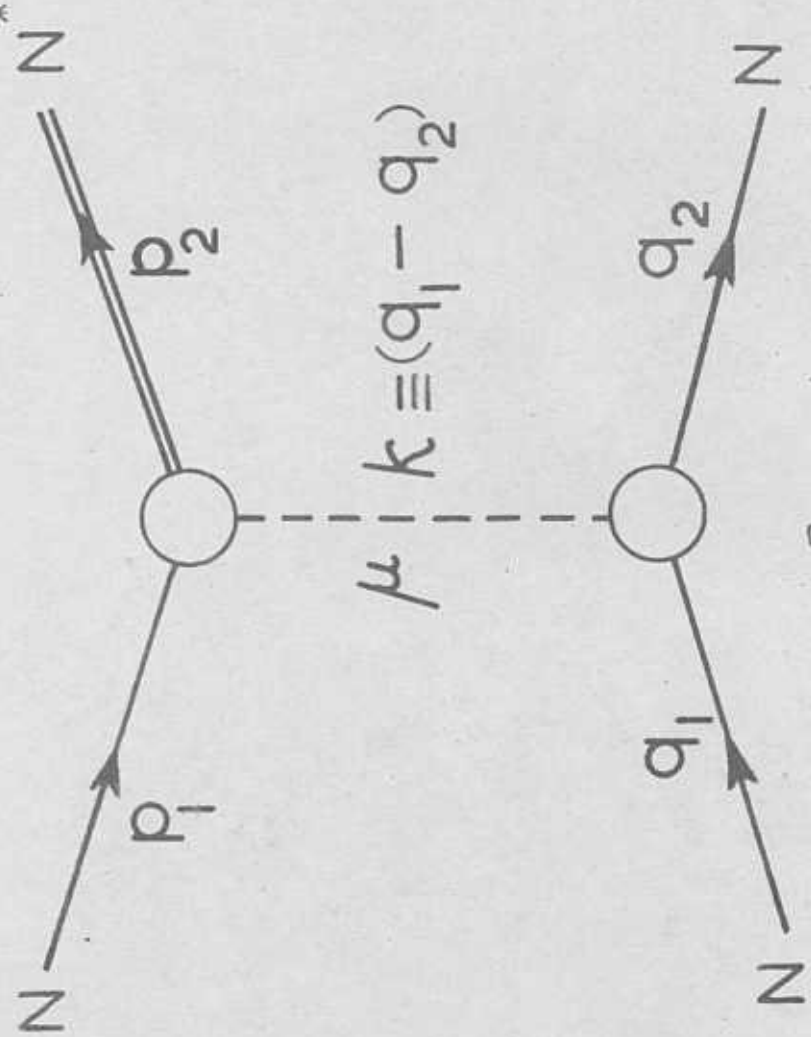

Fig. 2.

The symbol k_μ denotes the meson momentum, taken to be outgoing at an N-N- meson vertex and incoming at an N-N*- meson vertex, as shown in figure 2 . We would normally expect the second term in the NNV and NNA couplings to have a negative sign (as the momentum is outgoing at this vertex). The opposite sign has been introduced so as to ensure the correspondence $G_v = F_1(k^2)$ and $F_v = F_2(k^2)$, where F_1 and F_2 are the usual nucleon form factors[18]; this difference occurs because the $\sigma_{\mu\nu}$ used here is different from that used in writing the nucleon form factor as $[F_1 + C\sigma_{\mu\nu} k_\nu F_2]$ [18]. The coefficients g_i , F_i , G_i , etc. are, in general, functions of k^2 .

The propagators for spin 0 and spin 1 mesons are taken as

$$C\,(k^2 - \mu^2)^{-1} \qquad (2.10)$$

and

$$-C\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{\mu^2}\right)(k^2 - \mu^2)^{-1} \qquad (2.11)$$

respectively, while the projection operator for a spin 3/2 particle is takento be

$$P_{\mu\lambda}(p) = \frac{1}{3}\left[3\,\delta_{\mu\nu} - \frac{2p_\mu p_\lambda}{m^2} - \gamma_\mu \gamma_\lambda + \gamma.p\,\frac{(p_\mu \gamma_\lambda - p_\lambda \gamma_\mu)}{m^2}\right] \times$$

$$\times \frac{(\gamma.p + m)}{2m} \,. \qquad (2.12)$$

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18) e.g. see S.D.Drell and F.Zachariasen, Electromagnetic Structure of Nucleons, Oxford University Press, 1961.

Our γ -matrices are defined by writing the Dirac equation as

$$(i\gamma_\mu \partial_\mu + m)\,\psi_\lambda = 0, \quad \text{or} \quad (\gamma.p - m)\,\psi_\lambda(p) = 0 \qquad (2.13)$$

We define

$$\sigma_{\mu\nu} = \frac{1}{2i}\,[\gamma_\mu, \gamma_\nu] \qquad (2.13a)$$

The unpolarized differential cross-section in the c.m. system is given in terms of the T-matrix element by

$$\left(\frac{\partial\sigma}{\partial\Omega}\right) = \frac{1}{(2\pi)^2}\cdot\frac{1}{W^2}\cdot\frac{p_f}{p_i}\cdot m_1 m_2 M_1 M_2 \cdot \left[\frac{1}{4}\sum_{spins}|T|^2\right] \qquad (2.14)$$

where p_i , p_f are the initial and final c.m. momenta, and W is the total energy in the c.m. frame. Taking T as the sum of the matrix elements for the exchange of pseudoscalar, scalar, vector and axial vector particles, we obtain the following expressions for the production of such type of isobar ($P_{3/2}$ and $D_{3/2}$):

$$\frac{\partial\sigma^{(I)}}{\partial\Omega} = \frac{1}{(2\pi)^2}\cdot\frac{1}{W^2}\cdot\frac{p_f}{p_i}\cdot m_1 m_2 M_1 M_2 \left[\frac{1}{4}\sum_{spins}\Big|\sum_{I',\,r} \epsilon^{I}_{I'}\, T^{(I),\,r}_{I'}\Big|^2\right] \qquad (2.15)$$

where $T^{(I),\,r}_{I'}$ is the matrix element in the isospin state I for the exchange of a state with isospin I' .

The factors $\epsilon^{I}_{I'}$ arise from the isospin; they may be obtained from the crossing matrix, which is given in the next section.

The superscript r refers to the nature of the exchanged particle:

$$r = P\ ,\ S\ ,\ V \text{ or } A \qquad (2.16)$$

The expression in square brackets in equation (2.15) is the sum of the square terms and the interference terms, we shall keep only the square terms in our calculations and shall neglect the interference between different pole contributions.

We give below the differential cross-sections from the contribution of the various pole terms taken individually, using the notation

$$m = \tfrac{1}{2}(m_1 + m_2) \; ; \; M = \tfrac{1}{2}(M_1 + M_2) \; ; \; \delta = (m_1 - m_2)$$

$$\Delta = (M_1 - M_2) \; ; \text{ and } \; y = (s)^{-1} p_f \, p_i^{-1} (u^2 - t)^{-2} \qquad (2.17)$$

For the 'direct' exchange of mesons (i.e. as poles in the t-channel), the expressions for $\partial\sigma/\partial(\cos\theta^*)$ are the following:

A) Production of P 3/2 N*

(i) Exchange of a pseudoscalar (0^-) meson :

$$\frac{\partial\sigma_P}{\partial(\cos\theta^*)} = \left(\frac{\pi}{12}\right)\cdot\left(\frac{1}{4\pi} f_P^2\right)\left(\frac{1}{4\pi} G_P^2\right)\cdot \frac{y(4m^2 - t)^2}{m_2^2} \times (\delta^2 - t)(\Delta^2 - t) \qquad (2.18)$$

(ii) Exchange of a scalar (0^+) meson:

$$\frac{\partial\sigma_S}{\partial(\cos\theta^*)} = \left(\frac{\pi}{12}\right)\cdot\left(\frac{1}{4\pi} f_S^2\right)\left(\frac{1}{4\pi} G_S^2\right)\cdot \frac{y(4m^2 - t)}{m_2^2} \times (\delta^2 - t)^2 (4M^2 - t) \qquad (2.19)$$

(iii) Exchange of a vector (1^-) meson:

The matrix element for this may be rewritten in the form

$$\bar{\psi}_\mu(p_2)\left[g_V \gamma_5 \delta_{\mu\alpha} - f_V \gamma_5 k_\mu \gamma_\alpha - h_V \gamma_5 k_\mu k_\alpha\right] u(p_1) \times$$

$$\times \bar{u}(q_2)\left[(G_V + 2M F_V)\gamma_\alpha - 2Q_\alpha\right] u(q_1) \qquad (2.20)$$

We shall assume that the f and F coupling constants are smaller than the g and G coupling constants by a mass factor of the order of one or two nucleon masses. The largest contribution to the matrix element is then expected to be given by the term proportional to

$$g_V\left[G_V + 2M F_V\right]$$

which gives a differential cross-section

$$\frac{\partial \sigma_V}{\partial(\cos\theta^*)} = \left(\frac{2\pi}{3}\right)\cdot\left(\frac{1}{4\pi}g_V^2\right)\cdot\frac{1}{4\pi}\left|G_V + 2M F_V\right|^2 \cdot y\cdot(\delta^2 - t) \times$$

$$\times (2m_2^2)^{-1}\left[(s - m_2^2 - M_2^2)(s - m_1^2 - M_2^2) + t(s - 2m_2^2 - M_2^2)\right. \qquad (2.21)$$

$$\left. + m_2^2(M_1^2 + M_2^2 - 6M_1M_2)\right]$$

The complete expression for the differential cross-section from vector exchange is very long and will not be given here.

(iv) Exchange of an Axial Vector (1+) meson:

Here again only the largest term will be kept. This may be obtained by making the replacement

$$m_2 \longrightarrow -m_2 \quad ; \quad M_2 \longrightarrow -M_2 \qquad (2.22)$$

in the expression (2.21) (with the notation of figure 2)

B) Production of a $D_{3/2}$ N*.

The corresponding expressions for $\partial\sigma/\partial(\cos\theta^*)$ can be obtained by replacing

$$m_2 \rightarrow -m_2 \qquad (2.23)$$

in the expressions given above for the $P_{3/2}$ N* .

For the process we are considering, we must put $m_1 = M_1 = M_2$, $m_2 = M^*$, and $m_1 = M_1 = M_2 = M_N$, where M_N is the nucleon mass and M* the isobar mass.

The above expressions must be multiplied by the numerical (isospin) factors $(\epsilon^I_{I'})^2$ as in (2.15) . The values of $(\epsilon^I_{I'})^2$ are given by the following:

$$\underline{A} \qquad P_{3/2}N^*: (\epsilon^I_{I'})^2 = 1 \quad \text{always, as } I = I' = 1 . \qquad (2.24)$$

$$\underline{B} \qquad D_{3/2}N^*: (\epsilon^0_1)^2 = \frac{9}{4}\ ;\ (\epsilon^0_0)^2 = (\epsilon^1_0)^2 = (\epsilon^1_1)^2 = \frac{1}{4} .$$

The above expressions give the contributions of poles in the t-channel. The 'exchange' contribution from poles in the u -channel may be obtained by making the replacements

$$t \longleftrightarrow u\ ;\ m_1 \longleftrightarrow M_1 \qquad (2.25)$$

For different charge states, different combinations of the 'direct' and 'exchange' forces must be taken, as given in the next section. (Detailed derivation is given in the next chapter.)

3. Isospin

The consequences of isospin invariance for the process $N + N \longrightarrow N + N + \pi$ have often been discussed in terms of reaction channels with definite isospin[19]. We have analyzed these in terms of the exchange of states with definite isospin.

Since an isobar model is valid only when the final-state interaction between the nucleons is negligible, we shall neglect the antisymmetrization between the final nucleons. It is, of course, necessary to antisymmetrize between the initial nucleons. †

For N_1^* production only $I' = 1$ exchange is allowed, and also the reaction channel must have $I = 1$. For N_2^* production the crossing matrices $\alpha_{(t)}$ and $\alpha_{(u)}$ for exchanges in the t and u channels are the same as the crossing matrices for NN-scattering:

$$\alpha_{(t)} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \; ; \; \alpha_{(u)} = \begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad (3.1)$$

Using these, the following results are obtained for the the differential cross-sections, the correct normalization being as discussed by Salzman and Salzman in ref.[30]. The actual derivation of the following results is presented in the next chapter.

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19) e.g. see D.C.Peaslee, Phys. Rev. 94, (1954), 1085 and Phys. Rev., 95, (1954), 1580.

30) F.Salzman and G.Salzman, Phys. Rev., 121, (1961), 1541.

† Please see Note II at the end of this Chapter. (p.191)

1) pp collisions:

$$d^2\sigma(pp \to \pi^0 pp) = \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_a d\Gamma_a +$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_b d\Gamma_b \qquad (3.2)$$

$$d^2\sigma(pp \to \pi^+ np) = \frac{5}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_a d\Gamma_a$$

$$+ \frac{1}{6}\left[d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_b d\Gamma_b \qquad (3.3)$$

2) np collisions:

$$d^2\sigma(np \to \pi^0 np) = \frac{1}{3}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_a d\Gamma_a$$

$$+ \frac{1}{12}\left[2\,d\sigma_{(0)}(t) + 5\,d\sigma_{(1)}(t) + 5\,d\sigma_{(1)}(u)\right]_b d\Gamma_b \qquad (3.4)$$

$$d^2\sigma(np \to \pi^+ nn) = \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_a d\Gamma_a$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(u) + 4\,d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_b d\Gamma_b \qquad (3.5)$$

$$d^2\sigma(np \to \pi^- pp) = \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_a d\Gamma_a$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(t) + d\sigma_{(1)}(t) + 4\,d\sigma_{(1)}(u)\right]_b d\Gamma_b \qquad (3.6)$$

Here $d^2\sigma(pp \to \pi^0 pp)$ etc. denote the differential cross-section for a particular charge-state with respect to some appropriate set of variables.† The quantity $d\sigma_{(I')}(t)$ is the differential cross-section for the production of an isobar N^* by the exchange of a state with isospin I' in the t-channel, and $d\Gamma_a$ and $d\Gamma_b$ are the decay distributions of an N^*_1 and N^*_2 respectively. The subscripts a and b to the brackets denote the N^*_1 and N^*_2 contributions respectively.

On examining the expressions (3·2) to (3·6), it is evident that in pp collisions in the process $np \to np\pi^0$ and for all N^*_1 contributions, the isobar angular distribution is always forward-backward symmetric. However the N^*_2 contribution to the process $np \to nn\pi^+$ and $np \to pp\pi^-$ are not forward-backward symmetric.

Anticipating the results of sec. 7, we state that when the form factors at the vertices of fig. 2 are strongly damped at large momentum transfers, as would happen when the exchanged poles are Regge poles, then an exchange in the t-channel will give a forward peaking, whereas an exchange in the u-channel will give a backward peaking. When this strong damping is present, equations (3·2) to (3·6) give the following results.

In np collisions (i.e. neutrons incident on target protons in the lab.), the N^*_2 c.m. angular distribution as inferred from the reaction $np \to nn\pi^+$ has a forward/backward ratio of 4 : 1 for I = 1 exchange, while I = 0 exchange gives a

† These may be taken as the scattering angle for the reaction $p + p \to N^* + N$ and the decay angle for the decay $N^* \to N + \pi$.

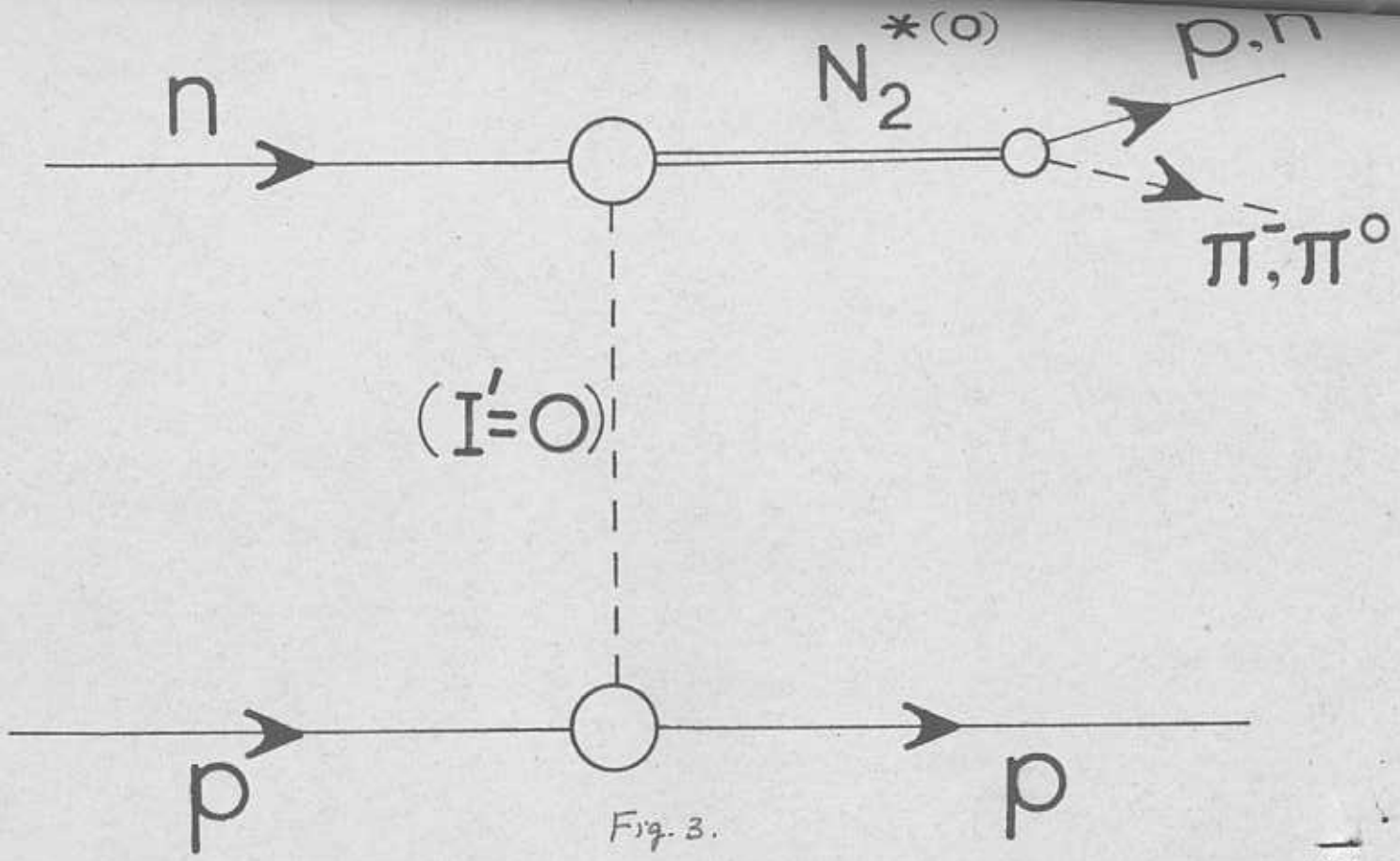

Fig. 3.

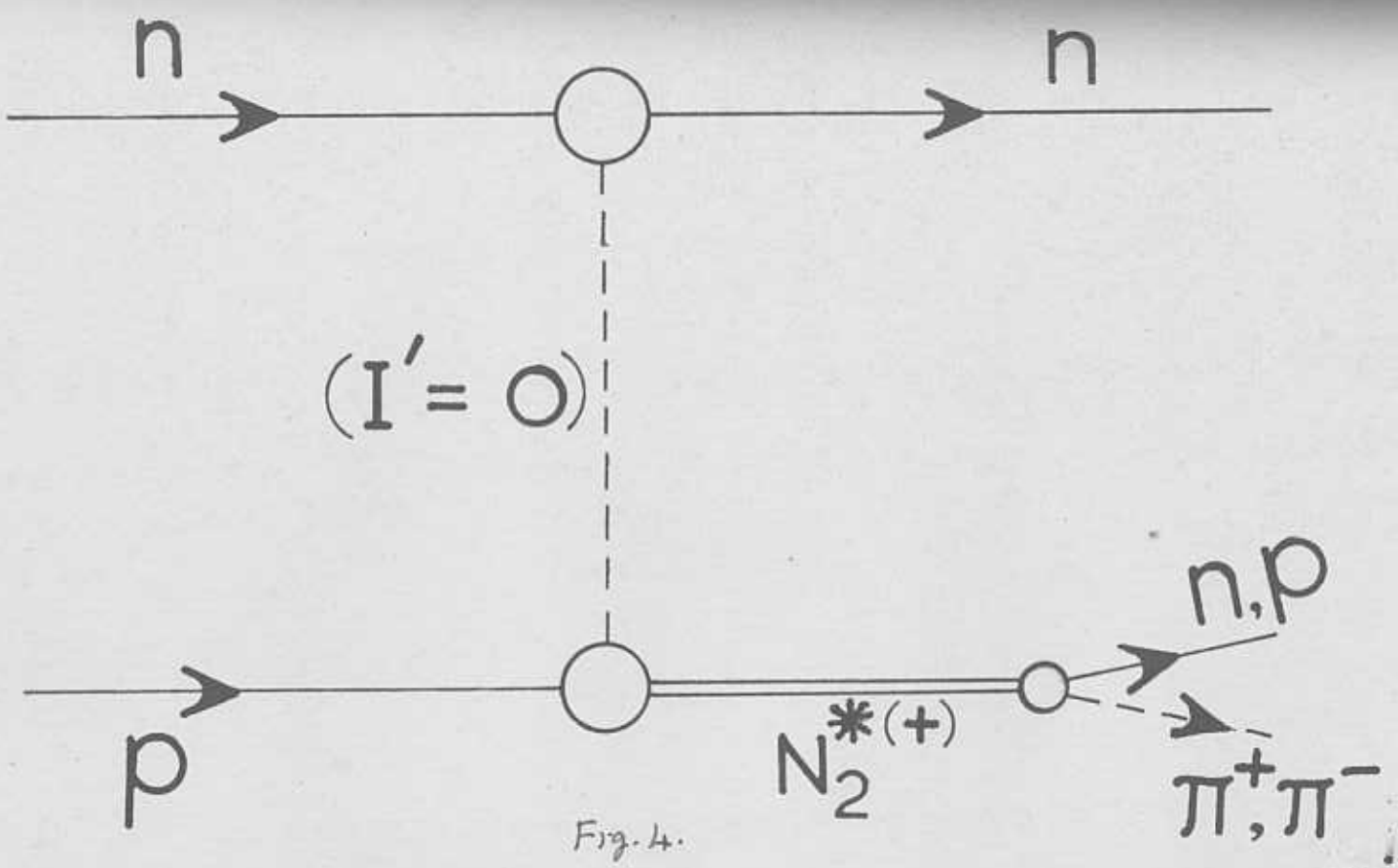

Fig. 4.

back-ward peak and a zero cross-section in the forward direction. For the N^*_2 distribution as deduced from the reaction $np \rightarrow pp\ \pi^-$ $I = 1$ exchange gives a forward/backward ratio of 1 : 4 while $I = 0$ exchange gives a forward peak and a zero cross-section in the backward direction.

At sufficiently high energies, when the decay pion momentum is almost parallel to the isobar momentum, this implies that forward production of π^+ and backward production of π^- in np collisions are a measure of $I = 1$ exchange, while forward production of π^- and backward production of π^+ are a measure of $I = 0$ exchange, provided the isobar is an $I = 1/2$ isobar.

This could be used for testing the assumption generally made that the fall in N^*_2 N^*_1 production at high energies is because of the diminishing importance of $I = 1$ exchange as compared to $I = 0$ exchange, which is not necessarily obvious.

The consequences noted above can be made evident from simple qualitative considerations as illustrated in fig. 3 and fig. 4. (Note that in the c.m. frame, N* produced at the upper vertex are peaked forward while those produced at the lower vertex are peaked backward.)

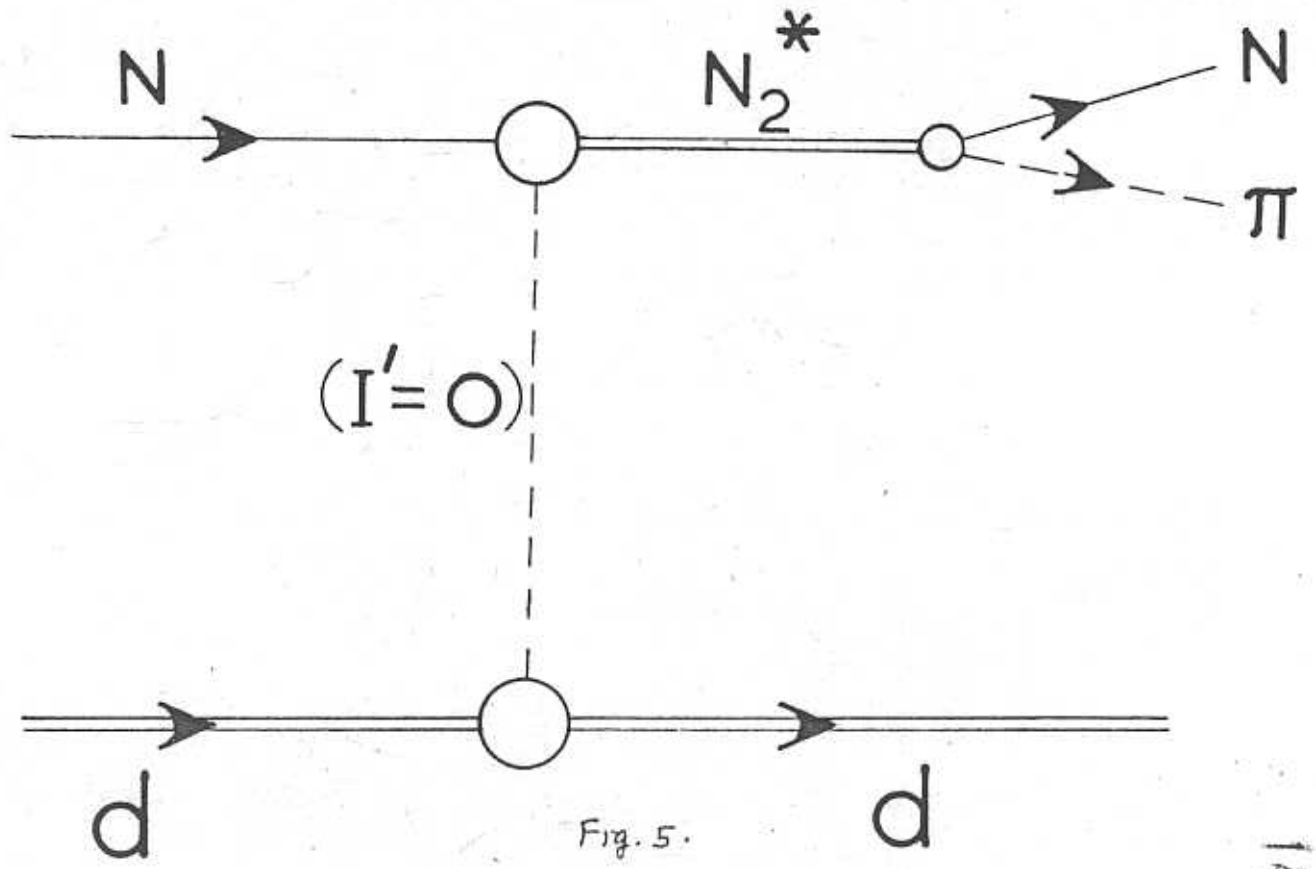

Fig. 5.

We shall conclude this section by briefly discussing how pionproduction by nucleons on deuterium may be used for estimating the importance of $I = 0$ exchange in the process $N + N \rightarrow N^* + N$.

When one selects events in which the deuteron is left in tact and in which the pion is produced along with the nucleon at the upper vertex (see fig. 5), only $I = 0$ exchange can contribute as the deuteron has zero isospin.

Such events may be isolated by observing the final deuteron and by suitably choosing the kinematics. A detailed discussion of pion production in $N - d$ collisions will be given in a later publication.

4. Coupling constants and form factors

In this section we state our assumptions about the various coupling constants involved and the different choices of form factors for which we have calculated the cross-sections and angular distributions.

The πNN^* coupling constants have been obtained as follows:

For the $P_{3/2}$ N^* the coupling constant $\frac{1}{4\pi} \cdot f^2$ as defined in equation (2.9) is related to the width $\Gamma(N_1^*)$ by

$$\Gamma(N_1^*) = \frac{1}{4\pi} \cdot f^2 \cdot p_N^3 \cdot (M_1^*)^{-1} \cdot \frac{4}{3} (E_N + M_N) \qquad (4.1)$$

while for the $D_{3/2}$ N^*_2, the relation is

$$\Gamma(N_2^*) = \frac{1}{4\pi} \cdot f^2 \cdot p_N^3 (M_2^*)^{-1} \cdot \frac{8}{3} \cdot (E_N - m_N) \qquad (4.2)$$

in an obvious notation.

We have taken the effective half-width of the N^*_1 as 120 Mev, from the $\pi - N$ scattering cross-section curve quoted by Melissinos et al[20]. For the N^*_2 we have assumed that the total width is about 130 Mev and that the partial width for $N^*_2 \rightarrow N + \pi$ is 0.77 of this, as deduced from the observed inelastic cross-section.[21]

$$\sigma_{inel.}(\pi N) \approx 14 \text{ m.b.} \qquad (4.3a)$$

and the resonant elastic cross-section

$$\sigma_{el.}(\pi N \rightarrow \pi N) = \frac{8\pi}{k^2} \approx 47.5 \text{ m.b.} \qquad (4.3b)$$

at the energy of the N^*_2 ++

These assumptions give

$$\frac{1}{4\pi} f^2_{\pi N N_1^*} \approx 0.1 \; ; \; \frac{1}{4\pi} f^2_{\pi N N_2^*} \approx 0.08 \qquad (4.4)$$

We have also assumed[7]

$$\frac{1}{4\pi} \cdot G^2_{\pi NN} = 14.4 \qquad (4.4a)$$

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20) A.C.Melissinos, T.Yamanouchi, G.G.Fazio, S.J.Lindenbaum and L.C.L.Yuan, Phys. Rev., 128 (1962), 2373.

21) P.Falk - Vairant and G.Valladas, Proceedings of the 1960 Annual International Conference on High Energy Physics at Rochester (University of Rochester, Rochester, 1960), 38.

++ We are grateful to Dr.M.Jacob for suggesting this method of finding the partial width for $N^*_2 \longrightarrow N + \pi$

Recently, values of 100 Mev have been quoted for the widths of the N^*_1 and the N^*_2 [22]. We have made our assumptions on the basis of available π-N cross-section curves, as our approximation consists in replacing the complete π-N interaction by an isobar.

A smaller value of the width would give total cross-sections that are smaller by a known factor.

For the ρ N N coupling constants, Islam and Preist's estimate from the forward peak in n p charge-exchange scattering at 2.04 Gev gives $\frac{1}{4\pi} G^2_{\rho NN} \approx 0.384$, $F_\rho / G_\rho \approx -5.5$ and $\frac{1}{4\pi} F^2_{\rho NN} \approx 11.6$ while Scotti and Wong's estimate gives $\frac{1}{4\pi} G^2_{\rho NN} \approx 1.27$, $\frac{1}{4\pi} F^2_{\rho NN} \approx 11.4$ at 400 Mev incident energy [7, 11]. Islam and Preist's estimate of $G_{\rho NN}$ varies with the incident energy suggesting that a Regge behaviour of the ρ pole is important even at energies below 3 Gev.[11] This may account for at least a part of the discrepancy between the two sets of estimates. As we require the relative sign of $G_{\rho NN}$ and $F_{\rho NN}$ we have used Islam and Preist's estimate, which gives

$$\frac{1}{4\pi} \left| G_{\rho NN} + 2 m_N F_{\rho NN} \right|^2 \approx 7.8 \qquad (4.5)$$

For the ηNN, ωNN, SNN and σNN coupling constants, we assume values suggested by Scotti and Wong's estimates:[7]

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22) e.g. see A.H.Rosenfeld, Tentative data on Strongly Interacting States (January 1963)

$$\frac{1}{4\pi} G^2_{NN\eta} \approx 12 \; ; \; \frac{1}{4\pi} G^2_{NNS} \approx \frac{1}{4\pi} G^2_{NN\sigma} \approx 3 \qquad (4.6a)$$

and

$$\frac{1}{4\pi} G^2_{NN\omega} \approx 2.8 \; ; \frac{1}{4\pi} F^2_{NN\omega} \approx 0 \qquad (4.6b)$$

To obtain the orders of magnitude of the inelastic coupling constants, we have assumed that

$$\frac{G^2_{\pi NN}}{f^2_{\pi NN^*}} \approx \frac{G^2_{\eta NN}}{f^2_{\eta NN^*}} \approx \frac{G^2_{\sigma NN}}{f^2_{\sigma NN^*}} \approx \qquad (4.7a)$$

$$\approx \frac{G^2_{\omega NN}}{g^2_{\omega NN^*}/M^2} \approx \frac{G^2_{f_{NN}}}{g^2_{f_{NN^*}}/M^2} \approx 144$$

and

$$\frac{1}{4\pi} G^2_{ANN} \cdot \frac{1}{4\pi} g^2_{ANN^*} \approx 1. \qquad (4.7b)$$

where M is a dimensional mass factor chosen as

$$M \approx 2 m_N. \qquad (4.8)$$

We then obtain

$$\frac{1}{4\pi} f^2_{\eta NN^*_2} \approx 0.08 \; ; \; \frac{1}{4\pi} f^2_{\sigma NN^*_2} \approx \frac{1}{4\pi} f^2_{SNN^*_2} \approx 0.02 \qquad (4.9a)$$

and

$$\frac{1}{4\pi} g^2_{\omega NN^*_2} \approx 3.5 \qquad (4.9b)$$

Owing to the ambiguity in the value of the effective $\rho N N$ coupling to be used in equation (4.7a) ,.We have assumed that the effective $\rho N N^*$ coupling constant is

$$\frac{1}{4\pi} \cdot g^2_{\rho NN^*} \approx 1 \qquad (4.10)$$

We believe that the actual coupling strength is likely to be larger than this, rather than smaller. These values of the coupling constants are taken only as giving the order of magnitude. We wish to stress that these values cannot be considered to be too large.

We have evaluated the differential and total cross-sections for different assumptions about the form factors:

(i) a constant form factor,

(ii) energy-independent form factors, and

(iii) energy-dependent form factors suggested by a Regge-pole picture.

At incident energies of 1.41 and 1.92 Gev, we have calculated only for (i) and (ii), while at energies of 2.85 and 8 Gev we have calculated for (i), (ii) and (iii) . At the lower energies, the Regge type of damping is not expected to become effective.

The energy-independent form factors (ii) have been chosen as follows:

For ρ and ω exchange, we have assumed that the isoscalar and isovector elastic (N - N - V) form factors

have the shape of the Hofstadter electromagnetic form factors.[23] The same shapes are also assumed for the inelastic (N - N* - V) form factors and the off-shell correction to the propagator has been assumed to be negligible. This gives[23]

$$F_{\rho NN}(t) \approx F_{\rho NN^*}(t) = -0.2 + 1.2\,(1-0.1t)^{-1} \qquad (4.11)$$

$$F_{\omega NN}(t) \approx F_{\omega NN^*_2}(t) = 0.44 + 0.56\,(1-0.214t)^{-1} \qquad (4.12)$$

For the axial vector couplings NNA and NN*A we have assumed the same form factors as for the vector couplings.

For the exchange of scalar and pseudoscalar particles, i.e. π , η , S and σ we have taken energy-independent form factors of the Ferrari-Selleri shape[3]

$$F_\pi(t) = 0.28 + 0.72\left[1 + {}^{(1-t)}/_{4.73}\right]^{-1} \qquad (4.13)$$

$$F_\eta(t) \approx F_S(t) \approx F_\sigma(t) = 1.144\, F_\pi(t) \qquad (4.14)$$

All these assumptions are made with the object of examining the consequendes of particular plausible shapes of the form factors. They are not claimed to be accurate descriptions of the matrix elements.

The pionic form factor has been normalized to unity at the pole $t = \mu^2$, as the pion coupling constants are generally defined thus. All the other form factors are normalized to unity

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23) R. Hofstadter and R.Herman, Phys . Rev. Lett., 6, (1961), 293.

at $t = 0$. This is because the corresponding coupling constants have been obtained from estimates based on physical NN scattering data (using constant vertex functions). As the pole at $t = \mu^2$ is far from the physical region when μ^2 is large (i.e. when the particle exchanged is heavy) it would not be correct to normalize the form factors to 1 at the pole. As $t = 0$ is much closer to the physical region (in NN scattering) it is more nearly correct to normalize $F(t)$ to 1 at $t = 0$. In particular, the ρ NN coupling constants have been obtained from forward scattering, where $t \approx 0$. The energy-dependent form factors (iii) have been obtained by assuming that the vertices are damped because of a Regge behaviour of the different poles. This is taken into account approximately by multiplying the amplitudes by the following 'form factors' (see for instance reference 7)

$$F_\pi(t) = \left(\frac{s}{2m_N^2}\right)^{\alpha'_\pi(m_\pi^2)\left[t - m_\pi^2\right]} \qquad (4.15)$$

and

$$F_i(t) = \left(\frac{s}{2m_N^2}\right)^{\alpha'_i(\mu_i^2)\cdot t} \quad ; i = \rho, \omega, \eta, \pi, S, \sigma, \text{ and } A. \qquad (4.16)$$

where the $\alpha'_i(\mu_i^2)$ are the slopes of the trajectories $\alpha_i(t)$ of the various poles at the physical mass $t = \mu_i^2$ and have all been assumed to be equal to $(50\ m_\pi^2)^{-1}$. The form factors are normalized as discussed earlier.

5. Total cross-sections

The main features of the observed total cross-sections for single-pion production in pp collisions are as follows[24,25]+

(i) For the process $pp \rightarrow np\pi^+$, $\sigma_{tot.}$ reaches a maximum of about 18 mb at about 1 GeV and then decreases, reaching about 11.5 mb at 2.85 GeV.

(ii) For the reaction $pp \rightarrow pp\pi^\circ$, $\sigma_{tot.}$ reaches a maximum of about 3.8 mb at 1 GeV and decreases to about 2.9 mb at 2.85 GeV.

(iii) The ratio $R = \dfrac{\sigma(np\pi^+)}{\sigma(pp\pi^\circ)}$ decreases from about 4.8 at 1 GeV. to about 3.9 at 2.85 GeV.

A successful model must explain

(a) the magnitudes of the total cross-sections,

(b) the decrease in their absolute magnitudes with increasing energy (above 1 GeV), and

(c) the decrease in the π^+/π° ratio with increasing energy.

As is well-known, the decrease in the π^+/π° ratio may be interpreted as caused by the increasing importance of N^*_2 production. The most obvious reason for the latter would be an increase in the relative contribution of the exchange of states with zero isospin.

In Tables 2, 3 and 4 are given N^*_1 and N^*_2 contributions to the (calculated) total cross-sections for the process $pp \rightarrow pn\pi^+$. The cross-sections for the other charge states are easily obtained using the relations (3.2) to (3.6) In the tables, the columns (i), (ii) and (iii) give cross-sections obtained with the form factors (i), (ii) and (iii)

+For Refs. 24 and 25, please see at the end of the chapter. (p.188)

Table 2

$\sigma_{Tot.}(pp \to pn\pi^+)$ in mb for N^*_1 and N^*_2 Production with π exchange and 'S' exchange. (i), (ii) and (iii) refer to the choice of form factors.

Incident Nucleon lab. energy (in Gev.)	π exchange						S exchange					
	N^*_1			N^*_2			N^*_1			N^*_2		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
1.41	59.5	8.34	--	0.3	0.08	--	1.2	0.19	--	0.49	0.08	--
1.92	66.0	8.1	--	1.8	0.22	--	1.6	0.22	--	0.93	0.17	--
2.85	75.0	7.64	2.25	3.3	0.34	0.08	2.1	0.26	0.02	0.98	0.18	0.21
8.0	117.0	10.1	0.23	9.8	0.83	0.01	4.3	0.46	0.001	0.88	0.125	0.03

Table 3.

$\sigma_{tot.}(pp \to pn\pi^+)$ in mb. for N^*_1 and N^*_2, production with f exchange and A exchange.

Incident nucleon lab. energy (in Gev)	f exchange						A exchange					
	N^*_1			N^*_2			N^*_1			N^*_2		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
1.41	2.8	0.04	--	0.65	0.0005	--	8.32	0.34	--	0.06	0.4×10^{-4}	--
1.92	4.2	0.055	--	2.64	0.025	--	8,08	0.37	--	0.16	1.25×10^{-4}	--
2.85	5.5	0.07	0.52	4.57	0.12	1.24	6.38	0.85	1.98	0.15	4×10^{-3}	0.02
8.0	10.3	0.09	0.46	8.22	0.44	1.63	6.92	0.80	1.51	0.17	9×10^{-3}	0.014

Table 4.

$\sigma_{tot.}(pp \to pn\pi^+)$ in mb. for N^*_2 production with I = 0 exchange

Incident nucleon lab. energy (in Gev.)	η exchange			σ exchange			ω exchange			A_o exchange		
	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)	(i)	(ii)	(iii)
1.41	0.16	0.003	--	0.7	0.13	--	0.3	$2.\times10^{-4}$	--	0.06	0.4×10^{-4}	--
1.92	0.7	0.1	--	1.32	0.21	--	1.2	0.011	--	0.16	1.25×10^{-4}	--
2.85	1.47	0.2	0.014	1.33	0.23	0.21	2.1	0.05	0.6	0.15	4×10^{-3}	0.02
8.0	5.46	0.6	0.001	0.91	0.18	0.07	4.0	0.2	0.8	0.17	9×10^{-3}	0.014

respectively defined in Section 4.

Before discussing the results, we note that our isobar model is expected to give cross-sections that are smaller than the actual case, as we are taking into account only the resonant $P_{3/2}$ and $D_{3/2}$ (π - N) interactions, whereas the kinamtics allow a final (π -N) pair to interact over an energy range which, for instance, extends to about 2 Gev (π N total energy in their own c.m.s.) for an incident nucleon energy of 2.85 Gev[3] . Thus most of the non-resonant interaction and the effect of the higher resonances are not taken into account in our model.†

The calculated cross-sections reveal the following features:

(a) With constant form factors, the cross-section from pion exchange alone is too large, as is well-known. Also, all the contributions give cross-sections that increase with energy between 1 and 3 Gev.

With OPE alone, N^*_2 production is much smaller than N^*_1 production. This is partly because of the isospin factors, which suppress N^*_2 production in pp reactions. In np reactions, the isospin factors favour N^*_2 production. For OPE , it is still less important than N^*_1 production, but for the other exchanges it is more important. For instance, for ρ exchange in $np \longrightarrow np\ \pi^0$ the N^*_2 contribution is about 2.4 times as large as the N^*_1 contribution. Also, $I = 0$ exchanges produce the N^*_2 but not the N^*_1 .

† A small part of the non-resonant interaction has been taken into account, as we have estimated the widths directly from the observed total cross-sections, as explained earlier.

(b) Although individual contributions other than single-pion-exchange are small in pp reactions, they can add to give an appreciable contribution which is, for instance, about 1/3 of the OPE contribution at 2.85 Gev , with constant form factors. In np reactions, contributions other than OPE are more than half the OPE contribution at 2.85 Gev.

(c) With the energy-independent form factors (ii), all cross-sections are much reduced, but in different proportions. With the form factors we have selected (see section 4) contributions other than OPE are reduced more and now contribute about 1/4 as much as OPE for the reaction $pp \longrightarrow pn\ \pi^+$. It is interesting to note that the various cross-sections now add to about 10 mb at 2.85 Gev; this is probably not of any particular significance because the cross-section now gradually increases with energy between 1 and 3 Gev , contrary to the observed decrease. The OPE contribution alone decreases slowly, but more slowly than was obtained by FS for the 33 resonance region. We do not know which feature of our isobar approximation is responsible for this difference. The point we wish to make is that contributions other than OPE are still appreciable, unless all other coupling constants turn out to be much smaller, which seems unlikely.

(d) When all the contributions to the total cross-section are taken into account, then the most natural way to get the fairly rapid observed decrease in σ_{tot}. would be to assume

energy-dependent form factors of the form (iii), for instance. The form factors (iii) give a total cross-section of about 6.6 mb for the reaction $pp \longrightarrow pn\,\pi^+$ of which only about 2.3 mb comes from the OPE. The 'A' meson and the ρ meson both give important contributions.

(e) Withthe energy-dependent form factors (iii), the total cross-sections decrease with energy between 2.85 and 8 Gev, whereas the energy independent form factors (ii) give increasing cross-section except for OPE. Experiments on the variation of the cross-section between 3 and 10 Gev would be useful in determining how strongly the form factors are damped.

(f) The contribution of the I = 0 exchanges, ω, η and σ to $\sigma_{tot.}$ ($pp \longrightarrow p\,n\,\pi^+$) at 2.85 Gev is about 5 % with the form factors (ii) and about 4 % with the form factors (iii). For the reaction $np \longrightarrow np\,\pi^o$, I = 0 exchange is more important when the form factors have the shapes (iii). I = 0 exchange now contributes about 4.5 % and 10 % with the form factors (ii) and (iii) respectively. We have not included the contribution of the f^o and of the Pomeranchuk trajectory, which are expected to be important at higher energies.

To sum up the results of this section, firstly there seems to be no reason to suppose that all contributions other than OPE are negligible. With the lack of information about the coupling constants and form factors, we cannot make definite statements about the relative importance of different mechanisms.

The important contributions other than the π would seem to come from the ρ, from an $I = 1$ axial vector meson, from the η and from $I = 0$ and $I = 1$ scalar mesons. N_2^* production and $I = 0$ exchange appear to be relatively unimportant in pp reactions only because here the N_2^* is suppressed by the isospin factors. Their importance becomes apparent in np reactions.

The actual form factors would probably have a shape between the shapes (ii) and (iii). A decrease of all cross-sections with increasing energy is obtained most easily with energy-dependent form factors of the type of (iii).

Measurements required are 1) on the variation of $\sigma_{tot.}$ between 3 and 10 GeV, and 2) the values of $\sigma_{tot.}$ for the different charge-states in np reactions between 1 GeV. and 10 GeV.

6. C.M. Energy spectra

In this section we discuss briefly the energy spectra of the final nucleons and pion in the process $N + N \rightarrow N + N + \pi$ as obtained in our isobar model. The general expressions for the energy spectra in different charge-states have been derived in the next chapter in terms of the exchange of states with definite isospin. Lindenbanm and Sternheimer[+] had given the general analysis in terms of reaction channels with definite isospin; we go further in giving the analysis in terms of the isospin of the exchanged states.

The final nucleon energy spectrum is a superposition of the spectrum of the decay nucleon and that of the recoil nucleon, the relative weights depending on the charge-states and the isospin of the exchanged state, as shown in the next chapter.

The energy spectrum of the recoil nucleon is essentially a reflection of the Lorentz transform of the mass distribution of the isobar. For an isobar with a definite mass, i.e., a δ-function mass spectrum, the recoil nucleon energy spectrum will also be δ-function spectrum. In general, the recoil nucleon energy spectrum may be obtained as follows. For a

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[+] Ref. the second paper quoted in 1).

given total energy W, the cross-section $\sigma_{tot.}(M^*)$ for producing an isobar of mass M^* is found out as shown in section 2. This is also the cross-section for producing a recoil nucleon with energy

$$W_2 = \frac{W^2 + m_N^2 - M^{*2}}{2M^*} \qquad (6.1)$$

For a given value of W, there corresponds to each value of M^*, a particular value of W_2. If $f(M^*)$ be the isobar mass distribution, normalized such that

$$\int f(M^*)\, dM^* = 1 \qquad (6.2)$$

the integration being over the complete mass spectrum, then the projection of $\sigma_{tot.}(M^*)\, f(M^*)$ on to the variable W_2 gives the energy spectrum of the recoil nucleon.

The energy spectra of the decay nucleon may be obtained as follows:

For a given value of the isobar mass M^*, the decay nucleon spectrum will be given by the well-known expression[1)]

$$\frac{\partial \sigma}{\partial E_N}(M^*) = \frac{\sigma_{tot.}}{P_N} \cdot \frac{\left[1 - v_{N^*}^2\right]^{1/2}}{2 v_{N^*}}, \qquad (6.3)$$

where v_{N^*} is the velocity of the isobar in the overall c.m. frame,

$$v_{N^*} = \frac{p_2}{E_2} . \qquad (6.4)$$

and P_N is the momentum of the decay nucleon in the rest frame of the isobar.

To obtain the decay nucleon spectrum for an isobar with a mass spectrum $f(M^*)$ the energy spectrum $\frac{\partial \sigma}{\partial E_N}(M^*)$ must be integrated over the part of the isobar mass spectrum which contributes to the production of a decay nucleon with energy E_N .

We would then obtain

$$\frac{\partial \sigma}{\partial E_N} = \int_a^b dM^* f(M^*) (1 - v_{N^*}^2)^{1/2} (2 v_{N^*} P_N)^{-1} \qquad (6.5)$$

where a and b are the extreme values of M^* that can give (on decaying) a nucleon with a given energy E_N , where the isobar has a total energy

$$E_2 = \frac{W^2 + M^{*2} - m_N^2}{2W} \qquad (6.6)$$

The limits a and b may be found out from a plot of $E_N^{max.}$ and $E_N^{min.}$ against M^* , where

$$(E_N)_{min.}^{max} = (1 - v_{N^*}^2)^{-1/2} \left[\varepsilon_N \pm v P_N \right] \qquad (6.7)$$

are the maximum and minimum values of E_N obtainable from the decay of an isobar with a given mass M^* and a given velocity v_{N^*}.

Here ε_N is the total energy of a decay nucleon in the rest frame of the isobar:

$$\varepsilon_N = \frac{1}{2M^*}\left(M^{*2} + m_N^2 - m_\pi^2\right); \quad P_N^2 = \varepsilon_N^2 - m_N^2. \quad (6.8)$$

The decay pion spectrum may be found out in a similar way, replacing the parameters relating to the decay nucleon by the corresponding ones for the decay pion.

These overall energy spectra (that is, the spectra integrated overall angles) are <u>not</u> very sensitive to the mechanism of production. They depend on this mechanism only because

different mec hanisms give isobars with mass spectra that are distorted in different ways. This distortion can show itself in the energy spectrum from an isobar emitted at particular angles where $\partial\sigma/\partial(\cos\theta^*)$ is most sensitive to the isobar mass. This effect is largely smoothed out when integrated over all angles.

The c.m. energy spectra can give two types of information:

i) about the relative probabilities of N^*_1 and N^*_2 production, and

ii) about deviations from the basic assumptions of the isobar model.

To illustrate how such information may be obtained we have given in figs. 6 to 10 , the calculated energy spectra for an incident nucleon energy of 2.85 Gev. For simplicity, these have been calculated assuming an undistorted Breit-Wigner mass distribution with a half-width of 100 Mev for each isobar and by integrating the energy spectra over this mass distribution. The spectra of the recoil nucleon and the decay nucleon have been superposed with the weights given by the isospin factors.

In the figures the full lines refer to N^*_1 production and broken lines to N^*_2 production through I = 1 exchange. For pp πr reactions , the contribution to N^*_2 production from I = 0 and I = 1 exchanges are weighted equally by the Isospin factors while for np reactions I = 0 exchange is weighted one-fifth as much as I = 1 exchange. The figures do <u>not</u> show the relative magnitudes of the N^*_1 and N^*_2

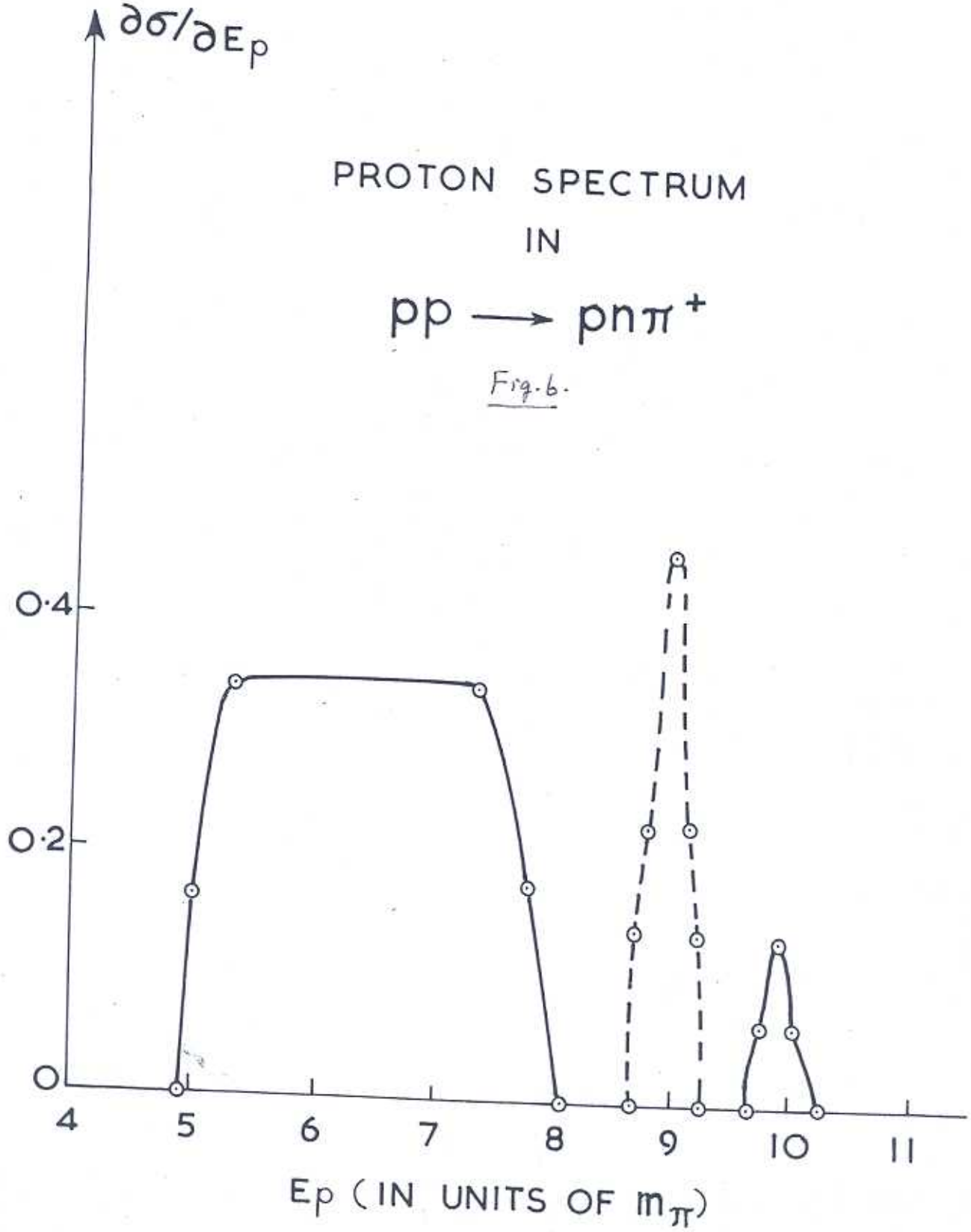

$\partial\sigma/\partial E_p$
PROTON SPECTRUM
IN
$pp \longrightarrow pn\pi^+$
0·4
0·2
0
4
5
6
7
8
9
10
11
E_p (IN UNITS OF m_π)

Fig. 6.

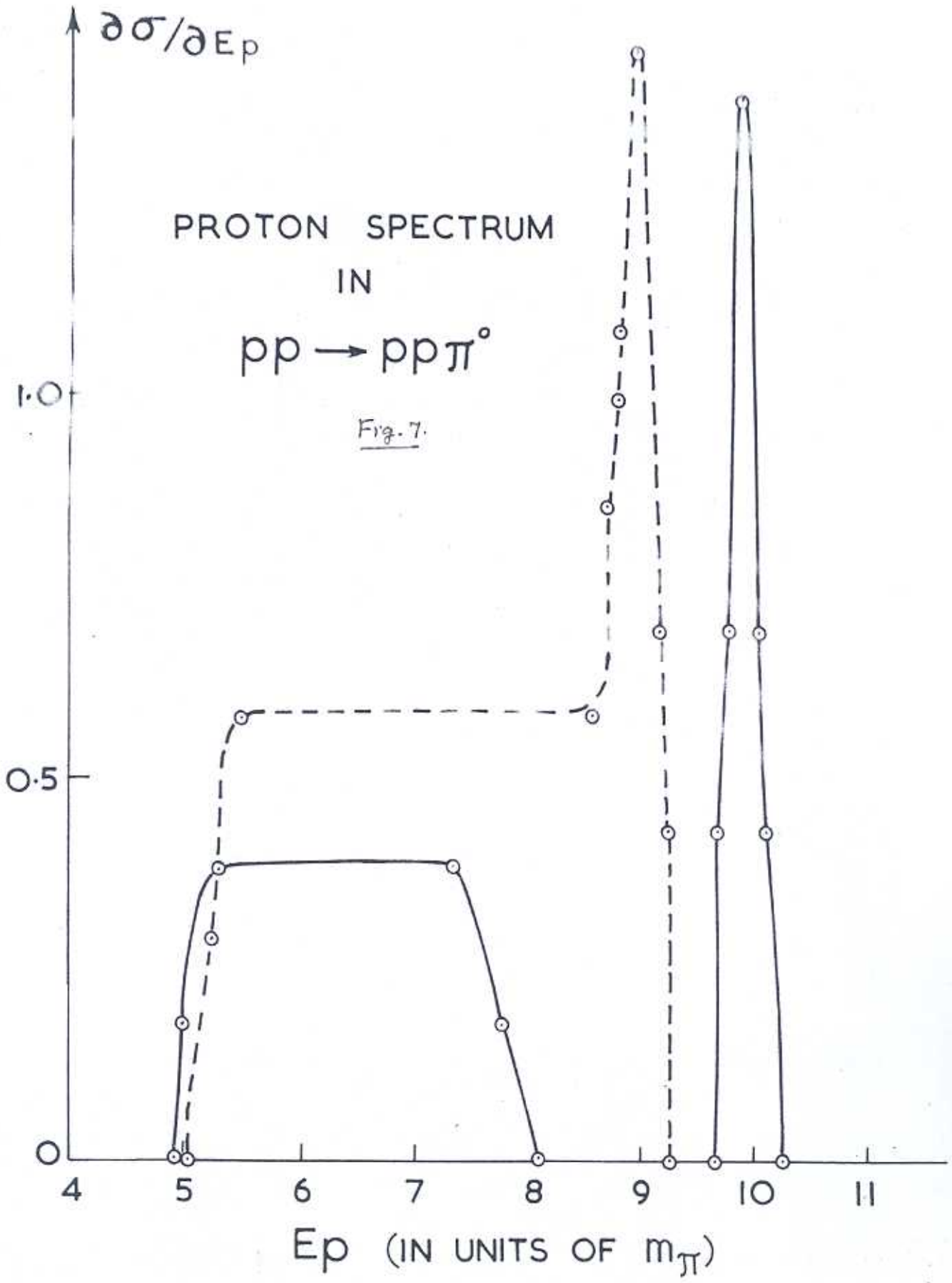

∂σ/∂Ep
PROTON SPECTRUM
IN
pp → ppπ°
1·0
0·5
0
4
5
6
7
8
9
10
11
Ep (IN UNITS OF mπ)

Fig. 7.

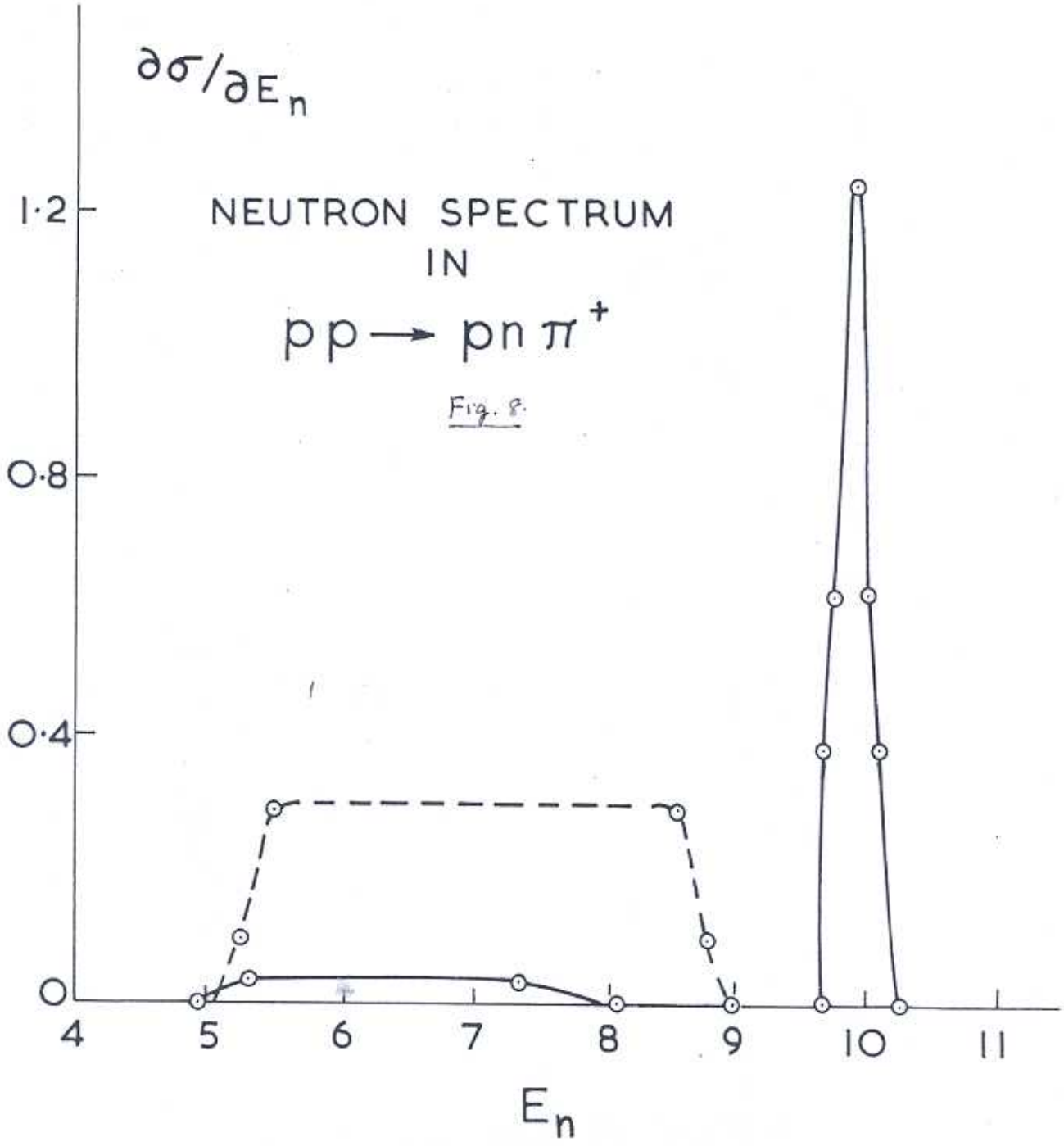

$\partial\sigma/\partial E_n$
NEUTRON SPECTRUM
IN
$pp \rightarrow pn\pi^+$
1·2
0·8
0·4
0
4
5
6
7
8
9
10
11
E_n

Fig. 8.

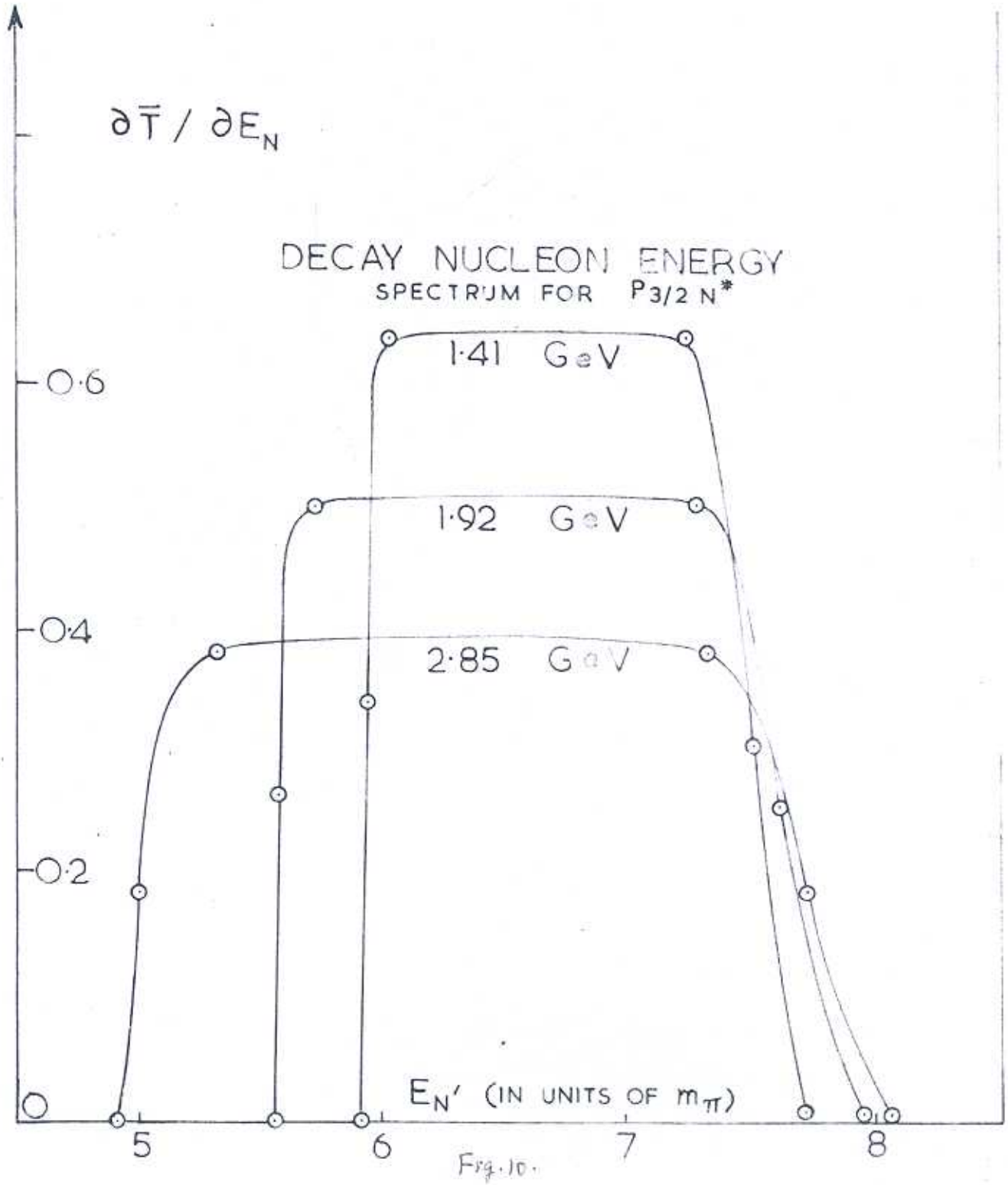

∂T̄ / ∂E_N
DECAY NUCLEON ENERGY
SPECTRUM FOR P3/2 N*
1·41 GeV
1·92 GeV
2·85 GeV
0·6
0·4
0·2
0
5
6
7
8
E_N' (IN UNITS OF m_π)

Fig. 10.

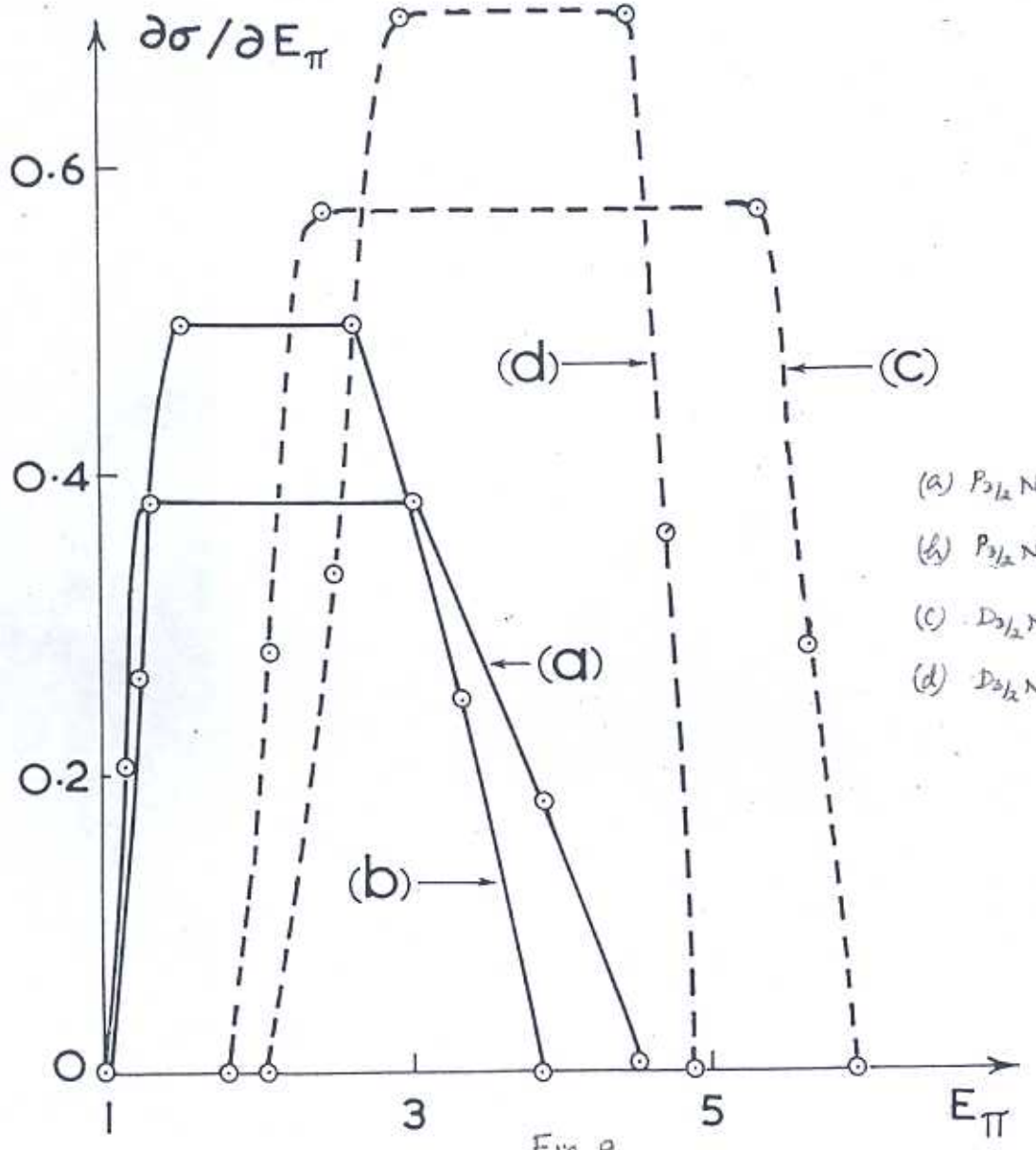

$\partial\sigma/\partial E_\pi$
0·6
0·4
0·2
0
1
3
5
E_π
(a)
(b)
(c)
(d)
(a) $P_{3/2}N^*$: 2·85 Gev.
(b) $P_{3/2}N^*$: 1·92 Gev.
(c) $D_{3/2}N^*$: 2·85 Gev.
(d) $D_{3/2}N^*$: 1·92 Gev.

Fig. 9.

contributions or of I = 1 and I = 0 exchange, which depend of course on the relative cross-sections.

The observed spectra may be analysed to give information about these relative production cross-sections, provided measurements are made on different charge states in pp and np reactions.

In fig. 9 are given the pion c.mm m. energy spectra at 2.85 Gev and 1.92 Gev incident nucleon energy in the laboratory. As the incident energy increases, the spectrum becomes flatter and spreads out towards higher values of the pion energy. The decay nucleon spectrum, on the other hand, flattens and spreads out towards lower values of the energy as shown in fig. (10) .

These calculations are of course idealized because of the assumption of a Breit-Wigner mass spectrum for the isobar. With a more refined calculation, it would be possible to detect any important deviations from the isobar model by comparison with experiment. The most interesting of these deviations is the distortion of the final-state phase space by overlapping resonances.

7. C.M. Angular Distributions

Information about the isobar angular distribution in single isobar production is important because such information, when taken together with measurements of the polarization correlation and other measurements that give information about the spin of exchanged states, can enable deductions to be made about the form factors for different exchanges. These will be discussed in Section 8 .

To isolate single isobar production, one may proceed as follows: As a first step, if one neglects double pion production from the 2π decay of a single N^*_2, then the problem is ~~to~~ same as isolating / ~~isolate~~ the single-pion production process. This may be done, as suggested by Melissinos **et al**[20], by measuring the π^- production in pp collisions (which is only possible when more than 1 pion is produced) and subtracting an equivalent amount from the π^0 and π^+ production processes (assuming that in the production of 2π, 3π, etc., π^+, π^-, and π^0 production are alike, apart from isospin factors.) (Similar considerations may be used for taking into account $2\pi^-$ production from a single N^*_2).

The pion angular distribution in the single-pion production process may be used for inferring the isobar angular distribution as shown by Lindenbaum and Sternheimer[1], when only one isobar is produced. When the N^*_1 and N^*_2 are produced with comparable probability, one must first separate the N^*_1 and N^*_2 contributions, which may be done by taking suitable linear combinations of the differential cross-sections for different charge-states, as is obvious from equation (3.2) to (3.6). The N^*_1 and N^*_2 angular distributions can be obtained separately.

More direct information on the N* angular distribution may be obtained by measuring nucleon energy spectrum as a function of the angle and deducing the angular distribution of the

sharp peak in the nucleon spectrum, which comes from the recoil nucleon. The angular distribution of this peak, reflected according to $\theta \longrightarrow (\pi - \theta)$ gives the isobar c.m. angular distribution for isobar production.

A combination of the two methods can be expected to give reliable information about the isobar angular distribution.

To save space, we give the calculated angular distributions only for one energy viz., 2.85 Gev. These are given for the different mechanisms in figs. 11 to 21. These will provide a basis for interpreting data on N* angular distributions when such data become available.

The different curves (a) - (f) in the figures refer to different assumptions about the form factors; (a) and (d) are for constant form factors (i), (b) and (e) for the energy-independent form factors (ii) and (c) and (f) for the form factors (iii), these form factors being given in Section 4.

The salient features of these angular distributions are summarised below.

Figs. 11 to 16 give the N^*_1 angular distributions in all pp and np reactions and the N^*_2 distributions in pp reactions and in the reaction $np \longrightarrow p n \pi^0$. All these are forward-backward symmetric.

With constant form factors N^*_1 production with π and 's' exchange is almost isotropic (see figs. 11 and 13);

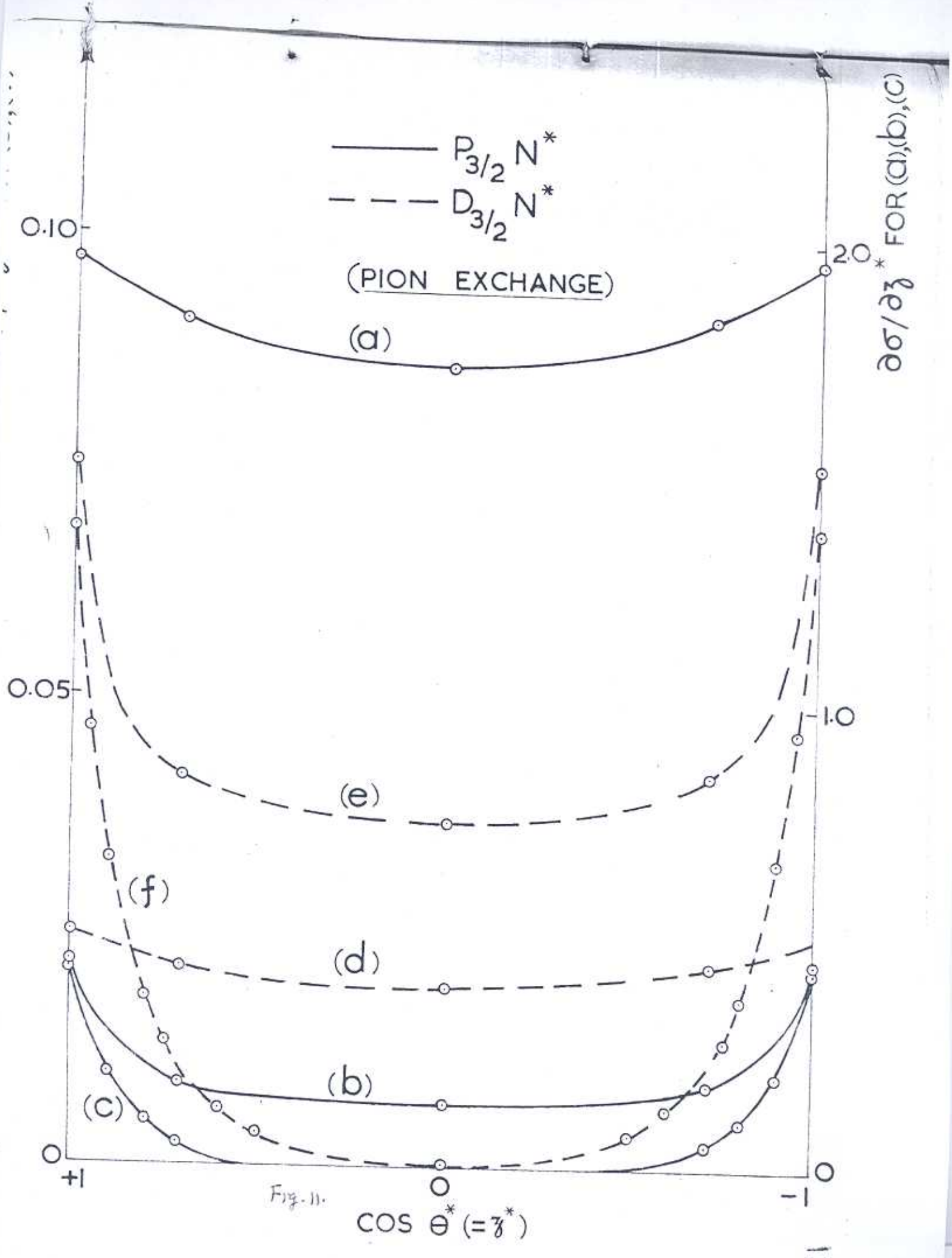

$P_{3/2}\ N^*$
$D_{3/2}\ N^*$
(PION EXCHANGE)
(a)
(b)
(c)
(d)
(e)
(f)
0.10
0.05
0
2.0
1.0
0
+1
0
−1
$\cos\theta^*\ (=z^*)$
$d\sigma/dz^*$ FOR (a),(b),(c)

Fig. 11.

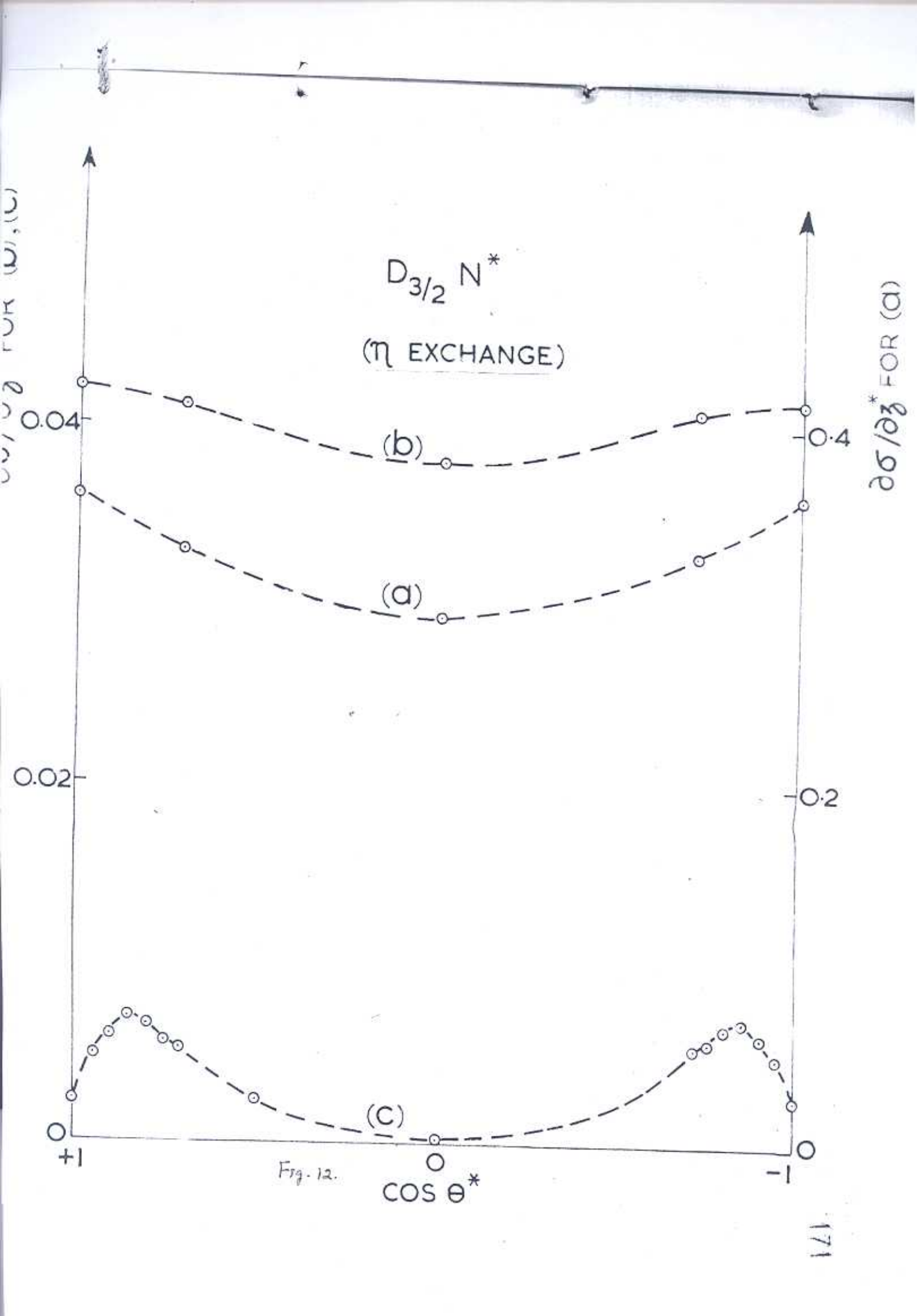

$D_{3/2}$ N^*
(η EXCHANGE)
0.04
0.02
(b)
(a)
(c)
0·4
0·2
0
+1
0
−1
cos θ*
$\partial\sigma/\partial z^*$ FOR (a)

Fig. 12.

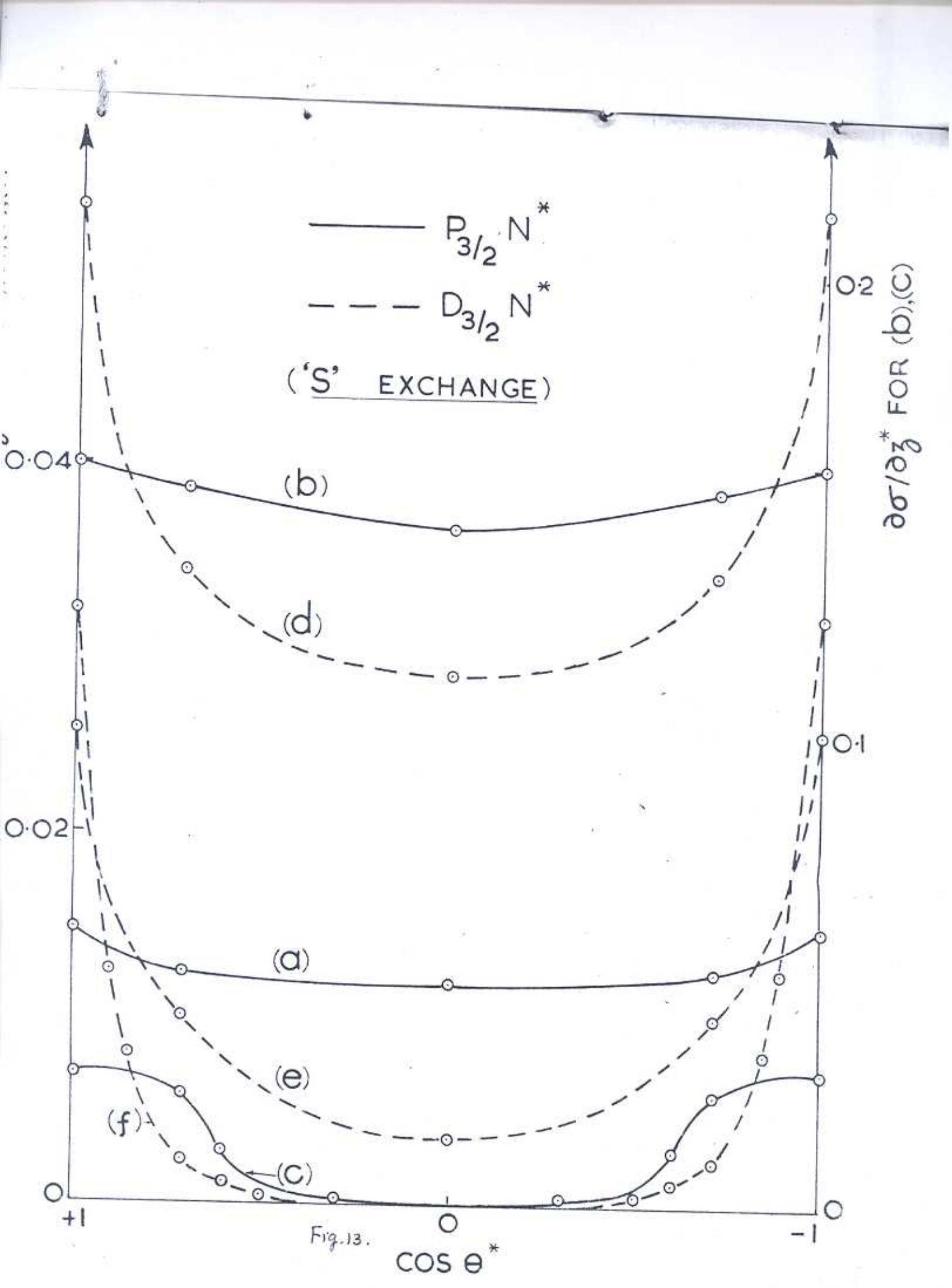

$P_{3/2}\,N^*$
$D_{3/2}\,N^*$
('S' EXCHANGE)
0·04
0·02
0
0·2
0·1
0
(b)
(d)
(a)
(e)
(f)
(c)
+1
0
−1
COS θ*
$\partial\sigma/\partial\Omega^*$ FOR (b),(C)

Fig. 13.

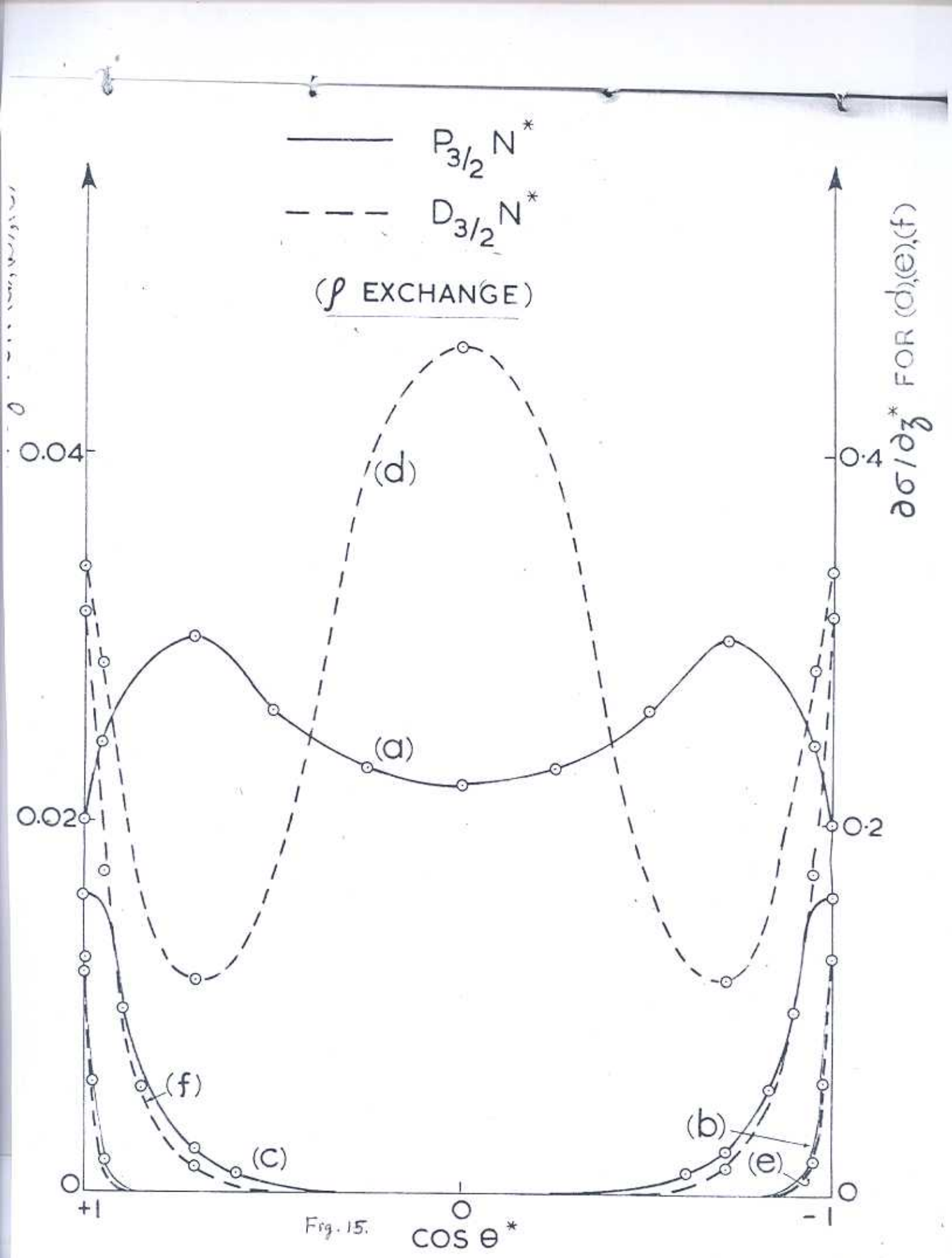

$P_{3/2}N^*$
$D_{3/2}N^*$
(ρ EXCHANGE)
0.04
0.02
0
0·4
0·2
0
(a)
(b)
(c)
(d)
(e)
(f)
+1
0
−1
COS θ*
$\partial\sigma/\partial z^*$ FOR (d),(e),(f)

Fig. 15.

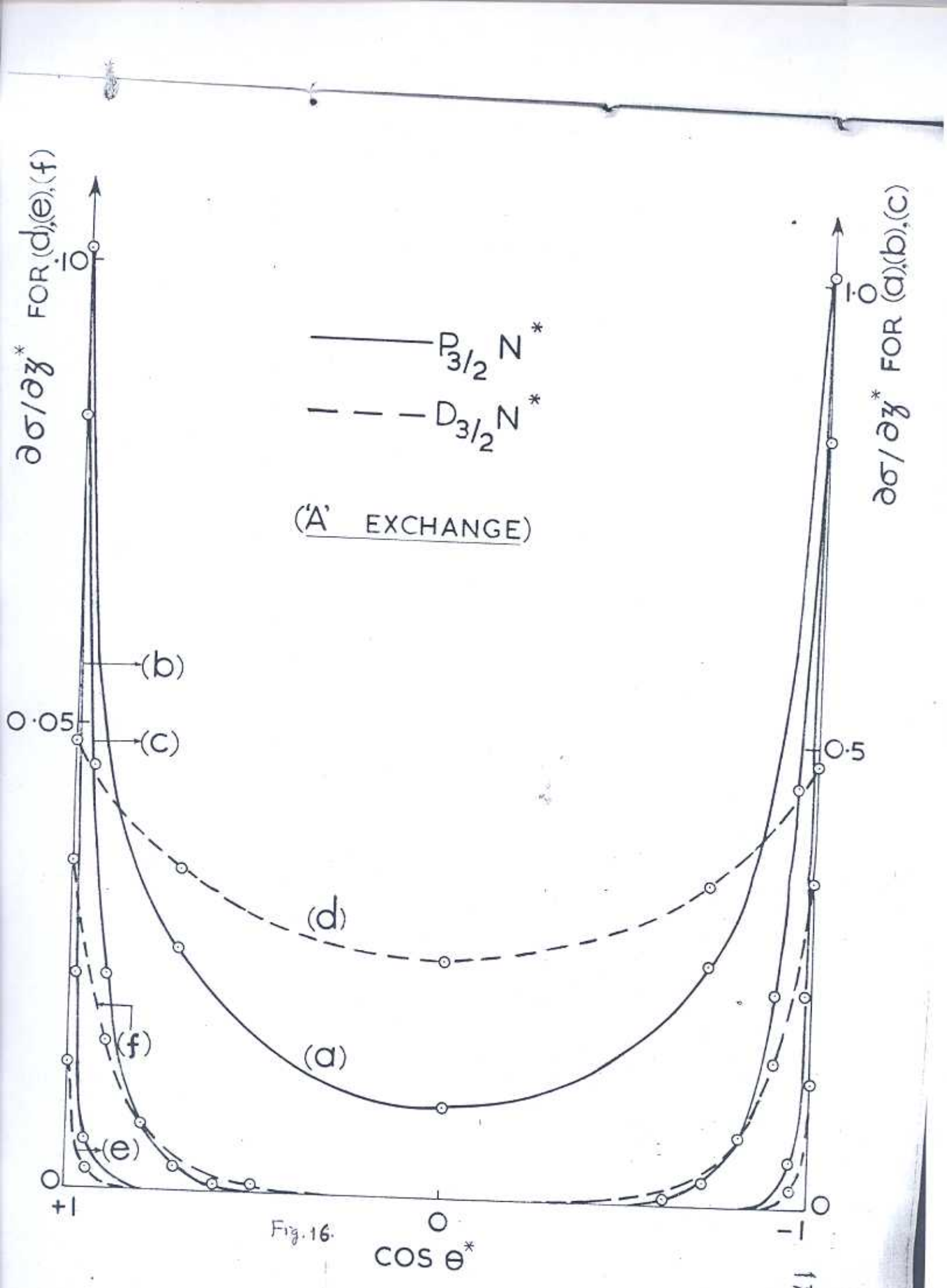

$\partial\sigma/\partial\chi^*$ FOR (d),(e),(f)
$\partial\sigma/\partial\chi^*$ FOR (a),(b),(c)
$P_{3/2}\,N^*$
$D_{3/2}\,N^*$
('A' EXCHANGE)
(a)
(b)
(c)
(d)
(e)
(f)
·10
0·05
1·0
0·5
0
+1
0
−1
$\cos\theta^*$

Fig. 16.

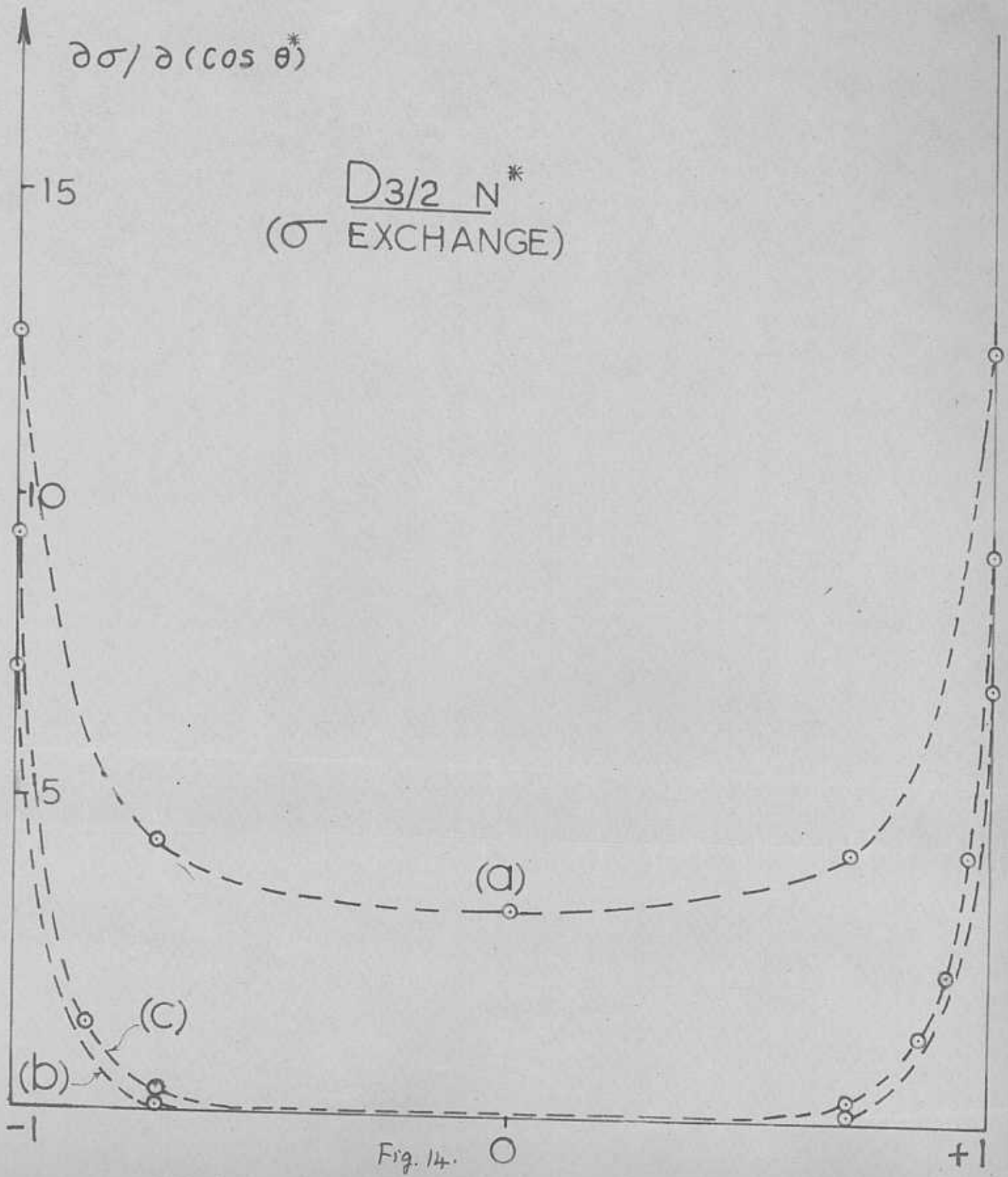
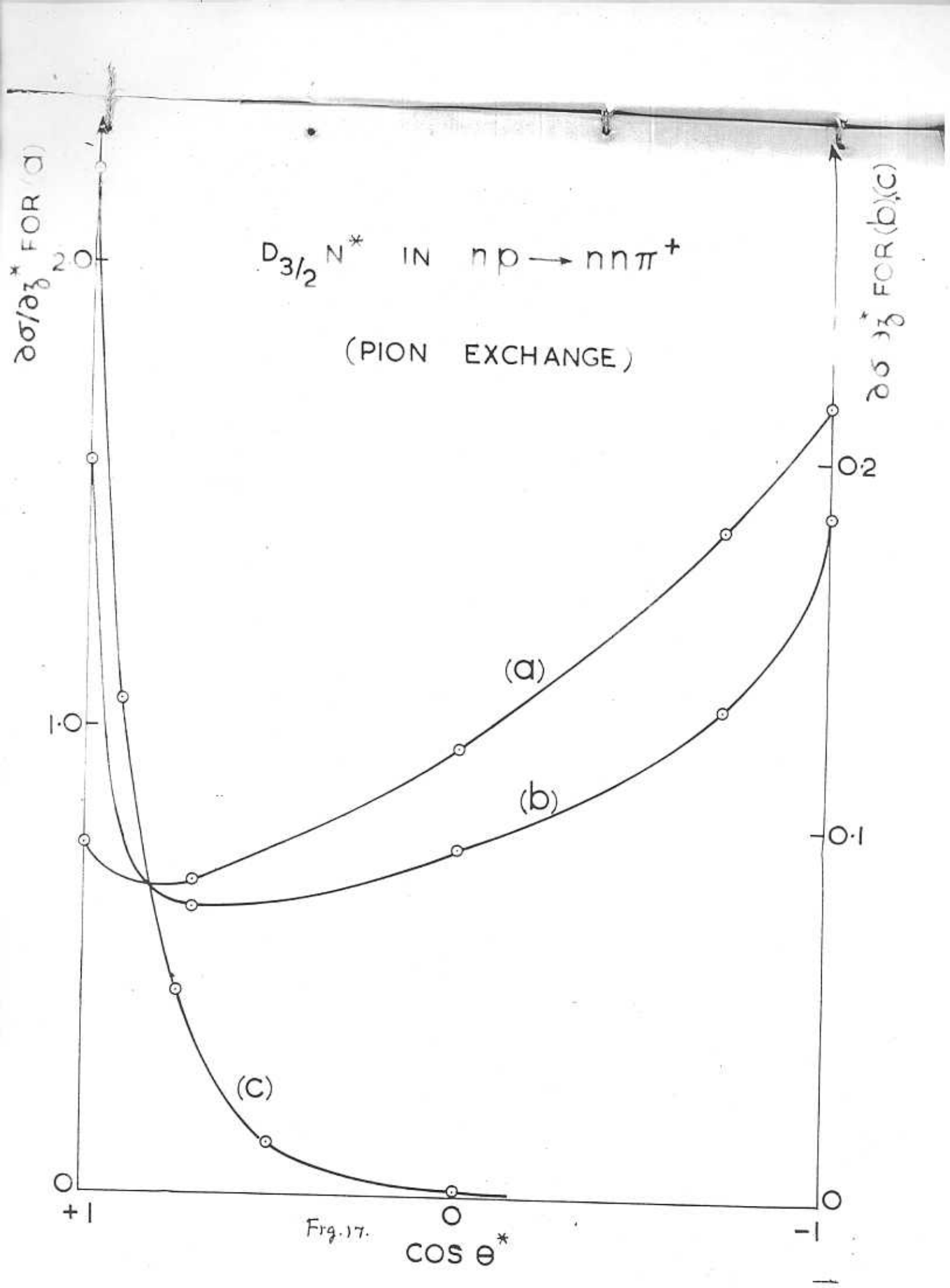


Fig. 14.

with ρ exchange it has a peak at 45° and 135° as in fig. (15 a) while 'A' exchange gives a sharp peaking at θ = 0° and 180° , (fig. 16 a). For N^*_2 production with constant form factors, π and η exchange give a nearly isotropic distribution (fig. 11 and 12); 'S' and 'A' exchange give a peaking at θ = 0° and 180° , more so 'S' exchange (as in fig. 13) while ρ exchange gives the shape in fig. 15 b with peaks at 0° and 180° . ω exchange gives an N^*_2 distribution with a much sharper peak than S exchange at θ = 0° and 180° as shown in fig. (14) .

With the form factors (ii) and (iii) given in section 4, all peakings become very sharp and forward-backward peakings appear in most contributions. The exceptions are (1) N^*_1 production with 'S' exchange where the form factor (ii) still does not lead to much peaking (fig. 13 b) and for which the form factor (iii) gives a broad forward peak (fig. 13 c) and (2) N^*_2 production with η exchange, where the form factor (ii) (fig. 12 b) gives a more nearly isotropic distribution than a constant form factor while a Regge damping (iii) gives rise to a peak near θ = 30° and 150° (fig. 12 c). The peakings are sharpest for ρ and 'A' exchange.

In figs. 17 to 21 are shown the N^*_2 angular distributions as inferred from the reaction $np \longrightarrow nn\ \pi^+$ (This may be obtained experimentally by obtaining the N^*_1 contribution alone by combining the two pp reactions and subtracting it


$D_{3/2}$ N^* IN $np \rightarrow nn\pi^+$
(PION EXCHANGE)
$\partial\sigma/\partial\beta^*$ FOR (a)
$\partial\sigma/\partial\beta^*$ FOR (b)(c)
2·0
1·0
0
0·2
0·1
0
(a)
(b)
(C)
+1
0
−1
cos θ*

Frg. 17.

$D_{3/2}$ N^* IN $n\,p \rightarrow n\,n\,\pi^+$
(η EXCHANGE)
$\partial\sigma/\partial\Omega$ FOR (a)
$\partial\sigma/\partial\Omega$ FOR (b), (c)
0·4
0·2
0
0·04
0·02
0
(a)
(b)
(c)
+1
0
−1
$\cos\theta^*$

Fig. 18.

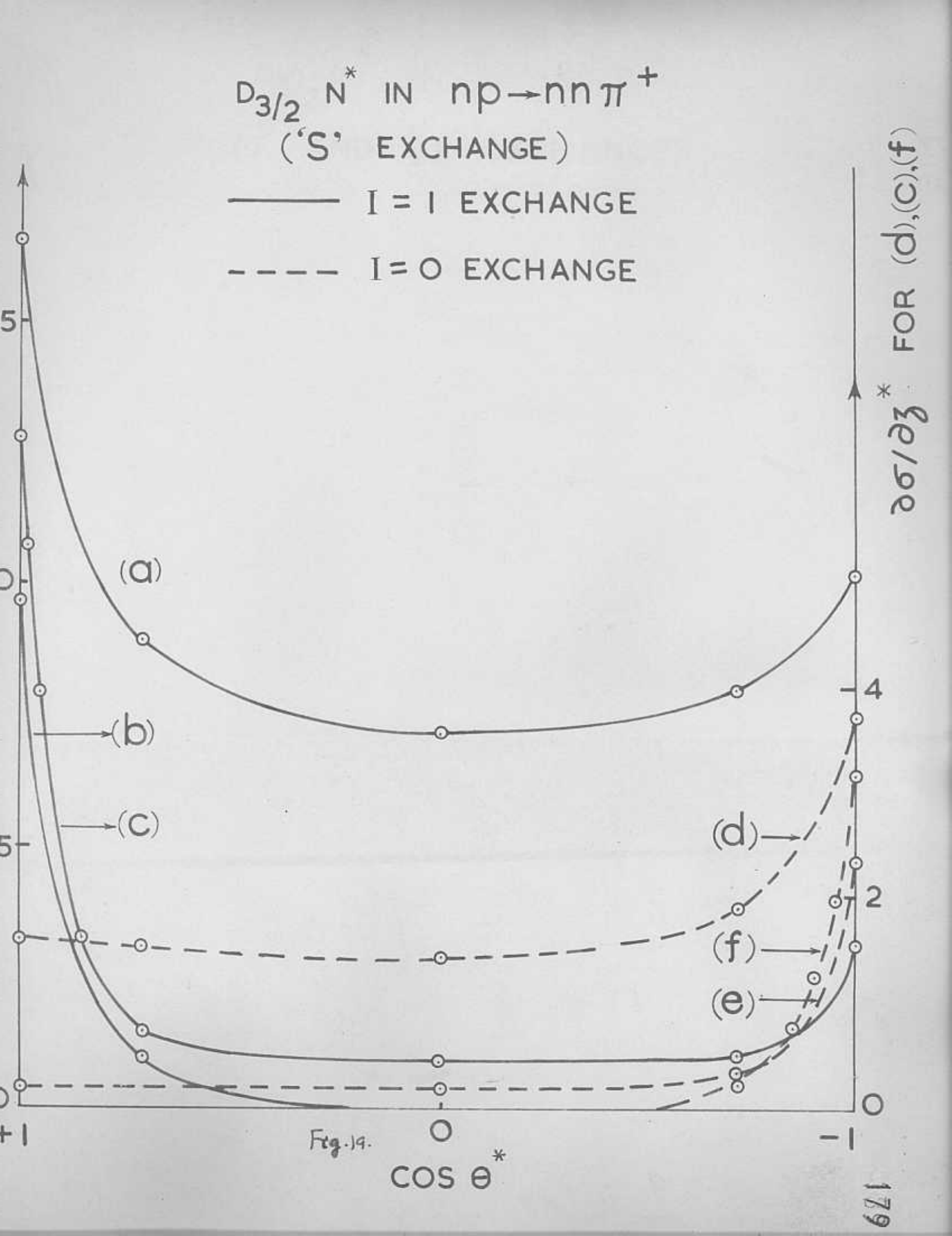
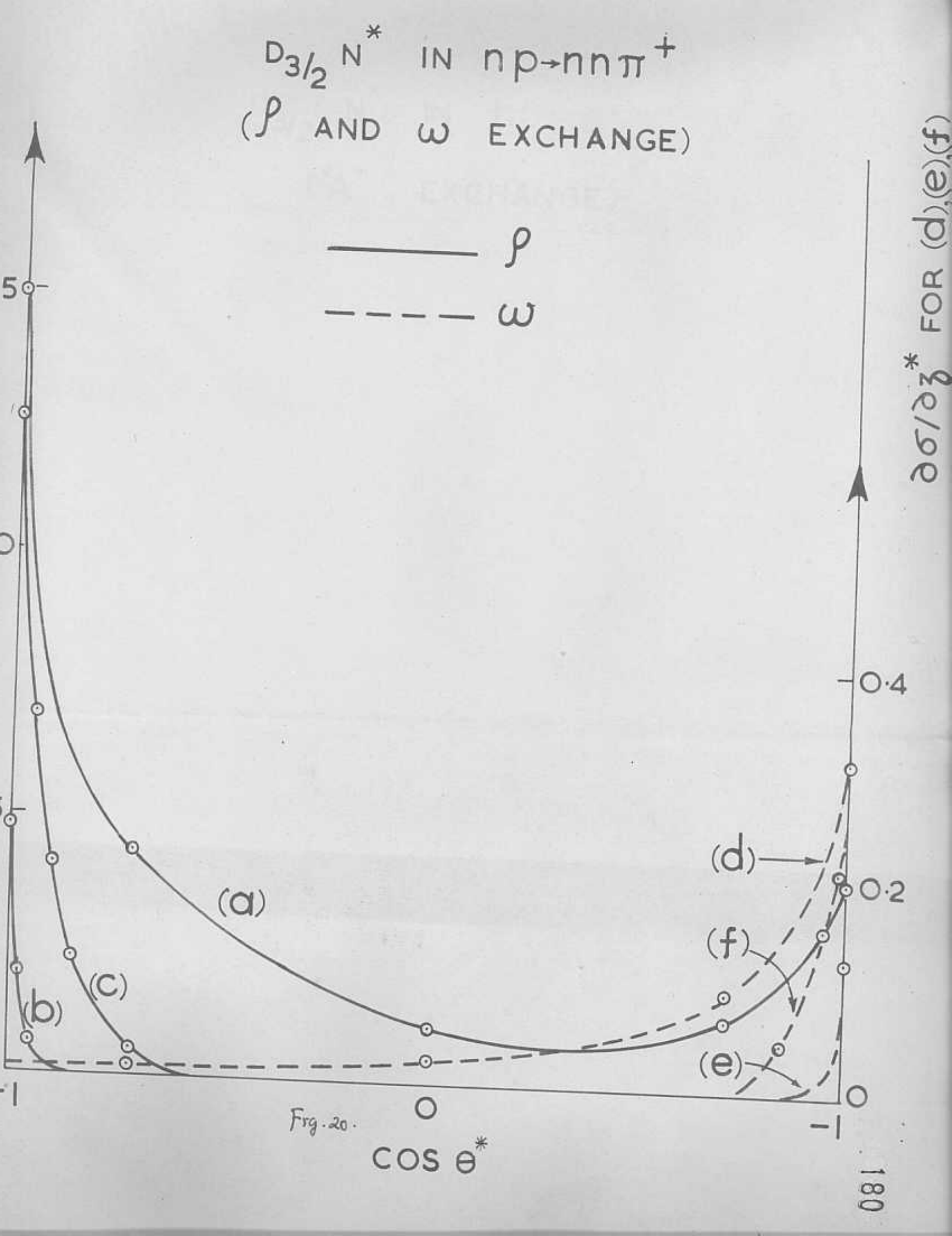
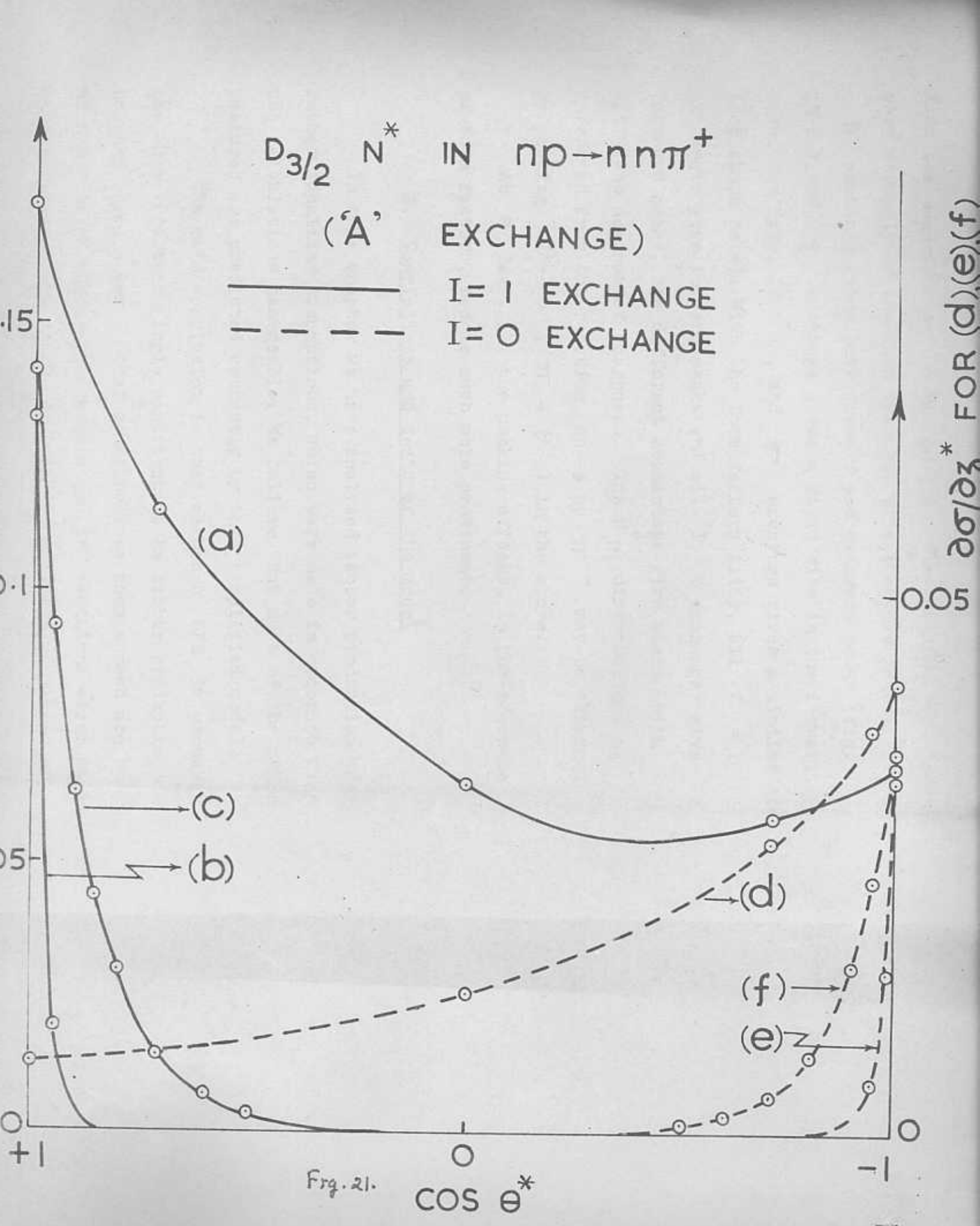

$D_{3/2}$ N^* IN $np \rightarrow nn\pi^+$
('S' EXCHANGE)
I = 1 EXCHANGE
I = 0 EXCHANGE
(a)
(b)
(c)
(d)
(e)
(f)
$\partial\sigma/\partial\beta^*$ FOR (d),(c),(f)
4
2
0
+1
0
−1
$\cos\theta^*$

Fig. 19.



$D_{3/2}N^*$ IN $np \to nn\pi^+$
(ρ AND ω EXCHANGE)
ρ
ω
$\partial\sigma/\partial z^*$ FOR (d),(e),(f)
0·4
0·2
0
(a)
(b)
(c)
(d)
(e)
(f)
-1
0
-1
$\cos\theta^*$

Fig. 20.



$D_{3/2}$ N^* IN $np \to nn\pi^+$
('A' EXCHANGE)
I = 1 EXCHANGE
I = 0 EXCHANGE
(a)
(b)
(c)
(d)
(e)
(f)
.15
0.1
05
0
+1
0
-1
0.05
0
$\partial\sigma/\partial\xi^*$ FOR (d),(e),(f)
$\cos\theta^*$

Fig. 21.

from the reaction $np \longrightarrow nn\,\pi^+$). The peakings here depend very strongly on the form factors. With the form factor (ii), π exchange gives both forward and backward peaks (fig. 17 b) and η exchange gives a rapid rise in the forward direction (fig. 18 b), and σ exchange gives a similar rise (not shown here). With the form factors (iii), all $I = 1$ exchanges give forward peaks and all $I = 0$ exchanges give backward peaks; the different mechanisms give peaks with different degrees of sharpness. The N^*_2 distributions as inferr ed from the reaction $np \longrightarrow pp\,\pi^-$ may be obtained by putting $\theta \longrightarrow (\pi - \theta)$ in the above.

At 8 Gev , all the peaking effects, in the presence of form factors, become much more prominent.

8. Conclusions and tests of the model

In this chapter we have analysed isobar production under rather idealized assumptions, which were made in order to keep the calculations manageable. We believe that most of the gross features are predicted correctly by this simplified model.

The main conclusion is that although OPE is probably the most important s ingle contribution to isobar production between 1 and 3 Gev , other contributions when summed are by no means negligible. Measurements on pp reactions alone will tend to give a wrong indication of the importance of the N^*_2

production and of the exchange of states with zero isospin, as in these reactions N^*_2 production is suppressed by the isospin factors alone. To understand the relative importance of T = 1/2 isobars in single-pion production in NN collisions in general, extensive measurements on np reactions are necessary.

Shortcomings of our present calculation are (1) the neglect of interference between pole contributions, (2) the neglect of the higher nucleon isobars, and (3) the neglect of Pomeranchuk exchange and f^o exchange.

Concerning (1), we note that contrary to the statement made by Ueda,[9] the interference between A and P exchange (and S and V exchange) are not the only interference terms allowed (in addition to the interference between the exchanges of like mesons). Many more interference terms are allowed when one considers interference between a pole in the t-channel and a pole in the u-channel.

Experiments that would test the presence of the various possible mechanisms are the following:

Confining ourselves to single-isobar production and assuming that this has been isolated as discussed in section 7, the first task is to get an accurate estimate of the relative contributions of the N^*_1 AND N^*_2, which may be obtained by measurements of the c.m. energy spectra, of the distribution of charge states, and of the distribution in Q-value of (π - N) pairs in the final state. If individual events are

studied, then useful information may be obtained by an analysis of effective-mass and missing-mass distributions[26], although these have to be interpreted carefully because the pion can resonate with either nucleon in the final state.

Next the c.m. angular distribution $\partial\sigma/\partial(\cos\theta^*)$ of the isobars must be obtained as discussed in section 7. Further information about the momentum-transfer dependence of the cross-section may be obtained by measurements of the laboratory energy spectrum of the final nucleons at various angles, combined with an assumed isobar decay spectrum. This, together with $\partial\sigma/\partial(\cos\theta^*)$ will give a general indication of how strongly the form factors are damped at large momentum transfers, information about which can also be obtained by observing the variation of σ_{tot}. with the incident energy.

The next step would be to separate the contributions of $I = 0$ and $I = 1$ exchange. This may be done by experiments on deuterium, as discussed in section 3 (see fig. 5) which give the $I = 0$ exchange contribution. A detailed calculation of this will be given separately. By subtraction one may then deduce the contribution of $I = 1$ exchange in the reaction $NN \longrightarrow N^* N$.

By comparing the $I = 0$ and $I = 1$ exchange contributions to the N^*_2 angular distributions in $np \longrightarrow nn\ \pi^+$ (or $np \longrightarrow pp\ \pi^-$) with each other and with the calculated

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26) e.g. see B.Maglic, 'On the NewResonances' Lectures at the Institute of MathematicalSciences, Madras, May 1962.

distributions (see fig.17 to 21) more definite information about the form factors may be obtained. Measurements at different energies would show whether the form factors vary with energy; if they do, this may be taken as evidence for a Regge damping.

In order to obtain more information about the types of exchange that dominate, it is necessary to have some knowledge of the angular momenta of the exchanged states. Knowledge of this may be obtained by

a) measuring the variation of the cross-section ~~with~~ with the incident energy;

b) measuring the various polarization correlations; and

c) observing correlations of the type discussed by Treiman and Yang[27] and by Sakurai[28].

We shall briefly elaborate on a) here; some aspects of (b) will be discussed in detail in the ~~next~~ VIII th chapter. Regarding (b) we merely remark that there are comparatively simple tests which can detect whether the exchange of states with non-zero spin is important. For instance, if one performs an experiment with a polarized nucleon beam incident on a polarized target, then if the angular distribution or the total cross-section shows a dependence on the relative orientation of the polarization vectors of the 2 nucleons in the initial state, then this

27) S.B.Treiman and C.N.Yang, Phys. Rev.Lett.8 (1962) 140

28) L.Stodolsky and J.J.Sakurai, Phys. Rev. Lett.11 (1963) 90

is evidence that there is an exchange of a state with non-zero spin. The type of correlation expected for ρ -exchange is given in a later chapter, where we also discuss the correlations between the polarizations of an initial nucleon and a final nucleon.

Regarding (a), it is well known that on writing the amplitude as a sum over the partial waves in the crossed channel, the differential cross-section may be expressed in the form

$$\frac{\partial\sigma}{\partial(\cos\theta)} = \text{Const.} \frac{1}{W^2} \cdot \frac{p_f}{p_i} \cdot \sum_{J=0}^{2J_{max.}} a_J(t) s^J, \quad (8.1)$$

where $J_{max.}$ is the maximum angular momentum exchanged[29]. The expressions for the differential cross-section given in Sect.2 illustrate this.

By measuring the variation of the differential cross-section at a fixed value of the invariant momentum transfer t with the incident energy, or equivalently s , one may obtain information of the following type:

i) If the function $\frac{p_i}{p_f} \cdot \left[\partial\sigma/\partial(\cos\theta) \right]$ can be fitted by a polynomial in s, for fixed t , and if the degree of this polynomial is independent of t , then one

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29) M. Gourdin and A. Martin: CERN pre-print (1962).

may infer that the exchange of few non-reggeized poles can explain the data. The maximum angular momentum exchanged can be directly obtained from (8.1).

If the value of $J_{max.}$ required to fit the observed variation of $\partial\sigma/\partial(\cos\theta)$ with s (for fixed t) changes slowly with t, then this suggests that the reaction is still dominated by a few poles, but that one or more of these lie on a Regge trajectory.

The presence of odd powers of s in (8.1) and the magnitudes of their coefficients give information about the strength of the interference between the different pole contributions and can confirm whether these interference terms are really negligible, as we have assumed them to be.

Information about the spin of the exchanged particle may be combined with a closer examination of the angular distributions to infer whether particular mechanisms such as ρ exchange or f^0 exchange are important.

The experiments outlined above, when compared with more refined calculations, should throw light on the mechanisms involved in isobar production.

Additional References

24) V.E.Barnes et al: Phys. Rev. Lett. 7, 288 (1961).

W.J.Fickinger et al: Phys. Rev. Lett. 7, 196 (1961).

G.A.Smith et al: Phys. Rev. 123, 2160 (1961).

25) Morris, Fowler and Garrison: Phys. Rev. 103, 1472 (1956)

M.M.Block et al: Phys. Rev.103, 1484, (1956)

W.B.Fowler et al: Phys. Rev.103, 1489 (1956)

NOTE I

In this Note, we briefly discuss the NN_1^*V and NN_1^*A couplings given in (2.9)

When one considers the possible angular momentum states of a $(\bar{N}N^*)$ system, one finds that a vector particle can only be obtained with $(\ell=0^+, S=1^-)$, $(\ell=2^+, S=1^-)$ or $(\ell=2^+, S=1^-)$, where ℓ is the orbital angular momentum and S is the total spin of the $(\bar{N}N^*)$ system. Similarly, an axial vector particle can only be obtained with $(\ell=1^-, S=1^-)$, $(\ell=1^-, S=2^-)$, or $(\ell=3^-, S=2^-)$.

The vector couplings may therefore be of three forms:

$g_{\mu\nu}\gamma_5$, $\gamma_\mu(i\gamma\cdot k)\gamma_5(ik_\nu)$, and $(ik_\mu)(ik_\nu)\gamma_5$.

Noting that $(\gamma\cdot k)$ in the second term can be replaced by a real constant on using the Dirac equation (2.13) one obtains the terms in NN_1^*V.

For the axial vector coupling, the possible terms are

$g_A(i\gamma\cdot k)g_{\mu\nu}$, $i\gamma_\mu k_\nu$ and $-(i\gamma\cdot k)k_\mu k_\nu$

which give the terms in NN_1^*A.

It is for this reason that the second term in the NN^*V (or NN^*A) coupling does not have a factor i relative to the other two terms, although it has an odd power of the momentum k. This peculiarity is due to our choice of γ-matrices, as given by the Dirac equation (2.13).

In our calculations we have kept only the g_V and g_A terms and the above considerations are not put to test. However, when one keeps all the terms in the couplings, the importance of putting the 'i' factors correctly, becomes evident.

NOTE II

In antisymmetrizing the initial two-nucleon state, one must be careful about the normalization. The correct normalization has been discussed by Salzman and Salzman[30]. The net result is that the total matrix element for N_1^* production is the contribution of the poles in the t - channel minus the contribution of the poles in the u - channel, without any additional factor of $1/\sqrt{2}$. For N_2^* production each of the pole contributions must be multiplied by the appropriate element of the crossing matrix, as shown in the next chapter.

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30) F. Salzman and G. Salzman, Phys. Rev. 121 (1961) 1541.

Captions to Figures

Fig.1 Notation

Fig.2 Single-meson exchange diagram in $NN \to N^* N$.

Fig.3 $I = 0$ exchange in $np \to pp\pi^-$ and $np \to np\pi^0$

Fig.4 $I = 0$ exchange in $np \to nn\pi^+$ and $np \to np\pi^0$.

Fig.5 'Elastic' N_2^* production on deuterium.

Fig.6 Proton spectrum in $pp \to pn\pi^+$. This and all the other figures are not meant to give the relative contributions of the N_1^* and N_2^* or of $I = 0$ and $I = 1$ exchange.

Fig.7 Proton spectrum in $pp \to pp\pi^0$.

Fig.8 Neutron spectrum in $pp \to pn\pi^+$

Fig. 9 Decay pion energy spectrum in $NN \longrightarrow NN\pi$. The curves (a) and (b) refer to N_1^* production at 1.92 and 2.85 GeV. respectively, while the curves (c) and (d) refer to N_2^* production at the same energies.

Fig.10 Decay Nucleon energy spectrum at incident energies of 1.41, 1.92 and 2.85 Gev. for $P_{3/2}$ N_1^*.

Fig.11 N_1^* and N_2^* angular distribution in pp reactions (and the reaction $np \rightarrow np\pi^0$) with π exchange. The units are arbitrary.

Fig.12 N_2^* angular distribution in pp reactions with η exchange.

Fig.13 N_1^* and N_2^* angular distributions in pp reactions with S_1 and S_0 exchange. (S_1 and S_0 denote $I = 1$ and $I = 0$ scalar mesons with assumed masses $m_{S_1} \approx m_{S_0} \approx$ 550 Mev)

Fig.14 N_2^* angular distribution in pp reactions with σ exchange.

Fig.15 N_1^* and N_2^* angular distributions in pp reactions with ρ exchange. (m_ρ = 750 Mev)

Fig.16 N_1^* and N_2^* angular distributions in pp reactions with 'A' exchange (m_A =750 Mev)

Fig.17 N_2^* angular distribution in $np \to nn\pi^+$ with π exchange.

Fig.18 N_2^* angular distribution in $np \to nn\pi^+$ with η exchange.

Fig.19 N_2^* angular distribution in $np \to nn\pi^+$ with S_0 and S_1 exchange.

Fig.20 N_2^* angular distribution in $np \to nn\pi^+$ with ρ and ω exchange.

Fig.21 N_2^* angular distributions in $np \to nn\pi^+$ with A_0 and A_1 exchange ($m_{A_0} = m_{A_1} =$ 750 Mev).

CHAPTER VII

ISOSPIN ANALYSIS OF ISOBAR PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

Abstract

In this chapter, we derive the expressions for the differential and total cross-sections for the different charge states in terms of the exchange of states with definite isospin. The analysis is an extension of that given by Lindenbaum and Sternheimer, who give the total cross-sections and energy-spectra for different charge-states in terms of reaction channels with definite isospin.

ISO-SPIN ANALYSIS OF ISOBAR PRODUCTION IN NUCLEON-NUCLEON COLLISIONS+

In the previous chapter, single-isobar production in N-N collisions was considered in a model in which the forces were approximated by the exchange of the different multi-pion resonances. In this chapter, we shall first derive the expressions for the differential and total cross-sections for the different charge-states in terms of the exchange of states with definite isospin and in the second part of the chapter we shall also derive the energy spectra for the different charge states. The analysis is an extension of that given by Lindenbaum and Sternheimer[1], who give the total cross-sections and energy-spectra for different charge-states in terms of reaction channels with definite isospin.

+ Alladi Ramakrishnan, K. Raman and R.K.Umerjee (to be published)

1) S.J.Lindenbaum and R. Sternheimer, Phys. Rev. 105 (1957), 1874; Phys. Rev. 123 (1961), 333.

We are interested in the reactions

$$N + N \longrightarrow N_1^* + N \qquad (1)$$

and

$$N + N \longrightarrow N_2^* + N \qquad (2)$$

and the subsequent decays

$$N_{1,2}^* \longrightarrow \pi + N \qquad (3)$$

states with a definite charge (or third component of isospin, I_π) and a definite isospin are given by the following

$$|N_1^* N\rangle_{1,1} = \frac{\sqrt{3}}{2} |N_1^{*(++)} n\rangle - \frac{1}{2} |N_1^{*(+)} p\rangle$$

$$|N_1^* N\rangle_{1,0} = \frac{1}{\sqrt{2}} |N_1^{*(+)} n\rangle - \frac{1}{\sqrt{2}} |N_1^{*(0)} p\rangle \qquad (4)$$

$$|N_2^* N\rangle_{1,1} = |N_2^{*(+)} p\rangle$$

$$|N_2^* N\rangle_{1,0} = \frac{1}{\sqrt{2}} |N_2^{*(+)} n\rangle + \frac{1}{\sqrt{2}} |N_2^{*(0)} p\rangle \qquad (5)$$

$$|N_2^* N\rangle_{0,0} = \frac{1}{\sqrt{2}} |N_2^{*(+)} n\rangle - \frac{1}{\sqrt{2}} |N_2^{*(0)} p\rangle$$

For the decay of the N_1^* and N_2^*, the branching into the different charge-states is given by the following

$$N_1^{*(++)} = |\pi^+ p\rangle$$

$$N_1^{*(+)} = \sqrt{\tfrac{2}{3}}\,|\pi^0 p\rangle + \sqrt{\tfrac{1}{3}}\,|\pi^+ n\rangle$$

$$N_1^{*(0)} = \sqrt{\tfrac{1}{3}}\,|\pi^- p\rangle + \sqrt{\tfrac{2}{3}}\,|\pi^0 n\rangle \qquad (6)$$

$$N_1^{*(-)} = |\pi^- n\rangle$$

$$N_2^{*(+)} = -\tfrac{1}{\sqrt{3}}\,|\pi^0 p\rangle + \sqrt{\tfrac{2}{3}}\,|\pi^+ n\rangle$$

$$N_2^{*(0)} = -\sqrt{\tfrac{2}{3}}\,|\pi^- p\rangle + \sqrt{\tfrac{1}{3}}\,|\pi^0 n\rangle \qquad (7)$$

Consider pp collisions resulting in the charge-states π^+pp and π^+np.

(i) $pp \to \pi^0 pp$

The amplitude for this process may be written as

$$T(pp \to \pi^0 pp) =$$

$$= \langle \pi^0 pp \,|\, N_1^* N \rangle_{1,1} \langle N_1^* N \,|\, pp \rangle_{A(1,1)} +$$

$$+ \langle \pi^0 pp \,|\, N_2^* N \rangle_{1,1} \langle N_2^* N \,|\, pp \rangle_{A(1,1)} \qquad (8)$$

where the suffixes represent the quantities $(I, I_{\pi\gamma})$. For pp collisions these can have only the values $(1,1)$. The subscript A denotes the antisymmetrization in the initial nucleons.

The first term in (8) may be expressed as

$$\langle \pi^0 pp \,|\, N_1^{*(+)} p \rangle \langle N_1^{*(+)} p \,|\, pp \rangle_{A(1,1)}$$

$$= \langle \pi^0 p \,|\, N_1^{*(+)} \rangle \langle N_1^{*(+)} p \,|\, N_1^* N \rangle_{1,1} \mathcal{M}_A^{(1)}(N_1^*)$$

$$= \sqrt{\tfrac{2}{3}}\, D(N_1^*) \cdot (-\tfrac{1}{2})\, \mathcal{M}_A^{(1)}(N_1^*)$$

$$= -\frac{1}{\sqrt{6}} D(N_1^*)\, \mathcal{M}_A^{(1)}(N_1^*) \qquad (9)$$

In the first step we have used the isobar assumption from which it follows that only the state $|N_1^{*(+)}p\rangle$ and not the state $|N_1^{*(++)}n\rangle$ can contribute to the production of a $|\pi^0 pp\rangle$ final state. In the second step also, we have used the isobar assumption according to which one of the final protons is merely a spectator in the transition

$$\langle \pi^0 pp \,|\, N_1^{*(+)} p\rangle \quad .$$

In evaluating the quantity $\langle \pi^0 pp | N_1^* N\rangle_{1,1}$ we have neglected the symmetrization between the final protons and have put this quantity equal to $\langle \pi^0 p | N_1^{*(+)}\rangle \langle N_1^{*(+)} p | N_1^* N\rangle_{1,1}$

$\mathcal{D}(N_1^*)$ is the dynamical part of the decay matrix element for the decay $N_1^* \longrightarrow N + \pi$ in which the isospin factor is not included; and $\mathcal{M}_A^{(1)}(N_1^*)$ is the dynamical part of the matrix element for the transition $N+N \longrightarrow N_1^* + N$ in the isospin state $I = 1$, antisymmetrixed in the initial nucleons (as denoted by the subscript A)

$$\mathcal{M}_A^{(1)}(N_1^*) = \langle N_1^* N \,|\, NN\rangle_A^{(I=1)} \qquad (10)$$

The total isospin of the reaction channel will be always put as a superscript and that of the exchanged state as a subscript to the amplitude, as in section 2 of the previous chapter.

The second term in (8) may be similarly expressed as

$$\langle \pi^0 p | N_2^{*(+)} \rangle \langle N_2^{*(+)} p | N_2^* N \rangle_{I,I} \mathcal{M}_A^{(I)}(N_2^*)$$

$$= -\frac{1}{\sqrt{3}} D(N_2^*) \mathcal{M}_A^{(I)}(N_2^*) \qquad (11)$$

where

$$\mathcal{M}_A^{(I)}(N_2^*) = \langle N_2^* N \,|\, NN \rangle_A^{(I=1)} \qquad (12)$$

Equation (8) then becomes

$$T(pp \to \pi^0 pp)$$

$$= -\frac{1}{\sqrt{6}} D(N_1^*) \mathcal{M}_A^{(I)}(N_1^*) -$$

$$\frac{-1}{\sqrt{3}} D(N_2^*) \mathcal{M}_A^{(I)}(N_2^*) \qquad (13)$$

N_1^* production can proceed only through the exchange of $I = 1$ states and the antisymmetrized production matrix element may be written as

$$\mathcal{M}_A^{(I)}(N_1^*) = \left[\mathcal{M}_{(I)}(t) - \mathcal{M}_{(I)}(u) \right]_{N_1^*} \qquad (14)$$

where $\mathcal{M}_{(I)}(t)$ is the matrix element for the exchange of an $I=1$ state in the t channel and $\mathcal{M}_{(I)}(u)$ is the matrix element for $I=1$ exchange in the u channel.

For N_2^* production the exchange of both $I=1$ and $I=0$ states contribute and we obtain

$$\mathcal{M}_A^{(I)}(N_2^*) = \left[\mathcal{M}^{(I)}(t) - \mathcal{M}^{(I)}(u)\right]_{N_2^*}$$

$$= \left[\mathcal{M}_{(0)}(t) - \mathcal{M}_{(0)}(u) + \mathcal{M}_{(1)}(t) - \mathcal{M}_{(1)}(u)\right]_{N_2^*} \qquad (15)$$

where in the last step we have used the crossing matrices for exchanges in the t and u channels, as given in section 3 of the previous chapter.

(13) then becomes

$$T(pp \to \pi^0 pp) = -\frac{1}{\sqrt{6}} D(N_1^*)\left[\mathcal{M}_{(1)}(t) - \mathcal{M}_{(1)}(u)\right]_{N_1^*} -$$

$$-\frac{1}{2\sqrt{3}} D(N_2^*)\left[\mathcal{M}_{(0)}(t) - \mathcal{M}_{(0)}(u) + \mathcal{M}_{(1)}(t) - \mathcal{M}_{(1)}(u)\right]_{N_2^*} \qquad (16)$$

with the notation used above.

The differential cross-section for the process $pp \to \pi^0 pp$ may then be expressed as

$$d^2\sigma(pp \to \pi^0 pp)$$

$$= \frac{1}{6}\left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u)\right]_{N_1^*} dT(N_1^*) +$$

$$+ \frac{1}{3}\left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u)\right]_{N_2^*} dT(N_2^*)$$

$$= \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} dT(N_1^*) +$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + \right. \qquad (17)$$

$$\left. + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_2^*} dT(N_2^*)$$

In equation (17) the differential cross-section $d^2\sigma(pp \to \pi^0 pp)$ is with respect to two variables, one of which may be thought of as the N^* production angle θ^*_λ in the reaction $pp \to N^*N$ and the other as the angle of emission α of the decay nucleon in the decay process $N^* \to N + \pi$

In equation (17) , $\left[d\sigma(\,)\right]_{N^*}$ in the right hand side is the differential cross-section for the process $pp \longrightarrow N^*N$ with reference to $\eta^*_\lambda = \cos\theta^*$ and $dT(N^*)$ is the decay distribution of the N^* with reference to the decay angle. α . $d\sigma_{(0)}(t)$ is the contribution

of $I = 0$ exchange in the t channel, and so on.

The total cross-section is obtained by integrating over the angles which gives (17)

$$\sigma_{tot}(pp \to \pi^0 pp)$$

$$= \frac{1}{6}\sigma_{(1)}(N_1^*)\,T(N_1^*) +$$

$$+ \frac{1}{12}\left[\sigma_{(0)}(N_2^*) + \sigma_{(1)}(N_2^*)\right] T(N_2^*) \qquad (18)$$

where we have defined

$$\sigma_{(I)}(N_1^*) \equiv \int d\pi^* \left[d\sigma_{(I)}(t) + d\sigma_{(I)}(u)\right]_{N_1^*} \quad etc. \qquad (18.a)$$

(ii) $pp \to \pi^+ n p$.

The expressions may be derived exactly as for the reaction $pp \to \pi^0 pp$ above. We obtain

$$T(pp \to \pi^+ n p)$$

$$= \langle \pi^+ n p \,|\, N_1^* N \rangle_{I,I}\, \mathcal{M}_A^{(I)}(N_1^*) +$$

$$+ \langle \pi^+ n p \,|\, N_2^* N \rangle_{I,I}\, \mathcal{M}_A^{(I)}(N_2^*)$$

$$= \Big[\langle \pi^+ p \,|\, N_1^{*(++)} \rangle \langle N_1^{*(++)} n \,|\, N_1^* N \rangle_{A,(I,I)} +$$

$$+ \langle \pi^+ n \,|\, N_1^{*(+)} \rangle \langle N_1^{*(+)} p \,|\, N_1^* N \rangle_{A,(I,I)}\Big] +$$

$$+\langle \pi^+ n \,|\, N_2^{*(+)} \rangle \langle N_2^{*(+)} p \,|\, N_2^* N \rangle_{A,(1,1)}$$

$$= \left[D(N_1^*) \tfrac{1}{2}\sqrt{3}\, M_A^{(1)}(N_1^{*(++)} n) + \tfrac{1}{\sqrt{3}} D(N_1^*)(-\tfrac{1}{2}) M_A^{(1)}(N_1^{*(+)} p) \right] +$$

$$+\sqrt{\tfrac{2}{3}}\, D(N_2^*) M_A^{(1)}(N_2^*)$$

$$= D(N_1^*) \left[\frac{\sqrt{3}}{2} M_A^{(1)}(N_1^{*(++)} n) - \frac{1}{2\sqrt{3}} M_A^{(1)}(N_1^{*(+)} p) \right]$$

$$+ D(N_2^*) \left[\sqrt{\tfrac{2}{3}}\, M_A^{(1)}(N_2^*) \right] \qquad (19)$$

where the two contributions to N_1^* production as given in the first bracket, <u>are incoherent</u> (because the particles in the final state of the reaction $N + N \longrightarrow N^* + N$ are differentiy) and therefore do <u>not</u> add to $\left(\frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{3}}\right) M_A^{(1)}(N_1^*)$

We next use (14) to write (19) in terms of $I = 1$ exchanges in the t and u channels. The results for the differential and total cross-sections are the following

$$d^2\sigma(pp \to \pi^+ n p)$$

$$= \frac{5}{6} \left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u) \right]_{N_1^*} dT(N_1^*) +$$

$$+ \frac{2}{3} \left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u) \right]_{N_2^*} dT(N_2^*)$$

$$= \frac{5}{6}\left[d\tilde{\sigma}_{(1)}(t) + d\tilde{\sigma}_{(1)}(u)\right]_{N_1^*} d\Gamma(N_1^*) +$$

$$+ \frac{1}{6}\left[d\tilde{\sigma}_{(0)}(t) + d\tilde{\sigma}_{(0)}(u) + d\tilde{\sigma}_{(1)}(t) + d\tilde{\sigma}_{(1)}(u)\right] d\Gamma(N_2^*) \; ; \qquad (20)$$

$$\tilde{\sigma}_{tot}(pp \to \pi^+ n p)$$

$$= \frac{5}{6}\,\tilde{\sigma}_{(1)}(N_1^*)\;\Gamma(N_1^*) +$$

$$+ \frac{1}{6}\left[\tilde{\sigma}_{(0)}(N_2^*) + \tilde{\sigma}_{(1)}(N_2^*)\right]\;\Gamma(N_2^*) \qquad (21)$$

where a definition similar to (18a) has been used.

We next consider np collisions. An additional complication here is that, in contrast to the situation in pp collisions, the reaction can proceed in both the $I=0$ and $I=1$ channels. In general we may write the amplitude for the process

$$n + p \longrightarrow N^* + N$$

as

$$\mathcal{T}(np \to N^* N)$$

$$= \langle N^* N | (N^* N)_{1,0} \rangle \langle (NN)_{1,0} | np \rangle \mathcal{M}_A^{(1)}(N^*) +$$

$$+\langle N^*N|(N^*N)_{1,0}\rangle\langle(NN)_{1,0}|n\rho\rangle\mathcal{M}_A^{(0)}(N^*)$$

$$=\frac{1}{\sqrt{2}}\langle N^*N|(N^*N)_{1,0}\rangle\mathcal{M}_A^{(1)}(N^*) \quad -$$

$$-\frac{1}{\sqrt{2}}\langle N^*N|(N^*N)_{1,0}\rangle\mathcal{M}_A^{(0)}(N^*) \qquad (22)$$

where $(NN)_{1,0}$ and $(N^*N)_{1,0}$ are states of $(N+N)$ and (N^*+N) respectively with $I=1$ and $I_3=0$.

Now consider the different charge-states.

(i) $n\rho \longrightarrow \pi^0 n\rho$

$$T(n\rho\longrightarrow\pi^0 n\rho) = T^{(1)}+T^{(0)} \qquad (23)$$

where

$$T^{(1)}=\sum_{i=1,2}\frac{1}{\sqrt{2}}\left[\langle\pi^0\rho|N_i^{*(+)}\rangle\langle N_i^{*(+)}n|(N_i^*N)_{1,0}\rangle\mathcal{M}_A^{(1)}(N_i^{*(+)}n)+\right.$$

$$\left.+\langle\pi^0 n|N_i^{*(0)}\rangle\langle N_i^{*(0)}\rho|(N_i^*N)_{1,0}\rangle\mathcal{M}_A^{(1)}(N_i^{*(0)}\rho)\right]$$

$$=\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{3}}\left[M_A^{(1)}(N_1^{*(+)}n)-M_A^{(1)}(N_1^{*(0)}\rho)\right]D(N_1^*)+\right.$$

$$+\frac{1}{\sqrt{6}}\left[-M_A^{(1)}(N_2^{*(+)}n) + M_A^{(1)}(N_2^{*(0)}p)\right]D(N_2^*)\Bigg\} \qquad (24)$$

and

$$T^{(0)} = -\frac{1}{\sqrt{2}}\Big[\langle\pi^0 p|N_2^{*(+)}\rangle\langle N_2^{*(+)}n|(N_2^*N)_{1,0}\rangle M_A^{(0)}(N_2^{*(+)}n) +$$

$$+\langle\pi^0 n|N_2^{*(0)}\rangle\langle N_2^{*(0)}p|(N_2^*N)_{1,0}\rangle M_A^{(0)}(N_2^{*(0)}p)\Big]$$

$$= -\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{6}}\left[-M_A^{(0)}(N_2^{*(+)}n) - M_A^{(0)}(N_2^{*(0)}p)\right]D(N_2^*)\right\}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{6}}\left[M_A^{(0)}(N_2^{*(+)}n) + M_A^{(0)}(N_2^{*(0)}p)\right]D(N_2^*) \qquad (25)$$

The amplitude (23) may now be written as

$$T(np\to\pi^0 np)$$

$$= \frac{1}{\sqrt{6}}\left[M_A^{(1)}(+n) - M_A^{(1)}(op)\right]_{N_1^*}\cdot D(N_1^*) +$$

$$+\frac{1}{2\sqrt{3}}\Big\{\left[-M_A^{(1)}(+n) + M_A^{(0)}(+n)\right]_{N_2^*} +$$

$$+\left[M_A^{(1)}(op) + M_A^{(0)}(op)\right]_{N_2^*}\Big\}\cdot D(N_2^*) \qquad (26)$$

where

$$(+n)\equiv(N^{*(+)}n)\ ;\ \text{and}\ (op)\equiv(N^{*(0)}p)$$

Before obtaining the differential cross-section we must express $\mathcal{M}_A^{(I,O)}(N^*N)$ in terms of exchanges in the t and u channels. Denoting the initial nucleons by 1 and 2 we have

$$\mathcal{M}_A^{(I)}(12\to N_1^*N) \equiv \mathcal{M}^{(I)}(12\to N_1^*N) - \mathcal{M}^{(I)}(21\to N_1^*N)$$

$$\equiv \left[\mathcal{M}^{(I)}(t) - \mathcal{M}^{(I)}(u)\right] = \left[\mathcal{M}_{(I)}(t) - \mathcal{M}_{(I)}(u)\right] \qquad (27)$$

† $$\mathcal{M}_A^{(I)}(N_2^*) \equiv \mathcal{M}^{(I)}(12\to N_2^*N) - \mathcal{M}^{(I)}(21\to N_2^*N)$$

$$= \tfrac{1}{2}\left[\mathcal{M}_{(0)}(t) - \mathcal{M}_{(0)}(u)\right] +$$

$$+ \tfrac{1}{2}\left[\mathcal{M}_{(I)}(t) - \mathcal{M}_{(I)}(u)\right] \qquad (28)$$

† $$\mathcal{M}_A^{(0)}(N_2^*) \equiv \mathcal{M}^{(0)}(12\to N_2^*N) - \mathcal{M}^{(0)}(21\to N_2^*N)$$

$$= \tfrac{1}{2}\left[\mathcal{M}_{(0)}(t) + \mathcal{M}_{(0)}(u)\right] -$$

$$- \tfrac{3}{2}\left[\mathcal{M}_{(I)}(t) + \mathcal{M}_{(I)}(u)\right] \qquad (29)$$

The co-efficients here follow from the crossing matrices (cf. section 3 of the previous chapter).

The amplitude (26) then becomes

† Here $\mathcal{M}_A^{(k)}(N_2^*)$ means $\mathcal{M}_A^{(k)}(1+2\to N_2^*N)$

$$T(np \to \pi^0 np)$$

$$= \frac{1}{\sqrt{6}} \left\{ \left[M_{(1)}(t) - M_{(1)}(u) \right]_{(+n)} - \left[M_{(1)}(t) - M_{(1)}(u) \right]_{(op)} \right\}_{N_1^*} D(N_1^*) +$$

$$+ \frac{1}{2\sqrt{3}} \left\{ \left[M_{(0)}(u) - 2M_{(1)}(t) - M_{(1)}(u) \right]_{(+n)} + \right. \qquad (30)$$

$$\left. + \left[M_{(0)}(t) - M_{(1)}(t) - 2M_{(1)}(u) \right]_{(op)} \right\}_{N_2^*} D(N_2^*)$$

The differential cross-section is now obtained as

$$d^2\sigma(np \to \pi^0 np) =$$

$$= \frac{1}{3} \left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u) \right]_{N_1^*} d\Gamma(N_1^*) +$$

$$+ \frac{1}{12} \left[|M_A^{(0)} - M_A^{(1)}|^2 + |M_A^{(0)} + M_A^{(1)}|^2 \right]_{N_2^*} d\Gamma(N_2^*)$$

$$= \frac{1}{3} \left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u) \right]_{N_1^*} d\Gamma(N_1^*) +$$

$$+ \frac{1}{12} \left[2\, d\sigma_{(0)}(t) + 5\, d\sigma_{(1)}(t) + 5\, d\sigma_{(1)}(u) \right]_{N_2^*} d\Gamma(N_2^*)$$

(31)

while the total cross-section is given by

$$\sigma_{tot.}(np \to \pi^0 np) = \frac{1}{3}\sigma^{(1)}(N_1^*)\,\Gamma(N_1^*) +$$

$$+ \frac{1}{6}\sigma^{(0)}(N_2^*)\,\Gamma(N_2^*)$$

$$= \frac{1}{3}\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*) + \frac{1}{12}\left[\sigma_{(0)} + 5\,\sigma_{(1)}\right]_{N_2^*}\Gamma(N_2^*) \qquad (32)$$

Proceeding in an analogous way, the following results are obtained for the other-charge-states in np collisions.

(ii) $np \to \pi^+ nn$.

$$d^2\sigma(np \to \pi^+ nn) =$$

$$= \frac{1}{6}\left[d\sigma^{(1)}(t) + d\sigma^{(1)}(u)\right]_{N_1^*} d\,\Gamma(N_1^*) +$$

$$+ \frac{1}{12}\left[|\mathcal{M}_A^{(1)} - \mathcal{M}_A^{(0)}|^2\right]_{N_2^*} d\,\Gamma(N_2^*)$$

$$= \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} d\,\Gamma(N_1^*) +$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(u) + 4\,d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right] d\,\Gamma(N_2^*) \qquad (33)$$

$$\sigma_{tot.}(np \to \pi^+ nn) = \frac{1}{6}\sigma^{(1)}(N_1^*)\,\Gamma(N_1^*) +$$

$$+\frac{1}{12}\left[\sigma^{(0)}+\sigma^{(1)}-2\,\mathrm{Re}.\,M_A^{(0)*}M_A^{(1)}\right]_{N_2^*}T(N_2^*)$$

$$=\frac{1}{6}\sigma_{(1)}(N_1^*)\,T(N_1^*)+\frac{1}{12}\left[\frac{1}{2}\sigma_{(0)}+\frac{5}{2}\sigma_{(1)}\right]_{N_2^*}T(N_2^*) \qquad (34)$$

(iii) $np \to \pi^- pp$

$$d^2\sigma(np\to\pi^-pp) = \frac{1}{6}\left[d\sigma^{(1)}(t)+d\sigma^{(1)}(u)\right]_{N_1^*}dT(N_1^*)+$$

$$+\frac{1}{12}\left[|M_A^{(1)}+M_A^{(0)}|^2\right]_{N_2^*}dT(N_2^*)$$

$$=\frac{1}{6}\left[d\sigma_{(1)}(t)+d\sigma_{(1)}(u)\right]_{N_1^*}dT(N_1^*)+$$

$$+\frac{1}{12}\left[d\sigma_{(0)}(t)+d\sigma_{(1)}(t)+4\,d\sigma_{(1)}(u)\right]_{N_2^*}dT(N_2^*) \qquad (35)$$

$\sigma_{tot}(np\to\pi^-pp)$

$$=\frac{1}{6}\sigma^{(1)}(N_1^*)\;T(N_1^*)+$$

$$+\frac{1}{12}\left[\sigma^{(0)}+\sigma^{(1)}+2\,\mathrm{Re}\left(M_A^{(0)*}M_A^{(1)}\right)\right]_{N_2^*}T(N_2^*)$$

$$=\frac{1}{6}\sigma_{(1)}(N_1^*)\,T(N_1^*)+\frac{1}{12}\left[\frac{1}{2}\sigma_{(0)}+\frac{5}{2}\sigma_{(1)}\right]_{N_2^*}T(N_2^*) \qquad (36)$$

In their paper, Lindenbaum and Sternheimer have introduced as unknown parameters the relative magnitude $(k_1)^{1/2}$ and phase φ_0 of the $I = 0$ and $I = 1$ amplitudes for N_2^* production in np collisions. Once we assume the mechanisms, these parameters can however be calculated.

In our notation

$$\frac{\mathcal{M}_A^{(0)}(np \to N_2^* N)}{\mathcal{M}_A^{(1)}(np \to N_2^* N)} = (k_1)^{1/2} e^{i\varphi_0} \qquad (37)$$

From (28) and (29) one may then obtain k_1 and φ_0 in terms of the dynamical matrix elements for $I=0$ and $I=1$ exchange. We consider two special cases below.

Case (a): $I = 0$ exchange dominates (and $I = 1$ exchange is negligible).

Then

$$\mathcal{M}_A^{(0)} = \frac{1}{2}\left[\mathcal{M}_{(0)}(t) + \mathcal{M}_{(0)}(u)\right]$$

$$\mathcal{M}_A^{(1)} = \frac{1}{2}\left[\mathcal{M}_{(0)}(t) - \mathcal{M}_{(0)}(u)\right] \qquad (38.a)$$

and

$$k_1 = \frac{\left|\mathcal{M}_{(0)}(t) + \mathcal{M}_{(0)}(u)\right|^2}{\left|\mathcal{M}_{(0)}(t) - \mathcal{M}_{(0)}(u)\right|^2} . \qquad (38.b)$$

$$\varphi_0 = \arg\left[\frac{M_{(0)}(t) + M_{(0)}(u)}{M_{(0)}(t) - M_{(0)}(u)}\right] \qquad (38.C)$$

If we neglect the interference terms, we obtain $k_1 \approx 1$. Noting that $M_{(0)}(t)$ and $M_{(0)}(u)$ are relatively real when only pole terms are considered, as in our model, we infer that φ_0 must be 0° or 180° according to the absolute signs of $M_{(0)}(t)$ and $M_{(0)}(u)$ and their relative magnitudes.

Case (b) : I = 1 exchange dominates (amd I = 0 exchange is negligible.)

$$M_A^{(0)} = -\frac{3}{2}\left[M_{(1)}(t) + M_{(1)}(u)\right]; \quad M_A^{(1)} = \frac{1}{2}\left[M_{(1)}(t) - M_{(1)}(u)\right] \qquad (39.a)$$

$$k_1 = \frac{9\left|M_{(1)}(t) + M_{(1)}(u)\right|^2}{\left|M_{(1)}(t) - M_{(1)}(u)\right|^2}. \qquad (39.b)$$

$$\varphi_0 = \arg\left\{3\left[\frac{M_{(1)}(t) + M_{(1)}(u)}{M_{(1)}(t) - M_{(1)}(u)}\right]\right\} = 0^\circ \text{ or } 180^\circ. \qquad (39.C)$$

depending on the relative magnitudes of $M_{(1)}(t)$ and $M_{(1)}(u)$ and their absolute signs.

In each case, k_1 and φ_0 may be obtained once the matrix elements are evaluated.

The important features of the differential cross-sections derived here were discussed in section 3 of the previous chapter.

Now we shall derive the energy spectra for the different charge-states in terms of the exchange of states with definite isospin. We shall derive both the differential energy spectra (i.e. the energy spectra at a given angle.) and the overall energy spectra (integrated over all angles).

First consider pp collisions.

(i) $pp \longrightarrow \pi^+ p n$.

The N_1^* contribution to the amplitude for producing a proton of given energy E_p will be given by

$$\left[\langle \pi^+ n | N_1^{*(+)} \rangle \langle N_1^{*(+)} p | (NN)_{1,1} \rangle \langle (NN)_{1,1} | pp \rangle \right]_N +$$

$$+ \left[\langle \pi^+ p | N_1^{*(++)} \rangle \langle N_1^{*(++)} n | (NN)_{1,1} \rangle \langle (NN)_{1,1} | pp \rangle \right]_{N'}$$

$$= -\frac{1}{2\sqrt{3}} \left[D(N_1^*) \mathcal{M}_A^{(1)}(N_1^*) \right]_N + \frac{\sqrt{3}}{2} \left[D(N_1^*) \mathcal{M}_A^{(1)}(N_1^*) \right]_{N'} \qquad (40)$$

where the first bracket with subscript N is the amplitude for the proton coming out as the 'recoil' nucleon and the second bracket with subscript N' is the amplitude for the proton coming out as the 'decay' nucleon.

The N_1^* contribution to the differential proton energy spectrum is then

$$\frac{\partial^2\sigma}{\partial\Omega^*\partial E_p} = \frac{1}{12}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} \cdot \frac{\partial\bar{\Gamma}}{\partial E_N}\,\Gamma(N_1^*) \;+$$

$$+\frac{3}{4}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} \cdot \frac{\partial\bar{\Gamma}}{\partial E_{N'}}\,\Gamma(N_1^*) \qquad (41)$$

where $\frac{\partial\bar{\Gamma}}{\partial E_N}$ and $\frac{\partial\bar{\Gamma}}{\partial E_{N'}}$ are the normalized energy spectra of the 'recoil' nucleon and the 'decay' nucleon respectively. Here the reaction can proceed only in the $I = 1$ channel and only through $I' = 1$ exchange and therefore

$$d\sigma^{(1)}(N_1^*) = d\sigma_{(1)}(N_1^*)$$

The N_1^* contribution to the overall energy spectrum is given by

$$\frac{\partial\sigma}{\partial E_p} = \frac{5}{6}\,\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*)\left[0.1\,\frac{\partial\bar{\Gamma}}{\partial E_N} + 0.9\,\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*} \qquad (42)$$

The N_2^* contribution to the amplitude for producing a proton of given energy E_p will be given by

$$\left[\langle\pi^+ n\,|\,N_2^{*(+)}\rangle\langle N_2^{*(+)} p\,|\,(NN)_{1,1}\rangle\langle(NN)_{1,1}\,|\,pp\rangle\right]_N$$

$$= \sqrt{\frac{2}{3}}\left[D(N_2^*)\,M_A^{(1)}(N_2^*)\right]_N \qquad (43)$$

where we note that the proton can be produced only as the recoil nucleon.

The N_2^* contribution to the differential proton energy spectrum is, using (28)

$$\frac{\partial^2\sigma}{\partial\pi^*\partial E_p}(N_2^*) =$$

$$= \frac{1}{6}\left[d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_2^*} \cdot \frac{\partial\bar{I}}{\partial E_N}(N_2^*) \cdot I(N_2^*) \qquad (44)$$

while the N_2^* contribution to the overall energy spectrum is

$$\frac{\partial\sigma}{\partial E_p}(N_2^*) = \frac{2}{3}\sigma^{(1)}(N_2^*)\, I(N_2^*) \cdot \frac{\partial\bar{I}}{\partial E_N}(N_2^*)$$

$$= \frac{1}{6}\left[\sigma_{(1)} + \sigma_{(0)}\right]_{N_2^*} \cdot I(N_2^*) \cdot \frac{\partial\bar{I}}{\partial E_N}(N_2^*) \qquad (45)$$

The differential and overall proton energy spectra in the reaction $pp \to \pi^+ n p$ are finally obtained as

$$\frac{\partial^2\sigma}{\partial\pi^*\partial E_p}(pp \to \pi^+ n p)$$

$$= \frac{5}{6}\left\{d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right\}_{N_1^*} \cdot \left[0.1\frac{\partial\bar{I}}{\partial E_N} + 0.9\frac{\partial\bar{I}}{\partial E_{N'}}\right]_{N_1^*} \cdot I(N_1^*) +$$

$$+ \frac{1}{6}\left\{d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right\}_{N_2^*} \cdot \left[\frac{\partial\bar{I}}{\partial E_N}\right]_{N_2^*} \cdot I(N_2^*)$$

(46.a)

$$\frac{\partial \sigma}{\partial E_p}(pp \rightarrow \pi^+ n p)$$

$$= \frac{5}{6}\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*)\left[0.1\frac{\partial \bar{\Gamma}}{\partial E_N} + 0.9\frac{\partial \bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*} +$$

$$+ \frac{1}{6}\left\{\sigma_{(0)} + \sigma_{(1)}\right\}_{N_2^*} \cdot \Gamma(N_2^*)\cdot\left[\frac{\partial \bar{\Gamma}}{\partial E_N}\right]_{N_2^*} \qquad (46.6)$$

The neutron spectrum may be similarly obtained as

$$\frac{\partial^2 \sigma}{\partial \pi_\beta^* \partial E_n}(pp \rightarrow \pi^+ n p) =$$

$$= \frac{5}{6}\left\{d\sigma_{(0)}(t) + d\sigma_{(1)}(u)\right\}_{N_1^*}\left[0.9\frac{\partial \bar{\Gamma}}{\partial E_N} + 0.1\frac{\partial \bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*}\Gamma(N_1^*)$$

$$+ \frac{1}{6}\left\{d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right\}_{N_2^*}\left[\frac{\partial \bar{\Gamma}}{\partial E_N}\right]_{N_2^*}\cdot\Gamma(N_2^*)$$

(47.a)

$$\frac{\partial \sigma}{\partial E_n}(pp \rightarrow \pi^+ n p) =$$

$$= \frac{5}{6}\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*)\left[0.1\frac{\partial \bar{\Gamma}}{\partial E_N} + 0.9\frac{\partial \bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*} +$$

$$+\frac{1}{6}\left\{\tilde{\sigma}_{(0)}+\tilde{\sigma}_{(1)}\right\}_{N_2^*}\cdot \bar{\Gamma}(N_2^*)\left[\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right]_{N_2^*} \qquad (47.6)$$

The pion energy spectrum is given by

$$\frac{\partial^2\sigma}{\partial\gamma^*\partial E_\pi}(pp\to\pi^+np) =$$

$$=\frac{5}{6}\left\{d\tilde{\sigma}_{(1)}(t)+d\tilde{\sigma}_{(1)}(u)\right\}_{N_1^*}\cdot\left[\frac{\partial\bar{\Gamma}}{\partial E_\pi}\right]_{N_1^*}\cdot\bar{\Gamma}(N_1^*)+$$

$$+\frac{1}{6}\left\{d\tilde{\sigma}_{(0)}(t)+d\tilde{\sigma}_{(0)}(u)+d\tilde{\sigma}_{(1)}(t)+d\tilde{\sigma}_{(1)}(u)\right\}_{N_2^*}\left[\frac{\partial\bar{\Gamma}}{\partial E_\pi}\right]_{N_2^*}\cdot\bar{\Gamma}(N_2^*) \qquad (48.a)$$

$$\frac{\partial\sigma}{\partial E_\pi}(pp\to\pi^+np) =$$

$$=\frac{5}{6}\tilde{\sigma}_{(1)}(N_1^*)\,\bar{\Gamma}(N_1^*)\,\frac{\partial\bar{\Gamma}}{\partial E_\pi}(N_1^*)+$$

$$+\frac{1}{6}\left\{\tilde{\sigma}_{(0)}+\tilde{\sigma}_{(1)}\right\}_{N_2^*}\cdot\bar{\Gamma}(N_2^*)\cdot\frac{\partial\bar{\Gamma}}{\partial E_\pi}(N_2^*)\,. \qquad (48.6)$$

(ii) $pp \to \pi^0 pp$

The results for this reaction are the following

$$\frac{\partial^2\sigma}{\partial\Omega_\pi^* \partial E_p}(pp \to \pi^0 pp) =$$

$$= \frac{1}{3}\left\{d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right\}_{N_1^*} I(N_1^*)\left[0.5\frac{\partial\bar{I}}{\partial E_N} + 0.5\frac{\partial\bar{I}}{\partial E_{N'}}\right]_{N_1^*} +$$

$$+ \frac{1}{6}\left\{d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right\} I(N_2^*) \times$$

$$\times\left[0.5\frac{\partial\bar{I}}{\partial E_N} + 0.5\frac{\partial\bar{I}}{\partial E_{N'}}\right]_{N_2^*} \qquad (49.a)$$

$$\frac{\partial\sigma}{\partial E_p}(pp \to \pi^0 pp) =$$

$$= \frac{1}{3}\sigma_{(1)}(N_1^*)\, I(N_1^*)\left[0.5\frac{\partial\bar{I}}{\partial E_N} + 0.5\frac{\partial\bar{I}}{\partial E_{N'}}\right]_{N_1^*} \qquad (49.b)$$

$$+ \frac{1}{6}\left\{\sigma_{(0)} + \sigma_{(1)}\right\}_{N_2^*} I(N_2^*)\left[0.5\frac{\partial\bar{I}}{\partial E_N} + 0.5\frac{\partial\bar{I}}{\partial E_{N'}}\right]_{N_2^*}$$

$$\frac{\partial^2\sigma}{\partial\Omega_\pi^* \partial E_\pi}(pp \to \pi^0 pp) =$$

$$= \frac{1}{6}\left\{d\sigma_{(1)}(\epsilon) + d\sigma_{(1)}(u)\right\}_{N_1^*} I(N_1^*)\left[\frac{\partial\bar{I}}{\partial E_\pi}\right]_{N_1^*} +$$

$$+\frac{1}{12}\left\{d\sigma_{(0)}(t)+d\sigma_{(0)}(u)+d\sigma_{(1)}(t)+d\sigma_{(1)}(u)\right\}_{N_2^*}\times$$

$$\times\,\Gamma(N_2^*)\left[\frac{\partial\bar{\Gamma}}{\partial E_\pi}\right]_{N_2^*} \qquad (50.a)$$

$$\frac{\partial\sigma}{\partial E_\pi}(pp\to\pi^0pp) =$$

$$=\frac{1}{6}\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*)\frac{\partial\bar{\Gamma}}{\partial E_\pi}(N_1^*)+$$

$$+\frac{1}{12}\left[\sigma_{(0)}+\sigma_{(1)}\right]_{N_2^*}\cdot\Gamma(N_2^*)\,\frac{\partial\bar{\Gamma}}{\partial E_\pi}(N_2^*) \qquad (50.b)$$

We next consider np collisions. As pointed out in the earlier part of this chapter, we now have an extra complication because the reaction can proceed through both the $I=0$ and $I=1$ channels.

(i) $np\to\pi^0np$

The N_1^* contribution is non-zero only for the $I=1$ channel and is obtained as for pp collisions; it will be given only in the final expressions.

The N_2^* contribution to the $I = 0$ amplitude for producing a proton of energy E_p is

$$\left[\langle \pi^0 n | N_2^{*(0)} \rangle \langle N_2^{*(0)} p | (NN)_{0,0} \rangle \langle (NN)_{0,0} | np \rangle\right]_N +$$

$$+ \left[\langle \pi^0 p | N_2^{*(+)} \rangle \langle N_2^{*(+)} n | (NN)_{0,0} \rangle \langle (NN_{0,0} | np \rangle\right]_{N'}$$

$$= \frac{1}{2\sqrt{3}} \left[\mathcal{M}_A^{(0)}(0p)\, D(N_2^*)\right]_N + \frac{1}{2\sqrt{3}} \left[\mathcal{M}_A^{(0)}(+n)\, D(N_2^*)\right]_{N'} \qquad (51)$$

where

$$(0p) \equiv (N_2^{*(0)} p); \text{ and } (+n) \equiv (N_2^{*(+)} n)$$

The two terms in (51) are incoherent.

Similarly, the N_2^* contribution in the $I = 1$ channel is given by

$$\left[\frac{1}{2\sqrt{3}} \mathcal{M}_A^{(1)}(0p)\, D(N_2^*)\right]_N - \left[\frac{1}{2\sqrt{3}} \mathcal{M}_A^{(1)}(+n)\, D(N_2^*)\right]_{N'} \qquad (52)$$

We note that in (51) , the two terms are mutually incoherent, and so also in (52) ; however, the first term in (51) and the first term in (52) are mutually coherent and so also the second terms.

The total amplitude for producing a proton of given energy is then just

$$\frac{1}{2\sqrt{3}} \left\{\left[\mathcal{M}_A^{(0)}(0p) + \mathcal{M}_A^{(1)}(0p)\right] D(N_2^*)\right\}_N +$$

$$+\frac{1}{2\sqrt{3}}\left\{\left[\mathcal{M}_A^{(0)}(+n)-\mathcal{M}_A^{(1)}(+n)\right]D(N_2^*)\right\}_{N'} \qquad (53)$$

The N_2^* contribution to the differential proton energy spectrum is then given by

$$\frac{\partial^2\sigma}{\partial\Omega^*\partial E_p}(N_2^*)=\frac{1}{12}\left[\left|\mathcal{M}_A^{(0)}+\mathcal{M}_A^{(1)}\right|^2\right]_{N_2^*}\Gamma(N_2^*)\cdot\frac{\partial\bar{\Gamma}}{\partial E_N}+$$

$$+\frac{1}{12}\left[\left|\mathcal{M}_A^{(0)}-\mathcal{M}_A^{(1)}\right|^2\right]_{N_2^*}\Gamma(N_2^*)\cdot\frac{\partial\bar{\Gamma}}{\partial E_{N'}} \qquad (54)$$

Using (28) and (29) and neglecting the interference terms between the exchanges in the t and u channels, we obtain the differential energy spectrum as

$$\frac{\partial^2\sigma}{\partial\Omega^*\partial E_p}(np\to\pi^0 np)=$$

$$=\frac{1}{3}\left[d\sigma_{(1)}(t)+d\sigma_{(1)}(u)\right]_{N_1^*}\Gamma(N_1^*)\left[0.5\frac{\partial\bar{\Gamma}}{\partial E_N}+0.5\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*}+$$

$$+\frac{1}{12}\left\{\left[d\sigma_{(0)}(t)+d\sigma_{(1)}(t)+4\,d\sigma_{(1)}(u)\right]\frac{\partial\bar{\Gamma}}{\partial E_N}+\right.$$

$$\left.+\left[d\sigma_{(0)}(u)+4\,d\sigma_{(1)}(t)+d\sigma_{(1)}(u)\right]\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right\}_{N_2^*}\Gamma(N_2^*) \qquad (55.a)$$

and the overall proton energy spectrum as

$$\frac{\partial\sigma}{\partial E_p}(np\to\pi^0 np)=$$

$$=\frac{1}{3}\sigma_{(1)}(N_1^*)\,\Gamma(N_1^*)\left[0.5\frac{\partial\bar{\Gamma}}{\partial E_N}+0.5\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right]_{N_1^*}+$$

$$+\frac{1}{12}\left[\sigma_{(0)}+5\sigma_{(1)}\right]_{N_2^*}\cdot\Gamma(N_2^*)\left[0.5\frac{\partial\bar{\Gamma}}{\partial E_N}+0.5\frac{\partial\bar{\Gamma}}{\partial E_{N'}}\right]_{N_2^*} \qquad (55.b)$$

The neutron energy spectrum is similarly obtained by interchanging the coefficients of $\frac{\partial \bar{I}}{\partial E_N}$ and $\frac{\partial \bar{I}}{\partial E_{N'}}$ in (55·a) and (55·6) ; the latter equation is left unaltered, so that the overall proton and neutron energy spectra are the same.

The pion energy spectrum is given by

$$\frac{\partial^2 \sigma}{\partial \Omega^* \partial E_\pi}(np \to \pi^0 np) =$$

$$= \frac{1}{3}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} \cdot I(N_1^*) \frac{\partial \bar{I}}{\partial E_\pi}(N_1^*) +$$

$$+ \frac{1}{12}\left[d\sigma_{(0)}(t) + d\sigma_{(0)}(u) + 5\,d\sigma_{(1)}(t) + 5\,d\sigma_{(1)}(u)\right]_{N_2^*} \times$$

$$\times I(N_2^*) \frac{\partial \bar{I}}{\partial E_\pi}(N_2^*) \qquad (56\cdot a)$$

$$\frac{\partial \sigma}{\partial E_\pi}(np \to \pi^0 np) =$$

$$= \frac{1}{3}\sigma_{(1)}(N_1^*)\, I(N_1^*) \frac{\partial \bar{I}}{\partial E_\pi}(N_1^*) +$$

$$+ \frac{1}{12}\left[\sigma_{(0)} + 5\sigma_{(1)}\right]_{N_2^*} \cdot I(N_2^*) \cdot \frac{\partial \bar{I}}{\partial E_\pi}(N_2^*) \qquad (56\cdot 6)$$

(ii) $np \longrightarrow \pi^+ nn$

The results are the following

$$\frac{\partial^2 \sigma}{\partial \pi^* \partial E_n}(np \to \pi^+ nn) =$$

$$= \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*} \cdot I(N_1^*) \times$$

$$\times \left[0.5 \frac{\partial \bar{I}}{\partial E_N} + 0.5 \frac{\partial \bar{I}}{\partial E_{N'}}\right]_{N_1^*} +$$

$$+ \frac{1}{6}\left[d\sigma_{(0)}(t) + 4 d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_2^*} \cdot I(N_2^*) \times$$

$$\times \left[0.5 \frac{\partial \bar{I}}{\partial E_N} + 0.5 \frac{\partial \bar{I}}{\partial E_{N'}}\right]_{N_2^*} \qquad (57.a)$$

$$\frac{\partial \sigma}{\partial E_n}(np \to \pi^+ nn) =$$

$$= \frac{1}{6}\left[\sigma_{(1)}\right]_{N_1^*} I(N_1^*)\left[0.5 \frac{\partial \bar{I}}{\partial E_N} + 0.5 \frac{\partial \bar{I}}{\partial E_{N'}}\right]_{N_1^*} +$$

$$+ \frac{1}{12}\left[\sigma_{(0)} + 5\sigma_{(1)}\right]_{N_2^*} \cdot I(N_2^*) \times$$

$$\times \left[0.5 \frac{\partial \bar{I}}{\partial E_N} + 0.5 \frac{\partial \bar{I}}{\partial E_{N'}}\right]_{N_2^*} \qquad (57.b)$$

$$\frac{\partial^2\sigma}{\partial\Omega^*\partial E_\pi}(np\to\pi^+nn) =$$

$$= \frac{1}{6}\left[d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_1^*}\cdot I(N_1^*)\cdot\frac{\partial\bar{I}}{\partial E_\pi}(N_1^*) +$$

$$+\frac{1}{6}\left[d\sigma_{(0)}(t) + 4d\sigma_{(1)}(t) + d\sigma_{(1)}(u)\right]_{N_2^*}\cdot I(N_2^*)\cdot\frac{\partial\bar{I}}{\partial E_\pi}(N_2^*) \qquad (58.a)$$

$$\frac{\partial\sigma}{\partial E_\pi}(np\to\pi^+nn) =$$

$$= \frac{1}{6}\sigma_{(1)}(N_1^*)\, I(N_1^*)\,\frac{\partial\bar{I}}{\partial E_\pi}(N_1^*) +$$

$$+\frac{1}{12}\left[\sigma_{(0)}+5\sigma_{(1)}\right]_{N_2^*}\cdot I(N_2^*)\,\frac{\partial\bar{I}}{\partial E_\pi}(N_2^*) \qquad (58.b)$$

(iii) $np\to\pi^-pp$

The differential proton and pion energy spectra are obtained by interchanging the coefficients of $d\sigma_{(1)}(t)$ and $d\sigma_{(1)}(u)$ in the N_2^* contribution in (57.a) and (58.a) while the overall spectra are the same as (57.b) and (58.b).

CHAPTER VIII

POLARIZATION EFFECTS IN THE GENERALIZED SINGLE PARTICLE EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS.

Abstract

Some of the polarization effects are studied for the process of single-isobar production in nucleon-nucleon collisions in a model in which the production amplitude is approximated by the pole contributions. Only those effects are studied which do not involve the isobar polarization and which involve the polarization of only two of the nucleons -- either the two initial nucleons or an initial nucleon and the recoil nucleon; this is done using covariant projection operators. The most general dependence of the differential cross-section on the polarizations is derived, and the six polarization correlation functions are defined and evaluated as functions of the scattering angle; numerical results are obtained for π and ρ exchange. The type of information that can be obtained from the polarization correlations is discussed.

POLARIZATION EFFECTS IN THE GENERALIZED SINGLE PARTICLE EXCHANGE MODEL FOR SINGLE PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS+

1. Introduction

In the two previous chapters we have made a detailed analysis of the total cross-sections, c.m. angular distributions and c.m. energy spectra for the processes of isobar production in nucleon-nucleon collisions[1)]. In the present chapter which is a continuation of work presented in Chapter VI, we analyse some of the polarization effects in these processes of isobar production.

The main object of studying the polarization effects is to gain some insight into the mechanisms involved in isobar production. Apart from this, such a study is also of intrinsic interest as an example of polarization effects in a single-particle exchange model.

The polarization effects in the process

$$N + N \longrightarrow N + N^{*} \qquad (1.1)$$

may be grouped conveniently into two broad classes:

(i) Effects which do not involve the polarization of the final isobar;

(ii) Effects related to the polarization of the isobar, the

+ Alladi Ramakrishnan, K.Raman and R.K.Umerjee
Nuclear Physics (to be published)

1) A.Ramakrishnan, K.Raman and R.K.Umerjee, Isobar production in Nucleon-Nucleon Collisions (Nuclear Physics - in press)

latter being inferred from measurements on the decay distribution of the isobar and from measurements on the polarizations and angular correlations in the 3-particle final state.

In this chapter we study the first class of polarization effects. We hope to examine (ii) in a subsequent publication.

We shall describe the polarization of the nucleons in terms of the covariant projection operators introduced by Michel and Wightman[2]. The advantage is that all calculations may be performed simply in terms of traces of products of γ -matrices without going explicitly into the 2-component form. The polarization formalism is briefly described in section 2, where we also derive the most general dependence of the differential cross-section on the polarization vectors when only two of the nucleons are polarized, i.e. either (i) both the initial nucleons are polarized and one does not observe the polarization of the final nucleons, or (ii) one of the initial nucleons is polarized and the other unpolarized, and one observes the polarization of the 'recoil' nucleon alone. (The 'recoil' nucleon may be distinguished by a measurement of the energy spectra; as is evident from chapter VI).

In section 3 we examine the dependence of the differential cross-section on the relative orientation of the polarization vectors when both the incident and target nucleon are polarized (completely) and the final polarizations are not observed.

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2) L. Michel and A.S.Wightman, Phys. Rev., 98, (1955), 1190; See also, A. Bouchiat and L.Michel, Nucl. Phys., 5, (1958), 416.

Expressions for the six correlation functions are derived. The numerical results are given directly for the angular distribution for seven different choices of the relative orientation of the polarization vectors, for N_1^* and N_2^* production through ρ exchange. For pion exchange, there is no dependence on the initial polarizations when the final polarizations are not observed.

In sections 4 and 5 are given the polarization correlation functions for N_1^* production through π and ρ exchanges when only one of the initial nucleons is polarized and one observes the polarization of the recoil nucleon. All the correlation functions are evaluated as functions of the scattering angle for an incident nucleon (laboratory) kinetic energy of 2.85 Gev. The results given in this chapter all refer to this energy; they are evaluated both for constant form factors and for energy-independent form factors of the shapes assumed in Chapter VI.

In section 6 the conclusions of the model are briefly discussed.

2. Polarization formalism

We shall describe the polarization effects in terms of the covariant projection operators, introduced by Michel and Wightman[2].

Briefly, the pure state of a nucleon will be described by a polarization vector n_μ which is a Lorentz pseudovector with the properties,

$$n_\mu n_\mu = -1, \quad \text{and} \quad (p.n) = 0 \tag{2.1}$$

where p is the 4-momentum of the nucleon. In the rest frame of the nucleon, the fourth component of n_μ is zero:

$$n_0 = 0 \tag{2.2}$$

and in this frame $\vec{n}$ is the direction of the spin of the nucleon.

In order to obtain n_μ in any frame C in terms of its components in the rest frame, i.e.,

$$n_\mu = (0, \vec{n}) \tag{2.3}$$

We make the Lorentz transform with velocity $\vec{v}$ that takes the particle from its rest frame to the frame C. If the particle has 4-momentum

$$p = (E, \vec{p}) \tag{2.4}$$

in the frame C, then any 4-vector $A = (A_0, \vec{A})$ in the rest frame goes over into the 4-vector $A^C = (A^C_0, \vec{A}^C)$ in

the frame C according to

$$\vec{A}^c = \vec{A} + \frac{\vec{p}}{m}\left[\frac{(\vec{p}\cdot\vec{A})}{E+m} - A_o\right] \quad ;$$

$$A_o^c = \frac{E}{m}A_o + \frac{(\vec{p}\cdot\vec{A})}{m} \quad , \tag{2.5}$$

where m is the mass of the particle, and where the velocity $\vec{v}$ of the Lorentz transform has been taken to be

$$\vec{v} = -\frac{\vec{p}}{E} \tag{2.6}$$

The transform of the 4-vector n_μ in the frame C is then given by

$$\vec{n}^c = \vec{n} + \frac{\vec{p}}{m(E+m)}\cdot(\vec{p}\cdot\vec{n}) \quad ;$$

$$n_o^c = \frac{(\vec{p}\cdot\vec{n})}{m} \, . \tag{2.7}$$

The projection operator for the nucleon with polarization n_μ is just

$$\frac{1}{2}\left[1 + i\gamma_5(\gamma_\mu n_\mu)\right] \tag{2.8}$$

In this chapter we consider only the situation where two of the nucleons are polarized, i.e., either

(i) the two initial nucleons are polarized and the polarizations of the isobar and recoil nucleon are <u>not</u> observed,

or (ii) one of the initial nucleons is polarized and the other unpolarized, and the polarization of the recoil nucleon <u>is</u> observed (while the polarization of the isobar is <u>not</u> observed).

By the recoil nucleon is meant the final nucleon that is produced directly and not through the decay of the isobar. Although the final nucleons are identical, it is still possible to call one of them the recoil nucleon and the other the decay nucleon by observing their energy spectra (at each angle). As shown in chapter (VI), the sharply peaked part of the energy spectrum is associated with the recoil nucleon and the rounded part of it with the decay nucleon.

When two of the nucleons are polarized, say with polarization 4-vectors n_1 and n_2 (and momenta q_1 and q_2), then the most general dependence of the differential cross-section on n_1 and n_2 may be obtained by imposing the following restrictions:

1) The differential cross-section $\partial\sigma/\partial\Omega$ must be linear in n_1 and linear in n_2.

2) The differential cross-section must be a real quantity.

3) The orthogonality conditions $(q_1 \cdot n_1) = 0$ and $(q_2 \cdot n_2) = 0$ are satisfied.

4) The differential cross-section must be a scalar, because parity is conserved. (Note that n_1 and n_2 are Lorentz pseudo-vectors.)

We first note that the last condition is a necessary consequence of parity conservation but is not sufficient to ensure parity conservation; even when parity is not conserved, the differential cross-section will be a scalar when the interference terms between the scalar and pseudoscalar parts of the matrix element vanish.

The requirements[1)and 2)] imply that every term of the differential cross-section must be linear in n_1 and linear in n_2. This is because the 4-vector n_μ appears as in_μ in the projection operator, and therefore any term depending on n_1 alone or n_2 alone must be pure imaginary.

The most general dependence of the differential cross-section on n_1 and n_2 is then a linear combination of the following six terms:

$$1 \; ; \; (n_1 \cdot n_2) \; ; \; (n_1 \cdot p_1)(n_2 \cdot p_2) \; ;$$

$$(n_1 \cdot p_1)(n_2 \cdot p_1) \; ; \; (n_1 \cdot p_2)(n_2 \cdot p_1) \; ; \tag{2.9}$$

and $\epsilon_{\alpha\beta\delta\nu}(n_1)_\alpha (p_1)_\beta (p_2)_\delta \epsilon_{\tau\sigma\lambda\nu}(n_2)_\tau (q_1)_\sigma (p_2)_\lambda$. (2.9)

where p_1 and p_2 are the momenta of the other two nucleons. (Fig. 1)

The co-efficients of these will be functions of the energy and scattering angle. It is easily verified that all other scalar combinations are linearly dependent on the six scalars given in (2.9) .

In the c.m. frame, the six scalars may be chosen as follows:

$$1 \; ; \; (\vec{n}_1 \cdot \vec{n}_2) \; ; \; (\vec{n}_1 \cdot \hat{q}_1)(\vec{n}_2 \cdot \hat{q}_2) \; ;$$

$$(\vec{n}_1 \cdot \hat{q}_2)(\vec{n}_2 \cdot \hat{q}_1) \; ; \; (\vec{n}_1 \cdot \hat{q}_1)(\vec{n}_2 \cdot \hat{q}_1) \; ;$$

$$(\vec{n}_1 \cdot \hat{q}_1 \times \hat{q}_2)(\vec{n}_2 \cdot \hat{q}_1 \times \hat{q}_2) \qquad (2.10)$$

Here $\hat{q}_1, \hat{q}_2$ are unit vectors, and n_1, n_2 are the polarization vectors in the rest frames of the corresponding nucleons.

The polarization correlation functions are most conveniently defined as coeffients of the six linearly independent functions of the angles specifying $\vec{n}_1$ and $\vec{n}_2$ that enter into the differential cross-section.[5]

Choosing the z axis along $\vec{q}_1$, and the xz plane as the reaction plane, and denoting the directions of $\vec{n}_1$, $\vec{n}_2$ and $\vec{q}_2$ by

$$(\Theta_1, \Phi_1), (\Theta_2, \Phi_2) \quad \text{and} \quad (0,0) \tag{2.11}$$

respectively, we infer from (2.10) that the differential cross-section can depend on the directions of $\vec{n}_1$ and $\vec{n}_2$ only through the following six functions:

$$1 ; \cos\Theta_1 \cos\Theta_2 ; \sin\Theta_1 \sin\Theta_2 \cos(\Phi_1 \pm \Phi_2) ;$$

$$\cos\Theta_1 \cdot \sin\Theta_2 \cos\Phi_2 ; \sin\Theta_1 \cos\Phi_1 \cos\Theta_2 \tag{2.12}$$

The restriction to these functions of the angles (and their linear combinations) is a consequence of the requirements 1) to 4) stated earlier.

A more convenient method of specifying the correlation functions which makes it easier to evaluate them, is obtained by choosing one of the vectors $\vec{n}_1$, $\vec{n}_2$, say $\vec{n}_1$, along each of the three coordinate axes in turn, and treating the direction of $\vec{n}_2$ as the variable direction (Θ, Φ). For each of the three orientations of $\vec{n}_1$, the differential cross-section can then depend on (Θ, Φ) at most through the functions

$$1 ; \cos\Theta ; \sin\Theta \cos\Phi ; \text{and } \sin\Theta \sin\Phi . \tag{2.13}$$

For each particular orientation of $\vec{n}_1$ (parallel to one of the coordinate axes), only two or three of the functions (2.13) are present as indicated below:

$$\vec{n}_1 = \vec{e}_z : \quad 1 \; ; \; \cos\Theta \; ; \; \sin\Theta\cos\Phi \; ;$$
$$\vec{n}_1 = \vec{e}_x : \quad 1 \; ; \; \cos\Theta \; ; \; \sin\Theta\cos\Phi \; ;$$
$$\vec{n}_1 = \vec{e}_y : \quad 1 \; ; \qquad\qquad \sin\Theta\sin\Phi \; . \qquad (2.14)$$

The coefficient of the term independent of (Θ, Φ) is just the unpolarized differential cross-section (except for a possible factor of 1/2) , and is always the same. Therefore, there are just six polarization correlation functions.

We stress again that $\vec{n}_1$ and $\vec{n}_2$ refer to the polarization vectors of two of the nucleons, each in its own rest frame. Given a polarized nucleon beam, a measurement of its polarization enables one to specify the direction (Θ, Φ) of its polarization vector in its rest frame, with respect to a coordinate system fixed to the rest frame.

For an emergent nucleon beam, one must measure the polarization of the beam for every angle θ of emergence. From this measurement, the angle (Θ, Φ) specifying the polarization direction of the beam is specified by making a Lorentz transformation along $\vec{p}$ that brings the nucleon

to rest, and measuring (θ, Φ) with respect to a coordinate system in which the y direction is perpendicular to the plane of reaction and the direction of $\vec{p}$ is $(0, 0)$.

In practice, of course, the nucleons are only partially polarized. The only difference then is that

$$0 > n_\mu n_\mu > -1 \qquad (2.15)$$

instead of (2.1a).

When the use of polarized nucleon targets becomes more common[3], it would be convenient to take the target nucleon as the one whose polarization $(\vec{n}_1)$ is taken parallel to the x, y, and z axes in turn, as the target nucleon is at rest in the laboratory frame.

An important point to remember in connection with the polarization is that when one of the initial nucleons is polarized and the other unpolarized, then in general, the recoil nucleon will be only partially polarized and its polarization vector will have a magnitude less than unity. In special cases, however as in the case of single-pion-exchange as discussed in section 4, it is still possible to have a situation where the recoil nucleon is completely polarized.

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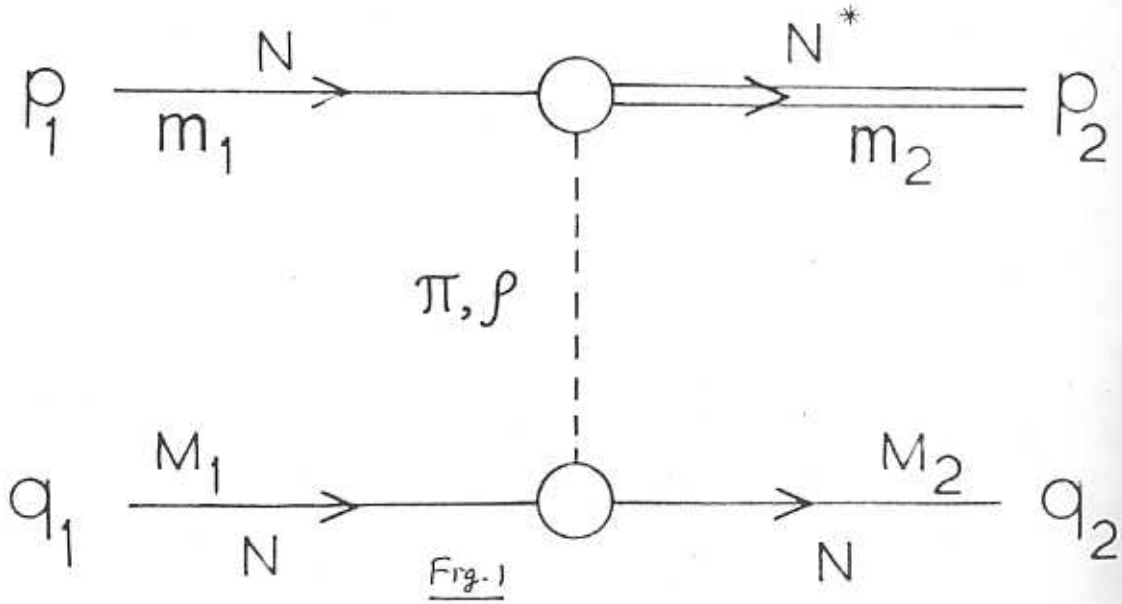
3) For a discussion of polarized nucleon targets, see E. Segre, A talk on 'some recent experimental data', at the Institute of Mathematical Sciences, Madras, Nov. 17, 1962.



Frg. 1

3. Incident nucleon and target nucleon both polarized

In this section we shall study what may be termed the 'initial-state polarization correlations'. Here, one performs an experiment in which a polarized nucleon beam is incident on a polarized nucleon target and examines how the different measurable quantities depend on the relative orientation of the polarization vectors in the initial state.

The quantities that can be measured are the angular distributions and the polarizations of the outgoing nucleons. In this chapter we restrict ourselves to the effects obtained by having two of the nucleons polarized, as discussed in the last section. Therefore in this section we shall examine only the angular distributions of the (final) 'recoil' nucleon.

Throughout this section we shall denote the polarization vector of the target nucleon (with 4-momentum q_1) by n_A and that of the incident nucleon (with 4-momentum p_1) by n_B.

First consider 1-pion exchange. The matrix element for production of a $P_{3/2}$ N_1^* is given by

$$\left[f_{\pi N N_1^*} \cdot \bar{\psi}_\alpha (p_2)(p_2 - p_1)_\alpha \, u(p_1) \right] \times$$

$$\times \left[G_{\pi NN} \, \bar{u}(q_2) \gamma_5 \, u(q_1) \right] \cdot \frac{i F_\pi (t)}{(k^2 - \mu^2)} \qquad (3.1)$$

where $F_\pi(t)$ is an unknown form factor which was discussed in chapter (VI). This form factor probably represents the inadequacy of the OPE model.

The differential cross-section averaged over the final polarizations is obtained as

$$\frac{\partial\sigma_\pi}{\partial(\cos\theta^*)} = \frac{\pi}{12}\cdot\frac{1}{W^2}\cdot\frac{p_f}{p_i}\cdot f^2_{\pi NN^*_1}\cdot G^2_{\pi NN}\times\frac{1}{(4\pi)^2}\times$$

$$\times\frac{|F_\pi(t)|^2}{(t-\mu^2)^2}\times\frac{[(m_1+m_2)^2-t]^2}{m_2^2}\times[(m_1-m_2)^2-t][(m_1-M_2)^2-t] \qquad (3.2)$$

The expression (3.2) is the same as the expression for the unpolarized cross-section. Thus when the exchanged particle has zero spin, averaging over the final polarizations is equivalent to averaging over the initial polarizations also and this averages out the dependence of the matrix element on the relative orientation of the initial polarization vectors. This is to be expected, as a spinless (exchanged) particle does not have the additional degree of freedom required to carry a 'memory' of the polarization at one vertex into the other vertex and therefore cannot give rise to any dependence on the relative orientations of the polarizations at the two vertices.

On the other hand, when the particle exchanged has non-zero spin, the differential cross-section averaged over the final spins will depend on the orientation of the initial spins,

as we shall see below for ρ exchange. Thus, in principle a method of estimating the importance of the exchange of states with non-zero spin is to use a polarized nucleon target and a polarized incident nucleon beam and examine whether the angular distribution depends on the relative orientations of the polarization vectors of the incident nucleons. Whether the method is practicable depends on the magnitude of the effect.

As it seems likely that the state with non-zero spin whose exchange is most important is the ρ meson, we have obtained the numerical results for ρ exchange alone.

Taking the matrix element for ρ exchange as[1)]

$$\bar{\tau}_\mu(p_2)\left[g_v \gamma_5 \delta_{\mu\nu} - f_v \gamma_5 k_\mu \gamma_\alpha - h_v \gamma_5 k_\mu k_\nu\right] u(p_1) \times$$

$$\times (-i)\frac{\left[g_{\nu\alpha} - \frac{k_\nu k_\alpha}{m_\rho^2}\right]}{(t-m_\rho^2)} \cdot \bar{u}(q_2)\left[G_v \gamma_\alpha + i F_v \sigma_{\alpha\beta} k_\beta\right] u(q_1) \cdot F_\rho(t) \qquad (3.3)$$

and keeping only the terms that are in the lowest order in the derivative couplings, as in chapter (VI), we obtain $\frac{\partial\sigma}{\partial\Omega}$ for N_1^* production through ρ exchange (for a pole in the t-channel).

$$\frac{\partial\sigma_\rho(N_1^*)}{\partial(\cos\theta^*)} = \frac{2\pi}{3}\cdot\frac{g_v^2}{4\pi}\,\frac{\left|G_v+(M_1+M_2)F_v\right|^2}{4\pi}\cdot \times$$

$$\times\left|F_\rho(t)\right|^4\cdot(t-m_\rho^2)^{-2}\cdot\left[X+Y+Z\right] \qquad (3.4)$$

where $[F_\rho(t)]$ is the form factor in the matrix element for ρ exchange, and X , Y , and Z are defined by the following expressions:

$$X = [(m_2-m_1)^2-t]\left\{\frac{t(2m_2^2+M_2^2-s)}{2m_2^2} - \right.$$

$$-\frac{1}{2}(M_1^2+M_2^2-6M_1M_2)-\frac{1}{2m_2^2}(s-m_2^2-M_2^2)\times$$

$$\left.\times(s-m_1^2-M_2^2)\right\} \quad ; \qquad (3.5)$$

$$Y = 2(n_A\cdot n_B)\Big[(m_1-m_2)(M_1-M_2)s-(m_2M_1+m_1M_2)t$$

$$+\left\{-m_2M_2(m_1^2+m_1^2)-m_1M_1(m_2^2+M_2^2)\right.$$

$$\left.+m_1M_2(m_2^2+M_1^2)+m_2M_1(m_1^2+M_2^2)\right\}\Big]$$

$$+2\left[M_1(q_2\cdot n_B)-M_2(q_1\cdot n_B)\right]\times$$

$$\times\left[m_2(p_1\cdot n_A)-m_1(p_2\cdot n_A)\right] \quad ; \qquad (3.6)$$

$$Z = \frac{2}{m_2}\Big[M_2 \epsilon_{\alpha\beta\delta\mu}(p_1)_\alpha (n_B)_\beta (p_2)_\delta \epsilon_{\tau\sigma\lambda\mu} \times$$

$$\times (q_1)_\tau (n_A)_\sigma (p_2)_\lambda -$$

$$- M_1 \epsilon_{\alpha\beta\delta\mu}(p_1)_\alpha (n_B)_\beta (p_2)_\delta \epsilon_{\tau\sigma\lambda\mu}(q_2)_\tau (n_A)_\sigma (p_2)_\lambda\Big] \quad (3.7)$$

In order to obtain the observed differential cross-section, one must add to (3.4) the contribution of a pole in the u-channel , which may be done by symmetrizing (3.4) with respect to $\theta = 90^\circ$.

In our example, of course,

$$\begin{aligned} m_1 &= m_N \\ M_1 &= m_N \\ M_2 &= m_N \\ \text{and } m_2 &= M^* \end{aligned} \quad (3.8)$$

where m_N and M^* are the masses of the nucleon and isobar respectively. We have written the above equations for general masses so that the corresponding expressions for $D_{3/2}$ N^* and for axial vector exchange may be obtained merely by changing

the signs of the appropriate masses. Thus the differential cross-section for the production of a $D_{3/2}$ N* by ρ exchange is obtained from equations (3·4) - (3·7) by the replacement

$$m_2 \rightarrow - m_2 \qquad (3·9)$$

while the cross-section for the production of a $P_{3/2}$ N* by the exchange of an axial vector meson is obtained by replacing

$$m_2 \rightarrow -m_2$$

$$\text{and} \quad M_2 \rightarrow - M_2 \qquad (3·10)$$

in equations (3·4) - (3·7) . Finally, the replacement

$$M_2 \rightarrow - M_2 \qquad (3·11)$$

gives the differential cross-section for N^*_2 production by the exchange of an axial vector meson.

In evaluating the expressions (3·3) and (3·4) - (3·7) , we have used the relations (A-1), - (A·3) given in the appendix.

As discussed in section 2 , we define the polarization correlation functions by taking the polarization vector $\vec{n}_A$ of the target nucleon (assumed to be completely polarized) along $\vec{e}_y$, $\vec{e}_x$ and $\vec{e}_y$ in turn in the rest frame of the target nucleon. The polarization vector $\vec{n}_B$ of the incident nucleon is taken to be in the direction (Θ, Φ) (in the rest frame of the incident nucleon).

We then obtain the following results:

Case (a): $\vec{n}_A = \vec{e}_y \; ; \; \vec{n}_B \| (\Theta, \Phi)$

$$\frac{\partial \sigma_\rho(N_1^*)}{\partial(\cos\theta^*)} = \frac{2\pi}{3} \cdot \frac{1}{W^2} \cdot \frac{p_f}{p_i} \cdot \frac{g^2_{\rho NN_1^*}}{4\pi} \cdot \frac{\left| G_{\rho NN} + 2M_N F_{\rho NN} \right|^2}{4\pi} \times$$

$$\times \left| F_\rho(t) \right|^4 \times \left[A_\rho(\theta, s) + B_\rho(\theta, s) \cos\Theta + \right.$$

$$\left. + C_\rho(\theta, s) \sin\Theta \cos\Phi \right] \qquad (3.12)$$

where

$$A_\rho(\theta, s) = \frac{\left[(m_2 - m_1)^2 - t \right]}{(t - m_\rho^2)^2} \times$$

$$\times\Big\{ \frac{t(2m_2^2+M_2^2-s)}{2m_2^2} - \frac{1}{2}\left(M_1^2+M_2^2-6M_1M_2\right)$$

$$-\frac{1}{2m_2^2}\left(s-m_2^2-M_2^2\right)\left(s-m_1^2-M_2^2\right)\Big\} \quad ; \qquad (3.13)$$

$$B_\rho(\theta,s) = \frac{2}{(t-m_\rho^2)^2}\Big[-\frac{(q_1^2+\omega_1 E_1)}{m_1 M_1}\, y +$$

$$+\frac{1}{M_1 m_1}\Big\{m_1(E_2 q_1+\omega_1 q_2\cos\theta) - m_2 q_1\sqrt{s}\Big\}\times$$

$$\times\Big\{M_1(E_1 q_2\cos\theta+\omega_2 q_1) - M_2 q_1\sqrt{s}\Big\}$$

$$+\frac{m_1 M_1 q_2^2\sin^2\theta}{m^2}\left(\frac{\sqrt{s}\,\omega_1}{M_1}-M_2\right)\Big] \quad ; \qquad (3.14)$$

and

$$C_\rho(\theta,s) = \frac{2q_2\sin\theta}{(t-m_\rho^2)^2}\times$$

$$\times\Big[m_1(E_2 q_1+\omega_1 q_2\cos\theta) - m_2 q_1\sqrt{s} +$$

$$+\frac{(M_1M_2-\omega_1\sqrt{s})}{m_2}\cdot\left(q_2 E_1\cos\theta - q_1 E_2\right)\Big] \quad ; \qquad (3.15)$$

where

$$\gamma = (m_1 - m_2)(M_1 - M_2)s - (m_2 M_1 + m_1 M_2)t +$$

$$+ m_1 M_2 (m_2^2 + M_1^2) + m_2 M_1 (m_1^2 + M_2^2)$$

$$- m_2 M_2 (m_1^2 + M_1^2) - m_1 M_1 (m_2^2 + M_2^2). \qquad (3.16)$$

Case B (b) : $\vec{n}_A = \vec{e}_x \; ; \; \vec{n}_B \parallel (\Theta, \Phi)$

The functions A , B , and C in (3.12) are now replaced by a , b , and c , where

$$a_\rho(\theta, s) = A_\rho(\theta, s) \qquad (3.17a)$$

$$b_\rho(\theta, s) = \frac{2 m_1 q_2 \sin\theta}{(t - m_\rho^2)^2} \times$$

$$\times \left[\frac{M_1}{m_1} (E_1 q_2 \cos\theta + \omega_2 q_1) - \frac{M_2}{m_1} \sqrt{s}\, q_1 + \right.$$

$$\left. + \frac{M_2}{m_2} (\omega_1 q_2 \cos\theta + E_2 q_1) - \frac{M_1}{m_2} \sqrt{s}\, q_2 \cos\theta \right] \qquad (3.17b)$$

and

$$C_\rho(\theta,s) = \frac{2}{(t-m_\rho^2)^2} \times \Big[-y + q_2 \sin\theta \times$$

$$\times \Big\{ M_1 (E_1 q_2 \cos\theta - \omega_2 q_1) - M_2 (E_1 - \omega_1) q_1 \Big\} +$$

$$+ \frac{(q_2 E_1 \cos\theta - E_2 q_1)}{m_2} \times$$

$$\times \Big\{ M_1 \sqrt{s}\, q_2 \cos\theta - M_2 (\omega_1 q_2 \cos\theta + E_2 q_1) \Big\} \Big] \qquad (3.17c)$$

Case (c) : $\vec{n}_A = \vec{e}_y$; $\vec{n}_B \parallel (\Theta, \Phi)$

The square brackets in (3.12) are now replaced by

$$A_\rho(\theta,s) + C_\rho(\theta,s) \sin\Theta \sin\Phi \qquad (3.18)$$

where

$$\mathcal{A}_p(\theta, s) = A_p(\theta, s) \qquad (3.18a)$$

and

$$\mathcal{C}_p(\theta, s) =$$

$$= \frac{2}{(t-m_p^2)^2} \cdot \left[-y + \frac{M_1}{m_2} q_2 \sqrt{s} \left(E_1 q_2 - q_1 E_2 \cos\theta \right) + \right.$$

$$\left. + \frac{M_2}{m_2} \left\{ q_1^2 E_2^2 - q_1^2 q_2^2 \sin^2\theta - E_1 \omega_1 q_2^2 - q_1 q_2 E_2 (E_1 - \omega_1) \cos\theta \right\} \right] \qquad (3.18b)$$

The six correlation functions are then obtained by symmetrizing with reference to $\theta = 90^\circ$ the functions $A(\theta, s)$, $B(\theta, s)$, $C(\theta, s)$, $b(\theta, s)$ $c(\theta, s)$ and $\mathcal{C}(\theta, s)$.

In evaluating numerical results for both initial nucleons polarized, we have proceeded a little differently. In an experiment one would set the target nucleon with its polarization vector $\vec{n}_A$ along one of the coordinate axes and perform the experiment with the incident beam polarization

set at a definite angle. We have therefore evaluated the polarized differential cross-section, i.e., the isobar angular distribution in the overall c.m. frame, for 7 different orientations of the polarization vectors of the initial nucleons (in their respective rest frames). These will be denoted as cases (i) to (vii), as given below, where $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_n$ are unit vectors along the x, y and n axes. The $\vec{n}$ axis is taken along the incident nucleon momentum $\vec{p}_1$ and the x— axis in the plane of the (two-body) reaction.

Case (i) $\vec{n}_A \parallel \vec{q}_1$; $\vec{n}_B \parallel \vec{p}_1$

(Both nucleons longitudinally polarized).

Case (ii) $\vec{n}_A \parallel \vec{q}_1$; $\vec{n}_B \parallel (-\vec{p}_1)$

(Both nucleons longitudinally polarized).

Case (iii) $\vec{n}_A \parallel \vec{q}_1$; $\vec{n}_B \parallel \vec{e}_x$

(Target nucleon longitudinally polarized, incident nucleon transversely polarized in the plane of reaction).

Case (iv) $\vec{n}_A \parallel \vec{q}_1$; $\vec{n}_B \parallel \vec{e}_y$

(Target nucleon longitudinally polarized in the plane of reaction; incident nucleon polarized perpendicular to the plane of reaction).

Case (v) $\vec{n}_A \parallel \vec{e}_x$; $\vec{n}_B \parallel \vec{e}_y$

(Target nucleon transversely polarized in the plane of reaction; incident nucleon polarized perpendicular to the plane of reaction).

Case (vi) $\vec{n}_A \parallel \vec{e}_x$; $\vec{n}_B \parallel \vec{e}_x$.

(Both incident and target nucleon transversely polarized in the plane of reaction).

Case (vii) $\vec{n}_A \parallel \vec{e}_y$; $\vec{n}_B \parallel \vec{e}_y$.

(Both incident and target nucleon polarized perpendicular to the plane of the reaction).

For each case, one may easily evaluate the functions (3.4) so as to obtain the differential cross-sections.

The angular distributions for constant form factors, i.e.

$$F_\pi(t) \approx 1 \; ; \; F_\rho(t) \approx 1 . \qquad (3.19)$$

are plott ed in fig. 2 for the production of a $P_{3/2}$ N_1^* (with mass 1238 Mev) by π and ρ exchange, at an incident nucleon energy of 2.85 Gev. In fig. 2 , the chain lines give the angular distribution for π exchange; the other curves (i) to (vii) are for Cases (i) to (vii) with ρ exchange. (The π exchange curve is the same as the unpolarized angular distribution as noted in the last section). Cases (iv) and (v) give the same angular distributions.

The angular distributions for Cases (i) to (vii) have been calculated for the production of a $D_{3/2}$ N^* also. The angular distributions for different orientations of the initial polarizations are very similar to each other; hence only the cases showing some appreciable difference have been plotted. For pp reactions and the reaction $np \to np\ \pi^o$ the angular distributions are given in fig. 3. For the reaction $np \longrightarrow nn\ \pi^+$, they are given in fig. 4. [The asymmetry in the latter reaction comes from the isospin combinations. (see chapter VI)].

These are for constant form factors. We have seen in chapter (VI) that in order to obtain a description of the process that gives total cross-sections in at least rough agreement with experiment, it is necessary to multiply each pole contribution by a form factor that damps the matrix element at large momentum transfer. In that chapter, we had obtained the effect of energy-independent form factors as well as of energy-dependent form factors that could be considered as a method of approximating the Regge behaviour of the exchanged particle. We had surmised there that the actual form factors would probably be somewhere in between the two alternatives studied there.

In figures 5 and 6 are shown the angular distributions with polarized incident and target nucleons when we assume that the damping at large momentum transfers may be approximated by energy-independent form factors of the same shapes as assumed in chapter VI, i.e.,

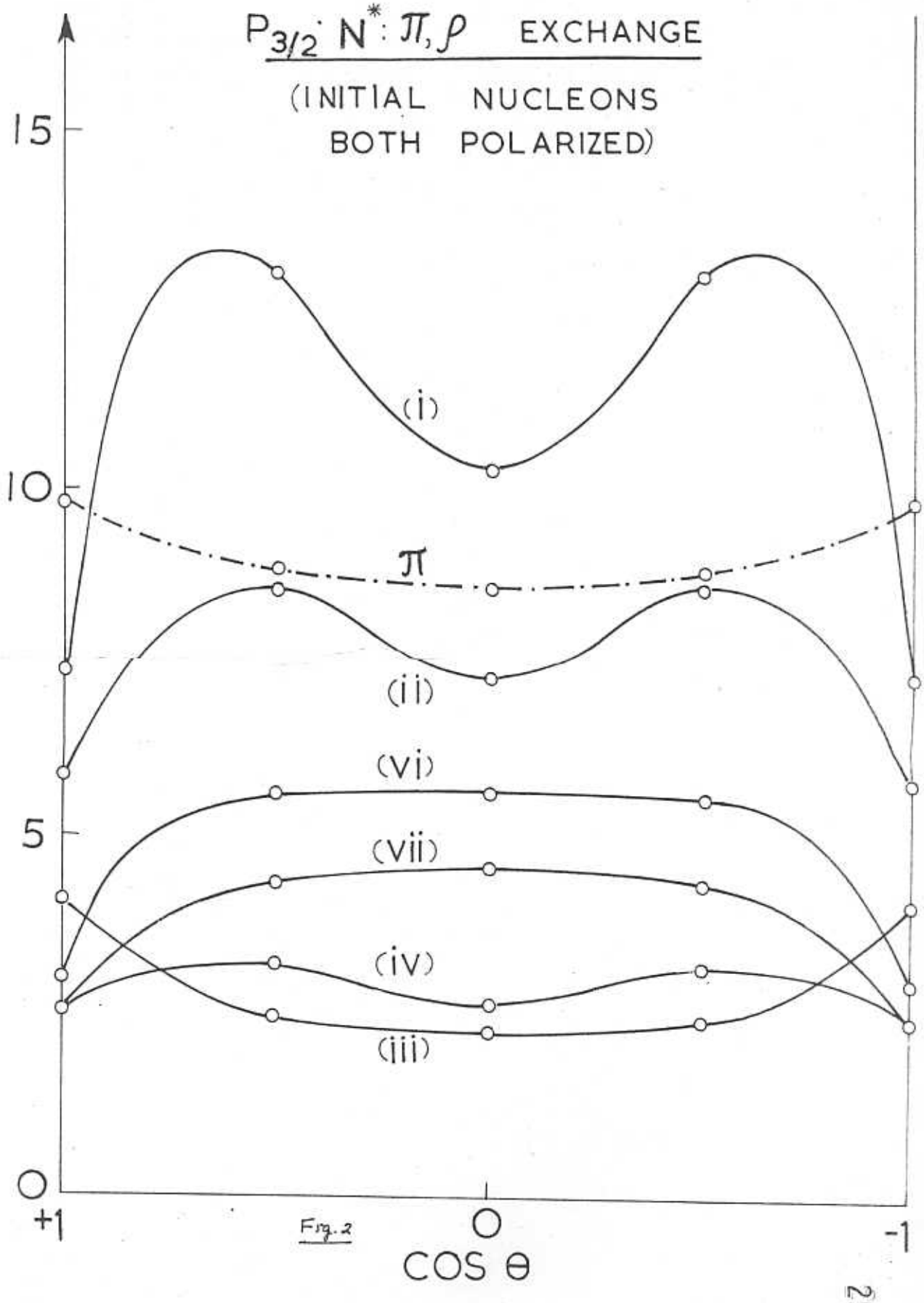

$P_{3/2}$ N^* : π, ρ EXCHANGE
(INITIAL NUCLEONS BOTH POLARIZED)
15
10
5
0
(i)
π
(ii)
(vi)
(vii)
(iv)
(iii)
+1
0
-1
COS θ

Fig. 2

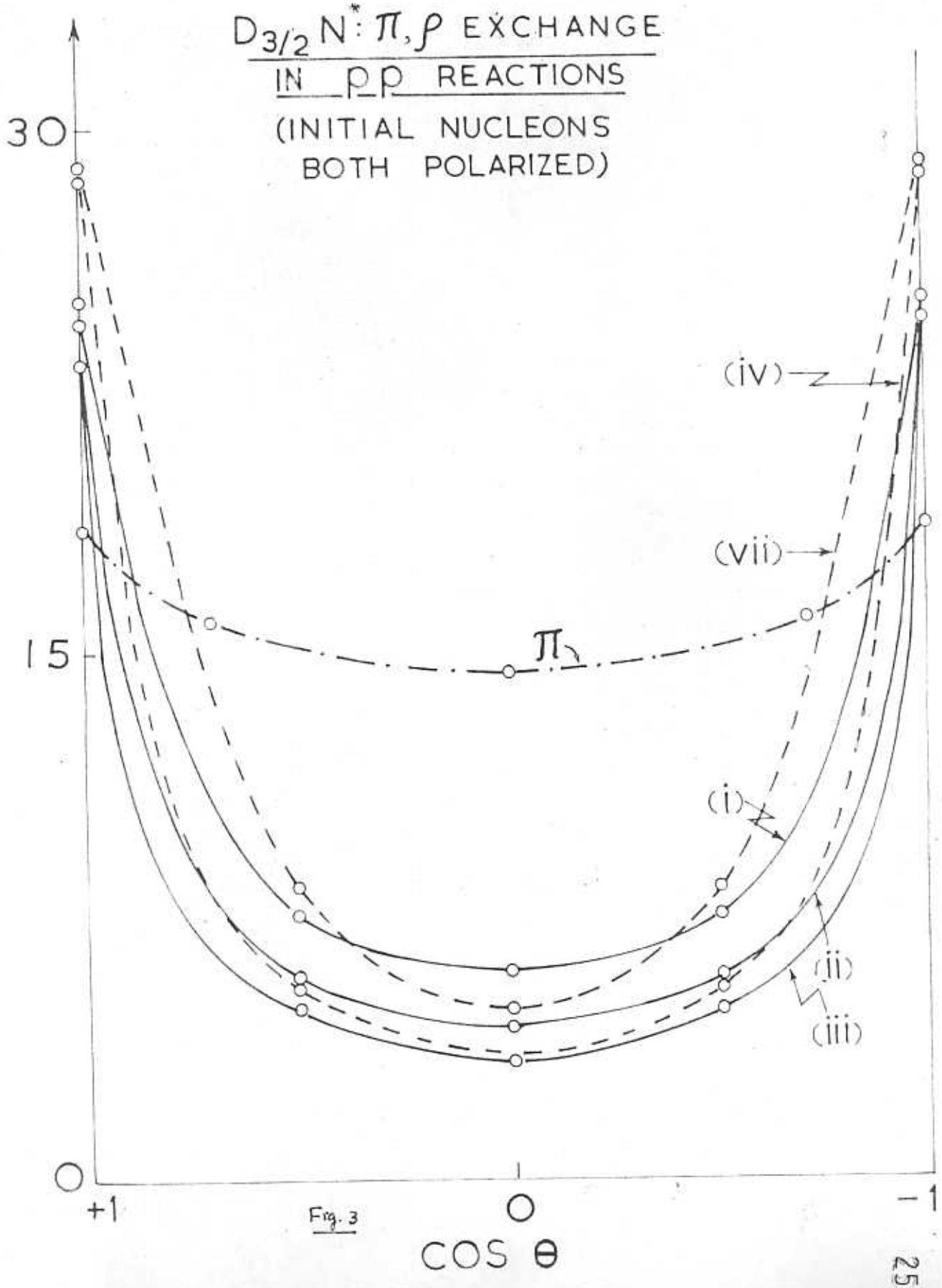

$D_{3/2}\ N^*: \pi, \rho$ EXCHANGE
IN pp REACTIONS
(INITIAL NUCLEONS
BOTH POLARIZED)
30
15
0
(iv)
(vii)
π_2
(i)
(ii)
(iii)
+1
0
−1
COS θ

Fig. 3

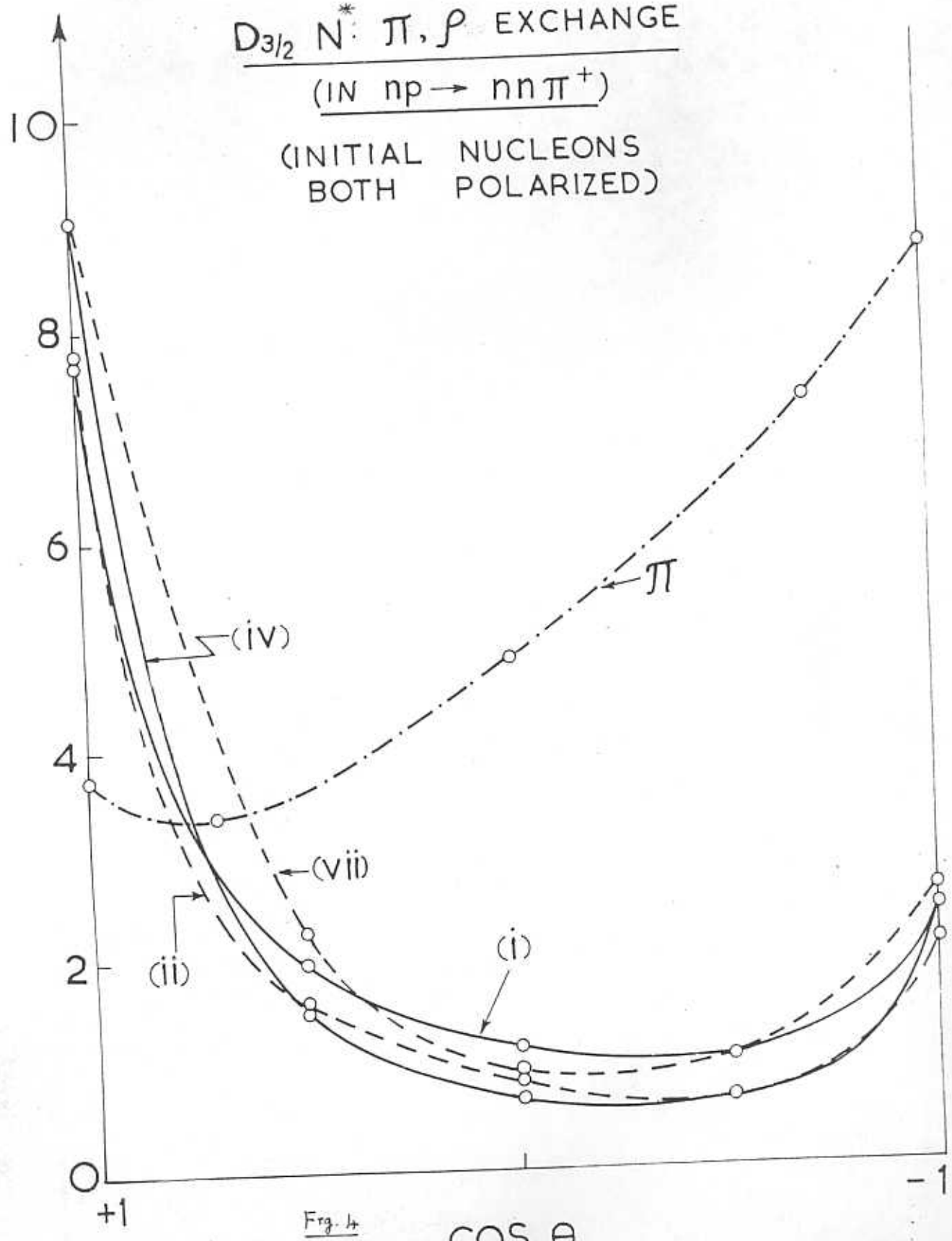

$D_{3/2}$ N^*: π, ρ EXCHANGE
(IN $np \rightarrow nn\pi^+$)
(INITIAL NUCLEONS BOTH POLARIZED)
10
8
6
4
2
0
+1
-1
COS θ
π
(iv)
(vii)
(ii)
(i)

Fig. 4

$$F_{\pi}(t) = 0.28 + 0.72\left[1 + \frac{(t - m_{\pi}^2)}{4.73}\right]^{-1} \quad ; \qquad (3.20)$$

$$F_{\rho}(t) = -0.2 + 1.2\left[m_{\pi}^2 - 0.1t\right]^{-1} \qquad (3.21)$$

The function F_{π} (t) is just the form factor assumed by Ferrari and Selleri.[4]

It is evident from the results that with the form factor (3.21) for ρ exchange, which is the same as the Hofstadter isovector form. factors of the nucleon[5] , the damping of large momentum transfers is strong enough to smoothen out most of the qualitative differences between the angular distributions for different relative orientations of the polarization vectors. However, one important effect that remains is the variation of the <u>total</u> cross-section with varying relative orientation of the polarizations which are evident from figs. 5 and 6 , and from figs. 2, 3 and 4.. The areas under the different curves give the relative magnitudes of the total cross-sections.

4) E. Ferrari and F.Selleri, Nuovo Cim. 27 (1963), 1450

5) R.Hofstadter and Herman, Phys. Rev.Lett., 6, (1961), 293.

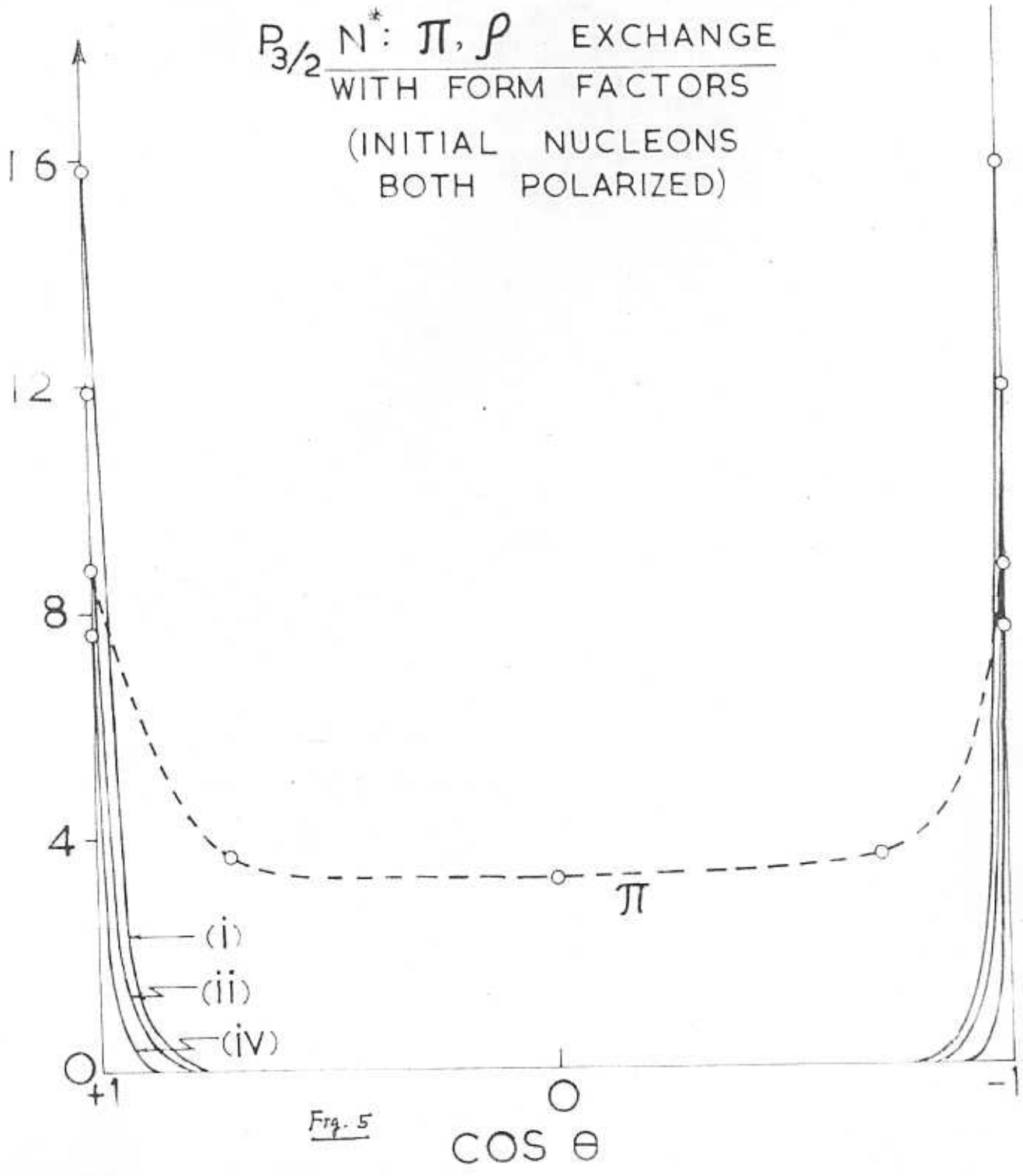
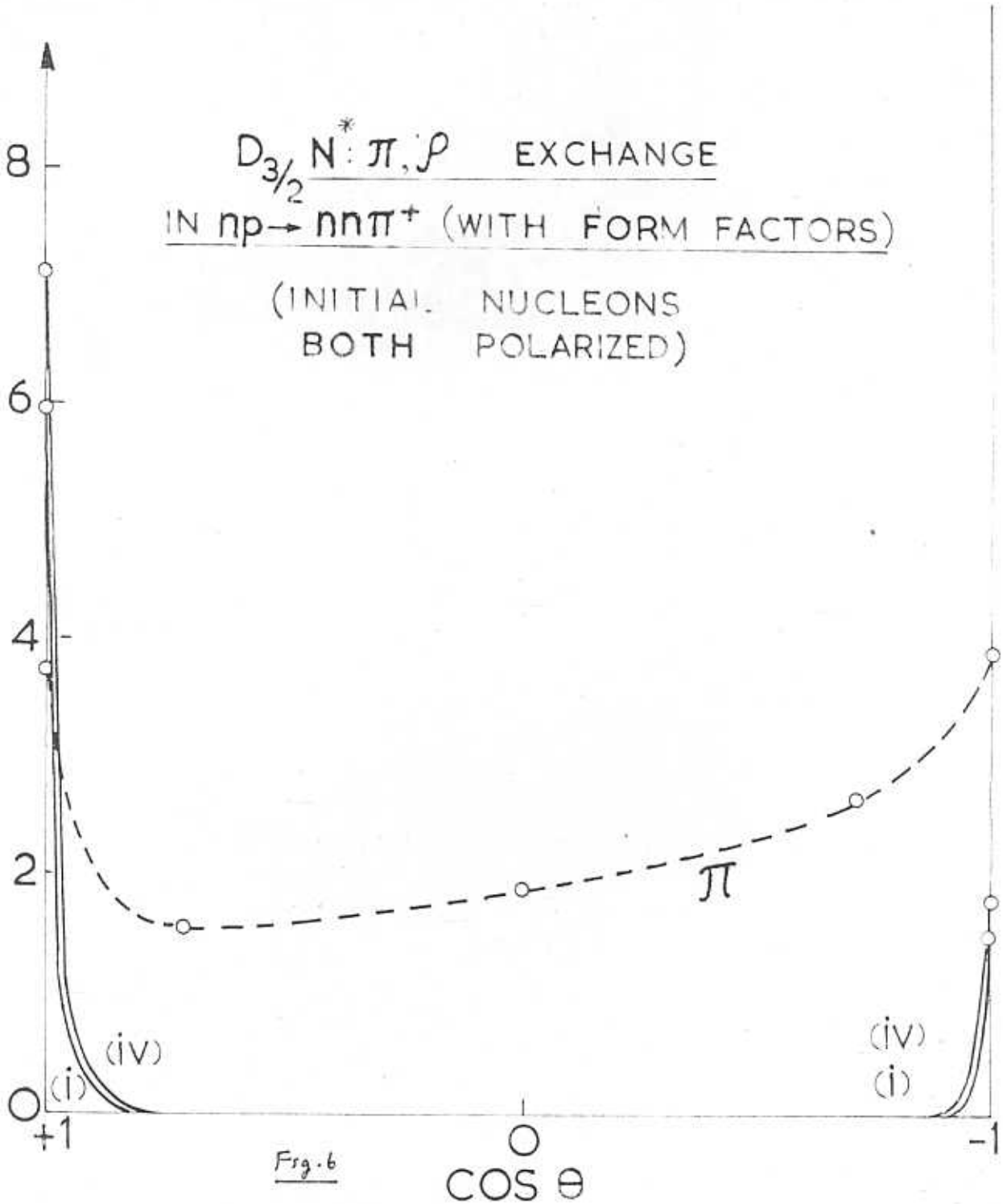

$P_{3/2}$ N^*: π, ρ EXCHANGE
WITH FORM FACTORS
(INITIAL NUCLEONS
BOTH POLARIZED)
16
12
8
4
0
(i)
(ii)
(iv)
π
+1
0
−1
COS θ

Fig. 5


$D_{3/2}$ N^*: π, ρ EXCHANGE
IN $np \to nn\pi^+$ (WITH FORM FACTORS)
(INITIAL NUCLEONS BOTH POLARIZED)
8
6
4
2
0
+1
0
-1
COS θ
π
(i)
(iv)
(iv)
(i)

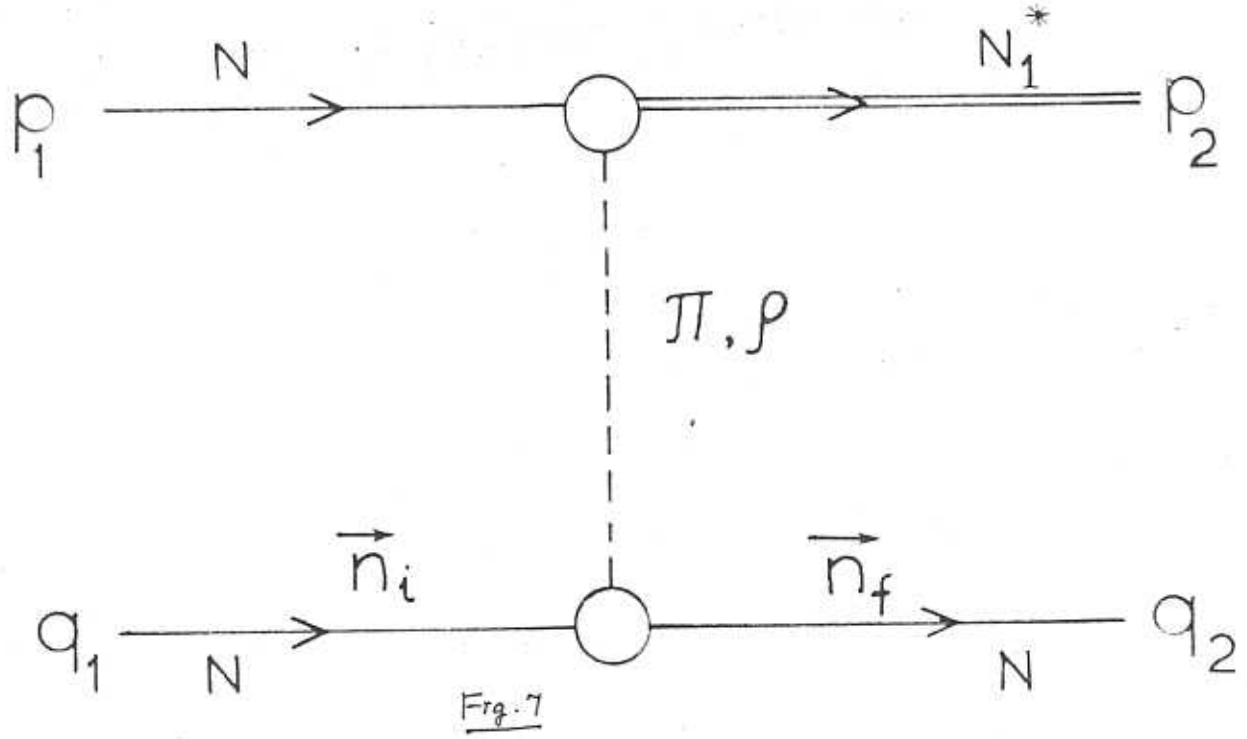
Frg. 6

4. Polarisation correla tions with one initial nucleon polarized : Pion exchange.

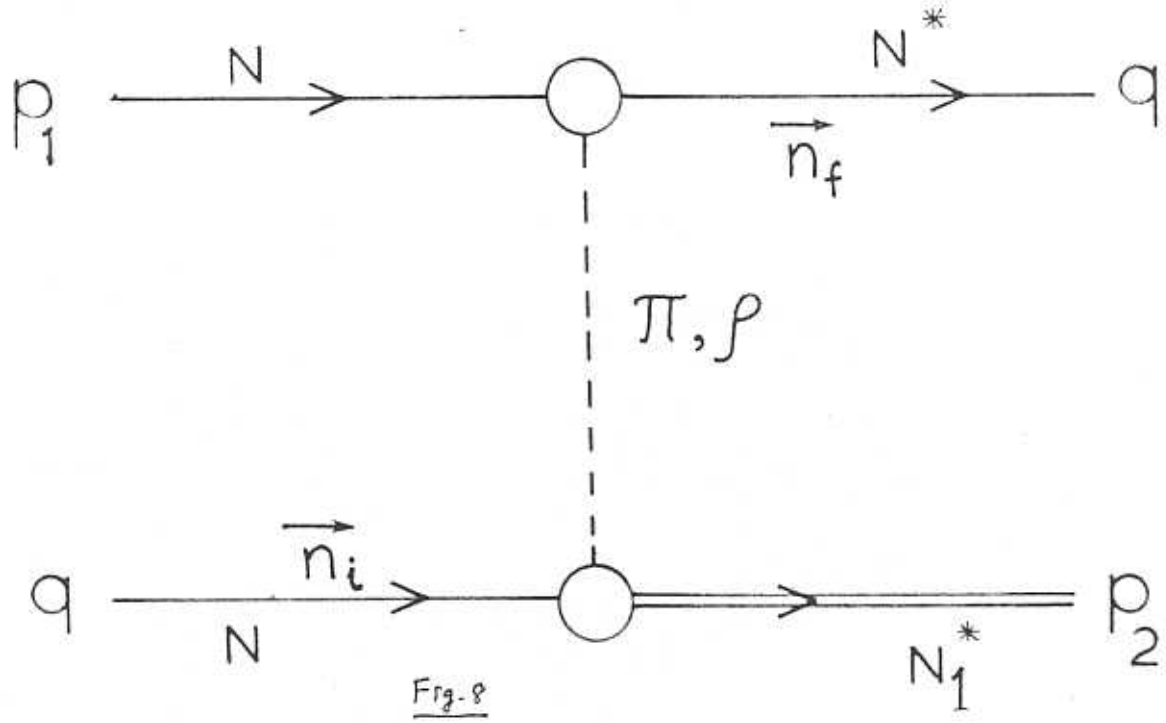
In the last section we have examined the dependence of the differential cross-section ϖ on the relative orientation of the nucleon polarizations in the initial state. We found that although in principle this could give information about the spin of the exchanged state, the presence of strongly damped form factors at the vertices would smoothen out most of the variations in the differential cross-section.

A different type of experiment would be one in which the correlation between the polarization vectors of an initial nucleon and a final nucleon is measured. The simplest situation theoretically is that in which one of the initial nucleons, which we shall take as the target nucleon, is polarized, and the other initial nucleon is unpolarized and one observes the polarization of the 'recoil nucleon'. One could equally well take an unpolarized nucleon target and a polarized nucleon beam; the analysis remains unchanged.

The first observation is that the recoil nucleon may come out either at the same vertex as the initially polarized nucleon (see fig. 7) or at the same vertex as the initially unpolarized nucleon (see fig. 8). These two contributions will be referred to as (I) and (II) respectively; the observed correlations will be the sum of those for (I) and (II).


N
N_1^*
p_1
p_2
π, ρ
$\vec{n}_i$
$\vec{n}_f$
q_1
N
N
q_2

Frg. 7


N
N^*
p_1
q
$\vec{n}_f$
π, ρ
$\vec{n}_i$
q
N
N_1^*
p_2

Frg. 8

An important difference between the contributions (I) and (II) is the following. In (I), the polarization correlation will be non-zero (i.e. the differential cross-section will depend on the relative orientation of the two polarizations), irrespective of the spin of the exchanged particle, as we are interested in the polarization correlation between two nucleons at the same vertex. The correlation functions will of course be different when the exchanged particle has different spins.

In (II), on the other hand, the correlationxx of interest is that between the polarizations of nucleons at different vertices. This will be non-zero only when the exchanged particle has a non-zero spin, for the same reasons as discussed in the last section.

When the exchanged particle has spin zero, in particular when it is a pion, all the correlation functions are non-zero for the contribution (I), whereas only $A(0,s)$, $a(0,s)$, and $\mathcal{A}(0,s)$ are non-zero for the contribution (II).

An important simplification that occurs for spin-zero exchange, is that, if $\vec{n}_c$ is the target nucleon polarization and $\vec{n}_\ell$ the recoil nucleon polarization, then for

$$(n_c)_\mu \, (n_c)^\mu = -1,$$

we have $(n_f)_\mu (n_f)^\mu = -1$ for the contribution (I) and $(n_f)_\mu = 0$ for the contribution (II), i.e. the magnitude of n_f is immediately known for both contributions.

For the exchange of states with non-zero spin, the recoil nucleon will be partially polarized, when one of the initial nucleons is unpolarized (or partially polarized), and therefore the magnitude of n_f is not known immediately for either the contribution (I) or (II). In order to find out the magnitude of n_f, it is necessary to calculate the density matrix of the recoil nucleon.

The reason for this difference may be summarized in the statement that the exchange of a spin-zero pa_rticle can transmit no information about the orientation of spins or the impureness of a state at one vertex into the other vertex, while the exchange of a particle with non-zero spin will necessarily transmit this information.

In this section, we shall discuss the polarization correlations for single-pion exchange, while in the next section we discuss them for the exchange of a ρ-meson.

Denoting the target nucleon polarization by the vector $\vec{n}_c$ and the recoil nucleon polarization by the vector $\vec{n}_f$, the following expression is obtained for the differential cross-section from pion exchange. (Note that for the contribution II with pion exchange, only the functions $A(\theta, s)$, a, and $\mathcal{A}$ are non-zero; the correlation functions B, C, b, c, and $\mathcal{C}$ are all zero.)

$$\frac{\partial \sigma_{\pi}(N_1^*)}{\partial(\cos\theta^*)} = \frac{\pi}{12}\cdot\frac{1}{W^2}\cdot\frac{p_f}{p_i}\cdot\frac{f^2_{\pi NN^*_L}}{4\pi}\cdot\frac{G^2_{\pi NN}}{4\pi}\cdot\frac{1}{8m_2^2}\times$$

$$\times\frac{|F_{\pi}(t)|^2}{(t-m_{\pi}^2)^2}\cdot\left\{R_t\left[\frac{1}{2}\left[t-(M_1-M_2)^2\right](1+n_i\cdot n_f)+\right.\right.$$

$$\left.\left.+(q_1\cdot n_i)(q_2\cdot n_f)+(q_1\cdot n_f)(q_2\cdot n_i)\right]+\frac{1}{2}R_u\left[u-(m_1-m_2)^2\right]\right\} \quad (4.1)$$

where

$$R_t = \left[(m_1+m_2)^2-t\right]^2\left[(m_1-m_2)^2-t\right], \quad (4.1a)$$

and

$$R_u = \left[(M_1+m_2)^2-u\right]^2\left[(M_1-m_2)^2-u\right] \quad (4.1b)$$

The corresponding expressions for a $D_{3/2}$ N^* may be obtained by the replacement

$$m_2 \longrightarrow -m_2$$

We have evaluated the numerical results only for N^*_1 production for these and all subsequent expressions and shall discuss only these for the rest of the chapter.

The polarization correlation functions, defined as in sections 2 and 3, are given below.

Case (a). $\vec{n}_i = \vec{e}_y \; ; \; \vec{n}_f \parallel (\Theta, \Phi)$

(Target nucleon longitudinally polarized)

$$\frac{\partial \sigma_\pi (N_1^*)}{\partial (\cos \theta^*)} = \frac{\pi}{12} \cdot \frac{1}{\omega^2} \cdot \frac{p_f}{p_i} \cdot \frac{f^2}{4\pi} \cdot \frac{G^2}{4\pi} \cdot \frac{1}{8 m_2^2} \times$$

$$\times \left[A_\pi (\theta, s) + B_\pi (\theta, s) \cos \Theta + \right.$$

$$\left. + C_\pi (\theta, s) \sin \Theta \cos \Phi \right] \qquad (4.2)$$

where

$$A_\pi (\theta, s) = \frac{|F_\pi (t)|^2}{(t - m_\pi^2)^2} \cdot \left[R_t + R_n \right] \qquad (4.3a)$$

$$B_\pi (\theta, s) = \frac{|F_\pi (t)|^2}{(t - m_\pi^2)^2} \cdot R_t \times$$

$$\times \left[\frac{1}{2} \left\{ t - (M_1 - M_2)^2 \right\} G_1 + \right.$$

$$\left. + \frac{1}{M_1} \left(q_2 \omega_1 \cos \theta_1 - \omega_2 q_1 \right) G_2 \right] \qquad (4.3b)$$

and

$$C_{\pi}(\theta, s) = \frac{|F_{\pi}(t)|^2}{(t-m_{\pi}^2)^2 M_1 M_2} \cdot R_t \times$$

$$\times \left[\frac{1}{2} \left\{ t - (M_1 - M_2)^2 \right\} q_2 \left(q_1 - \frac{\omega_1 q_2 \cos\theta}{\omega_2 + M_2} \right) - \right.$$

$$\left. - \left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right) \left(q_2 \omega_1 \cos\theta - \omega_2 q_1 \right) \right] \qquad (4.3c)$$

Here

$$G_1 = \frac{\omega_1}{M_1} + \frac{q_2 \cos\theta}{M_1 M_2} \left(q_1 - \frac{\omega_1 q_2 \cos\theta}{\omega_2 + M_2} \right) \quad ; \qquad (4.4)$$

and

$$G_2 = q_1 - \frac{q_2 \cos\theta}{M_2} \left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right) \qquad (4.5)$$

The quantities R_t and R_a are given by equations (4.1a) and (4.1b) .

Case (b): $\vec{n}_i = \vec{e}_x \; ; \; \vec{n}_f \parallel (\Theta, \Phi)$

(Target nucleon transversely polarized in the plane of reaction.)

$$\frac{\partial \sigma}{\partial (\cos\theta^*)} =$$

$$= \frac{\pi}{12} \cdot \frac{1}{W^2} \cdot \frac{p_f}{p_i} \cdot \frac{f^2}{4\pi} \cdot \frac{G^2}{4\pi} \cdot \frac{1}{8M_2^2} \times \Big[a_\pi(\theta, s) +$$

$$+ b_\pi(\theta, s) \cos\Theta + C_\pi(\theta, s) \sin\Theta \cos\Phi \Big] \qquad (4.6)$$

where

$$a_{\pi}(\theta, s) = A_{\pi}(\theta, s) \qquad (4.6a)$$

$$b_{\pi}(\theta, s) = \overline{R}\; q_2 \sin\theta \times$$

$$\times \left[G_2 - \frac{q_2 \cos\theta \left\{ t - (M_1 - M_2)^2 \right\}}{2 M_2 (\omega_2 + M_2)} \right] \qquad (4.6b)$$

and

$$C_{\pi}(\theta, s) = -\overline{R} \left[\frac{1}{2} \left\{ t - (M_1 - M_2)^2 \right\} \times \right.$$

$$\left. \times \left\{ 1 + \frac{q_2^2 \sin^2\theta}{M_2(\omega_2 + M_2)} \right\} + \frac{q_2^2 \sin^2\theta}{M_2} \times \left\{ \omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right\} \right] \qquad (4.6c)$$

Here

$$\overline{R} = \frac{\left| F_{\pi}(t) \right|^2}{(t - m_{\pi}^2)^2} \cdot R_t \qquad (4.7)$$

Case (c) : $\vec{n}_i = \vec{e}_y \; ; \; \vec{n}_f \parallel (\Theta, \Phi)$

(Target nucleon polarized perpendicular to the plane of reaction).

$$\frac{\partial \sigma}{\partial (\cos \theta^*)} = \frac{\pi}{12} \cdot \frac{1}{W^2} \cdot \frac{p_f}{p_i} \cdot \frac{f^2}{4\pi} \cdot \frac{G^2}{4\pi} \cdot \frac{1}{8 m_2^2} \times$$

$$\times \left[\mathcal{A}_\pi (\theta, s) + \mathcal{C}_\pi (\theta, s) \sin \Theta \sin \Phi \right] \qquad (4.8)$$

where

$$\mathcal{A}_\pi (\theta, s) = A_\pi (\theta, s) = a_\pi (\theta, s) \qquad (4.8a)$$

and

$$\mathcal{C}_\pi (\theta, s) = -\frac{\overline{R}}{2} \cdot \left\{ t - (m_1 - m_2)^2 \right\} \qquad (4.8b)$$

The six independent correlation functions A_π , B_π , C_π , b_π , c_π and $\mathcal{C}_\pi$ for pion exchange have been evaluated and are shown in fig. 9, for constant form

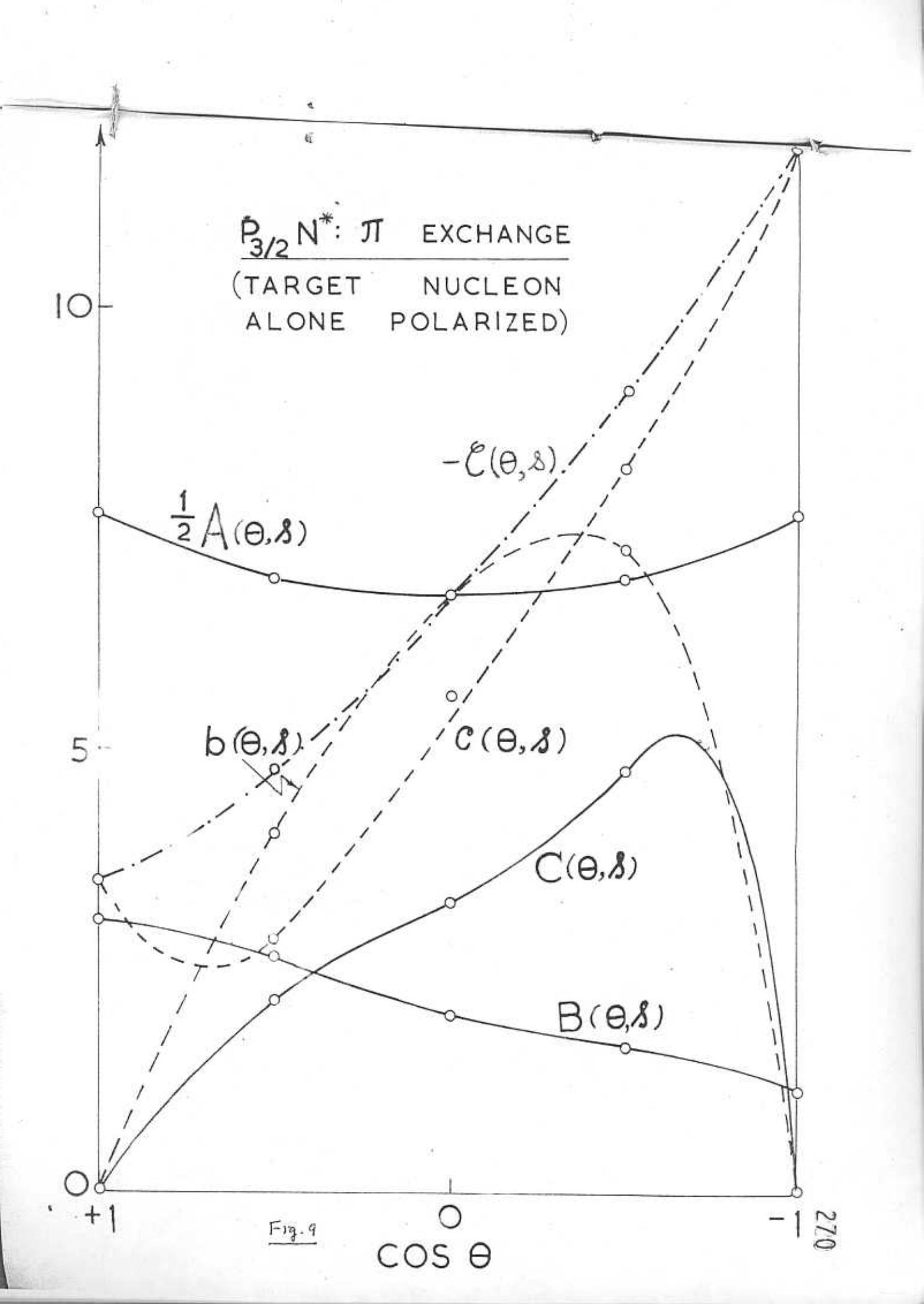

$P_{3/2}$ N^*: π EXCHANGE
(TARGET NUCLEON ALONE POLARIZED)
10
5
0
$-\mathcal{C}(\theta,s)$
$\frac{1}{2}A(\theta,s)$
$b(\theta,s)$
$C(\theta,s)$
$C(\theta,s)$
$B(\theta,s)$
+1
0
−1
COS θ
270

Fig. 9

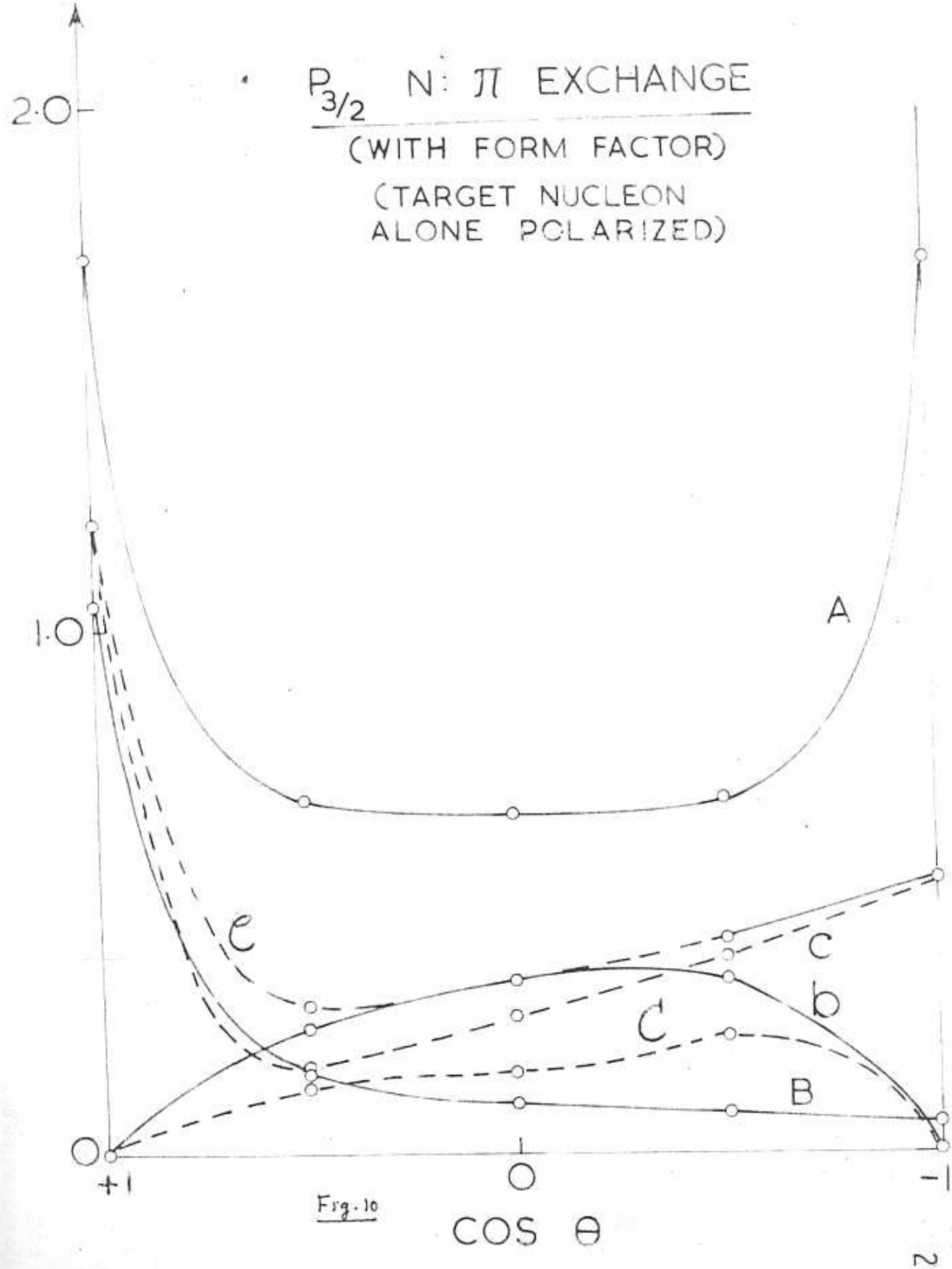

$P_{3/2}$ N: π EXCHANGE
(WITH FORM FACTOR)
(TARGET NUCLEON ALONE POLARIZED)
2·0
1·0
A
a
c
b
C
B
+1
0
−1
COS θ

Fig. 10

factors and in fig. 10 for the energy-independent form factor (3·20). These results will be compared with those obtained with ρ exchange at the end of the next section.

5. Polarization correlations with one initial nucleon polarized: ρ exchange

In this section, we discuss the polarization correlations for ρ exchange.

The complication involved here is that the magnitude of n_f will be different for the contributions (I) and (II) so that a knowledge of this magnitude is necessary for a complete knowledge of the correlation functions. In this chapter we shall not be calculating the magnitude of n_f, which requires the evaluation of the density matrix for the final state, owing to the complication involved. We hope to return to the question in a later publication.

Here we shall evaluate the contribution of (I) and (II) to the correlation functions separately. The observed polarization correlations will be a sum of these contributions, each weighted by the corresponding magnitude of n_f . .

The differential cross-section for N^*_1 production is given by

$$\frac{\partial\sigma_f(N_1^*)}{\partial(\cos\theta^*)} = \frac{2\pi}{3}\cdot\frac{1}{W^2}\cdot\frac{p_f}{p_i}\cdot\frac{g^2_{fNN_1^*}}{4\pi}\cdot\frac{\left|G_{fNN}+(M_1+M_2)F_{fNN}\right|^2}{4\pi}\times$$

$$\times\left[F_f(t)\right]^4\cdot\left[\frac{\frac{1}{2}\left[(m_1-m_1^2-t\right]}{(t-m_f^2)^2}P_I\left\{\alpha+\beta(n_i\cdot n_f)+\right.\right.$$

$$+(q_1\cdot n_f)(q_2\cdot n_i)+\frac{1}{m_2^2}\left[(M_1-M_2)^2-t\right](p_2\cdot n_i)(p_2\cdot n_f)-$$

$$-\frac{(s-m_2^2-M_2^2)}{m_2^2}(p_1\cdot n_i)(p_2\cdot n_f)-\frac{(s+t-m_2^2-M_1^2)}{m_2^2}\times$$

$$\left.\times(q_2\cdot n_i)(p_2\cdot n_f)\right\}+$$

$$+\frac{P_{II}}{(u-m_f^2)^2}\cdot\left\{\alpha'+\beta'(n_i\cdot n_f)+M_1M_2(p_1\cdot n_i)(p_2\cdot n_f)\right.$$

$$+m_1m_2(q_1\cdot n_f)(q_2\cdot n_i)-m_2M_2(q_1\cdot n_f)(p_1\cdot n_i)-$$

$$\left.\left.-m_1M_1(p_2\cdot n_f)(q_2\cdot n_i)+\frac{M_2}{m_2}F_1-\frac{M_1}{m_2}F_2\right\}\right] \qquad (5.1)$$

where,

$$\alpha = t\,\frac{(2m_2^2 + M_2^2 - s)}{2m_2^2} - \frac{1}{2}\left(M_1^2 + M_2^2\right) + 3M_1M_2$$

$$-\frac{1}{2m_2^2}\left(s - m_1^2 - M_2^2\right)\left(s - m_2^2 - M_2^2\right); \qquad (5.2)$$

$$\beta = t.\,\frac{(s - M_2^2)}{2m_2^2} - \frac{1}{2}\left(M_1 + M_2\right)^2 +$$

$$+\frac{1}{2m_2^2}\left(s - m_1^2 - M_2^2\right)\left(s - m_2^2 - M_2^2\right); \qquad (5.3)$$

$$\alpha' = \frac{t(s - 2m_2^2 - M_2^2)}{2m_2^2} -$$

$$- s\,\frac{(m_1^2 + 2m_2^2)}{2m_2^2} + \frac{1}{2}\left[m_2^2 + M_1^2 +\right.$$

$$\left. +6m_1M_2 + \frac{1}{m_2^2}\left(m_1^2 + m_2^2\right)\left(m_2^2 + M_2^2\right)\right]; \qquad (5.4)$$

$$\beta' = \frac{1}{2}\Big[(m_1 M_1 + m_2 M_2)s +$$

$$+ (m_1 m_2 + M_1 M_2)t - m_2 M_2 (m_1^2 + M_1^2)$$

$$- m_2 M_1 (M_1^2 + M_2^2) - M_1 M_2 (m_1^2 + m_2^2)$$

$$- m_1 M_1 (m_2^2 + M_2^2)\Big] \; ; \qquad (5.5)$$

$$F_1 = \varepsilon_{\alpha\beta\delta\mu} (q_1)_\alpha (n_i)_\beta (p_2)_\delta \times$$

$$\times \varepsilon_{\tau\sigma\lambda\mu} (p_1)_\tau (n_f)_\sigma (p_2)_\lambda \; ; \qquad (5.6)$$

$$F_2 = \varepsilon_{\alpha\beta\delta\mu} (q_1)_\alpha (n_i)_\beta (p_2)_\delta \times$$

$$\times \varepsilon_{\tau\sigma\lambda\mu} (q_2)_\tau (n_f)_\sigma (p_2)_\lambda \; . \qquad (5.7)$$

The functions F_1 and F_2 may be evaluated as indicated in the appendix.

In equation (5.1), the numbers $\mathcal{P}_{\text{I}}$ and $\mathcal{P}_{\text{II}}$ which lie between 0 and 1 , are the magnitudes of n_f for the contributions (I) and (II) respectively; they will be different for the three cases (a), (b) and (c) although we shall use the same notation.

The functions $A_\rho(\theta, s)$, B_ρ, etc. will be defined as in section 4 . We then obtain the correlation functions given below.

Case (a) $\vec{n_i} = \vec{e_\alpha}$; $\vec{n}_f \parallel (\oplus \Phi)$

$$A_\rho(\theta, s) = \left[R_t \alpha + R_u \alpha' \right] \tag{5.8a}$$

$$B_\rho(\theta, s) = \Bigg[R_t \Bigg\{ G_1 \beta + \frac{G_2}{M_1} U_1 + \frac{q_2 \sqrt{s} \cos\theta}{M_1 M_2 m_2^2} \cdot U_2 \Bigg\} + R_u \Bigg\{ G_1 \beta' + \Big[G_2 \frac{m_2}{M_1} + \frac{q_2 \sqrt{s} \cos\theta}{M_2} \Big] \Big[M_2 q_1 \sqrt{s} + m_1 (q_2 \omega_1 \cos\theta - \omega_2 q_1) \Big] -$$

$$- \frac{M_1 M_2}{m_2} q_2 \sin\theta \cdot S + \frac{M_1^2}{m_2} \cdot q_2^2 \sin^2\theta \cdot \sqrt{s} \Big\} \Big] \qquad (5.8b)$$

$$C_\rho(\theta, s) = \Big[R_t \Big\{ \beta \cdot \frac{q_2}{M_1 M_2} \cdot \sin\theta \left(q_1 - \frac{\omega_1 q_2 \cos\theta}{\omega_2 + M_2} \right) -$$

$$- \frac{q_2 \sin\theta}{M_1 M_2} \cdot \left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right) U_1 + \frac{q_2 \sqrt{s} \sin\theta}{M_1 M_2 m_2^2} \cdot U_2 \Big\} +$$

$$+ R_u \cdot \frac{q_2}{M_2} \cdot \sin\theta \Big\{ \frac{\beta'}{M_1} \left(q_1 - \frac{\omega_1 q_2 \cos\theta}{\omega_2 + M_2} \right) -$$

$$- \frac{M_1^2 M_2}{m_2} \cdot q_2 \sqrt{s} \cos\theta - \frac{M_1 M_2^2}{m_2} \cdot T +$$

$$+ \Big[M_2 q_2 \sqrt{s} + m_1 \left(q_2 \omega_1 \cos\theta - \omega_2 q_1 \right) \Big] \times$$

$$\times \Big[\sqrt{s} - \frac{m_2}{M_1} \left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right) \Big] \Big\} \Big] \qquad (5.8c)$$

where

$$R_t = \mathcal{P}_I \left| F_\rho(t) \right|^4 \cdot \frac{1}{2} \frac{\{(m_1 - m_2)^2 - t\}}{(t - m_\rho^2)^2} \qquad (5.9)$$

$$R_u = \mathcal{P}_{II} \cdot \frac{\left| F_\rho(u) \right|^4}{(u - m_\rho^2)^2} \qquad (5.10)$$

$$U_1 = M\left\{(q_2 \cdot n_i) - \frac{(s - m_2^2 - M_2^2)}{m_2^2}(p_2 \cdot n_i)\right\} ; \qquad (5.11)$$

$$U = -M_1\left\{(p_2 \cdot n_i)\left[(M_1 - M_2)^2 - t\right] - \right.$$

$$\left. -(q_2 \cdot n_i)\left[s + t - M_2^2 - m_1^2\right]\right\} ; \qquad (5.12)$$

$$S = q_2 \sin\theta\left\{E_1 + \frac{q_1 q_2 \cos\theta}{M_2} \cdot \left[\frac{\sqrt{s} + M_2}{\omega_2 + M_2}\right]\right\} ; \qquad (5.13)$$

$$T = (E_2 q_1 - E_1 q_2 \cos\theta) +$$

$$+ \frac{q_1 q_2^2 \sin^2\theta}{M_2} \cdot \left[\frac{\sqrt{s} + M_2}{\omega_2 + M_2}\right] ; \qquad (5.14)$$

and G_1 is defined by Eqn. (4.4)

Case (Bb): $\vec{n}_c = \vec{e}_x \; ; \; \vec{n}_f \parallel (\Theta, \Phi)$

$$a_f(\theta, s) = A_f(\theta, s) \; ; \qquad (5.15a)$$

$$b_f(\theta, s) = R_t \left\{ -\beta \frac{q_2^2 \sin\theta \cos\theta}{M_2(\omega_2 + M_2)} + G_2 \cdot V_1 + \right.$$

$$\left. + \frac{q_2 \sqrt{s}}{M_2 m_2^2} \cdot \cos\theta \; V_2 \right\} +$$

$$+ R_u \left\{ q_2 \sin\theta \left[-\frac{q_2 \cos\theta}{M_2(\omega_2 + M_2)} \beta' + \frac{M_1 m_1}{M_2} q_2 \sqrt{s} \cos\theta + \right. \right.$$

$$\left. + m_1 m_2 G_2 - \frac{M_1}{m_2} \sqrt{s} \left(E_2 q_1 + \omega_1 q_2 \cos\theta \right) \right] +$$

$$\left. + \frac{M_2}{m_2} \cdot S \cdot \left(E_2 q_1 + \omega_1 q_2 \cos\theta \right) \right\} \; ; \qquad (5.15b)$$

and.

$$C_f(\theta, s) = R_t \left[-\beta \left\{ 1 + \frac{q_2^2 \sin^2\theta}{M_2(\omega_2 + M_2)} \right\} - \right.$$

$$\left. - \frac{q_2 \sin\theta}{M_2} \left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2 + M_2} \right) V_1 + \frac{q_2 \sqrt{s} \sin\theta}{M_2 m_2^2} V_2 \right] +$$

$$+ R_{12}\left[-\beta'\left\{1+\frac{q_2^2 \sin^2\theta}{M_2(\omega_2+M_2)}\right\} + \right.$$

$$+ q_2^2 \sin^2\theta \cdot \frac{m_1}{M_2}\left\{M_1\sqrt{s} - m_2\left(\omega_1 - \frac{q_1 q_2 \cos\theta}{\omega_2+M_2}\right)\right\}$$

$$+ \left(E_2 q_1 + \omega_1 q_2 \cos\theta\right) \times$$

$$\left.\times\left\{\frac{M_2}{m_2} T + \frac{M_1}{m_2}\cdot q_2\sqrt{s}\cos\theta\right\}\right] \; ; \qquad (5.15c)$$

where

$$V_1 = \frac{(s-M_2^2)}{m_2^2}\cdot q_2 \sin\theta \; ; \qquad (5.16)$$

and

$$V_2 = -q_2 \sin\theta\left[s + M_1^2 - m_1^2 - 2M_1 M_2\right] \qquad (5.17)$$

Case (c) : $\vec{n_c} = \vec{e_y}$; $\vec{n_f} \parallel (\text{H}\,\Phi)$

$$\mathcal{A}_\rho(\theta,s) = A_\rho(\theta,s) = a_\rho(\theta,s) \; ; \qquad (5.18a)$$

and

$$\mathcal{C}_\rho(\theta,s) = -R_t\beta +$$

$$+ R_u\left[-\beta' + \frac{M_2}{m_2}\left\{E_2^2 q_1^2 - \omega_1 E_1 q_2^2 + \right.\right.$$

$$\left.\left. + (\omega_1 - E_1)\, q_1 q_2 E_2 \cos\theta \right\}\right.$$

$$\left. + \frac{M_1}{m_2} \cdot \sqrt{s} \cdot \omega_2 (\omega_1 q_2 + E_2 q_1 \cos\theta)\right] \qquad (5.18b)$$

The six ~~xxx~~ polarization correlation functions A_ρ, B_ρ, C_ρ, b_ρ, c_ρ, and $\mathcal{C}_\rho$, for ~~xhix~~ each of the contributions (I) and (II) to the differential cross ~~cross~~-section have been evaluated as a function of $\cos\theta$ for an incident nucleon energy of 2.85 Gev, and are shown in figs. 11 and 12 for a constant form factor, $F_\rho(t) \approx 1$, and in fig. 13 for the form factor (3.21).

Comparing the correlation functions for π and ρ exchange, as given in figs. 9 - 13, we note the following.

For constant form factors, a complete comparison cannot be made as we have not evaluated the magnitude of n_f. However, from the correlation functions for the separate contributions (I) and (II), we can make the following statements.

(i) The correlation functions $A_\rho(\theta,s)$, a and $\mathcal{A}$ are, of course, independent of n_f and therefore completely known.

(ii) When $\mathcal{P}_I$ and $\mathcal{P}_{II}$, the magnitudes of n_f for the contributions (I) and (II) are comparable, the differences between the correlation functions for π and ρ exchange will be most striking for the functions $C(\theta,s)$ and $\mathcal{C}(\theta,s)$ i.e., for the correlation functions involving the aximuth of the final nucleon polarization vector when the target nucleon is transversely polarized (with its polarization vector either in the plane of reaction or normal to it).

For the form factors (3.20) and (3.21) a comparison can be made even in the absence of a knowledge of $|n_f|$ i.e., of $\mathcal{P}_I$ and $\mathcal{P}_{II}$. In fig. 13, the curves A, B, C and b refer to the contribution (I); the functions c and $\mathcal{C}$ lie very close to B and have therefore not been shown. The functions A', B', C', b', c' and $\mathcal{C}'$ all refer to the contribution (II). The relative magnitudes would be as shown in fig.13 if

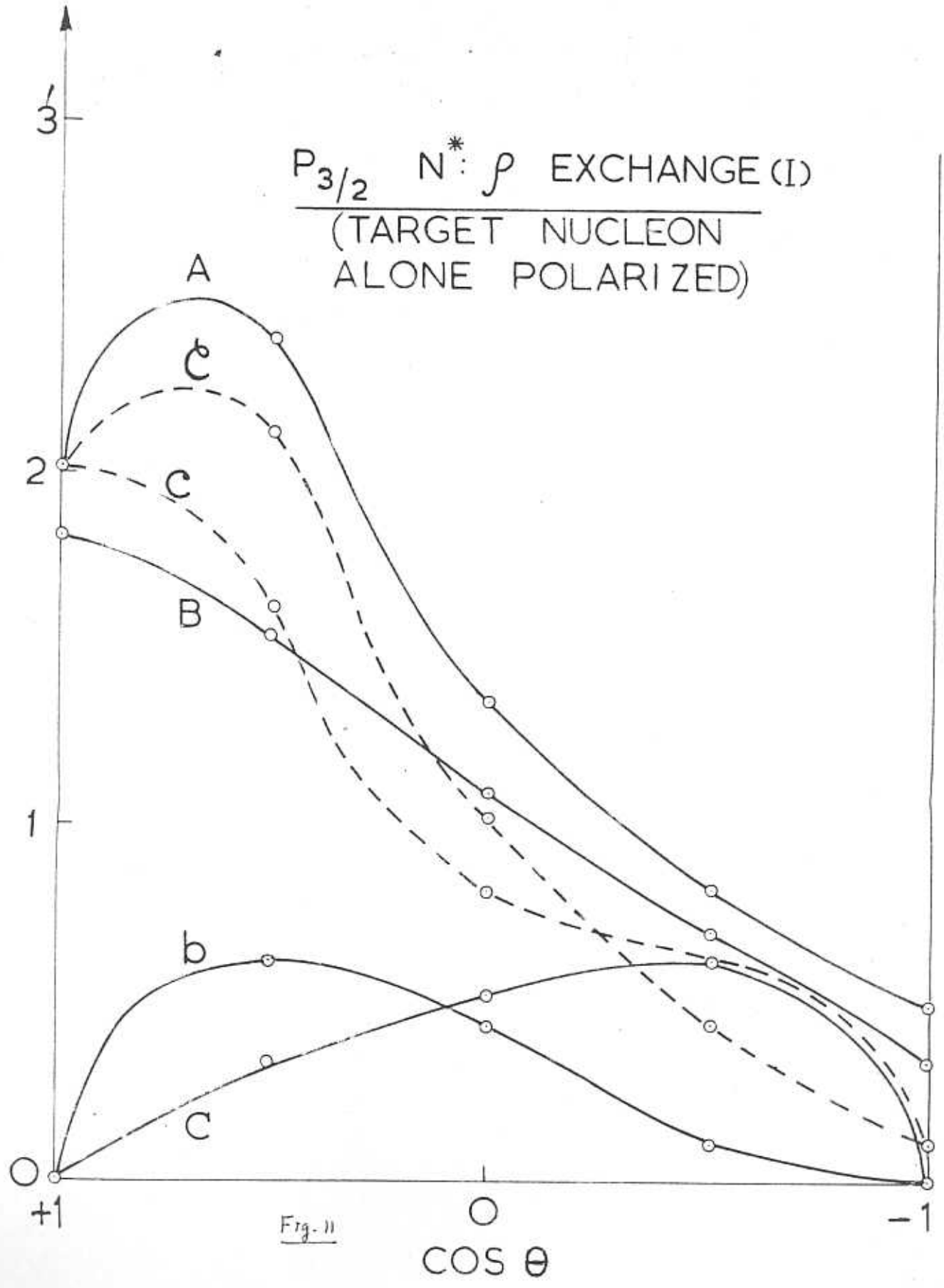

$P_{3/2}$ N^*: ρ EXCHANGE (I)
(TARGET NUCLEON ALONE POLARIZED)
A
c
c
B
b
C
3
2
1
0
+1
0
−1
COS θ

Fig. 11

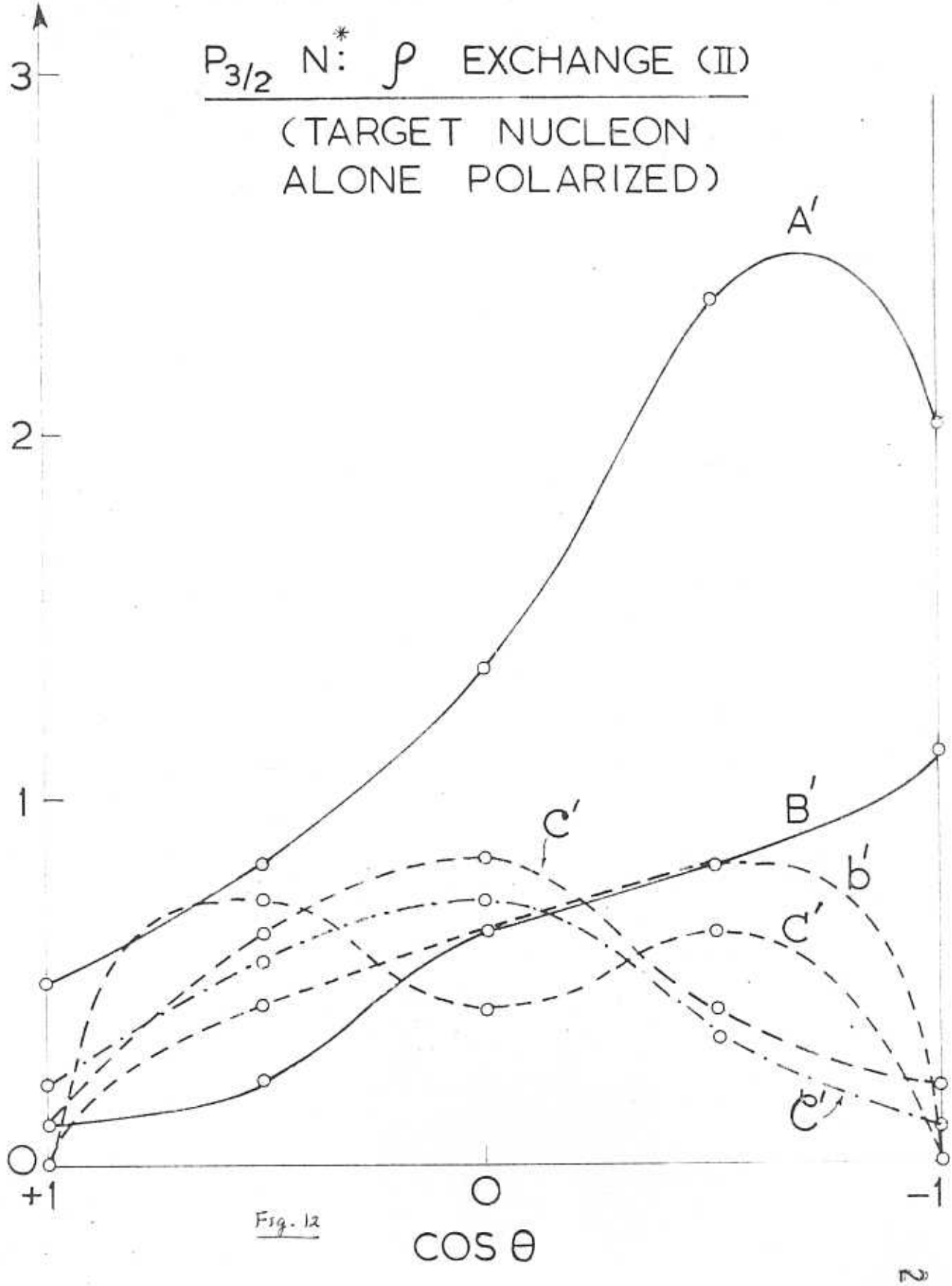
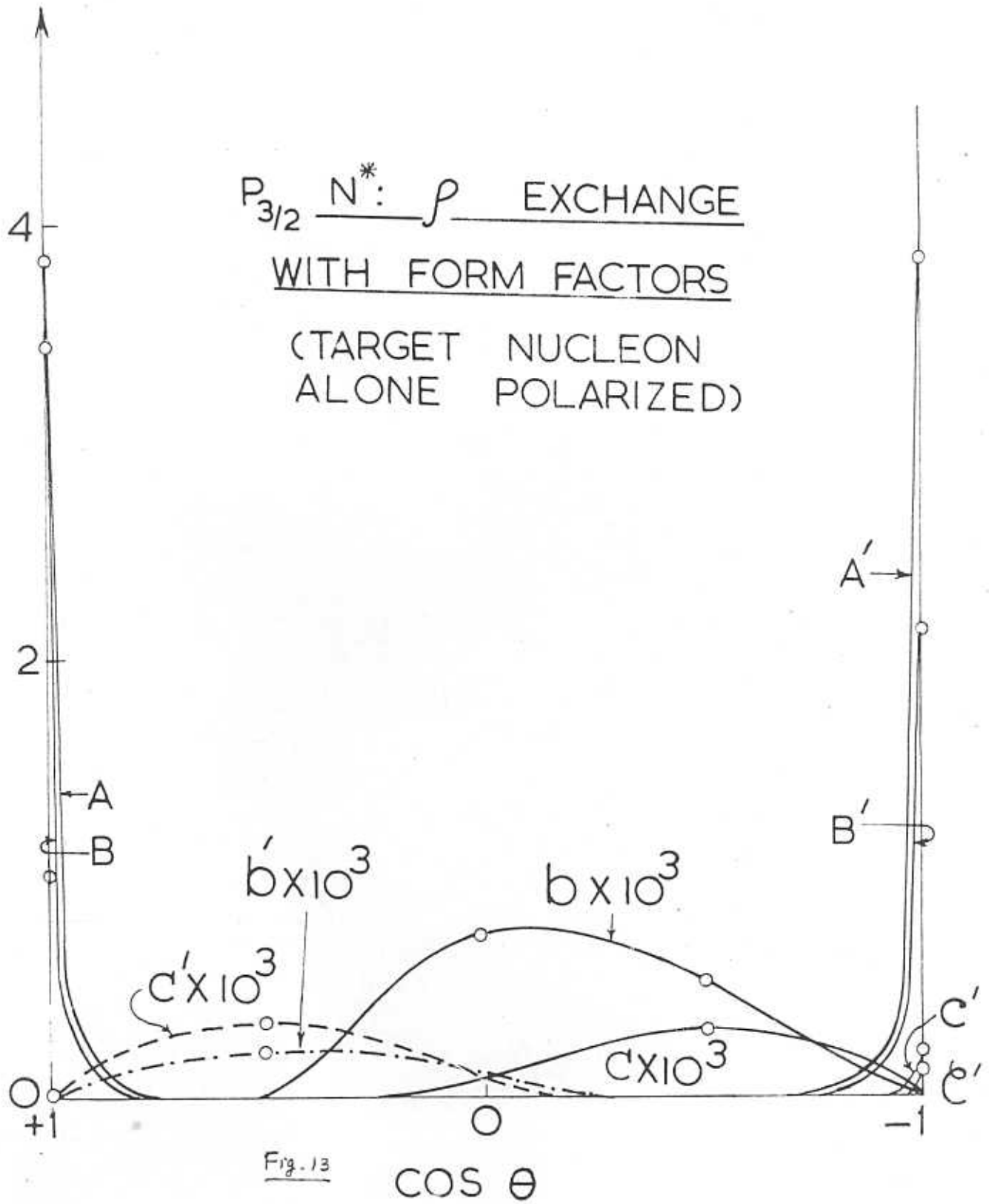

$P_{3/2}$ N* ρ EXCHANGE (II)
(TARGET NUCLEON ALONE POLARIZED)
A′
B′
C′
b′
C′
C′
3
2
1
0
+1
0
−1
COS θ

Fig. 12



$P_{3/2}$ N^*: ρ EXCHANGE
WITH FORM FACTORS
(TARGET NUCLEON
ALONE POLARIZED)
4
2
0
A
B
A′
B′
C′
C′
b′x10³
b x 10³
c′x10³
c x10³
+1
0
−1
COS θ

Fig. 13

$P_I = P_{II}$. Once $|nf|$ is evaluated for (I) and (II) the relative magnetidues can be immediately obtained. From figs. 10 and 13, it is seen that the most striking difference between the correlation functions for π and ρ exchange is that for ρ exchange, the correlation functions $A(\theta,s)$, A', B, B', C, C', G and G' are very sharply peaked in the forward or backward directions and are vanishingly small in the other directions. For pion exchange on the other hand, the functions continue to be appreciably large in directions other than $\theta = 0^\circ$ and 180°.

Some general conclusions that may be drawn from all these results are given in section 6.

6. Conclusions

In this chapter we have given a detailed discussion of those polarization effects in isobar production which involve the polarizations of only two of the nucleons.

The first observation to be made is that a study of the polarization effects will be most useful after some knowledge of the form factors governing the momentum-transfer dependence has been already obtained. The type of experiments from which such information may be obtained have been discussed in chapter VI .

A study of the polarization effects can give useful information about the spin of the exchanged states. Thus, if with both of the initial nucleons polarized, the angular

distribution and total-cross-section show no dependence on the relative orientations of the initial-state polarizations, one may conclude that the exchange of states of non-zero spin is relatively unimportant. One must be careful in interpreting the extent of dependence of the angular distribution on the relative orientation of the polarizations in terms of relative magnitudes of the exchange of states with zero spin and non-zero spin, as a strong damping of large momentum transfers tends to make the difference small. The dependence of the total cross-sections on the polarizations still gives useful information.

For a polarized target nucleon and unpolarized incident beam (or vice versa), a study of the polarization correlation between the target-nucleon and recoil nucleon can again give information about the spin of the exchanged state, as discussed in section 5. A previous knowledge of the form factors will enable a more precise evaluation of the results.

All our calculations have been done only for π and ρ exchange. The general expressions for the exchange of scalar and axial vector mesons can be obtained by changing the signs of the appropriate masses in the expressions given in this chapter, as indicated in section 2. We have obtained numerical results only for the exchanges that may be expected to be the most dominant, as detailed experimental data on the polarization correlations are not yet available.

In conclusion, we may state that the polarization correlations considered here when combined with a measurement of correlations of the type discussed by Treiman and Yang[6] and by Stodolsky and Sakurai[7] should give reliable information on the angular momentum of the exchanged states.

In this chapter we have considered situations in which one or both of the initial nucleons is completely polarized. A discussion of the effects when the nucleons are partially polarized, and an evaluation of the degree of polarization of the final nucleon which is, for instance, necessary for a complete discussion of ρ exchange, requires the evaluation of the density matrix for the final state.

We hope to discuss these questions as well as other types of polarization correlations in a later publication.

Appendix

In this Appendix we give some of the important relations used in the paper. We use the metric defined by $A.B = A_o B_o - \vec{A} . \vec{B}$.

The projection operator for a spin 3/2 N* on the mass-shell, with mass M and momentum p , has been taken as

6) S.B.Treiman and C.N.Yang, Phys. Rev. Lett., 8,(1962),140

7) L.Stodolsky and J.J.Sakurai, Phys.Rev.Lett.,11, (1963),90.

$$\rho_{\mu\lambda}(p) = \frac{1}{3}\left\{ 3 g_{\mu\lambda} - \frac{2 p_\mu p_\lambda}{M^2} - \gamma_\mu \gamma_\lambda + \right.$$

$$\left. + \frac{\not{p}\left(p_\mu \gamma_\lambda - p_\lambda \gamma_\mu\right)}{M^2} \right\} \cdot \left(\frac{\not{p} + M}{2M} \right) \qquad (A.1)$$

In obtaining the expressions (3.4) - (3.7) , we have used the relations

$$Tr \cdot \left[\gamma_5 \gamma_\mu \gamma_\nu \gamma_\lambda \gamma_\rho \right] = -4 \epsilon_{\mu\nu\lambda\rho}, \qquad (A.2)$$

and

$$\sum_{\mu,\lambda} \epsilon_{\alpha\beta\mu\lambda} \cdot \epsilon_{\tau\sigma\mu\lambda} = -2 \left[g_{\alpha\tau} g_{\beta\sigma} - g_{\alpha\sigma} g_{\beta\tau} \right] \qquad (A.3)$$

Here, $\epsilon_{\alpha\beta\mu\lambda}$ is the completely anti-symmetric Levi-Civita symbol.

In evaluating expressions of the form

$$N = \epsilon_{\alpha\rho\delta\lambda} A_\alpha B_\beta C_\delta \epsilon_{\mu\nu\sigma\lambda} a_\mu b_\nu c_\sigma, \qquad (A.4)$$

which occur in equations (3.7) and (5.7) , we proceed as follows:

From the identity

$$\epsilon_{\alpha\beta\delta\lambda} A_\alpha B_\beta C_\delta D_\lambda =$$

$$= D_0 (\vec{A}.\vec{B}\times\vec{C}) - \vec{D}.\left[A_0(\vec{B}\times\vec{C}) + B_0(\vec{C}\times\vec{A}) + C_0(\vec{A}\times\vec{B})\right] \qquad (A.5)$$

it follows that

$$L_\lambda = \epsilon_{\alpha\beta\delta\lambda} A_\alpha B_\beta C_\delta = (L_0, \vec{L}), \qquad (A.6)$$

where

$$L_0 = (\vec{A}.\vec{B}\times\vec{C}), \qquad (A.7)$$

and

$$\vec{L} = A_0(\vec{B}\times\vec{C}) + B_0(\vec{C}\times\vec{A}) + C_0(\vec{A}\times\vec{B}) \qquad (A.8)$$

Therefore the scalar (A.4), where A, B, C, a, b, c are all 4-vectors is given by

$$N = (\vec{A}.\vec{B}\times\vec{C})(\vec{a}.\vec{b}\times\vec{c}) -$$

$$- \left[A_0(\vec{B}\times\vec{C}) + B_0(\vec{C}\times\vec{A}) + C_0(\vec{A}\times\vec{B})\right] \times$$

$$\times \left[a_0(\vec{b}\times\vec{c}) + b_0(\vec{c}\times\vec{a}) + c_0(\vec{a}\times\vec{b})\right] \qquad (A.9)$$

Captions to Figures

Fig. 1. Notation

Fig. 2. Angular distribution for $P_{3/2}$ N^* for constant form factors and initial nucleons both polarized.

Fig. 3. Angular distribution for $D_{3/2}$ N^* in pp reactions and in $n\,p \longrightarrow n\,p\,\pi^o$, for constant form factors and initial nucleons both polarized.

Fig. 4 Angular distribution of $D_{3/2}$ N^* as inferred from the reaction $np \longrightarrow n\,n\,\pi^+$ for constant form factors and initial nucleons both polarized.

Fig. 5 Angular distribution of $P_{3/2}$ N^* with form factors and initial nucleons both polarized.

Fig. 6. Angular distribution for $D_{3/2}$ N^* as inferred from the reaction $n\,p \longrightarrow n\,n\,\pi^+$, with form factors and initial nucleons both polarized.

Fig. 7. The contribution (I) to the recoil nucleon polarization when only one initial nucleon is polarized.

Fig. 8. The contribution (II) to the recoil nucleon polarization when only one initial nucleon is polarized.

Fig. 9. Polarization correlation between target nucleon and recoil nucleon, for $P_{3/2}$ N^* production with π exchange and constant form factors.

Fig. 10. Polarization Correlation between target nucleon and recoil nucleon, for $P_{3/2}$ N^* production with π exchange and the form factor (3.20)

Fig. 11. The contribution (I) to the polarization correlation functions for $P_{3/2}$ N^* production with ρ exchange and constant form factors.

Fig. 12. The contribution (II) to the polarization correlation functions for $P_{3/2}$ N^* production with ρ exchange and constant form factors.

Fig. 13. Polarization correlation functions for $P_{3/2}$ N^* production with ρ exchange and the form factor (3.21) . The curves A, B, C, ... denote the contribution (I) , while the curves A', B', C' ... . denote the contribution (II).