39

39

MATSCIENCE REPORT 39

THEORIES OF GRAVITATION

(Based on the lectures delivered at MATSCIENCE)

J. V. NARLIKAR,

Visiting Scientist, Matscience

-ed by

Dr. R. VAS VAN

39

E INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-20. INDIA.

39

39

MATSCIENCE REPORT 39

THEORIES OF GRAVITATION

(Based on the lectures delivered at MATSCIENCE)

J. V. NARLIKAR,

Visiting Scientist, Matscience

~~~d by

Dr. R. VAS VAN

39

IE INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS-20. INDIA.
~~~

THE INSTITUTE OF MATHEMATICAL SCIENCES

MADRAS-20 (India)

MATSCIENCE REPORT 39

THEORIES OF GRAVITATION

(Based on the lectures delivered at MATSCIENCE)

Dr. J.V.Narlikar*

Prepared by

Dr. R.Vasudevan

---

* King's College, Cambridge University, Cambridge, U.K.

PART I

Contents

PART II

Contents.

* * * *

LECTURES ON GRAVITATION THEORIES*

Dr. J.V.Narlikar

***

1. Introduction

It is but proper to begin a talk on the theory of Gravitation by remembering the famous laws of Newton defining the gravitational force between the two masses m and m' as

$$F = G \frac{m m'}{r^2} \tag{1}$$

Along with this one should also bear in mind Newton's equation of motion given by $F = m\ddot{x}$ and the philosophy of 'action at a distance', by which the gravitational effects caused by the mass travelled with infinite velocity and affected the mass m instantaneously. In electromagnetism also a law similar to (1) described the force between two static charges $F = \frac{k q_1 q_2}{r^2}$ however, this was found inadequate to describe the phenomenon when charges were in motion. Even as early as 1846 Gauss emphasised that the action at a distance required modification in that this action should travel with a finite velocity. Hence when a charge moves the disturbance caused by its motion moves with a finite velocity i.e. the velocity of light. This calls for a field theory of interaction between charged particles and the most successful and brilliant culmination of the field concept was the formulation of Maxwell's Equations, in electrodynamics and their covariance under special relativistic transformations.

---

* Lectures delivered on 24th and 25th February, 1965 at the Institute of Mathematical Sciences, Madras-20.

2. The Electromagnetic Field

To derive Maxwell's equations from an action principle, taking into account the linearity of the equation, we can write for the action,

$$J = \sum \cdot \int m_a c\, da + \sum e/c \int_A A_i\, dx^i + \frac{1}{16\pi} \int F_{ik} F^{ki}\, d\Omega \tag{2}$$

where the charges are lagelled by a, b, ... and where m_e is the mass of the charge. The metric is given by

$$da^2 = \eta_{ik}\, dx^i dx^k \quad \text{with} \quad \eta_{ik} \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & +1 \end{bmatrix} \tag{3}$$

and

$$F_{ik} = \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \right)$$

A represents the vector potential of the field d_a represents the invariant interval in the world line of the particles and the summation extends over all the particles.

Consider variation of J and equate it to zero. If we assume the field to be given and vary the trajectory of the particle we get the equation of motion, relating the acceleration of the charges to the Lorentz force acting on them

$$m_a \frac{d^2 a^i}{d a^2} = e/c \, F^i_k \frac{d a^k}{d a} \tag{4}$$

Assuming the motion of the charges to be given and varying only the field that is only the potentials, we obtain Maxwell's equations,

$$F^{ik}{}_{,k} = \frac{4\pi}{c} J^i \tag{5}$$

Using the definitions of the Electromagnetic field tensors, we can verify that

$$\frac{\partial F_{ik}}{\partial x^l} + \frac{\partial F_{kl}}{\partial x^i} + \frac{\partial F_{li}}{\partial x^k} = 0 \tag{6}$$

Equations (5) and (6) completely determine the electromagnetic fields and are the fundamental equations of electrodynamics.

$$da^2 = g_{ik} \, da^i da^k$$

3. <u>Principle of Equivalence</u>.

Maxwell's equations are most elegant in their four dimensional form expressing their covariance under Lorentz transformations. They proved to the hit the invariance of the velocity of light, with which electrodynamic effects propagated. Carrying over the idea into gravitation, a field theory was formulated by Einstein. He postulated the ingeneous idea, that gravitation modified the space-time geometry of the universe and he took the general line-element

$$da^2 = g_{ik}\, da^i\, da^k \qquad (7)$$

where the g_{ik}'s are not constants like the η_{ik}'s but are function of the space-time at each point. Of course it is possible to diagonalise this g_{ik} into η_{ik} at only one point with $\frac{\partial g_{ik}}{\partial x^k} = 0$, around that point. This cannot be achieved at all points at once. (i.e.). We can work against gravity in an infinitesimal region around a point by putting an acceleration against it but cannot be do the same at all points simultaneously in a finite region. This is the essence and limitation of the <u>principle of equivalence</u>.

According to this principle, we can always replace a gravitational field by an accelerated frame locally in an infinitesimal region near a point. That is $g_{ik} = \eta_{ik}$ and $dg_{ik}/dx^l = 0$ near that point. However all the second derivatives of g_{ik}

can never be made to become zero by any transformation of coordinates. Therefore we have to construct a theory which uses invariants made out of the second derivatives of g_{ik}.

From the above, we see that an actual gravitational field cannot be eliminated over all space by any transformation, of coordinates i.e. the presence of gravitational field is such that the quantities g_{ik} cannot by any transformation be brought to their Galilean values, over all space-time. Such a space time is said to be curved or non-Eucledian and the ordinary laws of Eucledian geometry are not valid, and because of the strange metric properties of space the concept of definite distance between two given space points loses its meaning; remaining valid only for infinitesimal distances. Also the rate of a clock is different at different points in space in one and the same frame of reference in the presence of gravitational fields.

4. The Curvature Tensor

A Euclidean space, is characterised by a quantity called the curvature tensor. It ahppens that in a non-Euclidean space the parallel transport of a vector from one given point to another gives different results if the displacement is carried out over different paths. In particular it follows that if we displace a vector parallel to itself along some closed contour, then upon returning to the starting point, it will change. For such an infinitesimal closed contour, the change in a vector A_k is given by

$$\Delta A_k = \tfrac{1}{2} R^i_{\ klm} A_i \Delta f^{lm}$$

or

$$\Delta A^k = \tfrac{1}{2} R^k_{\ ilm} A^i \Delta f^{lm} \tag{8}$$

where Δf^{lm} is the area of the surface bounded by the contour and $R^i_{\ klm}$ is called the curvature tensor or the Riemann-Christoffel tensor. It also turns out that when the covariant derivative of a vector A_i, with respect to x^k and x^l are taken the result generally depends on the order of differentiation and

$$A_{i;k;l} - A_{i;l;k} = A_m R^m_{\ ikl} \tag{9}$$

Clearly in Euclidean space $R^{i}_{klm} = 0$ and parallel transport is a unique operation. The converse is also valid i.e. if $R^{m}_{ikl} = 0$ the space is Eucledian. The complete expression for $R_{iklm} = g_{in} R^{n}_{klm}$ is given by

$$R_{iklm} = \frac{1}{2}\left(\frac{\partial^2 g_{im}}{\partial x^k \partial x^l} + \frac{\partial^2 g_{kl}}{\partial x^i \partial x^m} - \frac{\partial^2 g_{il}}{\partial x^k \partial x^m} - \frac{\partial^2 g_{km}}{\partial x^i \partial x^l}\right) + g_{np}\left(\Gamma^{n}_{kl}\Gamma^{p}_{im} - \Gamma^{n}_{km}\Gamma^{p}_{il}\right) \qquad (10)$$

where Γ^{n}_{kl}'s are the Christoffel symbols defined by

$$\Gamma^{n}_{kl} = \frac{1}{2} g^{nm}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right) \qquad (11)$$

Thus R^{m}_{ilk} is a tensor made out of the second derivatives of the metric tensor and possesses well defined symmetry properties and contracting two of the indices gives a second rank tensor

$$R_{ik} = R^{l}_{ilk}, \quad R_{ik} = R_{ki} \qquad (12)$$

which is symmetric in its indices.

Finally contracting R_{ik} we obtain the invariant

(13)

which is called the scalar curvature of the space-time continuum. It was known that the R_μ satisfy other well-known symmetry properties and also obey what are called Bianchi identities. If we construct a general second rank tensor from the $R_{\mu\nu}$ it can be of the form

$$C_1 R_\mu^\nu + C_2 g_\mu^\nu R + C_3 g_\mu^\nu = B_\mu^\nu \qquad (14)$$

But choosing the constants here suitably it can be seen that

$$\left(R_\mu^\nu - \tfrac{1}{2} \delta_\mu^\nu R \right)_{,\nu} = 0 \qquad (15)$$

It is also thought-provoking to note that T_μ^ν which can be taken to represent the stress energy tensor of matter or radiation field or both fields together satisfy the conservation law of the form

$$T_{\mu,\nu}^{\nu} = 0 \qquad (16)$$

5. Einstein's Equations

Bearing the above ideas in mind, if we want to construct a variational principle to find the field equations in the presence of gravity, we should start with an action function,

$$J = J_G + J_F$$

where J_G is due to the gravitational part and J_F action due to all other fields existing. We write down the action for chafges in a combined Electromagnetic and gravitational field as

$$J = \sum_i \int m_a c\, da + A \int F_{ik} F^{ik} \sqrt{-g}\, d^4x + B \int A_i\, da^i + C \int R \sqrt{-g}\, d^4x \tag{17}$$

Here the most suitable values of A, B and C are obtained if we put

$$A = 1/16\pi c, \quad B = e/c \quad \text{and} \quad C = \frac{c^3}{16\pi G} \tag{18}$$

The choice of C is dictated by the fact that the field equations should reduce to the Poisson equation of the Newtonian Mechanics in the weak field approximation, G being the gravitational constant g is the determinant of the metric g_{ik} and it is important to note that $\sqrt{-g}\, d^4x$ is the invariant infinitesimal volume element in a curvilinear system, and not simply d^4x as in flat space.

Variation of the action J with respect to the metric tensor g_{ik} leads to the celebrated field equations of Einstein as given below:

$$\left.\begin{aligned} R_{ik} - \tfrac{1}{2} g_{ik} R &= \frac{-8\pi G}{c^4} T_{ik} \\ \text{or} \qquad R_i^{\ k} - \tfrac{1}{2} \delta_i^{\ k} R &= \frac{-8\pi G}{c^4} T_i^{\ k} \end{aligned}\right\} \tag{19}$$

Here T_{ik} is the sum of the stress energy tensor of both the matter field and the electromagnetic field put together. Varying the coordinates only we get the equations of motion in a gravitational field, which in the presence of the electromagnetic field are given by

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{kl} \frac{dx^k}{ds} \frac{dx^l}{ds} = \frac{c}{m_0 c^2} F^i_{\ k} \frac{dx^k}{ds} \tag{21}$$

which is the geodesic equation of a charged particle. The negative sign on the right hand side of the equation (19) is needed if gravitation is an attraction. But the equations by themselves do not prefer any particular sign for G. In an empty space $T_{ik} = 0$. Therefore,

$$R_{ik} - \tfrac{1}{2} g_{ik} R = 0 \tag{22}$$

This does not mean that in vacuum the space time is flat. Only if $R_{iklm} = 0$ can we say that the space is flat? It is remarkable to note that the field equations imply the equations of motion for localised sources. This statement leans heavily on the improved understanding of the relationship between covariance under arbitrary coordinate transformations and the structure of the field equations. The covariance of the theory of gravitation with respect to arbitrary coordinate transformations require that the field equations satisfy four differential identities- the contracted Bianchi identities since the source of the gravitational fields lies in the distribution of matter as described by matter tensor $T_{\mu\nu}$. The Bianchi identities impose restrictions on the matter tensor. Thus the distribution of matter cannot be arbitrarily assigned if a solution of the field equations has to exist. Conversely when equations (19) are satisfied the equations of motion follow quite easily. The solution of the field equations which are non-linear has been attempted for various boundary conditions and different matter density distributions. For a theory of such importance underlying the structure of space-time and as is sometimes suggested perhaps the structure of elementary particles as well-general relativity has led to remarkably few successful experiments. The four predictions the theory does make require an alomost impossible precision for any decissive measurement. Such precision has been realised only for three experiments in the past. Probably more will follow soon.

6. Experimental Verification[3)]

However, much this theory may be dished up by the elegance and cogency of mathematical arguments it might not have carried conviction, but for the spectacular experimental verification of its prediction in three cases. These are essentially due to the second order effects introduced in this theory, that could not have been thought of in the simple Newtonian theory. Considering the action in a non-relativistic form, we write for J

$$J = -mc \int \left(c\,dt - \tfrac{1}{2}\frac{\vec{V}}{c}\, d\vec{r} + \tfrac{1}{c}\,\phi\, dt \right) \tag{22}$$

Corresponding to this for a spherically symmetric gravitational field, the interval ds can expressed as:

$$ds^2 = \left(c^2 - \frac{2Gm}{r}\right) dt^2 - r^2\left(\sin^2\theta\, d\phi^2 + d\theta^2\right) - \frac{dv^2}{1 - \frac{2Gm}{c^2 r}} \tag{23}$$

For this metric employing the equation of motion for a test particle in the field of the sum, we end up with an equation of the type

$$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{A^2} + \frac{3GM}{c^2} u^2 \tag{24}$$

where $u = 1/r$. The effect of the second term in the equation is to yield the solution of a rotating ellipse, with the apses of the orbit advancing at the rate of δ radiam per revolution in the same sense as the revolution of the particle in the orbit.

$$\delta \quad \frac{6\pi G M}{c^2 a (1-\epsilon^2)}$$ where a is the semi major axis of the ellipse

(25)

The planet Mercury being nearest to the sun - can show a maximum effect. Taking intoaccount the perturbations due to other planets, this precession was calculated to be 41.25 ± 2.0 per century. The most recent estimate is 42.56 ± 0.94 per century.

For a light ray propagating in a gravitational field we know that $ds = 0$ and this implies that A in equation (24) goes to , and we have exactly the same equation as (24) without the first term on the right hand side. Integration of this equation leads to the result that the net angle between the initial and final directions would be

$$\Delta = \frac{4GM}{c^2 R} \tag{26}$$

For a light ray that is skirting round the sun, this foregoing predicts a deflection of 1.75. It has been demonstrated by Schiff that this result can also be obtained more directly without going into the detailed structure of general relativity, starting from the premises of the principle of equivalence alone. The procedure adopted is to photograph the position of a group stars in the angular vicinity of the sun, during the time of total eclipse and compare it with their positions in the absence of the gravitational field of the sun. There is a general agreement between the predictions and the observed results.

The third crucial test of the theory relates to the retardation of the clock rates in a gravitational field; considering the stationary clock at two points A and B for the same interval or coordinate time there correspond different proper times recorded by the clock due to difference in g_{44} at the two points. Considering atomic processes like emission of light taking place at A and at B, the ratio of the proper frequencies associated with the processes is given by

$$\frac{\nu_B}{\nu_A} = \frac{\sqrt{(-g_{44})A}}{\sqrt{(-g_{44})B}} \tag{28}$$

If the field at A is more negative than that at B, $\nu_B < \nu_A$ Thus the presence of a gravitational field causes a gravitational red shift of the spectral line emitted at a high field when observed at a low field. For the case of light emitted at the surface of the sun and observed at the earth

$$\frac{\Delta\nu}{\nu} = -2 \cdot 12 \times 10^{-6}$$

This is usually in terms of a velocity which will produce an equivalent Doppler shift. Though the experimental confirmation of the magnitude of this effect for solar radiation as well as from radiations from other stars has not been without ambiguities, phenomenal progress has been achieved in studying this effect by experiments utilising the remarkable discovery of Mossbauer effect. Mossbauer found that extremely sharp low energy γ radiation emitted by long lived isometric states of nuclei can be obtained recoil free since the recoil momentum is taken up by the solid as a whole, with the result that there is no Doppler shift. The lines can be so sharp that their absorbtion by another piece of the same substance can be destroyed by a relative speed of order of 1 cm/sec between the source and the absorber, i.e. by the introduction of a fractional frequency displacement of only about 10^{-10}. Fur experiments on the earth level difference of h cms near the surface of the earth, would result in a fractional shift $\frac{\Delta\nu}{\nu} = 1.09 \times 10^{-18} h$ Rebka, Ponnd and others used the resonant absorbtion of the 14.4 kev gamma radiation emitted by 10^{-7} sec Fe^{57} and after taking into account the temperature differences between the sources and the absorber and other inherent contribution to this effect verified this phenomenon to a great order of accuracy. For a level difference of 74 ft. the expected gravitational shift for a two-way passage was about 4.92×10^{-15} times the mean frequency. The experimental data yielded a net fractional shift of $(5.13 \pm 0.51) \times 10^{-15}$ cm

(4) Another experiment of great promises is to investigate the effect on the orientation of the spin axis of a gyroscope in the field of the rotating earth. (This has been discussed by Schiff in his lectures on Gravitation (4)). According to Newtonian theory such a gyroscope in the absence of bearing friction etc. should point indefinitely in its initial direction relative to space as determined by fixed stars. Both general and special relativity predict a precession of the spin axis, due to the following three reasons.

(1) According to the notion of parallel transport of a vector say the spin axis around a circle it will not return to its initial orientation, this effect being called goodetic precession

(2) In the vicinity of a rotating mass the inertial frame is dragged around slightly at a small fraction of the angular velocity of its mass. Thus a gyroscope even at rest near the earth withits axis not coanial with the earth will experience a change in orientation relative to fixed stars. This is referred to Lense-Thirring precession.

(3) A spinning object on the earth in motion experiencing non-gravitational forces will precess according to special relativity. For a gyroscope on the earth at the equator with spin axis normal to the earth's axis all the three effects have approximately the same value, .4 second of arc per year. To develop adequately stable gyroscopes, one involving superconducting spinning sphere magnetically supported has been investigated by Fairbraches et al.

VII. Mach's principle (5)

Very refined repetitions of Eotvos balance experiment reveal the complete identity of inertial and gravitational masses of a body. The concept of inertial dates back to Galileon. Newtom's second law applies to motions measured relative to inertial frames. If motion is measured relative to other frames additional forces like centrifugal forces come into play. These are also called inertial forces. Newton's water filled bucket experiment indicates that whenever rotation occurs relative to a specific reference frame, the surface of water become concave. This is an absolute effect. Thus according to Newton there is a 'true' acceleration and a 'true' rotation characterized by absence of other inertial forces. It turns out that this absolute reference frame of Newton relative to which inertial forces are observed is the fixed frame in which distant objects or the start of the universe remain constant and non-rotating. If the form of the dynamical law depends on the presence and the motion of distant matter a long range interaction between the latter and the objects in the laboratory must play an essential role. The essence of the Machian principle though rather speculative is that the inertia of a body is the direct outcome of its direct interaction with other bodies in the universe. Newton's law $F = ma$ connects the local force F exerted on the mass m with the inertial force ma which as we have seen is the result of the action on m of the rest of the universe. Thus though Newton's space has an absolute structure which is beyond the matter it contains, the view of Mach is based on a logical positivist philosophical stance. It asserts that physical concepts must be based on operational definitions through measurements. An empty universe according to Mach is devoid of physically measurable properties, and that the inertial properties must have their ultimate origin in matter contained in the universe. Although Einstein was motivated by considerations of Mach's ideas, the principle was not incorporated into general relativity. However, matter distribution does affect the geometry in the general relativity. A particularly nice effect is the Lense Thirring effect which showed that a rotating massive spherical shell acts to pull an inertial coordinate system inside the shell along with it partially. However, the difficulty in the Mach's principle within the frame work of General relativity is thrown into sharp focus by considering a universe empty except for a single test particle. The Einstein's field equation has a solution and determines the motion of the test particles even if T_{ik} is identically zero, by describing the gravitational effects as resulting from a tensor field. Einstein was able to exhibit the inertial force proportional to the acceleration of a particle as one of the force terms derived from the tensor field. As seen above in the absence of all matter, the metric tensor describes a flat space, and this flat space possesses inertial properties. Even Schwartzschild's famous solution is unsatisfactory from the point of view of Mach. As one moves to infinity and the mass source disappears in the distance the space becomes flat and continues to possess inertial properties in contradiction with the expectations of Mach. However, one way out of this difficulty is to say that we have to prescribe initial and boundary conditions for the field equations and all non-Machian solutions may be excluded in that way.

VIII. Stead State Cosmology

The coincidence of the inertial frame of Newton with that in which the distant parts of the Universe are non-rotating can be deduced from normal relativistic Cosmology. Though this may appear puzzling the reason for this is that relativistic cosmology contains two postulates which are extraneous to general theory of relativity. They are (1) Weyl's postulates (i.e.), the particles of the substratum representing the nebulae lie in the space time of the cosmos on a bundle of geodesics diverging from a point, in the (finitely or infinitely) distant past; (2) The cosmological principle. This states that the universe as a whole when looked upon from a large scale point of view is spatially homogeneous and isotropic. For all observers in the universe a common cosmical time may be adopted, so that at any given instant of this time, the metric of space-time continuum representing the development of our smeared out universe is everywhere the same. These assumptions after considerable investigations load to the Robertson-Walker line element of the form

$$ds^2 = dt^2 - s^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] \tag{29}$$

Now a transformation $r' = r\,s(t)$ $t' = t + \frac{1}{2}\frac{s'}{s}r'^2$ where $S = \exp(Ht)$, H being the Hubble's constant, for sufficiently small r leads for any local experiment to a metric of the special relativity type. θ and φ are not changed in this transformation. r', θ, φ, t' represent an inertial frame for a local experiment. Any rotation relative to this produces inertial forces. In other words this is the special frame indicated by the bucket experiment. Since φ and θ have not been changed for any t' -the distance particle of the universe are not rotating as required by Mach's principle. The usual method in cosmology is to take Einstein's field equations

$$R^{ik} - \frac{1}{2}g^{ik}R + \lambda g^{ik} = -kT^{ik} \tag{30}$$

and insert the values of g^{ik} from the line element (29) and find that T^{ik} can be expressible in terms of matter density pressure and flow vectors of matter in the universe. If we want to stick to Mach's principle we should read the equation from right to left and ask the question whether for a given T^{ik} do the equations lead uniquely to the line element (29)? Godel[5] has shown that for a normal form T^{ik} the solution for the line element is of a form which is fundamentally different and if we want a local special relativistic frame then the distant matter possess rotation and Mach's principle is not satisfied. Interpreted as above Mach's principle is not incorporated in General Relativity. Hence within the framework of the usual theory, what could be done is to take an initial space-like surface, and define coordinates on it to give the Robertson-walker line element and also specify the matter and Kinematical situations and the quantities $g_{\mu\nu}$ $\frac{\partial g_{\mu}}{\partial x^{\nu}}$, $\frac{\partial^2 g_{\mu\nu}}{\partial x^{\gamma}\partial x^{\mu}}$ consistent with the field equations to calculate the metric tensors off the initial surface. Specifications on the initial surface can be so chosen that the line element is of the Robertson Walker form everywhere. That is the initial boundary conditions were supposed to be imposed at the origin of the universe in such a way that it just happens that out of a number of possible solutions, the one with the required line element was chosen. Thus it may be said that Newton's concept of 'absolute space' has been replaced by initial boundary conditions on matter and the metric tensors.

The main goal of the steady state cosmology is to dispense with initial boundary conditions. In the steady state theory of Bondi and Gold, the perfect cosmological principle is used which states that the universe as a whole always remains in the steady state. Hence the coincidence we observe must be there always. In other words the perfect cosmological principle serves for the proper initial boundary conditions. Also there are no equation occurs.

In Hoyle's approach(7), Einstein's equations are modified through the introduction of a new field. Of course, in view of the curious situation surrounding Mach's principle, and the general role of boundary conditions modifications can be thought of in an infinity of ways. The simplest field that one can introduce is a scalar one denoted by C . In analogy with the theory of electromagnetism, the action function incorporating this field is written down as

$$J = \frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x - \sum m\int ds + \frac{1}{2}\int C_{,i}C^{,i}\sqrt{-g}\,d^4x$$

$$- \sum m \int C_i \frac{dx^i}{ds}\, ds \qquad (31)$$

where f is the coupling constant. The condition $\delta T = 0$ taken with respect to the metric tensors and independently with respect to the field C yield the equations

$$\left.\begin{aligned} R^{ik} - \tfrac{1}{2} g^{ik} R &= -\kappa \left[T^{ik} - f\left\{ C^i C^k - \tfrac{1}{2} g^{ik} C^l C_l \right\} \right] \\ C^i_{;i} &= \tfrac{1}{f} j^i_{;i} \qquad T^{ik}_{,i} = f C^i C^k_{;k} \end{aligned}\right\} \quad (32)$$

where j^i is the mass current. In equation (32) it is not necessary that both sides vanish separately since in this theory world lines of matter may end or begin at various points in space-time. With these field equations, provided we argue that in homogeneous isotropic case that C field is a function of t only, the solutions are consistent with Robertson-Walker line element; and for $C = 0$, they reduce to the usual cosmological equations. Even with $C^i_{;i}$ no significantly different solutions are obtained. If, however, $C^i_{;i} \neq 0$ possibility of creation or annihilation is allowed. We allow an infinitesimal perturbation and we obtain different class of solutions which as $t \to \infty$ reach a steady state solution. Also the matter density and the creation rate tend to a positive steady value per unite volume. i.e. in the asymptotic case creation and expansion are in exact balance. Further in the case $C^i_{,i} \neq 0$ it turns out that $C = t$ at all t. So reverting to Mach's principle we formulate the following problem. Set up surfaces $C =$ constant and define $t = C$. Suppose matter and velocity distribution is given on this C

surface. To permit a calculation off this surface, we specify, metric tensor and its derivatives. As C increases what happens? Using a perturbation calculation, Hoyle and Narlikar were able to show that if we take an initial line element, as C increases over any specified volume, the line element tends to a homogeneous isotropic form as time increase, provided that continuous creation acts in such a way as to smooth out any initial anisotropy or inhomogeneity over any specified proper volume. Like in the electrical circuits, once the generator is switched on and runs for a time the transients die away and a steady state is maintained. If the C field which is the driving force of the universe is present with $C^{i}_{;i} \neq 0$ the universe attains the observed regularity, irrespective of its initial boundary conditions.

PART II

I. Action at a Distance

The field concept has been so very successful in Electromagnetic theory that we have come to consider the field as a physical system in its own right interacting with particles. These ideas have considerably discouraged the general use of the tools which Gauss and Newton employed for analysis of forces, namely the concept of action at a distance (propagated with a finite velocity). However the field picture led to infinite self energy and to the necessity of mass renormalisation etc. It is becoming increasingly evident that it is possible to eliminate the concept of field and introduce only direct interaction between particles for description of different phenomena. The advantage is that a single particle in this approach has no self-energy. As is expounded by Wheeler and Feynman the formalism developed by Schwarzschild, Tetrode and Fokker in the case of electromagnetic theory makes no use of the notion of a field. Each particle moves in compliance with the principle of stationary action

$$J = -\sum m_a \int da - \sum_{a \neq b}\sum \tfrac{1}{2} c_a c_b \iint \delta(ab_i\, ab^i)\, \eta_{lm}\, da^l\, db^m \qquad (2.1)$$

Here we are now concerned with the time symmetric theory of action at a distance in a flat space-time. We therefore have η_{ik} diagonal $(-1, -1, -1, +1)$ and the a^{th} particle

has charge C_a and mass m_a and has coordinates (✡).
The line element .

(✡) $da^2 = \eta_{ik} da^i da^k$

and $(ab_i \, ab^k) = \eta_{ik}(a^i - b^i)(a^k - b^k)$ (2.2)

All of mechanics and electrodynamics is contained in this single variational principle without meeting with the difficulty of action of the charge on itself. However, in this formulation we have two other headaches, namely: (1) The field quantities come out to be the sum of half the advanced and half the retarded solutions of Maxwell's equations. To avoid the inclusion of advanced fields, so contrary to experience, Tetrode proposed to abandon the symmetry in the time of the elementary law of force. But this is no cure at all since (2) we have to explain the fact that the accelerated charge suffers a force of damping which is simultaneous with the moment of acceleration. Wheeler and Feynman resolved this impasse by taking into account the suggestion of Tetrode that the act of radiation has everything to do with the presence of the absorbers, in the universe.

II. Maxwell's Field from direct Particle Interactions

Let us now get back the Maxwell's equations from the varying of action J by altering the world line of particle a say $a^i(a)$ to $a^i(a) + \delta a^i(a)$. The $\delta(x b_\mu \, x b^\mu)$ occurring in the action is nothing but the Green's function $\bar{D}(x-b)$ which is symmetric in its variables, and in past and future.

i.e.

$$\bar{D}(x) = \tfrac{1}{2}\left[D^{ret}(x) + d^{adv}(x)\right] \tag{2.3}$$

We define the four potential of b at a point x by the function

$$A_m^b(x) = \int e_b \delta(x b_i \, x b^i)\, \eta_{mk}\, d b^k \tag{2.4}$$

From the variation of the action we obtain the equation of motion of the form given below yielding the usual Lorentz force on the right hand side.

$$m_a \frac{d^2 a^k}{d a^2} = e_a \sum F_l^{k(b)} \frac{d a^l}{d a} \tag{2.5}$$

where

$$F_{kl}^{(b)}(x) = \frac{\partial A_l^b(x)}{\partial x^k} - \frac{\partial A_k^{(b)}(x)}{\partial x^l} \tag{2.6}$$

represents the field of the charge b at X . Also making use of Dirac's identity

$$\eta^{ik}\frac{\partial^2}{\partial x^i \partial x^k} D^-(x-b) = -4\pi\,\delta(x^1-b^1)\,\delta(x^2-b^2)\,\delta(x^3-b^3)\,\delta(x^4-b^4) \tag{2.7}$$

we can easily find

$$\Box A_l^{(b)}(x) = 4\pi j_l^{(b)}(x) \tag{2.8}$$

where $j_l^{(b)}$ the current is defined as

$$j_l^{(b)}(x) = e_b \int_{-\infty}^{\infty} \delta(x-b)\,\eta_{lm}\frac{db^n}{db}\,db \tag{2.9}$$

Also one can easily see that $A_m^{(b)}$ satisfies the gauge conditions

$$\frac{\partial A^{m(b)}(x)}{\partial x^m} = -4\pi j_n^{(b)}(x) \tag{2.10}$$

and we get the Maxwell equations from

$$\frac{\partial F_n^{m(b)}(x)}{\partial x^m} = -4\pi J_n^{(b)}(x) \tag{2.11}$$

$$\frac{\partial F_{kl}^{(b)}}{\partial x^m} + \frac{\partial F_{lm}^{(b)}}{\partial x^k} + \frac{\partial F_{mk}^{(b)}}{\partial x^l} = 0 \tag{2.12}$$

The time symmetric nature of the theory is brought out by the fact that the solution $A_i^{(b)}$ is to be distinguished from all solutions of Maxwell's equations, by being half the sum of the advanced and retarded Lienard Wiechert potentials produced by the particle b. Let us write the solution for the field as

$$A_m^{(b)}(x) = \tfrac{1}{2} R_m^{(b)}(x) + \tfrac{1}{2} S_m^{(b)}(x) \tag{2.13}$$

Here

$$R^{m(b)} = \frac{e\, b^m}{b_\mu\, b x_\mu}$$

is the retarded potential evaluated at that point on the world line of b which intersects the light cone drawn from the point of observation into the past. $S_m^{(b)}(x)$ is a similar expression but evaluated at the point where the future light cone from X intersects the world line of b. The theory treats the advanced and retarded potentials in entirely symmetric fashion.

III. Radiation Damping

A charged particle on being accelerated sends out electromagnetic energy and itself loses energy. This loss interpreted as caused by a force acting on the particle, given in magnitude and direction by $\frac{2}{3}\frac{e^2}{c^3}$ where a is the rate of change of acceleration when the particle is moving slowly. The origin of this force of radiative reaction is not clear. Dirac advances no explanation for the origin of this damping force but supplies a well defined and relativistically invariant prescription to calculate its magnitude. From the motion of a particle calculate the field produced by it from Maxwell's equation with the boundary condition that at large distances, the field shall contain only outgoing waves.

In addition to the so defined retarded field of a particle calculate its advanced field. Define half the difference between the advanced and retarded fields as radiation field. This is finite everywhere. Evaluate it at the position of the charge and multiply it by the magnitude of the charge to obtain the force of radiative reaction. No physical explanation of this had come out till Feynman and Wheeler, took up the idea of Tetrode that the absorbers in the Universe are an essential element of the radiation process. Adopting this to the idea of action at a distance, it was assumed that (1) an accelerated charge does not radiate if there are no charges in space (2) the fields which act on the particle arise from other charges (3) These fields are sum of 1/2 retarded and 1/2 advanced Lienard-Wiechert solutions (4) Sufficiently large number of absorbers are present to absorb completely the radiation given off by the source. When a charge is accelerated a disturbance travels from the charge to the absorbers and each particle of the absorber is set in motion, and generates a field half advanced and half retarded. The sum of all the advanced effects of the absorbers evaluated in the neighbourhood of the sources, gives a field which has the property that it is independent of the absorbing medium and exerts a force on the charge which is finite and is simultaneous with the moment of acceleration. These absorbers therefore are the physical origin of the Dirac's radiation field. A simple minded way to derive the radiation reaction is to imagine that the accelerated charge sends out the retarded field of Maxwell's solution of our normal experience to the distant absorber at r_k which is given by

$$-\frac{c\vec{\alpha}}{r_k e^2} \sin(\vec{\alpha}, \vec{r}_k) \qquad (\alpha = \ddot{a}) \tag{2.14}$$

The typical particle of the absorber which has a charge C_k and mass m_k will experience an acceleration, and will generate a field which is half retarded and half advanced. The advanced part of this field will exert on the source a force simultaneous with the original acceleration. The component of this reactive force along the direction of the acceleration will be

$$\frac{\vec{\alpha} e^2}{2c^4} \frac{C_k^2}{m_k r_k^2} \sin^2(\vec{\alpha}, r_k) \tag{2.15}$$

To evaluate the total effect due to many particles we have to take into account the number of particles in the spherical shell

$r_k, r_k + dr_k$ as $4\pi N r_k^2 \, dr_k$ being the density of absorbers. The average value of the geometrical factor $\sin^2(\vec{\alpha}_k, r_k)$ will be 2/3. It is now necessary to properly add the effects due to all particles of a complete absorber with due allowance for their phase changes. For this we have to have the Fourier decomposition of $\vec{\alpha}$ as $\vec{\alpha} = \vec{\alpha}_0 e^{-i\omega t}$ and bear in mind that a disturbance of this frequency will experience in a medium of low density a refractive index

$$n = 1 - 2\pi N c_k^2 / m_k \omega^2 \tag{2.16}$$

The radiative reaction which reaches the source from a depth γ_k will have a phase lag behind the acceleration of the particle given by

$$\omega\left(\gamma/c - \frac{n r_k}{c}\right) = \frac{2\pi N C_k^2}{m_k C \omega} \tag{2.17}$$

The sum over contributions from all depths in the medium contributing to the total reactive force is given by

$$\frac{2e^2}{3c^3} \vec{\alpha} \int_0^\infty (2\pi) N C_k^2 / m_k C \; d r_k \; \exp\left(\frac{-i r 2\pi N C_k^2}{m_k C \omega}\right) \tag{2.18}$$

$$= \frac{2}{3} \frac{e^2}{c^2} \vec{\alpha}$$

Also it can be seen that the radiation field obtained at distances a number of wavelengths away from the source is given by

$$\left[-\frac{c\vec{\alpha}_0}{2rc^2} \, e^{c i \omega r/c - i\omega t} + \frac{c\vec{\alpha}_0}{2\pi c^2} \exp\left(\frac{-i\omega r}{c} - i\omega t\right)\right] \tag{2.19}$$

equal to difference between half retarded and half advanced field which one calculates from the source itself. The advanced field of a single charge of the absorber can be symbolised as a sphere which is converging on the particle and which will collapse on it at the time when it is disturbed. The shrinking sphere just before touching the particle will appear as nearly a plane wave, which passes over it headed towards absorbers. The effect of all the absorbers is to visualise an array of plane waves all marching towards the source and marching on it in step. The result out of all this is the spherical envelope of all these planes waves converging on the source. The sphere converges on the source and then pours out again as a divergent sphere. An observer will get the impression that this divergent wave originated from the source. Hence the elementary retarded field can be written in the form

$$\begin{pmatrix}\text{retarded} \\ \text{field of} \\ \text{our experience}\end{pmatrix} = \left[\frac{1}{2}\ \text{retarded} + \frac{1}{2}\ \text{advanced}\right] + \left[\frac{1}{2}\ \text{retarded} - \frac{1}{2}\ \text{advanced}\right] \tag{2.20}$$

The second term is the radiation field of Dirac which combines with the field of the source itself to produce the usual observed full retarded field. From a more sophisticated argument we can see that

$$\sum_{k\neq a} \tfrac{1}{2}\left(F_{ret}^{k}\right) + \tfrac{1}{2}\left(F_{adv}^{k}\right) = \sum_{k\neq a} F_{ret}^{k} + \left(\tfrac{1}{2}F_{ret}^{a} - \tfrac{1}{2}F_{adv}^{a}\right) - \sum_{\text{all } k}\left(\tfrac{1}{2}F_{ret}^{(k)} - \tfrac{1}{2}F_{adv}^{(k)}\right) \tag{2.21}$$

and the third terms can be shown to be identically zero outside all absorbers by use of arguments concerning wave equations for potentials: This term is zero if one makes the assumption that all radiation emitted by all particles is absorbed by the absorbers. If a source has emitted radiation during a certain interval then a very long time afterwards $\sum_k F_{\mu\nu}^{k\,ret}$ is zero. The quantity $\sum_k F_{\mu\nu}^{k\,adv}$ is also zero, at the same time since the source is no longer emitting any radiation. Now because $\sum_k \left[F_{\mu\nu}^{k\,ret} - F_{\mu\nu}^{k\,adv} \right]$ satisfies Maxwell's equations it vanishes at all times if it vanishes at one time. The second term is that causes reaction damping. The equation of motion in the relativistic form can be written as

$$m_a a_n = C_a \sum F_{n\alpha}^{(k)} a^\alpha + \left(\frac{2C_a^2}{3c^2}\right)\left(\dot{a}_n \ddot{a}_\alpha - \dddot{a}_n \dot{a}_\alpha\right) a^\alpha \tag{2.22}$$

Hence an accelerated charge loses energy to the surrounding medium of absorbers. Why does radiation has this irreversible character, even in a formulation of electrodynamics which is symmetrical in time? If in the arguments above (since condition of absorption is symmetrical between advanced and retarded potentials) we reverse the roles of the two fields, we will arrive at an equation of motion with the radiative reaction with an opposite sign. (e.e.) the source will be supplied with energy by the absorber. However, since this does not happen, Wheeler and Feynmen concluded that irreversibility of the emission process is a phenomenon of statistical mechanics, connected with the asymmetry of initial conditions with respect to time.

IV. Retarded and advanced potentials in curved space time

All the formulae and derivations given above refer to flat space geometry. However, we can consider Riemanian surfaces[9] with the general metric tensor with line element

$$da^2 - g_{ik}\, da^i\, da^k$$

Consider a vector x^{iA} at A . From this we generate a vector by parallel transport along the geodesic from A. The vector at an arbitrary point B can be denoted by $\bar{x}^{iB}$ by a propagator $\bar{g}^{iA}_{\ iB}$

$$\bar{x}^{iB} = \bar{g}^{iA}_{\ iB}\, x^{iA} \tag{2.23}$$

where $\bar{g}^{iA}_{\ iB}$ acts as two point tensor. Looking at the elementary solutions of the scalar and vector wave equations

$$g^{ik}\phi_{;ik} = 0 \ ; \ g^{ik}A^{l}_{;ik} + R^{lm}A_m = 0 \tag{2.24}$$

the scalar and vector green's functions are of the form

$$\bar{G} = \frac{1}{8\pi}\left[\Delta^{1/2}\delta(\sigma) - V\theta(-\sigma)\right]$$

$$\bar{G}_{iAiB} = \frac{1}{8\pi}\left[\Delta^{1/2}\bar{g}_{iAiB}\,\delta(\sigma) - V_{iAiB}\,\theta(-\sigma)\right] \tag{2.25}$$

where $\sigma = \frac{1}{2}S^2_{AB}$ where S_{AB} is the distance between A and B and

$$\Delta = -\det\left[\sigma_{;i_A i_B}\right] / \det\left[\bar{g}_{i_A i_B}\right] \tag{2.26}$$

and θ is the Heaviside step function.

The first term represents the parallel propagation of the disturbance from A to B or vice versa, provided A and B are connected by a light signal. The second term arises from the curvature of space time, which causes the disturbance to spread inside the light cone. By examining the behaviour of the Green's function in the limiting case A $\rightarrow$ B and comparing it with the flat space Green's functions we obtain

$$g^{m_A k_A}\ \bar{G}_{;m_A k_A} = -(-\bar{g})^{\frac{1}{2}}\ \delta^4(A,B)$$

$$g^{m_A k_A}\ \bar{G}_{i_A i_B;m_A k_A} + R_{i_A}{}^{m_A}\ \bar{G}_{m_A i_B} = (-\bar{g})^{-\frac{1}{2}} g_{i_A i_B}\, \delta^4(A,B) \tag{2.27}$$

$$\bar{g} = \det$$

and $\delta^4(A,B)$ is the four dimensional delta function. With these modifications to the propagators $\delta(xa^{\mu}\, ra_{\mu})$ which occurred in equation (2.1), we can generalise the action function J to the form

$$J = -\sum_a m_a \int da - \sum\sum \tfrac{1}{2} c_a c_b \iint 4\pi\, \bar{G}_{i_A i_B}\, da^{i_A}\, db^{i_B} \tag{2.28}$$

For flat space time $V_{iAiB} = 0$, $\bar{g}_{iAiB} = \eta_{iAiB}$, $\Delta = 1$ $\sigma = \frac{1}{2}(ab_i\, ab^i)$. We see that (2.28) reduces to eq.(2.1). Here we obtain the four potentials

$$A_m^{(b)}(x) = \int 4\pi e_b \, \bar{G}_{m_x iB}\, db^{iB} \tag{2.29}$$

and the equation of motion of the particles a is given by

$$m_a\left[\frac{d^2a^i}{da^2} + \Gamma^i_{kl}\frac{da^k}{da}\frac{da^l}{da}\right] = e_a \sum_{b \neq a} F_k^{i(b)}(a)\frac{da^k}{da} \tag{2.30}$$

where

$$F_{ik}^{(b)}(x) = A_k^b(x)_{;i} - A_i^b(x)_{;k} \tag{2.31}$$

Because $\bar{G}_{iAiB}$ satisfy the inhomogeneous wave equation (2.27) the four potentials $A^{m_x(b)}$ satisfy

$$g^{i_x k_x} A^{m_x(b)}_{;i_x k_x} + R^{m_x}_{l_x} A^{l_x(b)} = -4\pi j^{m_x(b)} \tag{2.32}$$

where

$$j^{m_x(b)} = \int e_b \bar{g}^{m_x}_{iB} (-\bar{g})^{-\frac{1}{2}} \delta^4(x, B) \frac{db^{iB}}{db} db$$

We obtain the sets of Maxwell's equation in a generalised form

$$F^{mn}_{;m} = -4\pi j^n; \quad F_{kl;m} + F_{lm;k} + F_{mk;l} = 0 \tag{2.33}$$

Also the gauge condition is satisfied

$$A^{m_x (b)}{}_{;m_x} = 0$$

From the wave equation for A we obtain the solutions of $A_m^{(b)}(x)$ in terms of retarded and advanced functions; $R_m^{(b)}(x)$ and $S_m^{(b)}(x)$ respectively where

$$R^{(b)}_{m_x} = \frac{C_a \Delta^{1/2}\, \bar{g}_{m_x i_B} \left(db^{i_B}/db \right)}{\left[\sigma ; k_B \left(db^{i_B}/db \right) \right]} - C_b \int_{-\infty}^{b_{(-)}} V_{m_x i_B} \frac{db^{i_B}}{db}\, db \, . \qquad (2.33a)$$

$b_{(-)}$ denotes the value of b at the point where past light cone from X meets the world line of b. The function $V_{m_x i_B}$ gives contribution from the whole past section of the world line of b up to $b_{(-)}$ which is interpreted as scattering by the curvature.

V. "C" Field Theory

Modifications of this are needed when there are two or more geodesics joining two given points in space-time. This may happen when there is a dense region of matter which acts as a lens. The action function must in such cases include the sum of Green's functions over all distinct geodesics. The broken line indicates creation or annihilation at the end points. To take account of these and also from the demands of the steady state cosmology, Hoyle and Narlikar[16] introduced the C field and modified the action function. From this they are able to show that expansion of the universe and creation of particle should go together in a steady state cosmology and for the electromagnetic field the fully retarded potentials seem to be the only consistent solution in such an expanding steady state cosmology. Thus the riddle of retarded potentials of Electromagnetic field of an accelerated particle gets resolved. The asymmetry in local electrodynamics is related to the time asymmetry of the expanding universe.

The introduction of the C field seems necessary not only because of cosmological reasons. The results obtained by Hoyle and Narlikar suggest that the introduction of a non-zero C field into an implosion problem, prevents a massive object from collapsing into a singularity. The C field seems to have a repulsive gravitational effect in this situation. It is worth noting that the λ term in Einstein's field equations does give rise to a force of repulsion which however, fails to compete with the gravitational force of attraction at high densities. Since for the singularities to occur, we have broken lines, to prevent any disastrous consequences the ends should generate a field called the 'C' field and this field is a scalar. The vector electromagnetic field due to world line of a charge a is given by

$$A^{(a)}_{ix} = 4\pi e_a \int \bar{G}_{ixiA}\, da^{iA} \qquad (2.34)$$

Similarly, if we take the world line a to be a segment with ends at A_1 and A_2. A_2 being at a later time the contribution of the world line a to the total C field at the point X is defined by

$$C^a(x) = f^{-1}\left[\bar{G}(xA_2) - \bar{G}(xA_1)\right] \qquad (2.35)$$

we rewrite this as

$$C^a(x) = 1/f \int_{A_1}^{A_2} \bar{G}_{;iA}\, da^{iA} \qquad (2.36)$$

Therefore including the C field as a direct particle interaction the action J can be realised by writing it as

$$J^{(a)} = -m_a \int da - 4\pi e_a \sum e_b \iint \bar{G}_{iAiB}\, da^{iA} db^{iB} + f^{-1} \sum \iint \bar{G}_{;iAiB}\, da^{iA} db^{iB} \qquad (2.37)$$

Varying J , (by varying a), we obtain that " " lines satisfy

$$\frac{d^2 a^k}{da^2} + \Gamma^k_{lm} \frac{da^l}{da} \frac{da^m}{da} = \frac{e_a}{m_a} \sum F^{(b)kl} \frac{da^p}{da} \tag{2.38}$$

and at the ends A_1 and A_2,

$$m_a \frac{da^k}{da} + e_a \sum_{b \neq a} A^{(b)k} - \sum_{b \neq a} C^{(b)k} = 0 \tag{2.39}$$

The C field has no contribution to the usual equation of motion of a particle. There is however an effect at the end of a world line where the field permits energy and momentum to be conserved through recoil effects on other particles.

VI. Hoyle-Narlikar Theory of Gravitation

With the revival of action at a distance concept, in the case of electromagnetic field and the C-field as detailed above, there is a great incentive to put in direct particle interaction terms only in the action function

$$J = \frac{1}{16\pi G}\int R\sqrt{-g}\, d^4x - \sum_a m_a \int da - \sum\sum 4\pi e_a e_b \iint \bar{G}_{i_A i_B}\, da^{i_A} db^{i_B} + f^{-1}\sum_{a<b}\sum \iint \bar{G}_{;i_A i_B}\, da^{i_A} db^{i_B} \tag{2.40}$$

The second term in the above is derived from the concept of inertia of Galileo and since this was retained the first term had to be added. Hoyle and Narlikar[11] started out saying that both these terms are not plausible but should be replaced by a double integral term involving direct particle interactions. They assumed that mass must be a direct particle field and must arise from all other masses, to satisfy Mach's principle. Since m is a scalar it must arise from a scalar Green's function symmetric in its variables and therefore the mass function at a point X due to a is defined by

$$m^a(x) = -\lambda \int G(x, A)\, da \tag{2.41}$$

λ being the coupling constant. Omitting the C-field and electromagnetic field the action is

$$J = -\sum^{1/2} \int m_a \, da = \lambda \sum_{a<b} \sum \iint \tilde{G}(AB)\, da\, db \tag{2.42}$$

The wave function for $\tilde{G}(AB)$ has to be specified and this can be written as

$$g^{i_x k_x} \tilde{G}(x, A)_{;\, i_x k_x} + \mu R \tilde{G}(x, A) = -(-g)^{-1/2} \delta^4(x, A) \tag{2.43}$$

When the particle a is at rest when at A, we should expect $\int \tilde{G}\, da \sim 1/r$ at X, r being the 3 dimensional distance of X from A. Hence we require that $\lambda > 0$ and μ from analogy with electromagnetic field is taken as $1/6$. By varying g_{ik} in a given volume we get

$$P^{ik} = 0 \tag{2.44}$$

where P^{ik}s are symmetrical tensors and the above give the field equations.

To obtain the equations of motion we vary $X^i(a)$ and obtain

$$\frac{d}{da}\left(m_a \frac{da^i}{da}\right) + m_a \Gamma^i_{kl} \frac{da^k}{da} \frac{da^l}{da} - g^{ik} \frac{\partial m_a}{\partial a^k} = e_a \sum F^{(b)i}_k \frac{da^k}{da} \tag{2.45}$$

in which $m = \sum_{b \neq a} m^{(b)}(a^i)$

In the above geodesic property is lost, since the mass of a particle arises from all other particles in the Universe. However in some approximation m_a may be independent of coordinates and the geodesic properties can be regained.

Concentrating our attention on the wave equation for $\tilde{G}$ and putting $\delta J = 0$, we get field equations given below

$$\frac{d\mu}{\lambda}\left(R_{ik}-\frac{1}{2}g_{ik}R\right)\sum\sum_{a<b} m^{a} m^{(b)}$$

$$= -g_{ip}g_{kq}\,\underset{m}{T}^{pq} + \frac{2\mu}{\lambda}\sum\sum_{a<b}\Big[m_a\left(g_{ik}g_{(b)}^{pq}\, m_{b;pq} - m_{;ik}\right) + m^{(b)}\left(g_{ik}g^{pq}\, m^{a}_{;pq} - m^{a}_{;ik}\right)\Big]$$

$$+\frac{1}{\lambda}\sum\sum_{a<b}\Big[(1-2\mu)\left(m^{(a)}_{;i}\, m^{b}_{;k} + m^{a}_{;k}\, m^{(b)}_{;i}\right) \qquad (2.46)$$

$$- (1-4\mu)\, g_{ik}\, m^{(a)}_{;l}\, m^{(b)}_{;l}$$

Obtaining the scalar form of the above equation (2.46) and putting

$$T = \sum_a \int m_a\, \delta^{(4)}_{(x,A)}\, (-g)^{1/2}\, da \qquad (2.47)$$

we see that μ has to have a value 1/6. It can now be shown that Einstein's theory can be obtained from this by a smooth fluid approximation. In the smooth fluid approximation we take

$$\sum\sum m^{(a)} m^{(b)} = \tfrac{1}{2} m_0^2 \quad = \quad \text{(a constant)} \tag{2.48}$$

where $m(x) = \sum_a m^a(x) = m_0$. With this simplification the field equations reduce to

$$\tfrac{1}{2} m_0^2 (R_{ik} - \tfrac{1}{2} g_{ik} R) = -3\lambda T_{ik} \tag{2.49}$$

Noting that proper density

$$N(x) = \sum_a \int \delta^4_{(x,A)} [-\bar{g}(x,A)]^{-1/2} da \tag{2.50}$$

and $g^{pq} m_{;pq} + \tfrac{1}{6} R m = \lambda N$ and $T = mN$ (2.51)

we obtain

$$\tfrac{1}{6} R m_0 = \lambda N \approx \frac{\lambda T}{m_0} \tag{2.52}$$

Equation (2.49) is the same as Einstein's equation if

$$G = \frac{3}{4\pi} \frac{\lambda}{m_0^2} \tag{2.53}$$

(i.e.) if we adopt the convention that masses are positive G is is positive λ being only a scalar factor and greater than 0. In Eintein theory gravitation could be repulsive. Here gravitation should be attractive if it is viewed from particle physics and not in terms of fields. To obtain the numerical value which m_0 can take we start with the Robertson walker line element

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]_{k=0,\pm 1} \tag{2.54}$$

which by the use of the field equations give

$$3\,\frac{\dot{S}^2+k}{S^2} = \frac{6\lambda}{m_0^2}\,\rho$$

$$2\,\frac{\ddot{S}}{S} + \frac{\dot{S}^2+k}{S^2} = 0 \tag{2.55}$$

No expression for m_0 can emerge since the theory does not contain any universal length. The Green's function $\widetilde{G}$ has the dimension of $(\text{length})^{-2}$. A length can be introduced by the addition of a second term,

$$(\lambda f)^{-1} \sum_{a<b}\sum \iint \bar{G}_{;\,i_A i_B}\, da^{i_A}\, db^{i_B} \tag{2.56}$$

where $(\lambda f)^{-1}$ has the dimension of a length equal to H^{-2} where H^{-1} is the new length scale. H here may be later identified with Hubble's constant. The contribution from this term to δJ in the smooth fluid approximation is the following expression

$$-\frac{H^2}{\lambda}\int \delta g^{ik}\left[C_i C_k - \tfrac{1}{2} g_{ik} C^l C_l\right]\sqrt{-g}\, d^4x \tag{2.57}$$

and the field equations become

$$R^{ik} - \tfrac{1}{2} g^{ik} R = -\frac{6\lambda}{m_0^2}\left[T_m^{ik} - \frac{H^2}{\lambda}\left(c^i c^k - \tfrac{1}{2} g^{ik} c_l c^l\right)\right] \tag{2.58}$$

and $$R = \frac{6\lambda}{m_0^2}\left[T + \frac{H^2}{\lambda} c^l c_l\right] \tag{2.59}$$

R is no longer proportioned to $T = Nm$ but to $Nm + nm$ where n is the number of broken lines inside the limit volume and $c^i c_i = m_0^2$. This means that $n = \dfrac{H^2 m_0}{\lambda}$

The source equation becomes

$$\left(m_0 H^2/\lambda\right) c^i_{;i} = j^i_{;i}$$

where $$j^i_{;i} = m_0 N \frac{dx^i}{ds} \tag{2.60}$$

For the cosmological case considered,

$$\dot{c} = m_0 \qquad S = e^{Ht} \qquad N = \frac{m_0 H^2}{\lambda} = N_0 \tag{2.61}$$

We see that $N_0 = n$ that is in the asymptotic steady state the number of new particles created is equal to the number of particles present per unit volume, and the density is given by

$$\rho = \frac{3H^2}{4\pi G} = \frac{m_0^2 H^2}{\lambda} \tag{2.62}$$

where H is now the Hubble constant. In local problems where local proper density is N large compared to cosmological density N_0 the C-field terms may be neglected

$$T^{ik} = m_0 N \frac{dx^i}{ds} \frac{dx^k}{ds} \tag{2.63}$$

and $$R^{ik} - \tfrac{1}{2} g^{ik} R = -\frac{6\lambda}{m_0} N \frac{dx^i}{ds} dx = 6H^2 \frac{N}{N_0} \frac{dx^i}{ds}\frac{dx^k}{ds} \tag{2.64}$$

If the particles in the rest of the universe is changed nothing will happen in Newton's and Einstein's gravitational theories. As was indicated in Part I even if $T_{ik} = 0$ on the right hand side of Einstein's field equation i.e. even if all matter is removed from the Universe, the curvature of space-time can be assumed to exist. There is a paradox here. Namely if $R^{ik} - \tfrac{1}{2} g_{ik} R = 0$; $R = 0$ and $R^{ik} = 0$. But if R is zero and there are no particles the action

$$A \int R \sqrt{-g}\, d^4x + B \sum m \int ds = J \tag{2.65}$$

should itself be zero. Hence $R^{ik} = 0$ seem to have been produced out of nothing. In the present formulation such difficulties do not arise. In an empty universe the action is wiped out altogether and there are no equations left. For there to be any physical laws at all the number of particles should be at least two.

But here if N_0 is changed by a factor 2, it would grossly change the properties of the sun. This is because as seen from the above equation (2.63), the curvature (in the case of a reduction) is increased, i.e. gravitational field will be larger and greater solar flux may be necessary to maintain equilibrium of the Sun. So 'Take away half the distant parts of the Universe and the earth would be fried to crisp. All the above results flowed from the fact the first two terms of action J equation ( ) were collapsed into one direct interaction double integral expression. The next step is further simplification of the action by further collapsing of terms into each other. This may prove to be the path towards a united theory of gravitation and electricity.

References

(1) Einstein A., The meaning of relativity, Princeton University Press.

(2) Landau L.D. and Lifshitz E.M., The Classical theory of fields, Pergamon.

(3) Chiu H.Y. and Hollman W.F., Gravitation and Relativity, Benjamin, Inc. 1964.

(4) Schiff L.I., Gravitation lecture notes MATSCIENCE Report 1963.

(5) Dicke R. H., Article Proceedings of International Summer School, Verenna 1961.

(6) Godel K., Review of Mod. Phys. 21, 447 (1949).

(7) Hoyle F. and Narlikar J. V., Proc. Royal Soc. A. 277, 1.
" " A. 278, 465.

(8) Wheeler J.A. and Feynman R.P. 1945. Rev. Mod. Phys. 17, 156.
" 1949, Rev. Mod. Phys. 21, 424.

(9) DeWitt B.S. and Brehme R.W., Annals of Physics 9, 220 (1960).

(10) Hoyle F. and Narlikar J.V., Proc. Roy. Soc. 179, 282A (1964).

(11) Hoyle F. and Narlikar J.V., Proc. Roy. Soc. 191, 282A (1964).

****