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MATSCIENCE REPORT 1

PROCEEDINGS
OF THE
FIRST ANNIVERSARY SYMPOSIUM

(January 14 to 16, 1963)

THE RESONANT STATES OF ELEMENTARY PARTICLES.

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MATSCIENCE REPORT 1

(1)

PROCEEDINGS

OF THE

FIRST ANNIVERSARY SYMPOSIUM

(January 14 to 16, 1963)

IN HONOUR OF

PROFESSOR R. E. MARSHAK,

NIELS BOHR VISITING PROFESSOR FOR 1963 AT THE

INSTITUTE OF MATHEMATICAL SCIENCES, MADRAS.

Subject: ***THE RESONANT STATES OF ELEMENTARY PARTICLES.***

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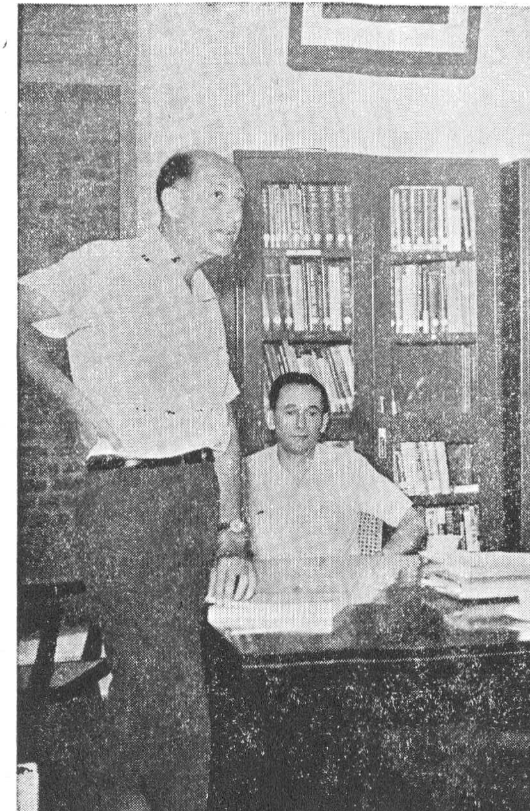
Professor Niels Bohr



Professors R. E. Marshak and L. I. Schiff

MATSCIENCE REPORT 1

The sponsors of the Institute are extremely grateful to Professor L. I. Schiff, Chairman, Department of Physics, Stanford University, California, U.S.A. and visiting professor at the Institute of Mathematical Sciences for releasing the Proceedings of the Symposium in honour of Professor R. E. Marshak, the first Niels Bohr Professor of the Institute, as the first "*MATSCIENCE REPORT*". They also wish to thank Professor N. Fukuda of the Tokyo University of Education, Tokyo, Japan, for unveiling the portrait of Professor Niels Bohr.

CONTENTS

INTRODUCTORY TALK

PROFESSOR ALLADI RAMAKRISHNAN

This symposium has been arranged as a tribute to our distinguished visitor Professor R. E. Marshak who so graciously accepted the Niels Bohr visiting professorship of our new Institute of Mathematical Sciences. There is a peculiar appropriateness in naming Professor Marshak as the first occupant of this chair. For the very conception of this Institute originated during my visit to America in the spring of 1956 when I had the opportunity to watch the proceedings of the Rochester Conference. At that time, to us in India, such a world characterised by high endeavour and creative achievement seemed beyond reach and almost beyond imagination. The fortuitous visit of Professor Niels Bohr to Madras and his sincere interest in the work of our group transformed a vision into a reality. And today we are assembled here in a spirit of dedication to that great ideal, expressed in the words of professor Niels Bohr, as the " worldwide co-operation in science which offers so great opportunities for promoting the understanding between all peoples ".

Were we not animated by this spirit we would not have had the courage or temerity to invite the sponsor of the foremost international conference in physics to participate in a symposium in which only a small group of young and inexperienced entrants are taking part. They have no claim to creative work in the subject under discussion, the resonant states of elementary particles. But I have asked them to make an effort in the hope that interest may be stimulated in our country and in particular in our city in a subject which holds the minds of the most gifted scientists in physics today.

The theory of elementary particles is now offering the greatest challenge since the birth of quantum mechanics. The problems are so complex and baffling that even the novice sometimes feels able to discuss with the savant without fear of being shown his place! For nobody knows whether we are dealing with a composite theory of elementary particles or an elementary theory of composite particles. It is a situation which can be treated as very encouraging to a young aspirant to a scientific career for there seems to be work to be done by every class of physicist from the humblest of calculators to the most exalted of creative scientists.

All our talks herein are expository. Here and there are some feeble strains of speculation and criticism which we place before our distinguished visitor as our first attempts in this field.

The best way, it seems to me, to start this symposium is to lay before you the maze of resonances* in the belief expressed as a preface to my own humble work, that there is a plan behind such an intricate maze which it is the task of the physicist to discern.

* See Table on the following page.

Particle	Established Quantum Number $I(J^{PG})$	Possible assignment: Quantum No. $I(J^{PG})$	Possible assignment: Regge Trajectory	Mass (Mev)	Γ (Mev)	Mass^2 $(\text{Bev})^2$	Dominant decays: Mode	%	Q (Mev)	P or P max. (Mev/c)
Vacuum ?	-	$0(2^{++})$	$+\omega_\alpha$	-	-	-	(Even no. of π) $\bar{K}$ K etc.			
η		$0\ (0^{-+})$	$\pm\omega_\beta$	548	10	.30	Neutrals	75	-	-
							$\pi^+\pi^-\pi^0$	25 ± 4	136	175
ω		$0\ (1^{--})$	$-\omega_\gamma$	782	15	.62	$\pi^+\pi^-\pi^0$	86	368	326
							$\pi^0\gamma$	14 ± 4	647	379
π π^0		$1(0^{--})$	$-\pi_\beta$	135	0	.018	$\pi^0\rightarrow2\gamma$	100	135	67
π $\pi^\pm$				140	0	.02	$\pi^\pm\rightarrow\mu\nu$	58	34	30
ρ		$1\ (1^{-+})$	$+\pi_\gamma$	750	100	.56	$\pi\pi$ (p-wave)	100	471	348
ξ	1(?).	$1(0^{+})$	$-\pi_\alpha$	560	15	.31	$\pi\pi$	?	290	245
K K^0		$1/2(1^-)$	K_β	498	0	.24	$K_1^0\rightarrow\pi^+\pi^-$	$2/3K_1$	219	206
K $K^\pm$				494	0		$K^\pm\rightarrow\mu\nu$	58	388	236
$K^*_{1/2}$ (888)		$1/2\ (1^-)$	K_γ	888	50	.78	$K\pi$ (p-wave)	100	$251(K^0\pi^-)$	283
$K^*_{1/2}$ (730)	$1/2(?)$	?	?	730	20	.53	$K\pi$	?	$101(K^-\pi^0)$	161

Table 2.

TENTATIVE DATA ON STRONGLY INTERACTING PARTICLES (A.H.Rosenfeld)

N $\begin{cases} n \\ p \end{cases}$	1/2(1/2$^+$)	N_α	940 938	0	.88	$pe^-\bar{\nu}$ –	100 –	.78 –	1.2 –
$N^*_{1/2}$ ("900" MeV)	1/2 (5/2 +)	N_α	1688	100	2.84	$N\pi$ (f wave)	?	610	572
$N^*_{1/2}$ ("600" MeV)	1/2 (3/2 −)	N_γ	1512	150	2.28	$N\pi$ (d wave)		434 (p)	450
$N^*_{3/2}$ (isobar)	3/2 (3/2+)	Δ_δ	1238	100	1.53	$N\pi$ (p wave)	100	160 (p)	233
$N^*_{3/2}$	3/2(J ≥ 5/2) ; 3/2(7/2+)	Δ_δ	1920	200	3.69	$N\pi$+ other	?	824 (p)	722
Λ	0(1/2+)	Λ_α	1115	0	1.24	π^-p	67	38	100
Y_0^*	0(j 5/2) 0(5/2+)	Λ_α	1815	120	3.29	KN+ others	?	383 (p)	541
Y_0^*	0(?) 0(1/2+) 0(1/2−)	Λ_β	1405	50	1.97	$\Sigma\pi$ $\Lambda 2\pi$	100	69 10	144 69
Y_0^*	0 (3/2−)	Λ_γ	1520	15	2.31	$\Sigma\pi$ (d wave)	60	194	267
						$\bar{K}N$ (d wave)	30	88	244
						$\Lambda 2\pi$	10	125	253
Σ^0	1 (1/2 +)	Σ_α	1191	0	1.42	$\Lambda\gamma$	100	76	74
Σ^-			1196	0	1.42	$n\pi^-$	100	117	192
Σ^+			1189	0	1.42	$n\pi^+$	50	110	185
Y_1^*	1(J 3/2) 1(3/2 +)	Σ_δ	1385	50	1.92	$\Lambda\pi$	98	135 ($\Lambda\pi^0$)	210
						$\Sigma\pi$	2 ± 2	49 ($\Sigma^-\pi^+$)	119
Y_1^*	1 (?) ?	?	1685?	?	2.85 ?	$\Lambda\pi$+ others.	?	435	459

Table 2 (contd.)

$\Xi \begin{cases} \Xi^0 \\ \Xi^- \end{cases}$	1/2(?)	$\frac{1}{2}(\frac{1}{2}+)$	Ξ_α	1311	0	1.72	$\Lambda\pi^0$	-	61	131
				1321			$\Lambda\pi^-$	-	66	138
Ξ^*	1/2(?) ?		?	1530	<7	2.34	$\Xi\pi$	100	74	148

Table 2 (concluded)

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SYMMETRIES AND RESONANCES

T.K. Radha

First of all let me apologise for talking on a subject in which I got interested only recently after hearing those inspiring lectures of Prof. Salam at Trieste ''On Lie groups and Lie Algebra''. So I may not do justice to the title of the talk but shall confine myself to those branches of the subject which I am able to appreciate if not understand completely.

I shall divide my talk into four parts

a) The need for higher Symmetries.

b) Charge conservation and charge Independence as Unitary Groups.

c) Resonances in Models built on

(i) SU_3 with Sakata model

(ii) SU_3 with Octet model

d) Global Symmetry and models built on ~~their~~ other groups

e) Some speculations

In the Appendix group theoretical concepts necessary for understanding this lecture are given.

a. We have just heard that the number of known resonant states of strongly interacting particles has increased enormously during the last two years. Before one takes up the question of the dynamical problem of the elementary forces responsible for the structure of these resonant states one must know all the internal symmetries and the corresponding quantum numbers.

For strong interactions we know that there are three exactly conserved quantum numbers 1) IsoSpin (I) 2) Baryon Number (N) 3) hyper charges (Y) or Strangeness (S). So then when one looks at the mass spectrum of elementary particles the natural question one asks now is ' Is their any higher symmetry (which perhaps is approximate as suggested by the mass spectrum again) of strong interaction which ofcourse incorporates I-spin, S, and N conversation ? Such an attempt has been made by Gell-Mann - Neeman[1,2] for the ''Octet model'' and by Salam and Ward[3] and others[4,5] for the ''Sakata Model''. It is interesting that certain predictions of ~~this theory has~~ these theories have been experimentally verified (e.g.) the existence of octet of vector mesons. Another parallel view is that of Sakurai[6] based on the vector theory of strong interactions - the basic idea is that in order to understand the existence of each conserved current, there should be introduced a vector field which interacts with this current and satisfies gauge invariance. The vector particles proposed were

1) I = 1 (ρ) coupled to I-spin current,
2) I = 0 (ω) coupled to hypercharge current,
3) I = 0 (B) coupled to Baryon current,
and 4) I = 1/2 (K^* ?) coupled to (Partially conserved) Strangeness changing currents.

b.

(i) Any unitary transformation is defined as[7]

$$\xi_\alpha \to \xi_\alpha' = U_{\alpha\beta}\,\xi_\beta \tag{i}$$

where ξ is an n-component complex vector and U satisfies

$$U^{\dagger} = U^{-1} \tag{2}$$

If $\zeta^{\alpha} = \zeta_{\alpha}^{*}$ then (1) leads to

$$|\zeta_\alpha|^2 = |\zeta_\alpha'|^2 \tag{3}$$

(i.e.) the transformation U leaves $|\zeta|^2$ invariant. Now conservation of charge can be expressed as invariance of all interactions under the unitary transformation $p \rightarrow e^{iex} p$ for all charged fields, ~~where~~ popere p is a one dimensional vector. Writing $\zeta = x + iy$ we find that $U = e^{iex}$ leaves $x^2 + y^2$ invariant - (i.e.) U corresponds to rotations in 2 dimensions(R_2)

(ii) Charge independence may be considered as generalization of 2 dimensional rotations to 3 dimensional rotations and requiring invariance of strong interactions under these rotations. These transformations can be related to the two dimensional unitary transformations U_2 (in complex space)

(e.g.)

$$\zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{say.} \tag{4}$$

Then

$$\zeta^{\alpha} = \begin{pmatrix} p^* \\ n^* \end{pmatrix} \tag{5}$$

The most general form of U_2 is

$$U_2 = e^{ix1} \, e^{i\tau\cdot\xi} \tag{6}$$

where 1 is the unit matrix and τ the 2 x 2 Pauli matrices. Invariance under $e^{i\chi 1}$ leads to baryon conservation while invariance under $e^{i\tau\cdot\chi} = SO_2$ leads to charge independence. For completeness we shall also write the following;

$$\mathrm{Tr}\ \tau_i \tau_j = 2\delta_{ij} \tag{7}$$

$$[\tau_i, \tau_j] = 2ie_{ijk}\tau_k \ ; \ \{\tau_i, \tau_j\} = 2\delta_{ij}1. \tag{8}$$

The I spin operator for $I=1/2$ particles has representation given by

$$I_i = \tau_i/2 . \tag{9}$$

For $I=1$, $[I_i, I_j] = ie_{ijk} I_k$ (10)

and I_i has elements $I_i^{jk} = -ie_{ijk}$. (3 x 3 matrices). (11)

To get the higher multiplets we have to merely use the tensor analysis given in the Appendix to get the irreducible representations

$$\pi_0^0 = \frac{1}{\sqrt{2}}\xi_\alpha \xi^\alpha = pp^* + nn^*/\sqrt{2}$$

$$\vec{\pi} = \xi_\alpha \xi^\beta - \frac{1}{2}\delta_\alpha^\beta(\xi^\gamma \xi_\gamma) = \begin{bmatrix} \frac{1}{2}(pp^* - nn^*) & pn^* \\ np^* & -\frac{1}{2}(pp^* - nn^*) \end{bmatrix}$$

$$= \begin{bmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{bmatrix} \tag{12}$$

Ofcourse we have identified the components of the basis of an irreducible representation of a group with a set of physical states having the same space-time properties. {(e.g.) spin, Relative parity, G parity ~~xxx~~ etc. and also $p^2=m^2$ is the same for all members of the multiplets}.

C

Unitary Symmetry

The generalization of charge independence (SU_2) to higher symmetry leads us to the group $U_3 = e^{i\,1\,\theta_k}\, e^{i\lambda_i\theta_i}$ where $i = 1, 2, \ldots, 8$. There are 3 x 3 — 1= 8 traceless hermitian matrices (~~like~~ similar to τ) given by

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix} \qquad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \qquad \lambda_8 = \begin{bmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -2/\sqrt{3} \end{bmatrix} \tag{13}$$

Further we have

$$Tr(\lambda_i \lambda_j) = 2\delta_{ij} \tag{14}$$

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k \tag{15}$$

$$\{\lambda_i, \lambda_j\} = \frac{4}{3}\delta_{ij} + 2d_{ijk}\lambda_k \tag{16}$$

f_{ijk} are real and totally antisymmetric and d_{ijk} are real and symmetric (i.e.) the f_{ijk} are odd under permutations of any two indices while the d_{ijk} are even. (eg)

ijk	f_{ijk}	ijk	d_{ijk}	ijk	d_{ijk}
123	1	118	$1/\sqrt{3}$	366	$-1/2$
147	$1/2$	146	$1/2$	377	$-1/2$
156	$-1/2$	157	$1/2$	448	$-1/2\sqrt{3}$
246	$1/2$	228	$1/\sqrt{3}$	558	$-1/2\sqrt{3}$
257	$1/2$	247	$-1/2$	668	$-1/2\sqrt{3}$
345	$1/2$	256	$1/2$	778	$-1/2\sqrt{3}$
367	$-1/2$	338	$1/\sqrt{3}$	888	$-1/\sqrt{3}$
458	$\sqrt{3}/2$	344	$1/2$		
678	$\sqrt{3}/2$	355	$1/2$		

(17)

First let us consider the Sakata model .

(i.e.)

$$\xi_\alpha = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix} \qquad \xi^\alpha = \begin{pmatrix} p^* \\ n^* \\ \Lambda^* \end{pmatrix} \tag{18}$$

These constitute a minimum set of particles necessary for manifestation of the I, B and S quantum numbers. Then U_3 transformation may be identified with

$$N \to e^{i\tau\cdot x} N$$

$$\Lambda \to e^{i\beta} \Lambda$$

$$\xi_\alpha \to \xi_\alpha \tag{19}$$

$$n \longrightarrow \Lambda \quad (\text{ also equivalent to } p \to \Lambda \ )$$

The higher multiplets can be found as follows:

we have the representation $D^3(1,0)$ for ξ_α. (See appendix)

Then all mesons have to be obtained from

$$D^3(1,0) \otimes D^3(0,1) = D^8(1,1) + D^1(0,0) \tag{20}$$

$$\left[M_\alpha{}^\beta = \xi_\alpha \xi^\beta - \tfrac{1}{3}\delta_\alpha{}^\beta (\xi^\gamma \xi_\gamma) \text{ and } M^\alpha_\alpha \right] \tag{21}$$

We may identify the representation $D^8(1,1)$ with pseudoscalar mesons or vector mesons. The I-spin contents give the particles as

		J=0, Mass (Mev)		J = 1	Mass (Mev)
I = 0	Y = 0	$\pi^{0'} = \eta$	(560)	ω	(782)
I = 1	Y = 0	$\vec{\pi}$	(140)	ρ	(750)
I=1/2	Y = 1	K	(494)	K^*	(888)
I=1/2	Y = -1	$\bar{K}$	(494)	$\bar{K}^*$	(888)

Okubo[8] has given the mass formula for any multiplet of SU_3 as

$$m = a + bY + c\left[I(I+1) - Y^2/4\right] \qquad (22)$$

This leads to the relation*

$$\frac{3m_\eta^2 + m_\pi^2}{4} = m_K^2 \qquad (23)$$

which gives m_η = 560 Mev which is in very good agreement with experiment. Similarly for the vector mesons

$$M = \begin{bmatrix} \frac{\rho(0)}{\sqrt{2}} + \frac{\omega(0)}{\sqrt{6}} & \rho(+) & K^{*(+)} \\ \rho^{(-)} & -\frac{\rho(0)}{\sqrt{2}} + \frac{\omega(0)}{\sqrt{6}} & K^{*(0)} \\ \overline{K}^{*(-)} & \overline{K}^{*(0)} & -2/\sqrt{6}\,\omega^{(0)} \end{bmatrix} \qquad (24)$$

---

* R.P Feynman has suggested that for bosons we have to take the formula for m^2 rather than for m.

we have

$$m_{K^*}^2 = \frac{m_\rho^2 + 3 m_\omega^2}{4} \quad \text{giving} \quad m_{K^*} \simeq 760 \text{ Mev} \tag{25}$$

which is not in agreement with the mass of the observed K^*: m_{K^*} = 888 Mev. It is interesting to note that we have got all bosons as composite systems of baryons and antibaryons. Let us now consider higher baryon multiplets in the Sakata model. We have

$$D^3(1,0) \otimes D^{3^*}(0,1) \otimes D^3(1,0) = D^{15}(2,1) + D^{6^*}(0,2) + D^3(1,0) + D^3(1,0) \tag{26}$$

This gives us

d	I	Y	Particles	Mass. (Mev)	J
3	0	0	Λ (Y^{***})	1115 (1520)	1/2(5/2+)
	1/2	1	N (N^{***})	940 ([illegible]	1/2(5/2+)
3	0	0	Y_0^{***}	1520	(3/2−)
	1/2	1	N^{***}	1512 [illegible]	(3/2−)
6	0	2	?		
	1/2	1	$N^*_{1/2}$(?)		
	1	0	Σ (?)	1190	1/2
15	1/2	−1	Ξ(?)	1315	1/2(?)
	3/2	1	N^*(?)	1238	3/2+
	1		[illegible]	1385	3/2+(?)
			(?)	1405	3/2+(
				?	
				?	

Thus we notice that the only place for the Ξ is in the $15'$ representation. and for Σ in the $\bar{6}$ or $\overline{15}$ representation. If it occurs with Y_1^* and N_1^* in this representation it should have spin 3/2. (and positive parity [illegible]). Further if we take that the Y_0^* (1405) has spin 3/2+ then we find that we should have a bound K^*N system with mass = 1150 Mev. (using Okubo's mass formula). and a $N_{1/2}^*$ resonance with mass 1268 Mev. with spin 3/2+.

Thus we find that the Λ and Σ do not belong to the same representation and also the Ξ should have spin 3/2 (?) . Alternative suggestion is that the fundamental particles are hidden and the 15 representation may include N, Λ, Σ, Ξ and 2 more particles.

(ii) Octet model

In this model the primitive objects are hidden. They are defined as

$$l_\alpha = \begin{pmatrix} \nu \\ e^- \\ \mu^- \end{pmatrix} \tag{27}$$

and

$$L_\alpha = [D^0 . D^+, S^+] \tag{28}$$

where (ν, e) and (D^0, D^+) form a doublet and μ and S are singlets. For this system we define the unitary spin F_i as

$$F_i = \lambda_i/2$$

with

$$[F_i, F_j] = i f_{ijk} F_k \tag{29}$$

Now form $\bar{L} \lambda_i l/\sqrt{2}$ with $i = 1, 2, \cdots, 8.$ (30)

This transforms like an irreducible representation of dimension 8.

We have the 8 x 8 matrices given by

$$F_i^{jk} = 2 f_{ijk} \qquad i, j, k = 1, \cdots 8 \tag{31}$$

We form

$$\tfrac{1}{2} \bar{L}(\lambda_1 - i\lambda_2) l \sim \Sigma^+ \sim D^+\nu$$

$$\tfrac{1}{2} \bar{L}(\lambda_1 + i\lambda_2) l \sim \Sigma^- \sim D^0 e^-$$

$$1/2\ \bar{L}(\lambda_3) l \sim \Sigma^0 \sim \frac{D^0\nu - D^+e^-}{\sqrt{2}}$$

$$1/2\ \bar{L}(\lambda_4 - i\lambda_5) l \sim p \sim S^+\nu$$

$$1/2\ \bar{L}(\lambda_6 - 2\lambda_7] l \sim n \sim S^+e^-$$

$$1/2\ \bar{L}(\lambda_6 + i\lambda_7) l \sim \Xi^0 \sim D^+\mu^-$$

$$1/\sqrt{2}\ \bar{L}(\lambda_8) l \sim \Lambda^0 \sim \frac{D^0\nu + D^+e^- - 2S^+\mu^-}{\sqrt{6}} \tag{32}$$

(6)

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2/\sqrt{6}\,\Lambda^0 \end{bmatrix} \tag{33}$$

Thus the eight known baryons form ane degenerate super-multiplet. w.r.t. unitary spin. If we introduce the (μ -e) mass difference the supermultiplet breaks up intd exactly known multiplets.

(i.e.) Assume $m_{D^0} = m_{D^+}$; $m_\nu = m_e$ and (34)

Then

$$m_N = m_S + m_e$$

$$m_\Lambda = \tfrac{1}{3}(m_D + m_e) + \tfrac{2}{3}(m_S + m_\mu)$$

$$m_\Sigma = m_D + m_e \tag{35}$$

$$m_\Xi = m_D + m_\mu$$

and we have

$$\frac{m_N + m_\Xi}{2} = \frac{3m_\Lambda + m_\Sigma}{4} \tag{36}$$

Which is in very good agreement with experiment. This also follows from the Okubo mass formula. Thus the baryons belong to the representation $D^8(1,1)$ To get the get the mesons we consider

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27 \tag{37}$$

The mesons may belong to the representation 8 as in the case of Sakata model. Then the matrices connecting the π_j are just the same as those connecting N_j .

(ie) $F_i{}^{jk} = -i f_{ijk}$

Now to couple 8 mesons invariantly to the eight baryons say by γ_5 we have

$$H = 2ig\,\bar{N}\gamma_5 O_i N \pi_i \qquad (38)$$

for which the relation $[F_i, O_j] = i f_{ijk} O_k$ holds (39)

The double occurence of 8 in the splitting shows there are two independent sets of 8 x 8 matrices O_i obeying (39). One may be F_i itself. For the other define $D_i^{jk} = d_{ijk}$. Then the D's also satisfy (39). The physical difference between these two couplings lies in the symmetry under the operation R which is not a member of the unitary group.

I am not discussing thesedetails since Prof. Marshak will be dealing with them in his talk. Let us consider the higher baryon multiplets in the Octet model. These also belong to

$$8 \otimes 8 = 1 \oplus 8 \oplus 8' \oplus 10 \oplus \overline{10} + 27.$$

8 and 8' correspond to D and F type couplings. We have [9]

$$(8;\ Y=1, T=1/2)^+ = \sqrt{3/20}\,(\sqrt{2}\,n\pi^+ + p\pi^0 + \sqrt{2}\,\Sigma^+K^0 + \Sigma^0K^+) - \tfrac{1}{\sqrt{20}}(p\chi + \Lambda K^+)$$

$$(8;\ Y=0\ T=0)^0 = \sqrt{1/5}\,(\Sigma^-\pi^+ + \Sigma^+\pi^- + \Sigma^0\pi^0 - \Lambda\chi) - 1/\sqrt{20}\,(pK^- + n\bar{K}^0 + \Xi^-K^+ + \Xi^0K^0)$$

$$(8;\ Y=0\ T=1)^+ = \sqrt{3/10}\,(p\bar{K}^0 + \Xi^0K^+) + \sqrt{1/5}\,(\Sigma^+\chi + \Lambda\pi^+)$$

$$(8;\ Y=-1, T=1/2)^- = \sqrt{3/20}\,(\sqrt{2}\,\Xi^0\pi^- + \Xi^-\pi^0 + \sqrt{2}\,\Sigma^-\bar{K}^0 + \Sigma^0K^-) - \sqrt{1/20}\,(\Xi^-\chi + \Lambda K^-)$$

$$(8';\ Y=1\ T=1/2)^+ = \sqrt{1/12}\,(\sqrt{2}\,\Sigma^+K^0 + \Sigma^0K^+ - \sqrt{2}\,n\pi^+ - p\pi^0) + \tfrac{1}{2}(\Lambda K^+ - p\chi)$$

$$(8';\ Y=0\ T=0)^0 = 1/2\,(pK^- + n\bar{K}^0 - (\Xi^-K^+ + \Xi^0K^0))$$

$$(8';\ Y=0\ T=1)^+ = \sqrt{1/3}\,(\Sigma^0\pi^+ - \Sigma^+\pi^0) + \sqrt{1/6}\,(p\bar{K}^0 - \Xi^0K^+)$$

$$(8';\ Y=-1,\ T=1/2)^- = 1/\sqrt{12}\,(\sqrt{2}\,\Sigma^-\bar{K}^0 + \Sigma^0K^- - \sqrt{2}\,\Xi^0\pi^- - \Xi^-\pi^0) + \tfrac{1}{2}(\Lambda K^- - \Xi^-\chi)$$

For '27' we have

$$(27;\ Y=0, T=2)^{++} = \Sigma^{+}\pi^{+}$$

$$(27;\ Y=0, T=1)^{+} = \sqrt{3/10}\,(\Sigma^{+}\chi + \Lambda\pi^{+}) - \sqrt{1/5}\,(p\bar{K}^{0} + \Xi^{0}K^{+})$$

$$(27;\ Y=0, T=0)^{0} = \sqrt{27/40}\,\Lambda\chi + \sqrt{3/40}\,(pK^{-} + nK^{0} + \Xi^{-}K^{+} + \Xi^{0}K^{0}) + \sqrt{1/120}\,(\Sigma^{+}\pi^{-} + \Sigma^{-}\pi^{+} + \Sigma^{0}\pi^{0})$$

$$(27;\ Y=1, T=3/2)^{++} = \sqrt{1/2}\,(p\pi^{+} + \Sigma^{+}K^{+})$$

$$(27;\ Y=1, T=1/2)^{+} = \sqrt{9/20}\,(p\chi + \Lambda K^{+}) + \sqrt{1/60}\,(p\pi^{0} + \Sigma^{0}\pi^{+} + \sqrt{2}\,n\pi^{+} + \sqrt{2}\,\Sigma^{+}$$

$$(27;\ Y=2, T=1)^{++} = pK^{+} \qquad (41)$$

Note that $(27;\ Y=-2, T=1)$, $(27;\ Y=-1, T=3/2)$ etc can be obtained from these by operation of R.

The '10' & '$\overline{10}$' are given by

$$(10;\ Y=1\ \ T=3/2)^{++} = \sqrt{1/2}\,(p\pi^{+} - \Sigma^{+}K^{+})$$

$$(10;\ Y=0, T=1)^{+} = \tfrac{1}{2}(\Sigma^{+}\chi - \Lambda\pi^{+}) + \sqrt{1/6}\,(p\bar{K}^{0} - \Xi^{0}K^{+}) - \tfrac{1}{\sqrt{12}}(\Sigma^{0}\pi^{+}$$

$$(10;\ Y=-1\ \ T=1/2)^{-} = \tfrac{1}{2}(\Lambda K^{-} - \Xi^{-}\chi) + \tfrac{1}{\sqrt{12}}\left(\Xi^{-}\pi^{0} - \Sigma^{0}K^{-} + \sqrt{2}(\Xi^{0}\pi^{-} - \Sigma^{-}\pi^{0})\right)$$

$$(10;\ Y=-2\ \ T=0)^{-} = \tfrac{1}{\sqrt{2}}\,(\Xi^{0}K^{-} - \Xi^{-}\bar{K}^{0}) \qquad (42)$$

$$(\overline{10};\ Y=-1, T=3/2)^{--} = \tfrac{1}{\sqrt{2}}\,(\Xi^{-}\pi^{-} - \Sigma^{-}K^{-})$$

$$(\overline{10};\ Y=0, T=1)^{+} = \tfrac{1}{2}(\Sigma^{+}\chi - \Lambda\pi^{+}) + \tfrac{1}{\sqrt{6}}(p\bar{K}^{0} - \Xi^{0}K^{+}) + \tfrac{1}{\sqrt{12}}(\Sigma^{0}\pi^{+} - \Sigma^{+}\pi^{0}$$

$$(\overline{10};\ Y=1, T=1/2)^{+} = \tfrac{1}{2}(\Lambda K^{+} - p\chi) + \tfrac{1}{\sqrt{12}}(p\pi^{0} + \sqrt{2}\,n\pi^{+} - \Sigma^{0}K^{+} - \sqrt{2}\,\Sigma^{+}\pi^{0})$$

$$(\overline{10};\ Y=2, T=0)^{+} = 1/\sqrt{2}\,(nK^{+} + pK^{0}).$$

$$(43)$$

Let us consider the 10 fold way. These may correspond to the following[10]

Y_1^*	I = 1	Y = 0	1385	J = 3/2 + ( )
N_1^*	I = 3/2	Y = 1	1238	J = 3/2 +
Ξ^*	I = 1/2	Y =-1	?	J = 3/2 + (?)
Ω	I = 0	Y =-2	?	J = 3/2 + (?)

It is interesting that there is a unique relation.

I = 1+Y/2 for this representation. Thus Okubo's mass relation reduces to

$$m = a + b\,y \tag{44}$$

This gives $m_{\Xi^*} = 1532$ Mev

and $m_\Omega = 1679$ Mev. (45)

The mass of Ξ^* is in very good. agreement with experiment while the Ω seems to be a stable particle (with S = -3) which cannot decay into $(\Xi + \bar{K})$ (threshold 1820 Mev) This is to be checked experimentally.

If we apply the mass formula for the imaginary masses also we get using

$$\Gamma(N_{3/2}^*) = 90 \text{ Mev}$$

$$\Gamma(\Omega) = 0 \tag{46}$$

We get $$\Gamma(Y_1^*) = 60 \text{ Mev}$$
$$\Gamma(\Xi^*) = 30 \text{ Mev} \qquad (47)$$

Which is fairly in agreement with experiments. The Ω can decay only by weak interaction ~~with~~ into

$$\Omega \rightarrow \Xi + \pi \qquad \text{or} \qquad \Xi^0 + e^- + \bar{\nu}_e \quad \text{etc}$$
$$\bar{K} + \Lambda$$
$$\bar{K} + \Sigma \qquad (48)$$

It is interesting to note that Eisenberg[11] predicted a mass of 1615 Mev for the ''new hyperon'' in 1954 if it decays into $\Lambda + \bar{K}$.

According to the equation (42) we find that N^* decay ratio

$$\frac{N_1^* \rightarrow p\pi^+}{N_1^* \rightarrow \Sigma^+ K^+} = 1$$

Which is not true since $\Sigma^+ + K^+$ channel is forbidden kinematically. Thus we find that unitary symmetry is violated serious, because of the huge mass difference ~~xxxx~~ between (π and K) and (p and Σ).

Also we find that for '10'

$$\frac{Y_1^* \rightarrow \Sigma + \pi}{Y_1^* \rightarrow \Lambda + \pi} \simeq 2/3 \; \frac{p_\Sigma^3}{p_\Lambda^3} \sim 16\,\% \qquad (49)$$

which experimentally in $\sim$ 0.

However if we invoke R invariance[12] $Y_1^* \rightarrow \pi + \Sigma$ vanishes. But this implies that $\overline{10}$ should also resonate leading to a Ξ^* reasonance with I = 3/2 and a bound $K^+ N$ system

The '27' fold way leads to the following resonances:

$I = 2$	$Y = 0$	Y_2^* resonance	1580 (?)(Observed by [illegible] et al)[13]
$I = 1/2$	$Y = 1$	ΛK resonance	1690 (?) ([illegible] and others)
$I = 0$	$Y = 0$	Y_0^*	1405
$I = 3/2$	$Y = -1$	Ξ^*	
$I = 3/2$	$Y = 1$	$N^*_{3/2}$	1920 (?)[14]
$I = 1$	$Y = 0$	Y_1^*	1685 (?)[14]
$I = 1$	$Y = 2$	K^+N	?

and a group of the K ($\bar{K}$)- ~~baryons~~ baryon and

χ- baryon resonances.

If Y_1^* (1385) belongs to this then $Y_1^* \rightarrow \Sigma\pi$ is automatically forbidden as can be seen from (41). The K^+p scattering shows no sign of a bump whatsoever corresponding to the K^+N resonance.

We have further for unitary symmetry with R symmetry (for D type coupling)

$$G^2_{\pi\Lambda\Sigma} = 4/3\ G^2_{\pi NN}. \qquad (a)$$

$$G^2_{\pi NN} = G^2_{\pi\Xi\Xi} \qquad (b)$$

$$G^2_{\pi\Sigma\Sigma} = 0 \qquad (c)$$

$$G^2_{K\Sigma N} = 3\ G^2_{K\Lambda N} \qquad (d)$$

$$G^2_{K\Lambda N} = G^2_{K\Xi\Lambda} \qquad (e)$$

$$G^2_{K\Sigma N} = G^2_{K\Xi\Sigma} \qquad (f)$$

$$G^2_{K\Sigma N} = G^2_{\pi NN}. \qquad (g) \qquad (50)$$

of these (b), (c), (e), and (f) follows from R invariance above. We see from (g) that K and π coupling to N is the same which contradicts the deductions from photoproduction experimental data from which we get that π coupling should be much stronger than K coupling . Perhaps the violation of unitary symmetry can be calculated to give

$$\frac{G^2_{\pi NN}}{G^2_{K\Sigma N}} \sim \frac{m_K^2}{m_\pi^2} \qquad (51)$$

The $J = {}^{3}/_{2}-$ resonances may be due to vector boson-baryon couplings just as P.S. meson - baryon coupling lead to $p_{3/2}$ resonances.

For this vector mesons we may write the decay widths as

$$\Gamma_{M \to K, \pi} = 2 \frac{\gamma^2_{MK,\pi} \; k_1^3}{4\pi \; m_M^2} \quad ; M = K, K^* \qquad (52)$$

$$\Gamma_{\rho \to 2\pi} = \frac{8}{3} \frac{\gamma^2_{\rho\pi\pi}}{4\pi} \frac{k_2^3}{m_\rho^2} \qquad (53)$$

Which for unitary symmetry will give

$$\frac{\Gamma_{M \to K\pi}}{\Gamma_{\rho \to 2\pi}} = \frac{3}{4} \frac{k_1^3}{k_2^3} \frac{m_\rho^2}{m_M^2} \qquad (54)$$

Then we get for $m_M = 880$ Mev assuming the ρ mass and width $\Gamma_{M \to K\pi} = 30$ Mev. While experimentally it is 50 Mev.[4]

For this we have

$$\frac{f^2_\omega}{4\pi} = \frac{3}{4} \frac{f^2_\rho}{4\pi} \qquad (54a)$$

C

Global symmetry was perhaps the first attempt at a higher symmetry.[15] It was assumed that th N, Λ , Σ and Ξ form a supermultiplet symmetrically coupled by very strong π interactions and degenerate but unsymmetrically coupled by moderately strong K interactions. The group theoretical arguments are based on G_{00}[16] Which has 8 x 8 unitary representation to which the baryons belong (i.e.)

$$B = \begin{bmatrix} N \\ \Xi \\ V \\ Z \end{bmatrix} = \begin{bmatrix} p \\ n \\ \Xi^0 \\ \Xi^- \\ \Sigma^+ \\ Y^0 \\ Z^0 \\ \Sigma^- \end{bmatrix} \qquad (55)$$

and $$G_0 = U_L \begin{bmatrix} aI & -bI & 0 & 0 \\ -b^*I & a^*I & 0 & 0 \\ 0 & 0 & a'I & b'I \\ 0 & 0 & -b'^*I & a'^*I \end{bmatrix} \qquad (56)$$

where
$$U_L = e^{i(l_1 U_1 + l_2 U_2 + l_3 U_3)} \qquad (57)$$

and $U_{1,2,3}$ Can be L, M or N (with corresponding small l, m, n)

Where $$L_1, L_2, L_3 = L_{i\,(i=1,2,3)} = \frac{1}{2}\begin{bmatrix} \sigma_i & & & \\ & \sigma_i & & \\ & & \sigma_i & \\ & & & \sigma_i \end{bmatrix} \qquad (58)$$

$$M_1 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \end{bmatrix} \qquad M_2 = \frac{1}{2} i \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & I & 0 \end{bmatrix} \qquad M_3 = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix}$$

$$N_1 = \frac{1}{2}\begin{bmatrix} 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad N_2 = \frac{i}{2}\begin{bmatrix} 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad N_3 = \frac{1}{2}\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

One gets

Resonance	Rep	Total Energy	Decay into	ω	Relative Partial width
$(N^*)^{++}_{3/2}$	$(3/2, 1/2, 0)$	1237	$\pi^+ + p$	1	1
$(Y^*)^+_1$		1385	$\pi^+ + \Lambda$	2/3	.5
			$\Sigma^+ + \pi^0$	1/6	.03
			$\Sigma^0 + \pi^+$	1/6	.03
$(Z^*)^{++}_2$		1539	$\pi^+ + \Sigma^+$	1	1.9 (?)
$(\Xi^*)^+_{3/2}$		1637	$\pi^+ + \Xi^0$	1	1.5 (?)

where the width were calculated by c.h. coefficients $\times \dfrac{q^3 E_B}{E_B + E_M}$

(q momentum of π in rest frame of resonance) and Energy (mass)

$$E^* = E_{N^*} + \alpha' L\cdot\Pi + \beta'(\Pi^2 - 3/4) + \gamma'(n_i - 1/2) \quad (59)$$

$$E = E_N + \alpha L \cdot M + \beta(n^2 - 3/4) + \gamma(n_3 - 1/2) \qquad (60)$$

This leads to same mass relation

$$\frac{3m_\Lambda + m_\Sigma}{4} = \frac{m_N + m_\Xi}{2} \qquad (61)$$

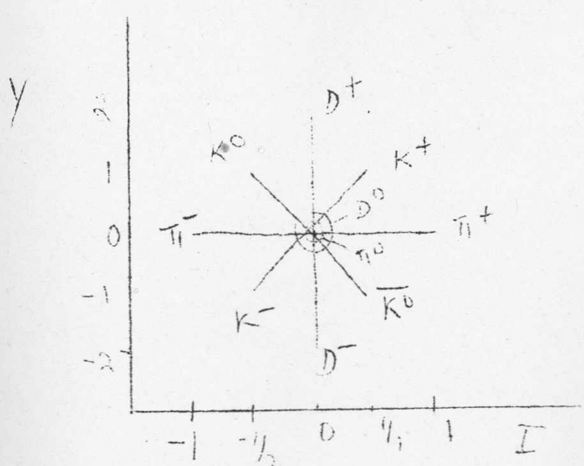
C_2

The basis for the five dimensional representation $D^5(0,1)$ may be taken as N, Λ, Ξ [17]. This naturally implies N, Λ, Ξ space time properties are the same but Σ may be different. For mesons we have

$$D^5(0,1) \otimes D^5(0,1) = D^1 \oplus D^{10} \oplus D^{14} \qquad (62)$$

(Notice ψ_a and ψ^a are equivalent for C_2).

Thus we have



Thus we have in addition to the usual octet of mesons with $Y = -2$, $\alpha - 2$ and $I = 0$. Similarly for vector mesons (resonances) also. For higher baryons multiplets

$$D^5(0,1) \otimes D^{10}(2,0) = 35' \oplus 10 \oplus 5 \qquad (63)$$

The Lowest representation which can accommodate Σ is $D^{(10)}$. This means we should have baryon resonances with $J - \frac{1}{2}$ and following quantum numbers

$I = 1/2$	$Y = 1$	πN res
$I = 1/2$	$Y = -1$	$\Xi\pi$ res
$I = 0$	$Y = 2, 0, -2$	NK res $\bar{K}N + \Xi K$ res $\Xi\bar{K}$ res.

Ofcourse the masses are not prescribed.

B2

For this $D^4(1,0)$ is the basic representation we are interested in with the basis given by N, Ξ. The Λ belongs to the singlet representation $D'(0,0)$. We have for K mesons

$$D^4(1,0) \otimes D'(0,0) = D^4(1,0) + D'(0,0) \qquad (64)$$

$D^4(1,0)$ thus gives K^+, K^-, K^0 & $\overline{K^0}$.

π meson and other baryons are got from

$$D^4(1,0) \otimes D^4(1,0) = D^{10} \oplus D^5 \oplus D' \qquad (65)$$

The '5' representation may include $\Sigma^{\pm,0}$ $(\pi^{\pm 0})$ and 2 other baryons $X^{\pm}(D^{\pm})$ with $T=0$ $Y=0$

Also Y_1^* may belong to representation '5'. Then $Y_1^* \rightarrow \Sigma + \pi$ is forbidden since Σ and π belong to '5' representation each and '5' ⊗ '5' does not contain a '5' again. But $Y_1^* \rightarrow \Lambda + \pi$ is allowed. Notice further that this representation does not require the π mass to be same as K mass.

G_2

$D^7(1,0)$. Basis may be taken as Σ, N, Ξ and Λ belong to $D^1(0,0)$. This can accomodate odd $\Sigma\Lambda$; Parity. Similarly we have

$$D^7(1,0) \longrightarrow K, \bar{K}, \pi \qquad D^1 \rightarrow \eta \qquad (66)$$

and

$$D^7(1,0) \longrightarrow K^*, \bar{K}^*, f \qquad D^1 \rightarrow \omega$$

For baryons resonances we have

$$D^7(1,0) \otimes D^7(1,0) = D^{27} \otimes D^{14} \oplus D^7 \oplus D^7 \qquad (67)$$

An $I = 3/2$ resonance is allowed only in the 14 and 27 representation. '14' fold way requires

$I = 3/2$	$Y = 1$	N^*
$I = 3/2$	$Y = -1$	Ξ^*
$I = 1$	$Y = 0$	?
$I = 0$	$Y = 2, 0, -2$	?

The $\Lambda\pi\ (Y_1^*)$ resonance belongs to representation 7. This multiplet does not contain $I = 3/2$. But may include $I = 1/2$, $\Xi\pi$ resonance. and $N\pi$, $I = 1/2$ resonance.

Sawyer[18] has recently considered global symmetry as invariance under the group operation $SU_3 \otimes SU_2$ which is a subgroup of SU_6. He take the basis as

$$B_6 = \begin{pmatrix} N \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} p \\ n \\ \Lambda \\ \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \tag{68}$$

Thus SU_3 will act on $\begin{pmatrix} N \\ Y \\ Z \end{pmatrix}$ while SU_2 will act on each N, Y, Z. To get the mesons we consider

$$B_6^* \otimes B_6 = 1 \oplus 35 \text{ under } SU_6 \tag{69}$$

$$= 1 \oplus 3 \oplus 8 \oplus 24 \text{ under } SU_3 \otimes SU_2 \tag{70}$$

The representation '3' gives the pseudoscalar π. (and ρ for vector states). '8' consists of $K, \eta\, \pi'$ (K^*, ω and f) and '24' consists of higher mass bosons. The merit of this group consists in the fact that π belongs to a different representation for K, η, π'.

For higher baryons multiplets, the lowest multiplet which contains a $T = 3/2$, $S = 0$ state is of dimension 12. Corresponding to ----

T	S	Resonance or Particle	Mass (m_π)
3/2	0	$N^*_{3/2}$	9
1	-1	Y_1^*	10
0	-1	Y_0^*	11
1/2	-2	Ξ	9.4
1/2	0	$N^*_{1/2}$	?

d. SOME SPECULATIONS

(i) First let us consider Okubo's mass relation

$$m = a + bY + c\left[T(T+1) - \frac{Y^2}{4}\right] \qquad (1)$$

Let us apply this to the representation '15' of the higher multiplets of baryons in the Sakata model. The I-spin, Y assignments for these are given by

$$\begin{array}{llll} \text{I} & T = 1/2 & Y = 1 & (2) \\ & T = 0 & Y = 0 \quad\quad T = \frac{Y}{2} & (3) \\ & T = 1 & Y = 2 & (4) \\ \hline \text{II} & T = 3/2 & Y = 1 & (5) \\ & T = 1 & Y = 0 \quad\quad T = 1 + \frac{Y}{2} & (6) \\ & T = 1/2 & Y = -1 & (7) \end{array}$$

It is interesting to note that for the first three ((2) to (4)) the relation $T = Y/2$ holds while for the second three ((5) to (7)) the relation is $T = 1 + \frac{Y}{2}$. Thus Okubo's mass relation immediately becomes a linear relation for these with

$$m_{I} = a + bY \tag{8}$$

and

$$m_{II} = a' + b'Y \tag{9}$$

Let us suppose that Y_1^* (1385) and $N_{3/2}^*$ (1238) belong to the second half with $J = 3/2+$. Then we can predict the mass of the third particle of II with $T = 1/2$ $Y = -1$ as

$$m(Y=-1, T=1/2) = 1532 \text{ MeV}. \tag{10}$$

which can be easily identified as the Ξ^* resonance <u>and not as the Ξ_- particle</u>. So the natural conclusion would be that if the Y_1^* and $N_{3/2}^*$ belong to the '15' representation then the Ξ should not be placed in the same multiplet but may have a place only in a further higher multiplet say from $3 \times 3^* \times 3 \times 3^* \times 3$ and it is quite probable that it will be grouped with particles with spin ½. A study of the irreducible representations of $3 \times 3^* \times 3 \times 3^* \times 3$ is being done.

It is worth while mentioning that to predict the mass of the 3rd particle in the division I (or II) it is necessary to know only 2 masses of the same group I (or II) in view of the linear relation between T and Y. Further knowing the 3 masses of the same group I (or (II) is not sufficient to give the mass of the other group though we have only 3 unknown in equation (1). So if we further assume that the

Y_0^* resonance with mass 1405 Mev. is also J - 3/2 +

then we got

$$m(Y=2, T=1) = 1131 \text{ MeV} \tag{11}$$

which should be a bound state of (K+ N). Also we get

$$m_{N^*}(Y=1, T=1/2) = 1265 \text{ Mev} \tag{12}$$

which is yet to be experimentally seen. Alternatively if we assume Y_0^* (1520 Mev.) and $N^*_{1/2}$ (1512 Mev) belong to '15' then all the particles of this group should have spin J - 3/2 −. This gives K^+N resonance with mass

$$m = 1504 \text{ MeV} \tag{13}$$

(ii) It is interesting to note $K^*_{T=1/2}$ (730 Mev)

$\varphi_{T=1}$ (560 Mev) and $\omega_{I=0}$ (752 Mev) satisfy the mass relation

$m_{K^*}^2 = \dfrac{m_\varphi^2 + 3m_\omega^2}{4}$ very well.

(iii) The particle belonging to the '27' fold way can also be split as follows:

$$\left.\begin{array}{ll} T=0 & Y=0 \\ T=1/2 & Y=1 \\ T=1 & Y=2 \end{array}\right\} T=\frac{Y}{2} \qquad \left.\begin{array}{ll} T=2 & Y=0 \\ T=1 & Y=-2 \\ T=3/2 & Y=-1 \end{array}\right\} T=2+\frac{Y}{2}$$

$$\left.\begin{array}{ll} T=1 & Y=0 \\ T=3/2 & Y=1 \\ T=1/2 & Y=-1 \end{array}\right\} T = 1 + \frac{Y}{2} \qquad (14)$$

so that knowing any two of the same group the third can be predicted by the linear mass relation.

(iii) In the 'Octet' model the higher baryon multiplets correspond to $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$.

We have already discussed about '10' in section (b). Let us consider the $\overline{10}$ now. We know that if Y_1^*, $N_{3/2}^*$ belong to the '10' representation there should be yet another state ~~in~~ baryon Ω with mass 1670 Mev. and a Ξ^* resonance with mass 1532 Mev . (Notice ~~ix~~ similarity with Sakata model '15' $\underline{\text{II}}$; same relation $T = 1 + \frac{Y}{2}$ holds for both). However the decay widths

$$\frac{\Gamma(Y_1^* \to \Sigma\pi)}{\Gamma(Y_1^* \to \Lambda\pi)} \sim 16\% \text{ according to unitary} \qquad (15)$$

symmetry, if Y^* belongs to '10'. We know experimentally it is almost $\sim 0\%$. This can be overcome by imposing R invariance. Now let us study the further consequences of R symmetry. Then As pointed out by Sakurai and Glashow if the '$\overline{10}$' resonate so should the $\overline{10}$. This leads to the following resonances

1) Ξ^* resonance with $I = 3/2$ $(RN^*_{3/2})$

2) $N^*_{1/2}$ " " $I = 1/2$ $(R\,\Xi^*_{1/2})$

3) Y_1^* " " $I = 1$ (RY_1^*)

4) $X\ (?)$ " " $I = 0$ $(R\Omega)$ (16)

We may predict the masses of 2 of these particles knowing the other two. We do not have experimental information on any of these except perhaps the Y_1^*. But wemay assume (using the effect of R-operation) that the mass of $\Xi^*_{3/2}(\Xi^*_{1/2})$ and $N^*_{3/2}(N^*_{1/2})$ transform in the same way as Ξ and N. This gives

$$m_{\Xi^*_{3/2}} \simeq 1735\ \text{Mev} \qquad m_{N^*_{1/2}} \simeq 1094\ \text{Mev}$$

Using this we get

$$m_{Y_1^*} \simeq 1415\ \text{Mev} \qquad m_{(K^+N)} \simeq 936\ \text{Mev}.$$

which again leads to a bound K^+N system.

(iv) In the Octet model we know that N, Λ, Σ and Ξ belong to the '8' representation of SU_3. Now the recently observed $N^*_{1/2}$ with mass 1685 Mev and Y^*_0 with mass 1815 Mev are supposed to be on the same Regge trajectory as the N and Λ respectively (if their spin is $J = \frac{5}{2}+$) with slope given by $\sim 1/50\, m_\pi^2$. Let us calculate the mass of the Y_1^* and Ξ^*. With $J = 5/2+$ which will be on the Σ and Ξ Regge trajectories respectively

For $\frac{d\alpha}{dt} \sim \frac{1}{5 c m_\pi^2}$ we get the following masses

B	Mass	B^*	Mass Predicted	Exptal value
N	940	$N^*_{1/2}$	1685	1688
Λ	1115	Y_0^*	1793	1815
Σ	1190	Y_1^*	1836	?
Ξ	1315	$\Xi^*_{1/2}$	1920	?

(19)

It would be interesting if these form belong to the same representation '8'. In this case using Okubo's mass relation and 3 of the above masses the fourth can be predicted. Let us assume the 1st three: (4)

$$1685 = a + b + c/2$$

$$1793 = a$$

$$1836 = a + 2c$$

This gives $m_{\Xi^*} = a - b + c/2 = 1923 \text{ MeV}$ (20)

which is in agreement with that given in the table (19). The experimental mass values for $N^*_{1/2}$ and Y_0^* are fairly in good agreement with the Regge trajectory values. One may thus expect two more $J = 5/2$ resonances at about the energies shown. both by symmetry arguments and from Regge trajectories.

We can generalize the above considerations for all Regge trajectories and may study in general the consequences of higher symmetries and the hypothesis of Regge poles together. One remark that can be made is that if $\frac{d\alpha}{dE} \sim$ constant (rather than $\frac{d\alpha}{dt(=E^2)} \sim \frac{1}{50 m_\pi^2}$) then Okubo's mass relation will hold for higher particles on the Regge Trajectory automatically. But if $\frac{d\alpha}{dt} \sim \frac{1}{50 m_\pi^2}$ then Okubo's mass relation will be satisfied by these higher mass particles in the Regge trajectory only if

$$\frac{m_N^2 + m_\Xi^2}{2} \sim \frac{3 m_\Lambda^2 + m_\Sigma^2}{4} \qquad (21)$$

which seems to be satisfied for N, Λ, Σ and Ξ.

APPENDIX.

Group theoretical concepts necessary for this lecture will be summarized here. We give the definitionof the following compact Lie groups as:

A_ℓ - order $(\ell^2 + 2\ell)$ — group of unitary unimodular matrices in complex space of $(\ell+1)$ dimensions.

B_ℓ - order $(2\ell^2 + \ell)$ group of orthogonal transformations (rotations) in a real space of $(2\ell+1)$ dimensions.

C_l — order $(2l^2+l)$ group of unitary matrices U in complex space of $2l$ dimensions.

G_2 — Exceptional group of order 14. [Take the case $l=2$

This implies there are two commuting matrices which ca n be diagonalized simultaneously and correspond to the number of conserved quantities. Any irreducibel representation of these groups can be labelled by means of two non-negative integers a, and q_2 which are related to Y & T for strong interactions.

We give below in tables the different groups and their dimensions in terms of q_1 and q_2 and also ~~the~~ I spin content for the different representations.

SU_3 Rep	d	I spin content	$\otimes D^3(1,0)$	$\otimes D^6(2,0)$	$\otimes D^8(1,1)$
$D(0,0)$	1	0	3	6	8
$D(1,0)$	3	$0, 1/2$	$6+3^*$	$10+8$	$15+6^*+3$
$D(0,1)$	3^*	$0, 1/2$	$8+1$	$15+3$	15^*+6+3^*
$D(2,0)$	6	$0, 1/2, 1$	$10+8$	$15'+15+6^*$	$24+15^*+6+3^*$
$D(1,1)$	8	$0, 1/2, 1/2, 1$	$15+6^*+3$	$24+15^*+6+3^*$	$27+10+10^*+8+8+1$
$D(0,3)$	10^*	$0, 1/2, 1, 3/2$			
$D(3,0)$	10	$0, 1/2, 1, 3/2$			
$D(2,1)$	15	$0, 1/2, 1/2, 1, 1, 3/2$			
$D(2,2)$	27	$0, 1/2, 1/2, 1, 1, 1, 3/2, 3/2, 2$			

$C_2 [B_2]$

Rep	d	J-spin	$\otimes D^4(1,0)$	$\otimes D^5(0,1)$	$\otimes D^{10}(2,0)$
$D(0,0)$	1	0	4	5	10
$D(1,0)$	4	$0, 0, \frac{1}{2}$ $[\frac{1}{2}, \frac{1}{2}]$	10+5+1	16+4	40+16
$D(0,1)$	5	$0, \frac{1}{2}, \frac{1}{2}$ $[0, 0, 1]$	16 + 4	14+10+1	35'+10+5
$D(2,0)$	10	$0, 0, 0, \frac{1}{2}, \frac{1}{2}, 1$ $[0, 1, 1, 1]$	20+16+4	35'+10+5	
$D(0,2)$	14	$0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1$ $[0, 0, 0, 1, 1, 2]$	40+16	35+30+5	

G_2

Rep.	d	I-Spin	$\otimes D^7(1,0)$	$\otimes D^{14}(0,1)$
$D(1,0)$	7	$\frac{1}{2}, \frac{1}{2}, 1$	27+14+7+1	64+27+7
$D(0,1)$	14	$0, 0, 0, 1, \frac{3}{2}$	64+27+7	77+77'+27+14+1
$D(2,0)$	27	$0, \frac{1}{2}, \frac{1}{2}, 1, 1, 1, \frac{3}{2}, \frac{3}{2}, 2$	77+64+27+14+7.	

Tensor Analysis of SO_3

The irreducible representations into which product representation breaks can be given as follows:

(i) $\psi_a \otimes \psi_b \sim \psi_{ab} \oplus \psi_{a,b}$

where $\psi_{ab} = (\psi_a \psi_b + \psi_b \psi_a)/2$

$\psi_{a,b} = (\psi_a \psi_b - \psi_b \psi_a)/2$

(ii) $\psi_a \otimes \psi^b = \left(\psi_a{}^b - \frac{1}{3}\delta_a{}^b \psi_c^c\right) \oplus \delta_a{}^b \psi_c^c$

(iii) $\psi_a \otimes \psi_b^b \times \psi^c \sim \psi_{ab,}{}^c \oplus \psi_{a,b}{}^c \oplus \psi_{ac}{}^c \oplus \psi_{cb}{}^c$

(iv) $\psi_a \otimes \psi_b \otimes \psi_c \sim \psi_{abc,} \oplus \psi_{ab,c} \oplus \psi_{ac,b} \oplus \psi_{a,b,c}$

where (eg) $\psi_{ab,c} = \frac{1}{4}(\psi_{abc} - \psi_{acb} + \psi_{bac} - \psi_{bca})$

with $\psi_{abc} = \psi_a \psi_b \psi_c$ etc.

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GROUP SYMMETRIES WITH R-INVARIANCE

Prof. R.E.Marshak

(invited talk)

GROUP SYMMETRIES WITH R-INVARIANCE.

R.E. Marshak.

In this talk, we will consider:

1) Group symmetries (including R symmetry),

2) Electromagnetic consequences of these symmetries,

3) R invariance applied to the particle resonances.

1) The physical motivation for these symmetries stems from the three conservation laws obeyed by strong interactions,

i.e. a) Strangeness (S)

b) I-spin (I)

3) Baryon number(B)

a) and c) can be considered as consequences of guage transformations G_1, G_2 while b) may be thought of as invariance under rotations in 3-dimensional space (R_3) which may be represented by the group operation SU_2. Thus we require the strong interactions to be invariant under the product off these transformations (i.e.) $G_1 \otimes R_3 \otimes G_2$ (represented by the group operation $U_1 \otimes SU_2 \otimes U_1'$). We may replace $SU_2 \otimes U_1'$ by U_2 and thus we require invariance under $U_1 \otimes U_2$. This is a subgroup of U_3 but strictly speaking not of SU_3. We will be considering the following two models :

i) The Sakata model,

ii) The octet model.

It is convenient to discuss the Sakata model within the framework of U_3 while the octet model may be discussed with reference to SU_3. This is because we may specify, I, B, S for the Sakata model with (p, n, Λ) while for the octet model we can only specify I and Y.

Global symmetry is based on rotations in 4-dimensional space (R_4).

We have the four doublets given by

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N_2 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \qquad N_3 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix} \qquad N_4 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}$$

The operation which connects

(N_1 , N_2) to (N_3, N_4) is R_3^I

while (N_1 , N_3) and (N_3 , N_4) are connected by R_3^y. $R_3^I \otimes R_3^y$ is equivalent to R_4 which is a subgroup of U_4 but not U_3.

We will follow the notation of Okubo and use f_1 , f_2 , f_3 as the parameters which characterize the different representations of U_3 , the dimensions of which are given by

$$d = 1/2 (f_1 - f_2 + 1) (f_1 - f_3 + 2) (f_2 - f_3 + 1)$$

Further we have

$$N = f_1 + f_2 + f_3 \text{ ; and } I = 1/2 (f_1' - f_2')$$

$$Y = (f_1' + f_2') - (f_1 + f_2 + f_3)$$

where f_1' and f_2' can take on all possible values given by

$$f_1 \geq f_1' \geq f_2 \geq f_2' \geq f_3$$

Rep	Dim	I	y
U_3 (0, 0, 0)	1	0	0
U_3 (1,0,0)	3	½, 0	1,0
U_3 (1,0,-1)	8	½, ½, 1,0	1, -1, 0, 0
U_3 (2,0,-1)	15	½, ½, 3/2	-1, 1, 1
		1, 1, 0	2, 0, 0
U_3 (1,1,-1)	6	½, 0, 1	1, 2, 0
U_3 (2,0,-2)	27	0, ½, ½, 3/2, 3/2	0, 1, -1, 1, -1,
		1, 1, 1, 2	0, 2, -2, 0.
U_3 (2,-1,-1)	10	½, 3/2, 1, 0	-1, 1, 0, -2
U_3 (1, 1, -2)	$\overline{10}$	½, 3/2, 1, 0	1, -1, 0, 2

In the Sakata model, the three primitive objects are taken to be (p , n , Λ) represented by U_3 (1, 0, 0) and the mesons belong to the representation U_3 (1, 0, -1) so that the higher baryon multiplets can be assigned to one of the decompositions of :

$$U^3 (1,0,0) \otimes U^8 (1,0,-1) = U^{15} (2,0,-1) \oplus U^6 (1, 1, -1) \oplus U^3 (1, 0, 0)$$

The possible identification of these particles has already been discussed in the lecture by Miss. Radha.

For the octet model, the representation U (1,0,-1) is taken to cover the known eight baryons N , Λ , Σ and Ξ . Higher baryon multiplets and mesons are formed from decompositions of the product representation:

[illegible]

$$U^{8}(1,0,-1) \otimes U^{8}(1,0,-1) = 2U^{8}(1,0,-1)$$
$$\oplus U^{1}(0,0,0) \oplus U^{27}(2,0,-2) \oplus U^{10}(2,-1,-1)$$
$$\oplus U^{\overline{10}}(1,1,-2)$$

The tenfoldway seems to be the promising one to accommodate the N_1^*, Y_1^*, and *. It has the unique relation between I and y given by $I = 1 + \frac{y}{2}$ and it consists of

State	I	Y	Mass (540)	J	Decay	
N_1^*	3/2	1	1238	3/2	N	
Y_1^*	1	0	1385	3/2		16 %
Ξ^*	1/2	-1	1532	?		
Ω	0	-2	1679	?	Stable	

where the masses of the * and have been calculated using Okubo's mass relation to first order (to be discussed later).

The basis of any representation can be written as ψ_μ which, for the Sakata model (for baryons), is :

$$\psi_\mu = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix} \qquad \psi^\mu = \begin{pmatrix} p^* & n^* & \Lambda^* \end{pmatrix}$$

Bosons which form an octet in the Sakata model are represented by the traceless tensor h^μ_ν. Thus a baryon-meson introduction may be written as

$$H_1 = \psi^\mu \gamma_5 \psi_\nu h^\nu_\mu$$

The boson-boson interaction can be written as (say for decay of vector bosons F^{μ}_{ν} into pseudoscalar mesons h^{μ}_{ν})

$$H_2 = F^{\mu}_{\nu}\left(h^{\nu}_{\lambda}\,\partial h^{\lambda}_{\mu} - \partial h^{\nu}_{\lambda}\, h^{\lambda}_{\mu}\right)$$

In the octet model, let us represent baryons by N^{μ}_{ν} and anti-baryons by M^{μ}_{ν} i.e. :

$$M^{1}_{2} = \overline{\Sigma^{+}}\,;\quad M^{2}_{1} = \overline{\Sigma^{-}}\,;\quad \frac{1}{\sqrt{2}}(M^{1}_{1}-M^{2}_{2}) = \overline{\Sigma^{0}}\,;\quad -\frac{3}{\sqrt{6}}M^{3}_{3} = \overline{\Lambda^{0}}$$

$$M^{3}_{1} = \overline{\Xi^{-}}\,;\quad M^{3}_{2} = \overline{\Xi^{0}}\,;\quad M^{1}_{3} = \overline{p}\,;\quad M^{2}_{3} = \overline{n}$$

and

$$N^{2}_{1} = \Sigma^{+}\,;\quad N^{1}_{2} = \Sigma^{-}\,;\quad \frac{1}{\sqrt{2}}(N^{1}_{1}-N^{2}_{2}) = \Sigma^{0}\,;\quad -\frac{3}{\sqrt{6}}N^{3}_{3} = \Lambda^{0}$$

$$N^{1}_{3} = \Xi^{-}\,;\quad N^{2}_{3} = \Xi^{0}\,;\quad N^{3}_{1} = p\,;\quad N^{3}_{2} = n$$

Now the usual R-operation is easily identified as the interchange of upper and lower indices of the tensor repre-sentation of the basis. Similarly, we may write (remembering that the π triplet is the analog of the Σ triplet, etc):

$$h^{1}_{2} = \pi^{-}\,;\quad h^{2}_{1} = \pi^{+}\,;\quad \frac{1}{\sqrt{2}}(h^{1}_{1}-h^{2}_{2}) = \pi^{0}$$

$$-\frac{3}{\sqrt{6}}h^{3}_{3} = \pi^{0\prime}(=\eta)\,;\quad h^{1}_{3} = K^{-}\,;\quad h^{3}_{1} = K^{+}\,;$$

$$h^{3}_{2} = K^{0} \qquad h^{2}_{3} = \overline{K^{0}}$$

In the octet model, we have the possibility of two baryon-boson couplings (remembering the occurrence of $U^{8}(1,0,-1)$ twice in the product representation of U (1,0,-1) $\otimes$ U (1, 0, -1) which can be written as

$$H_3 = ig\, M^{\mu}_{\nu}\gamma_5 N^{\nu}_{\lambda} h^{\lambda}_{\mu}$$

$$H_4 = ig'\, M^{\mu}_{\nu}\gamma_5 h^{\nu}_{\lambda} N^{\lambda}_{\mu}$$

Expanding

$$H_3 = ig\,\bar{N}\gamma_5(\underline{\tau}\cdot\underline{\pi})N + \frac{g}{\sqrt{2}}(\bar{\underline{\Sigma}}\gamma_5 \times \underline{\Sigma})\cdot\underline{\pi} + \frac{g}{\sqrt{6}}\left(i\bar{\underline{\Sigma}}\cdot\underline{\pi}\,\gamma_5\Lambda + c.c\right)$$

$$+\frac{g}{\sqrt{6}}\left[\bar{\Lambda}\gamma_5\Xi\tau_2 K + c.c\right] - \frac{\sqrt{6}}{3}g\left[i\bar{N}\gamma_5 K\Lambda + c.c\right]$$

$$-\frac{g}{\sqrt{2}}\left[\bar{K}\underline{\tau}\tau_2\bar{\Xi}\gamma_5\underline{\Sigma} + c.c\right] - \frac{ig}{\sqrt{6}}\pi^{0\prime}\left[2\bar{\Xi}\gamma_5\Xi + \bar{\Lambda}\gamma_5\Lambda - \bar{\underline{\Sigma}}\gamma_5\underline{\Sigma} - \bar{N}\gamma_5 N\right]$$

$$H_4 = \frac{g}{\sqrt{6}}\left[i\bar{\Lambda}\gamma_5\vec{\Sigma}\cdot\vec{\pi} + c.c\right] - \frac{g}{\sqrt{2}}(\bar{\underline{\Sigma}}\gamma_5\times\underline{\Sigma})\cdot\underline{\pi} - \frac{ig}{\sqrt{2}}\bar{\Xi}\gamma_5(\underline{\tau}\cdot\underline{\pi})\Xi$$

$$+\frac{g}{\sqrt{6}}\left[i\bar{N}\gamma_5 K\Lambda + c.c\right] - \frac{2g}{\sqrt{6}}\left[\bar{\Lambda}\gamma_5\Xi\tau_2 K + c.c\right]$$

$$+\frac{g}{\sqrt{2}}\left[i\bar{N}\gamma_5\underline{\tau}\cdot K\underline{\Sigma} + c.c\right] + \frac{ig\pi^{0\prime}}{\sqrt{6}}\left[\bar{\underline{\Sigma}}\cdot\gamma_5\underline{\Sigma} + \bar{\Xi}\gamma_5\Xi - \bar{\Lambda}\gamma_5\Lambda - 2\bar{N}\gamma\right.$$

where

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \Xi = \begin{pmatrix} -\Xi^0 \\ \Xi^- \end{pmatrix} \quad \underline{\Sigma} = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad \underline{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

The D and F couplings of Gell-Mann can be easily identified as $H_3 + H_4$ and $H_3 - H_4$ respectively with $g = g'$. As expected the $(\underline{\Sigma}\times\underline{\Sigma})\cdot\underline{\pi}$ terms gets cancelled in the D coupling and gives us $g^2_{\Sigma\Sigma\pi} = 0$ which, of course, is inconsistent with global symmetry. A similar statement can be made for F coupling, i.e. it is inconsistent with global symmetry.

Now let us consider meson-baryon scattering in the Sakata model. If $T^{\mu}_{\nu} = \psi^{\mu}\psi_{\nu}$, we may take a linear combination of the invariants of the form:

$$a T^{\mu}_{\nu} h^{\nu}_{\lambda} \tilde{h}^{\lambda}_{\mu} + b T^{\mu}_{\nu} \tilde{h}^{\nu}_{\lambda} h^{\lambda}_{\mu} + c T^{\mu}_{\mu} h^{\alpha}_{\beta} \tilde{h}^{\beta}_{\alpha}$$

The coefficients are determined by dynamical calculations. One gets, allowing a , b and c to be arbitrary :

$$\sigma(\pi^{+}+p) = \sigma(K^{+}+p)$$
$$\sigma(K^{-}+n) = \sigma(\pi^{+}+\Lambda)$$
$$\sigma(\pi^{0\prime}+p) = \frac{1}{3}\sigma(\pi^{0}+p) + \frac{2}{3}\sigma(K^{0}+p) \text{, etc}$$

In the octet model, the components of the '10' representation have the tensor notation given by

$$F^{\mu\nu}_{\alpha\beta}$$

which is symmetric in μ , ν and antisymmetric in α, β. The $\overline{10}$ representation $F^{\mu\nu}_{\alpha\beta}$ is antisymmetric in μ, ν and symmetric in α , β. Parenthetically, we remark :

$$R \text{ ' } 10 \text{ ' } = \text{ ' } \overline{10} \text{ '}$$

Further, the decay of ~~xx x~~ a baryon belonging to '10' into a baryon of '8' and a meson of '8' can be represented by :

$$S = F^{\mu\nu}_{\alpha\beta} M^{\alpha}_{\mu} h^{\beta}_{\nu}$$

This is unieque unlike the case when we have the decay of a baryon of '8' into a baryon of '8' and a meson of '8' for which we have two linearly independent interactions :

$$S_1 = M^\mu_\nu \, h^\nu_\lambda \, T^\lambda_\mu$$

$$S_2 = M^\mu_\nu \, h^\lambda_\mu \, T^\nu_\lambda$$

Thus it is possible to predict the decay probability of Y_1^* definitely only if it belongs to '10' and not to '8' because of the ambiguity (S_1 and S_2) in the latter case.

2. Electromagnetic Effects.

Let us consider the Sakata model first. We have the electromagnetic current given by:

$$j_\mu = \bar{p} \gamma_\mu p$$

This has the transformation property of T^1_1 where $T^\mu_\nu = \psi^\mu \psi_\nu$. Define $\langle S^\mu_\nu \rangle = a \delta^\mu_\nu T^\lambda_\lambda + b T^\mu_\nu$

Requiring the trace to vanish, we have

$$3 a + b = 0$$

and

$$\langle S^1_1 \rangle = a(\bar{p}p + \bar{n}n + \bar{\Lambda}\Lambda) - 3a\bar{p}p = a(\bar{n}n + \bar{\Lambda}\Lambda - 2\bar{p}p)$$

Now terms which have the same coefficient have the same form factors. Thus we have, in addition to the relation

$$\mu(n) = \mu(\Lambda) \quad ;$$

$$-\mu(p) = \mu(n)/2$$

But if we do not impose the tracelessness condition , we will have only $\mu(n) = \mu(\Lambda)$. In the octet model, we have:

$$j_\mu = \bar{p}\gamma_\mu p + \overline{\Sigma^+}\gamma_\mu \Sigma^+ - \overline{\Sigma^-}\gamma_\mu \Sigma^- - \overline{\Xi^-}\gamma_\mu \Xi^-$$

What we are interested in is not the space-time properties of j_μ but only the tensorial property in the unitary space (so we may forget the γ_μ for the present) :

$$j_\mu = M^1_3 N^3_1 + M^1_2 N^2_1 + M^2_1 N^1_2 - M^3_1 N^1_3$$

Define:

$$S^\mu_\nu = \left(M^\mu_\lambda N^\lambda_\nu - M^\lambda_\mu N^\nu_\lambda \right)$$

As far as electromagnetic properties are concerned, it is easy to see that we are interested only in S^1_1. (It is interesting to note that the expectation value $\langle S^1_1 \rangle$ gives the electromagnetic relations while the expectation value $\langle S^3_3 \rangle$ gives the mass relation between the particles of the octet) . The same relations hold for both the charge and magnetic form factors of the baryons. For spin 0 bosons, there are no magnetic form factors but only charge form factors. Consider

$$\langle S^\mu_\nu \rangle = a\delta^\mu_\nu (M^\alpha_\beta N^\beta_\alpha) + b\, M^\mu_\lambda N^\lambda_\nu + c\, M^\lambda_\nu N^\mu_\lambda$$

If we further impose the condition that the trace of this tensor is zero, we get the relation

$$3a + b + c = 0$$

Calculating

$$\langle S^1_1 \rangle = a M^\alpha_\beta N^\beta_\alpha + b\, M^1_\lambda N^\lambda_1 - (3a+b) M^\lambda_1 N^1_\lambda$$

for the octet model, we have under unitary symmetry :

$$\mu(\Sigma^+) = \mu(p)$$
$$\mu(\Lambda) = \frac{1}{2}\mu(n)$$
$$\mu(\Xi^0) = \mu(n)$$
$$\mu(\Xi^-) = \mu(\Sigma^-) = -[\mu(p) + \mu(n)]$$
$$\mu(\Sigma^0) = -\frac{1}{2}\mu(n)$$

with similar relations for the charge form factors. The implied relation $\mu(\Sigma^0) = \frac{1}{2}[\mu(\Sigma^+) + \mu(\Sigma^-)]$ follow simply from just charge independence. By applying the same arguments to the boson octet, we find:

$$F(K^0) = F(\overline{K^0})$$

where F is the charge form factor.

$\langle K^0 | j_\mu | K^0 \rangle = -\langle \overline{K^0} | j_\mu | \overline{K^0} \rangle$ by charge conjugation.

Thus $F(K^0) = -F(\overline{K^0})$ which, when combined with unitary symmetry yields $F(K^0)$ 0 . Also we have

$$F(K^+) = F(\pi^+)$$

This is quite natural since the difference in mass between π and K is neglected in the unitary symmetry model. It should also be pointed out that for the boson octet, the antiparticles are included in ~~xx~~ same representation - in contrast to the baryon octet.

To first order in the electromagnetic interaction and to all orders in unitary symmetry, one can deduce the relation

$$m_{\Xi^-} - m_{\Xi^0} = m_{\Sigma^-} - m_{\Sigma^+} + m_p - m_n$$

This equation predicts 5.4 Mev for $m_{\Xi^-} - m_{\Xi^0}$ whereas the latest experimental value is 4.7 ± 0.8 Mev. This good agreement may be accidental but at first sight it looks like a strong argument in support of the octet model for the baryons. Unfortunately, the same model does not work as well for the boson octet. We have noted earlier that

$$F(K^+) = F(\pi^+) \text{ and } F(K^0) = F(\pi^0) = 0.$$

This implies that $m_{K^+} > m_{K^0}$ since $m_{\pi^+} > m_{\pi^0}$ which is contrary to experiment. It will be recalled that the mass formula relating the masses of π, K and η (within the boson octet) does not work as well as the formula ~~relating the masses of~~ for N, Λ, Σ, and Ξ (within the baryon octet). The poorer result for the boson octet compared to the baryon octet may be a reflection of the fact that there are larger durations from written symmetry (due to the larger mass differences within the octet) for the bosons compared to the baryons.

In passing we note that the formula $\langle S^{\mu}_{2\nu} \rangle$ can also be used to derive the mass relation resulting from the "moderately strong" interactions, namely, we calculate $\langle S^3_3 \rangle$. This is seen immediately in the Sakata model since $S^3_3 = (m_\Lambda - m_N)\bar{\Lambda}\Lambda$ is the mass-splitting term. The octet model also leads to S^3_3 for the mass-splitting term. Using this fact, Okubo got the mass relation to the the first order in the moderately strong interaction :

$$m = a + bY + c\left[T(T+1) - Y^2/4\right]$$

For the '10' representation, this formula reduces to:

$m = a + bY$ because $I = 1 + Y/2$.

This yields the masses of Ξ^* and Ω as given before. Recently, Okubo has calculated the second order mass relation using the tensorial transformation properties of . The relation between masses now becomes quadratic in for the '10' representation but seems to yield almost the same mass for Ω as the first order mass relation. ~~Next~~

// Let us now consider R invariance in more detail. The essential properties of the R operation are that : $Y \to -Y$; $Q \to -Q$

This implies for the baryon and meson octets:

$$p \leftrightarrow \Xi^-$$
$$n \leftrightarrow -\Xi^0$$
$$\Lambda^0 \leftrightarrow \Lambda^0$$
$$\Sigma^+ \leftrightarrow \Sigma^-$$
$$\Sigma^0 \leftrightarrow \Sigma^0$$
$$\pi^+ \leftrightarrow \pi^-$$
$$\pi^0 \leftrightarrow \pi^0$$
$$K^+ \leftrightarrow K^-$$
$$K^0 \leftrightarrow \overline{K^0}$$
$$\eta \leftrightarrow \eta$$

where, by convention, the negative sign is taken for $\eta \leftrightarrow \Xi^0$ with all other signs taken as positive. Actually some of the other signs may be taken negative with different physical consequences for certain processes. Apart from the sign, in the octet model, the operation R merely interchanges the covariant and contravariant indices; hence R invariance implies that :

$$M^{\mu}_{\nu} = M^{\nu}_{\mu} \; ; \; h^{\mu}_{\nu} = h^{\nu}_{\mu}$$

R symmetry is not contained in unitary symmetry and therefore may be independently defined. But if we impose both unitary symmetry and R invariance simultaneously, we derive rather stringent consquences. Thus, consider the electromangetic current:

$$\langle j_\mu \rangle = \langle S^1_1 \rangle = M^1_\lambda N^\lambda_1 - M^\lambda_1 N^1_\lambda$$

Then

$$R \langle S^1_1 \rangle = -\langle S^1_1 \rangle$$

Thus we have $R\langle j_\mu \rangle = -j_\mu$ for all forms of R invariance since we always take diagonal matrix element of j_μ ; it follows that

$$\langle n | j_\mu | n \rangle = -\langle \Xi^0 | j_\mu | \Xi^0 \rangle \quad \text{or}$$

e.g.

$$\mu(n) = -\mu(\Xi^0)$$

Thus R combined with unitary symmetry forces $\mu(n) = 0$. In fact, most of the μ's vanish if we require R and U_3 invariance simultaneously.

For just R invariance (without unitary symmetry), we always have:

$$\langle \Lambda | j_\mu | \Lambda \rangle = -\langle \Lambda | j_\mu | \Lambda \rangle$$

and hence:

$$\mu(\Lambda) = 0$$

3. <u>The operation R and Particle Resonances.</u>

As an example of the consequences which can be drawn about the decays of "metastable" particles, we consider the η meson. Since η is taken as R_3 we can write $R\eta = \pm\eta$ wherein the sign is chosen to satisfy certain requirements (see below). Okubo and Marshak have considered three forms of $\bar{R}$ invariance and their properties are listed below (applied to the meson octet and the photon):

Boson	CR	CR'	CRI'
$\pi^\pm$	$\pi^\pm$	$\pi^\pm$	$\pi^\pm$
π^0	π^0	$-\pi^0$	$-\pi^0$
K^+	K^+	K^+	K^+
K^0	K^0	$-K^0$	$-K^0$
η	η	$-\eta$	$-\eta$
γ	γ	γ	γ

where C is the charge conjugation operator and can always be applied for the strong and electromagnetic interaction. It should be remarked that R goes with the D coupling and R'' with the F coupling of Gell-Mann and R' is inconsistent with global symmetry. Now let us consider η decay assuming η to be 0^{-+}. As far as the electromagnetic interaction is concerned, we have

$$\eta \to 3\pi \sim \alpha^2 \text{(proceeds by violating G parity)}$$
$$\to 2\pi + \gamma \sim \alpha$$
$$\to 2\gamma \sim \alpha^2$$
$$\to 2\gamma + \pi^0 \sim \alpha^2$$

where α is the fine structure constant. R invariance allows all four decays. But invariance under R' or R'' prohibits the $2\pi + \gamma$ and 2γ modes of decay. This selection rule only holds in the approximation where $\Delta = m_\Xi - m_N$ is set equal to zero; if we take account of $\Delta \neq 0$, the decay modes of η then are of the order :

$$\eta \to 3\pi \sim \alpha^2$$
$$\to 2\pi + \gamma \sim \alpha \Delta^2$$
$$\to 2\gamma \sim \alpha^2 \Delta^2$$
$$\to 2\gamma + \pi^0 \sim \alpha^2$$

It may be because of R' or R'' invariance that $\eta \to 3\pi$ is observed quite often compared to the next two decay modes. Similar considerations can be given for the decays of the vector meson octet (ρ , ω , $K^*, \overline{K^*}$) and of the baryon decet (N^* , Y_1^* , Ξ^* , Ω) if the concept of R invariance is extended in a natural way. It is conceivable that R invariance plus unitary symmetry is an interesting combination for decay processes but less interesting for the electromagnetic form factors. Or to put in another way, the corrections due to deviations from unitary symmetry plus R invariance may be more important for the electromagnetic form factors than for the decay processes.

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REGGE POLES AND RESONANCES

T,K. Radha

It is curious situation that just before this talk we have received information that at Brookhaven they have found that the diffraction peaks in π-p and K-p scattering do not shrink with energy while in the case of pp scattering it is much smaller than believed so far. The logrithmic shrinking of the diffraction peak was a unique prediction of the Regge Poles hypothesis for high energy scattering. Since the experimental data are not yet confirmed we shall proceed in the usual way and try to understand the possible reasons if the diffraction peak does not shrink. Since the title of the talk is with reference to Resonances, I shall not go into the details of the analytic properties of the functions involved.

Of course, by now we know that Regge Poles are generalized bound states and resonances in comlex angular momentum. So first we shall study the potential scattering in the complex angular momentum plane and then generalize it to high energy particle scattering also.

Potential scattering and poles in the complex Angular Momentum [3]

We start from the very beginning of quantum theory (i.e.) the Schrodinger equation

$$\Delta\psi(\vec{r}) + E\,\psi(\vec{r}) = V\,\psi(\vec{r}) \qquad (1)$$

Now to find the solution of (1) we split (for central symmetric potential) the wave function into a product of functions

$$\psi = \frac{R(r,l,E)}{r}\, P_l^m(z)\, e^{im\varphi} \qquad (2)$$

where $P_l^m e^{im\varphi}$ are spherical harmonics of angular momentum l (integers). R obeys

$$\frac{1}{2m}\frac{d^2R}{dr^2} + \left[E - V(r) - \frac{l(l+1)}{2\pi r^2}\right] R = 0 \qquad (3)$$

We can define the scattering amplitude once we know the solution of (1) with the following asymptotic behaviour as $r \to \infty$

$$\psi \sim e^{i\vec{k}\cdot\vec{r}} + f(E,\theta)\,\frac{e^{i\vec{k}\cdot\vec{r}}}{r} \qquad (4)$$

(θ is an gle between $\vec{k}$ and the $\longrightarrow$ in which we take the asymptotic limit $r \to \infty$. Therefore if $\vec{r} = r\cdot\vec{n}$ put this into $\psi(\vec{r})$ and make $r \to \infty$ while $\vec{n}$ is a fixed unit vector. Then $\vec{k}\cdot\vec{n} = k\cos\theta$). Then $d\Omega\,|f(z,E)|^2$ is then the probability of finding the particle scattered in the solid angle $d\Omega$ with outgoing momentum $\vec{k} = k\cdot\vec{n}$..

Then we expand

$$f(z,E) = \sum_{l=0}^{\infty} (2l+1) f_l(E)\, P_l(z) \qquad (5)$$

with

$$f(l,E) = e^{2i\delta_l(E)} - 1/2i = \frac{S_l(E)-1}{2i}$$

The phase shifts $\delta_\ell(E)$ of course depend on the potential .

Let us examine equations (2) and (5) a little carefully. It is obvious that angular momentum has been quantized and we have taken only integer values of ℓ . We need δ_ℓ only when ℓ is integer to know the scattering amplitude. This of course is a natural consequence of the limitation on $|\cos\theta| < 1$.

Now with the advent of the Mandelstam representation[4] this limitation had to be given up. True. One can make experimental verification only when $|\cos\theta| < 1$ - however the crossing properties implied by the relativistic Mandelstam representation relate πN scattering and $N\bar{N} \rightarrow \pi\pi$ (i.e.) $N\bar{N} \rightarrow \pi\pi$ is simply $\pi N \rightarrow \pi N$ viewed in a region considered unphysical according to $|\cos\theta| < 1$. Wherefore we have to use functions of hyperbolic angle instead of $P_\ell(\cos\theta)$. Thus we ~~xx~~ require noninteger and complex angular momenta. Now this concept is not totally unknown - this technique has been used for years in the discussion of diffraction phenomena and the theory of rainbow or propagation of waves around the earth.

The basic idea of the technique arises from a transformation due to Watson, of the Rayleigh-Foxen formula (5). This too is successful only if there exists an analytic function $f(\ell, k)$ of the complex variable which takes the value $f_\ell(k)$ when ℓ = integer . If so we have

$$f(z,E) = \frac{i}{2}\int_C \frac{(2\ell+1)\,d\ell}{\sin\pi\ell}\, P_\ell(-z)\, f(\ell,E) \qquad (7)$$

where C is defined as

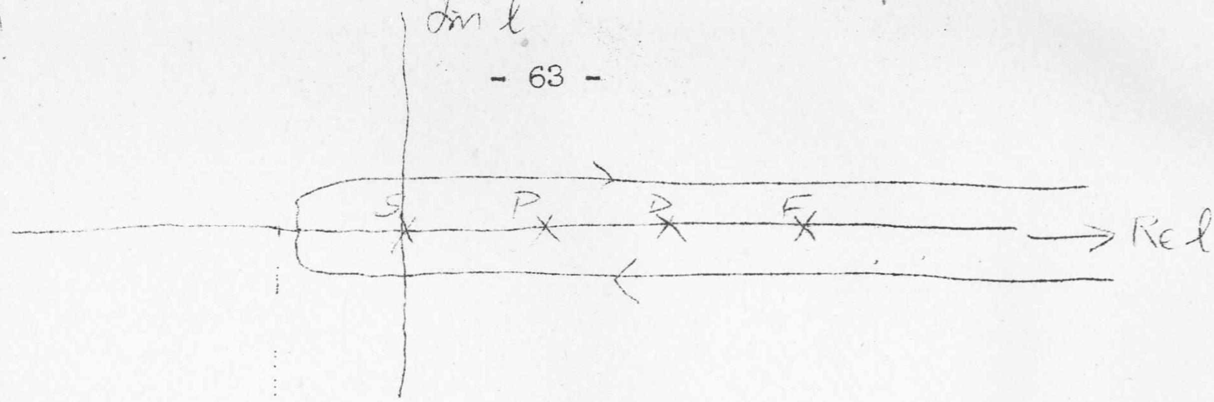


Fig (1)

The contour C avoids all the singularities of $S(\lambda, k)$ and encloses only the positive zeros of $\sin \pi \ell$. We can easily show that (7) reduces to (6) for

$$\frac{1}{\sin \pi \ell} \sim \frac{1}{\pi(\ell - n)(-1)^n} \qquad \text{when } \ell \sim n$$

$$= \frac{P_n(z)}{\pi(\ell - n)} \quad \text{and} \quad f(\ell, k) = f_\ell(k) \text{ for integer } \ell$$

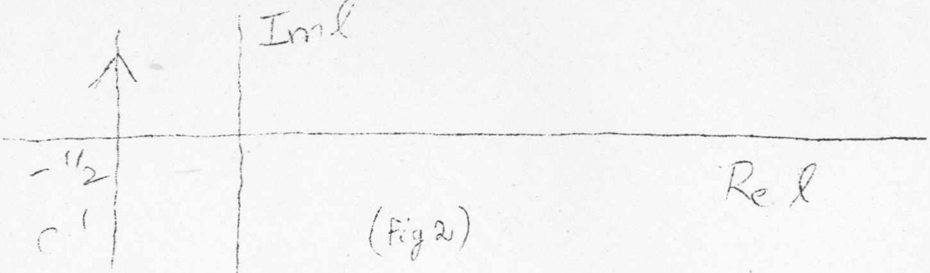
and by taking all the poles, we get the sum over n also. The region of convergence of (7) depends on $P_{\lambda - 1/2}$ and this gives an analytic continuation outside the Lehmann ellipse within which (5) was defined. Therefore now we have to determine the function $S(\ell, k)$ for general values of ℓ and k and study its analytic properties and asymptotic behaviour for large ℓ. Therefore we have to study the Schrodinger equation

$$\frac{d^2R}{dr^2} + k^2 R - \frac{\ell(\ell+1)}{r^2} R - V(r) R = 0 \tag{8}$$

Regge has showed for a superposition of Yukawa potentials[5]

$$V(r) = \int_\mu^\infty \frac{\sigma(\mu)\, e^{-\mu r}}{r}\, d\mu \quad \text{with} \int_\mu^\infty r V dr < M < \infty \tag{9}$$

that the scattering amplitude $S(\lambda, k)$ is a meromorphic function of ℓ in the half plane $Re\, \ell > -1/2$ with all poles of lying on the upper half of the ℓ plane. This enables him to shift the contour C into C'



(Fig 2)

We can thus write finally

$$f(z,E) = \frac{i}{2}\int_{-\frac{1}{2}-i\infty}^{-1/2+i\infty} \frac{(2\ell+1)\,d\ell}{\sin\pi\ell} f(\ell,E)\,P_\ell(-z)$$

$$+\sum_{n=1}^{N} \frac{\beta_N(E)}{\sin\pi\alpha_n(E)} P_{\alpha_n(E)}(-z) \qquad (10)$$

where $\beta_n(E)$ are the residues of $f(\alpha_n,E)$ at the poles $\ell=\alpha_n(E)$ in the complex ℓ plane. These poles which occur in the complex angular momentum plane are called the Regge Poles. (The Sommerfeld-Watson representation is strictly speaking valid only for positive kinetic energy but we may include bound states also by an analytic continuation in E[6]). The first term is called the 'background term'; it vanishes for $z\to\infty$,While the pole terms $\sim z^{\alpha_n(E)}$ as $z\to\infty$. Now we can consider the amplitude for bound and resonant states caused by Regge Poles (i.e) Consider

$$f(z,E) = \frac{\beta_1(E)\,P_{\alpha_1(E)}(-z)}{\sin\pi\alpha_1(E)} \qquad (11)$$

We can now project out any partial wave ($\ell\geq 0$ and integer) by using

$$\frac{1}{2}\int_{-1}^{1} P_\ell(z)\,P_\alpha(-z)\,dz = \frac{1}{\pi}\frac{\sin\pi\alpha}{(\alpha-\ell)(\alpha+\ell+1)} \qquad (12)$$

and therefore

$$f_\ell(E) = \frac{1}{\pi}\,\frac{\beta_1(E)}{(\alpha-\ell)(\alpha+\ell+1)} \qquad (13)$$

If $\alpha(E)$ for a particular E_r is ~~xxx~~ close to the integer m then we can expand it in the neighbourhood of E_r as

$$\alpha(E) \approx m + \left(\frac{d\,\mathrm{Re}\,\alpha}{dE}\right)_{E=E_r} (E-E_r) + i(\mathrm{Im}\,\alpha)_{E=E_r} \qquad (14)$$

and therefore

$$f(l,E) = \frac{1}{\pi}\,\frac{\beta(E_r)/(\alpha(E_r)+l+1)}{(m-l) + (E-E_r)\left(\frac{d\,\mathrm{Re}\,\alpha}{dE}\right)_{E_r} + i\,\mathrm{Im}\,\alpha(E_r)} \qquad (15)$$

which for $l = m$ has the familiar Breit-Wigner form

$$f(l,E) = \frac{\beta(E_r)/(2l+1)\frac{d\,\mathrm{Re}\,\alpha}{dE}}{(E-E_r+i\Gamma/2)} \qquad (16)$$

with a width

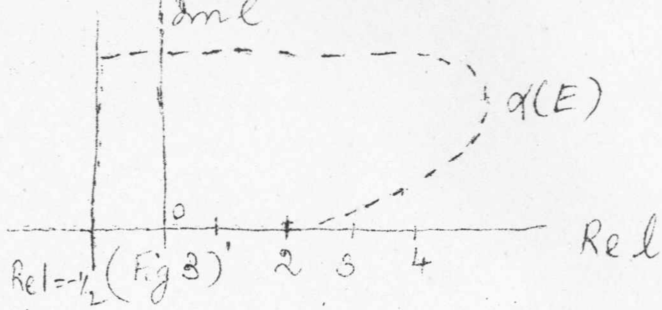
$$\Gamma = \mathrm{Im}\,\alpha(E_r)\Big/\left(\frac{d\,\mathrm{Re}\,\alpha}{dE}\right)_{E_r} \qquad (17)$$

Regge has also proved for a superposition of Yukawa potentials that $\mathrm{Im}\,\alpha$ is positive for $E>0$ and is 0 for $E<0$. Thus the Regge poles represent resonances with positive width when $E>0$ and bound states when $E<0$ with $\Gamma = 0$.

Movement of pole with energy and potential in the l plane.

(We do not know what happens to the left of $\mathrm{Re}\,\alpha = -1/2$) But for an attractive potential a particular pole passes through $\mathrm{Re}\,\alpha = -1/2$ at some negative E and moves to the right along the real axis as E increases. When it reaches the threshold energy the pole moves ~~with~~ into the upper half plane and continues its rightward movement further but eventually swings back and reaches $\mathrm{Re}\,\alpha = -1/2$ again. Lovelace has for different strength of potentials got the Regge trajectories.[7]

When $\alpha(E)$ passes through real ℓ = integer and $Im\,\ell = 0$ we have bound states and when $Re\,\ell$ = integer and $Im\,\ell$ = small +ve value we have resonances.



(Fig 3)

We see that as the potential strength increases the rightward movement of the pole will be extended and we have more bound states and resonances. The same trajectory may pass through (or near) different integer values of $Re\,\ell$ and thus we have one Regge pole giving rise to resonances and bound states in different angular momentum states ($Re\,\ell$ = 1, 2, 3 ...) at different energies.

So far we have not considered the exchange potential (for the space coordinates) in the Schrodinger equation. If we admit that also there are two possibilities - (i.e.) the case of symmetric (even) and antisymmetric (odd) solutions with

$$f(z,E) = \frac{\beta(E)}{\sin \pi\alpha(E)} \frac{1}{2}\left(P_{\alpha(E)}(-z) \pm P_{\alpha(E)}(z)\right) \quad (18)$$

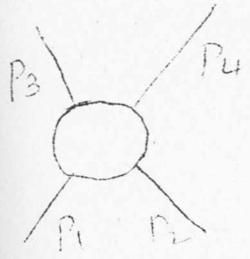
The Regge terms corresponding to physical states of even ℓ take the + sign and ~~in~~ if they have positive signature (Gell-Mann) likewise terms corresponding to physical states of odd ℓ have odd signature. Thus we have the distance between bound states (resonances) now $\Delta\ell = 2$, if each trajectory with specific signature is now considered.

2. Application to Theory of Elementary Particles.

We shall extend the concept of Regge poles to relativistic two-body scattering cases like

$$p_1 + p_2 \to p_3 + p_4$$

with



$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2 \qquad (1)$$

For equal mass case $\cos\theta_s = 1 - \dfrac{2t}{4M^2 - s} \quad \xrightarrow{t\to\infty} \sim t$

$$\cos\theta_t = 1 - \frac{2s}{4M^2 - t} \quad \xrightarrow{s\to\infty} \sim s \qquad (2)$$

The matrix element is given by

$$\langle f|S|i\rangle = \langle f|i\rangle + \frac{i}{(2\pi)^2}\,\frac{\delta(p_1 + p_2 - p_3 - p_4)}{\sqrt{16\,p_{10}\,p_{20}\,p_{30}\,p_{40}}} \qquad (3)$$

The differential cross-section is

$$\frac{d\sigma}{d(\cos\theta_s)} = \frac{|A|^2}{16\cdot 2\pi s} \qquad \frac{d\cos\theta_s}{dt} = \frac{1}{s} \qquad (4)$$

and by optical theorem

$$\sigma_{tot}(s) \sim \frac{1}{s}\,\mathrm{Im}\,A(s,0)_{t\to 0} \qquad (5)$$

<u>Experimental facts known so far are</u> :

1) $\sigma_{ab} = \sigma_{a\bar{b}}$ for $s \to \infty$

2) $\sigma_{ab}^{I} = \sigma_{ab}^{I'}$ "

3) All elastic scattering amplitudes show characteristic diffraction pattern with a forward peak.

4) Width of the peak shrinks logrithmically (?),

5) At high energies scattering amplitudes are purely imaginary.

(3) and (4) lead to

$$A_2(s,t) \sim \beta(t)\, s^{\alpha(t)} = \beta(t)\, e^{\alpha(t)\log s} \qquad (6)$$

and if $\alpha(0)=1$ then $\sigma_t \sim$ constant. The form (6) naturally makes one wonder whether we can extrapolate the Regge results of potential scattering to relativistic particle scattering also. So then we will consider the equation in the t channel

$$A(s,t) = f(z,E) = \sum_n \frac{\beta_n(t)\, P_{\alpha_n(t)}(-\cos\theta_t)}{\sin \pi\alpha_n(t)} \left\{ \frac{1 \pm e^{i\pi\alpha_n(t)}}{2} \right\}$$

$$+ \int^{l} \text{term which vanishes as } z \to \infty. \qquad (7)$$

As $s \to \infty$ (i.e.) $\cos\theta_t \to \infty$ we get

$$A(s,t) \sim \sum_n \frac{\beta_n(t)\, s^{\alpha_n(t)}}{\sin \pi\alpha_n(t)} \left\{ 1 \pm \frac{e^{-i\pi\alpha_n(t)}}{2} \right\} \qquad (8)$$

Let us suppose that one of the Regge poles dominate so that we have (say α_1)

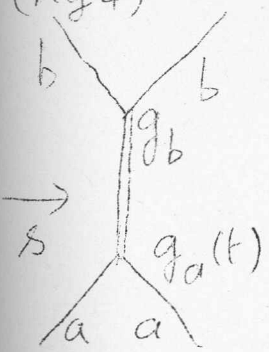
$$A(s,t) \sim \frac{\beta_1(t)}{\sin \pi\alpha_1(t)}\, s^{\alpha_1(t)} \left\{ \frac{1 \pm e^{-i\pi\alpha_1(t)}}{2} \right\} = F(t)\, s^{\alpha_1(t)} \qquad (9)$$

This as we all know corresponds to the unphysical region of the t-channel. Now to study the scattering amplitudes in the physical region we invoke crossing symmetry which is a special feature of relativistic scattering only. The same amplitude also represents scattering in the s channel for which the region $s \to \infty$ for fixed $|t|$ is in the physical region and gives the behaviour $F(t)\, s^{\alpha(t)}$. Further we have near a pole in the t-channel $t \to \max \alpha(t)$

$$A(s,t) = \frac{\beta(t_r)\, P_{\alpha(t_r)}(\cos\theta_t)}{\pi \operatorname{Re}\alpha'(t_r)\left[t - t_r + \dfrac{i \operatorname{Im}\alpha(t_r)}{\operatorname{Re}\alpha'(t_r)} \right]} \sim F(t)\, s^{\alpha(t)} \qquad (10)$$

Let us consider the corresponding dispersion ~~for~~ theoretic graph.

(Fig 4)



This gives

$$A(s,t) = \frac{g_a(t)\, g_b(t)}{t - m_r^2} P_\ell(\cos\theta_t) \sim F(t)\, s^\ell \tag{11}$$

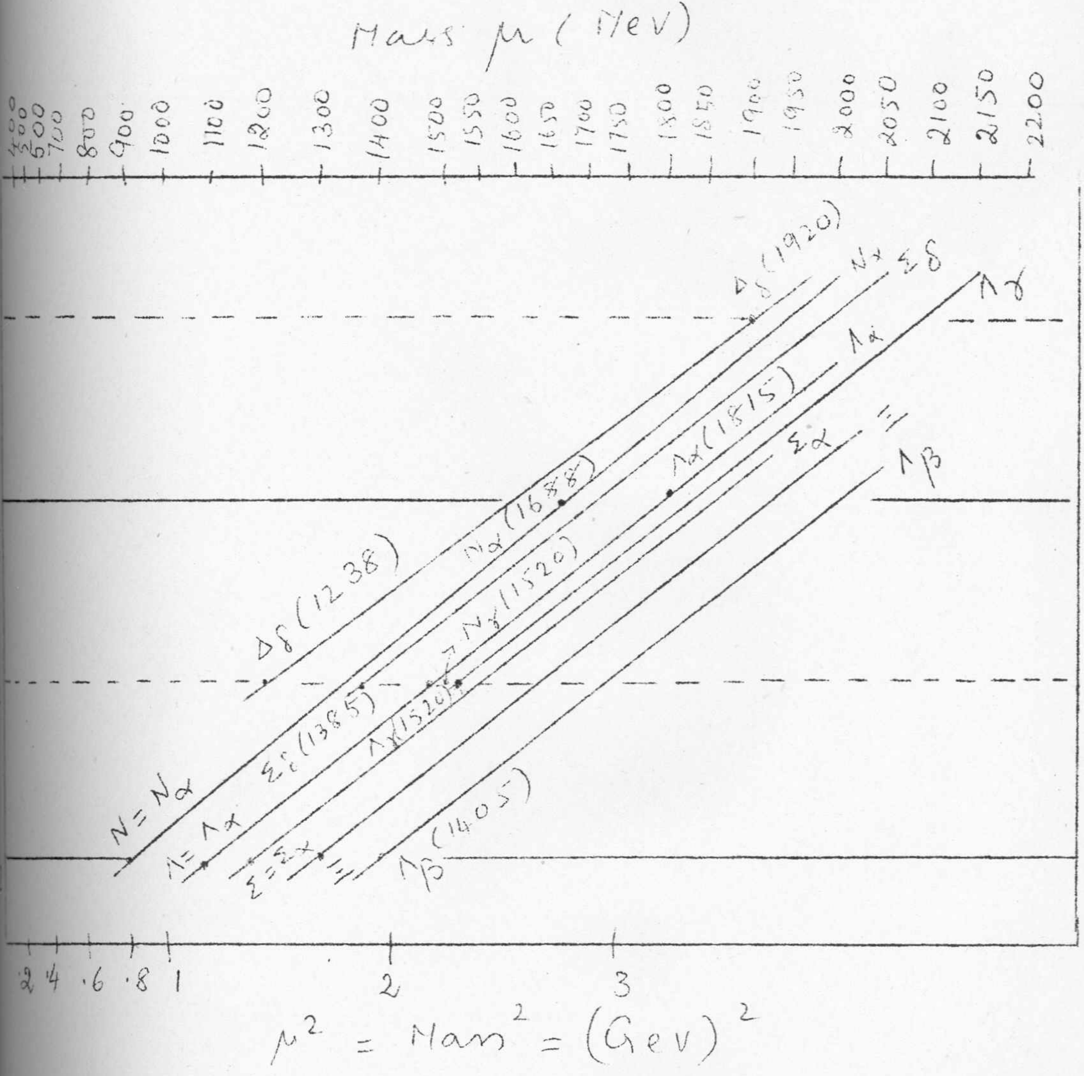
where ℓ is the angular momentum of X and $m_r^2 = t_r - \frac{i\Gamma}{2}$ where Γ ~~is~~ the width of the resonance. $\Gamma = \mathrm{Im}\,\alpha(t_r) / \mathrm{Re}\,\alpha'(t_r)$

Thus we have for asymptotic regions the behaviour given by

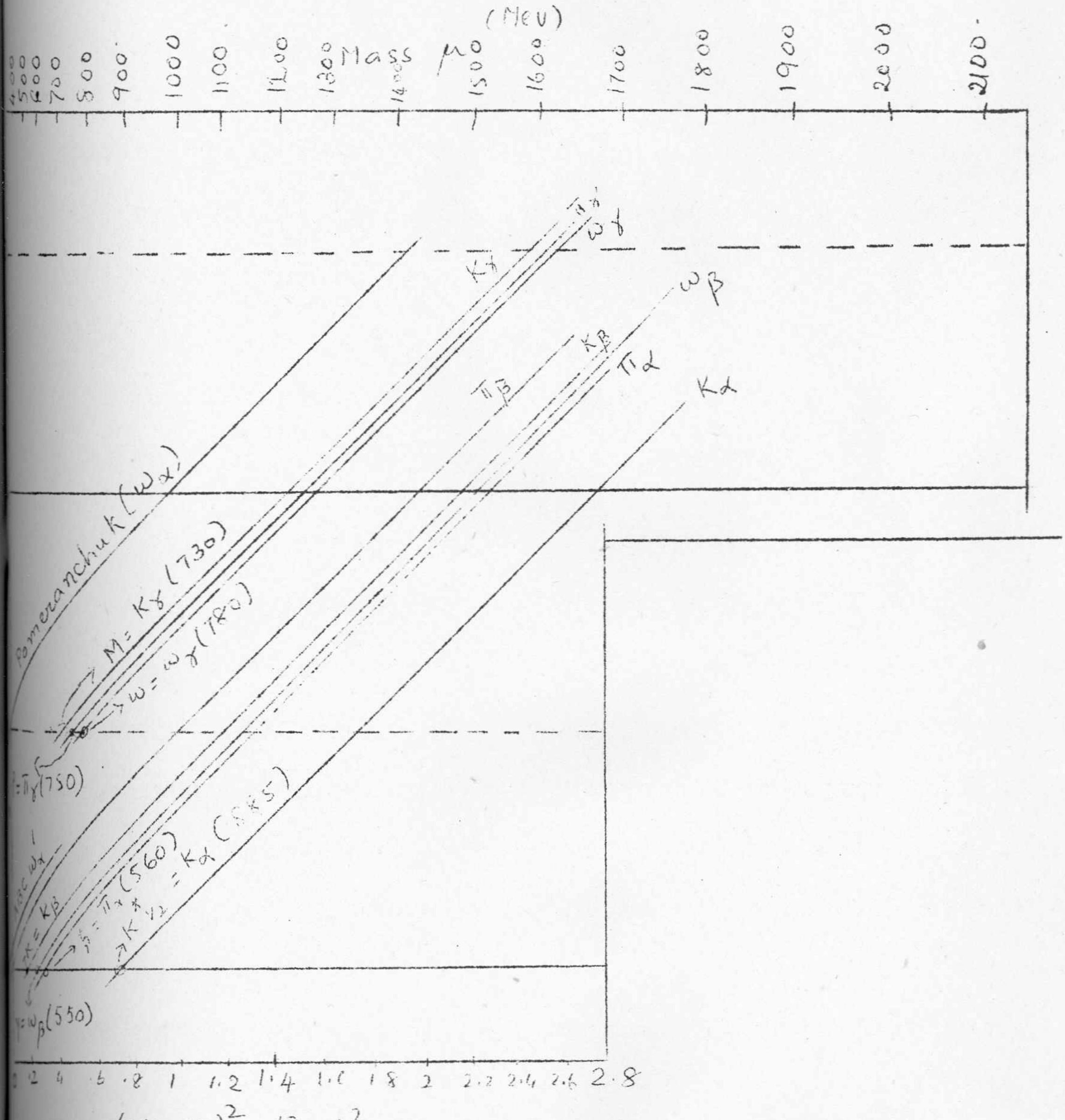
D.T. $\quad F(t)\, s^\ell \quad$ (12)

R.P. $\quad F(t)\, s^{\alpha(t)} \quad$ (13)

The two expressions coincide only at the pole. We also know that it is the s^ℓ behaviour which persists even at large s for $\ell > 1$ that leads to the divergence difficulties in dispersion theory. On the other hand, Froissart[9] has proved using Mandelstam representation that for unitarity to be satisfied the power of s should be ≤ 1 for $t \leq 0$. We at once realize that the Regge form may satisfy this criterion if $\alpha(t) < 1$ for $t \leq 0$ while for dispersive theory we need a damping. So we require that resonances and bound states with $\ell > 1$ are represented by Regge poles. Now it is tempting to speculate that even the harmless poles with $\ell = 0$ also lie on Regge trajectories in analogy to the dynamical resonances. This behaviour has been conjectured by Blankenbecker and Goldberger[10] for the nucleon and for all particles by Chew and Frautschi[11]. They have tried to fit all known particles into different Regge trajectories characterized by different quantum numbers, S , B , I and Parity (and signature).



Baryons and Baryon Resonances.



Mesons & Meson Resonances – Regge Trajectories

(See Fig. 5)

The two points N and N *** which can belong to the same trajectory give a slope $\sim \frac{1}{50 m_\pi^2}$ which according to the Regge formula

$$\frac{d\,(\ell + 1/2)^2}{dp^2} \simeq a^2 \text{ gives } \frac{1}{2 m_\pi} \text{ as the radius of}$$

interaction for the bound state.

Also[13] 1) N_{33}^* (1920 Mev) and N_1^* (1288 Mev) gives $\frac{1}{50 m_\pi^2}$.

2). Y_0^* (1815 Mev) Λ (1115 Mev) gives " .

It may be noticed that the trajectory called P or α_{vac} has ~~not~~ known particles on it. But one may easily identify that this should form the boundary beyond which no trajectory should exist according to Froissart's theorem if it passes through $\alpha(0) = 1$. From the diagram it is interesting to note that trajectories with lower quantum numbers (S , B , T) imply bigger $\alpha(0)$ and hence α_{vac} should have the simplest quantum numbers namely $S = B = T = 0$ of vacuum. The main features of this diagram are the (1) Prediction of an I = 0, J = 2 , B = 0 $S = 0$ resonance on the Pomeranchuk trajectory for which there seems to be experimental evidence now.[13]

(2) The "ghost" pole which corresponds to $\alpha = 0$ at $- 50\, m_\pi^2$ (- 1 Gev) (-ve mass^2).

We can easily see that from (8) if we assume that the $\alpha_{vac}(P)$ dominates over all other Regge (exchange) poles, we have for large s

$$A(s,0) \approx \lim_{t\to 0} \beta_p(t)\left[\frac{1+e^{i\pi\alpha_p(t)}}{\sin\pi\alpha_p(t)}\right] s^{\alpha_p(t)}$$

$$\approx i g s^{1} \qquad (14)$$

with $\alpha_p(0) = 1$

and therefore σ_{total} for s channel $= g$ = constant where $g = \beta_p(0)$

Thus we find that the vacuum trajectory leads us to the Pomernanchuk theorem for $s \to \infty$. If we write $\alpha_p(t) = 1 + t\alpha'_P(0)$ (15)

we get from

$$\frac{d\sigma}{dt} = F(t)\, s^{2(\alpha_p(t)-1)}$$

$$\approx F(t)\exp\left[2t\alpha'_p(0)\log s\right] \qquad (16)$$

with $\alpha'(0) > 0$. This is the physical region of the s channel (i.e.) $t<0$ indicates a lograthmic shrinking of the diffraction peak with energy . However if the Brookhaven experiments turn out to be correct it would contract this asymptotic behaviour of the differential cross-section and may indicate the existence of cuts or immovable singularities in the angular momentum plane which may cancel the logrithmic shrinking in the case of πp and Kp diffraction scattering.

Factorising princple of Gell-Mann.

For Fig. (9) we write the amplitude in ordinary perturbation calculation as in exp. (11) where we have factored the coupling constants at the two vertices. This factorising has been established in potential theory for theRegge poles by Gell-Mann[14] and by Girbor and Pomeranchunk[15] for analytically continued partial wave amplitude. According to this princople, we have if the same Regge pole (trajectory) P dominates, all high energy reactions ab, aa and bb then we have

$$\sigma_{ab}^{2} = \sigma_{aa} \cdot \sigma_{bb} \qquad (17)$$

For e.g. we should have

$$\sigma_{\pi\pi} = \sigma_{\pi N}^{2} / \sigma_{NN} \approx 15\,mb$$
$$\sigma_{KK} = \sigma_{KN}^{2} / \sigma_{NN} \approx 8\,mb \quad etc \qquad (18)$$

Thus all polarization effects in πN scattering vanish as and polarization experiments are valuable to find the interference of other poles with the α_p.

Let us consider the trajectories of other poles[14,16] resonances.

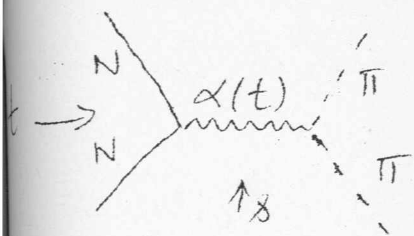
For this we consider linear combination of cross-sections in order to subtract out the influence of the vacuum pole. For Example, let us consider πN scattering. Then we have the A^+ and A^- amplitudes given by

$$A^{+} = \left(\tfrac{1}{3} A^{1/2} + 2 A^{3/2}\right)$$
$$A^{-} = \left(\tfrac{1}{3} A^{1/2} - A^{3/2}\right) \qquad (19)$$

By optical theorem

$$\tfrac{1}{2}\left[\sigma(\pi^{+}p) + \sigma(\pi^{-}p)\right] = \tfrac{1}{s}\, \mathrm{Im}\, A^{+}(t=0)$$
$$\tfrac{1}{2}\left[\sigma(\pi^{-}p) - \sigma(\pi^{+}p)\right] = \tfrac{1}{s}\, \mathrm{Im}\, A^{-}(t=0) \qquad (20)$$

We also know that A^+ represents $I=0$ $G=+1$ channel of $\pi\pi\to N\bar{N}$ and A^-, $I=1$, $G=+1$ of $\pi\pi\to N\bar{N}$



Thus we may have the vacuum and ABC pole contributing to A^+ and therefore we will have

$$\sigma(\pi^-p)+\sigma(\pi^+p)\sim a+b\,E^{-\left(1-\alpha_{ABC}(0)\right)}$$

If $\alpha_{ABC}(0)=0$

we get

$$\sigma(\pi^-p)+\sigma(\pi^+p)=a+b/E$$

Similarly
$$\sigma(\pi^-p)-\sigma(\pi^+p)\ \propto\ E^{-[1-\alpha_\rho(0)]} \tag{21}$$

We list below the contribution of the difference Regge trajectories to the various total cross-sections at high energies.

<u>Regge Poles contribution to various crosssection</u>

Crosssection	Regge Pole	Expected high E behaviour
$\sigma(\pi^-p)+\sigma(\pi^+p)$	P, ABC	$2\left[\sigma(\infty)+\pi\epsilon_{ABC}\,g_{\pi\pi ABC}\,g_{ppABC}\,(s/s_0)^{\alpha_{ABC}(0)-1}\right]$
$\sigma(\pi^-p)-\sigma(\pi^+p)$	ρ	$2\pi\epsilon_\rho\, g_{\pi\pi\rho}\, g_{pp\rho}\,(s/s_0)^{\alpha_\rho(0)-1}$
$\sigma(pp)+\sigma(\bar{p}p)$	P, ABC	$2\left[\sigma(\infty)-\pi\epsilon_{ABC}\,g^2_{ppABC}\,(s/s_0)^{\alpha_{ABC}(0)-1}\right]$
$\sigma(pp)-\sigma(\bar{p}p)$	ω,ρ	$2\left[\pi\epsilon_\omega\, g^2_{pp\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1}+\pi\epsilon_\rho\, g^2_{pp\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1}\right]$
$\sigma(pp)-\sigma(n,p)$	ρ	$2\pi\epsilon_\rho\, g^2_{pp\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1}$
$\sigma(pp)+\sigma(n,p)$	P, ABC, ω	$2\left[\sigma(\infty)+\pi\epsilon_{ABC}\,g^2_{ppABC}\,(s/s_0)^{\alpha_{ABC}(0)-1}-\pi\epsilon_\omega\, g^2_{pp\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1}\right]$
$\sigma(K^-p)-\sigma(K^+p)$	ω,ρ	$2\left[\pi\epsilon_\omega\, g_{KK\omega}\, g_{pp\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1}+\pi\epsilon_\rho\, g_{KK\rho}\, g_{pp\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1}\right]$

The π, η do not contribute at zero momentum transfer. The signs of the different terms differ channel representation attraction. ω and ρ (vectors) representation repulsive for identical particles and atraction between particle and anti-particle. Remaining signs are from isotopes.

<u>ω and P' trajectory.</u>

We can deduce from the table

$$\frac{1}{2}\left[\sigma_{\bar{p}p} - \sigma_{pp}\right] = \pi \epsilon_\omega g^2_{pp\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1} + \pi \epsilon_\rho g^2_{pp\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1} \quad (23)$$

$$\frac{1}{2}\left[\sigma(np) - \sigma(pp)\right] = \pi \epsilon_\rho g^2_{pp\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1} \quad (24)$$

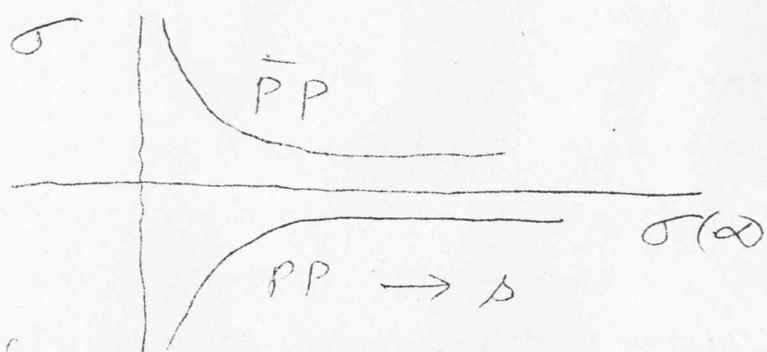
In the 10 Gev region $\frac{1}{2}\left[\sigma_{\bar{p}p} - \sigma_{pp}\right] = 20\,mb$ while $\frac{1}{2}\left[\sigma_{np} - \sigma_{pp}\right] = 2\,mb$ Therefore we may neglect the contribution of the ρ trajectory in (23) as can be deduced from (24) and concentrate on the ω in interpreting the $pp - \bar{p}p$ difference. Thus from experiments we find $\alpha_\omega(0) \simeq .4$ which agrees with the value given by the Chew and Frautschi diagram.

$$\sigma_{pp} = \sigma(\infty) - \pi \epsilon_\omega g^2_{pp\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1} \quad (25)$$

$$\sigma_{p\bar{p}} = \sigma(\infty) + \pi \epsilon_\omega g^2_{pp\omega}(s/s_0)^{\alpha_\omega(0)-1}$$

(i.e.) neglecting the ρ and ABC contribution.

Then why is $\sigma_{pp} \to$ Const (40 mb) beyond 10 Gev while $\sigma_{p\bar{p}}$ is not constant in this region. From (25) we expect



(Fig 7)

Perhaps σ_{pp} is constant means there is a third trajectory P' [17] which just cancels the imaginary part of the trajectory in the pp amplitude and adds to it for $p\bar{p}$ scattering. And this vacuum trajectory with $\alpha_{P'}(0) = \alpha_{\omega}(0) \approx .4$ would satisfy the above requirements (i.e.)

$$\sigma_{pp} = \sigma(P) - \sigma(\omega) + \sigma(P') \qquad (26)$$

$$\sigma_{p\bar{p}} = \sigma(P) + \sigma(\omega) + \sigma(P') \qquad (27)$$

If the ABC is responsible for this, then its trajectory cannot be simple like the others but may have a strange twist. Similar results for K^+p and K^-p total cross-sections hold: (i.e.) $\sigma_{K^+p} \approx 18\,mb$ above 5 Gev while K^-p is not. Thus the requirement for a P' seems more reasonable. Also this requires that the couplings of P' and ω to K and nucleons to be the same.

In the case of π^-p, π^+p scattering ω cannot contribute since

$$G = -1 \text{ for } \omega; \therefore \sigma(\pi^-p) + \sigma(\pi^+p) = \pi\epsilon_{\pi}\, g_{\pi\pi P}\, g_{NNP}\left(\frac{s}{s_0}\right)^{\alpha_{\pi}(0)-1} + P' \text{ contribution} \qquad (28)$$

$$\sigma(\pi^-p) - \sigma(\pi^+p) = \pi\epsilon_{\rho}\, g_{\pi\pi\rho}\, g_{NN\rho}\left(\frac{s}{s_0}\right)^{\alpha_{\rho}(0)-1} \qquad (29)$$

scattering and theory approach Pomeranchuk limit as contribution from P' vanishes as $s \to \infty$. Again one gets $\alpha_{P'}(0) \approx .4$ ~~0.4~~.

It was Igi[17] who first suggested the P' trajectory while treating non-charge exchange πN scattering he assumed the amplitude could be written as sum of two terms - one due to P and other giving an amplitude which decreases with s and satisfies an unsubtracted dispersion relation. But this gives a different scattering length from the observed value and hence we need either (1) another subtraction due to P which varies as $s^{\alpha(0)}$ where $\alpha(0) \sim .5$ close to P', or 2) admitting cuts in the angular momentum plane.

ρ trajectory.

To evaluate α_ρ we have to study $\sigma(pn)-\sigma(pp)$ or $\sigma(\pi^- p) - \sigma(\pi^+ p)$

as given in the table-(22)

Thus the ρ coupling to N is much weaker than the ω. The sign of $pn-pp$ difference is important and also the magnitude. $\sigma(pn) - \sigma(pp)$ has also contribution from π exchange which may be smaller than ρ. But $\sigma(\pi^- p) - \sigma(\pi^+ p)$ is due to ρ alone. This analysis leads to $\alpha_\rho(0) \simeq 0.3$. However $\Delta\sigma \approx 1.5$ from 10-20 Gev. This may be a violation of Pomeranchunk limit that $\Delta\sigma \to 0$ and in this case we may get $\alpha_\rho(0) = \alpha(t_r) = 1$ which indicates ~~our~~ an un-Reggised behaviour. Little is understood about the ρ.

Hahn[18] has shown that

$$\sigma_{pn-np} \sim 100 \text{ mb}$$

$$\frac{d\sigma_{np\to pn}}{d\Omega} \text{ is isotropic upto } |t| \sim 2 \text{ GeV}^2 \text{ at } E_{lab} = 25 \text{ GeV}$$

and this leads to a $\sigma_{np} - \sigma_{pp}$ which is an order of magnitude longer than that predicted by ρ alone. One wonders whether the π has a significant part to play in this. As a matter of fact an elastic unreggeised ~~are~~ π exchange graph of Ferrani and Sellen gives the observed value correctly.

Let us list a number of reactions in which the Regge pole hypothesis can be tested.

$$
\begin{array}{ll}
N: & \pi + N \longrightarrow N + \pi \\
 & \gamma + N \longrightarrow N + \pi \\
 & \pi + N \longrightarrow N + \omega \quad \text{etc} \\
Y = \Lambda, \Sigma & \pi + N \longrightarrow Y + K \\
 & K + N \longrightarrow N + K \\
 & \gamma + N \longrightarrow Y + K \quad \text{etc} \\
\pi & N + N \longrightarrow N + N \\
 & \gamma + N \longrightarrow \pi + N \\
 & \pi + N \longrightarrow \rho + N \\
 & K + N \longrightarrow K^{+} + N \\
 & N + N \longrightarrow N + N^{*} \\
K & \pi + N \longrightarrow K^{*} + Y \\
 & \gamma + N \longrightarrow K + Y \\
\omega & N + N \longrightarrow N + N \\
 & \pi + N \longrightarrow \rho + N
\end{array}
\qquad (30)
$$

Notice that we will have equality of differential cross-section for $\pi^- p \to K^0 \Lambda^0$ and $K^0 + p \to \pi^+ + \Lambda$ when the same pole dominates

Finally the present conclusions on the basis of no Regge cuts are

1) $\alpha_P(0) = 1$, $\alpha_{P'}(0) \approx .4$ $\quad \alpha_\omega(0) \approx .4$

2) ρ is weakly coupled and is unknown,

3) the fermion trajectories are unknown.

4) π to be explored.

Hahn experiment shows unreggised π behaviour. But Frautschi et al seem to deduce a reggised behaviour for π also from the analysis of $N + N \to N + N^{\times}$

Theoretical Developments.

1) Using elastic unitarity Mandelstam[19] and Gribov[20] have shown that the only singularities (energy dependent) of the analytically continued partial wave amplitudes $f_\ell(s)$ in the angular momentum plane are just simple poles. However this does not rule out the possibility of poles at fixed energy independent values of the angular momentum.

2) Eden[21] has also come to the same results by replacing the assumption of elastic unitarity by the condition of nonvanishing of the imaginary part of the trajectory $\ell(s)$ in the physical region.

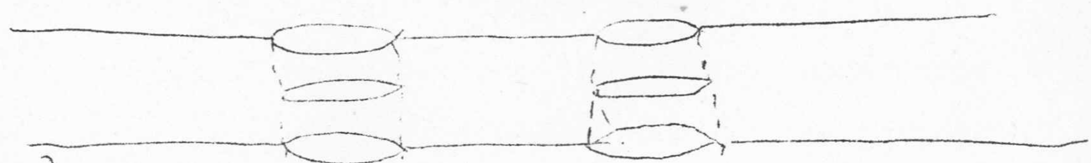
3) Lee and Sawyer[22] have shown that the Bethe Saltpeter scattering amplitude in the ladder approximation has atleast one Regge pole in the region $Re\,\ell > 3/2$ and further that the scattering amplitude has only poles in this region.

4) Using model field theory Amati et al[23] also arrived at the ~~Rxxxx~~ regge behaviour for the scattering amplitude including the

possibility of a "ghost" with $\alpha(t)=0$ for some $t<0$.

To avoid the "ghost", Gell-Mann has suggested that the coupling of the trajectory to any system is proportional to $\alpha(t)$ so that there is no residue at the "ghost". This is quite welcome because we need $\alpha(t=-\infty)<0$ to avoid subtractions in dispersion relations and we must have $\alpha(t)=0$ at some t if we want $\alpha(0)=+1$ which we require for constant total cross-sections. This behaviour also exists in the case of Regge poles for nuclei,[24] with ground state as $J>3/2$, say 2. Then before we go to negative values of α we will have to pass the $\alpha=0$ state, but this cannot exist and so we again assume that the coupling goes to 0, $C(t)=0$

Amati et al considered that the ghost is due to the fact that scattering amplitude is not unitary in the s-channel. So they added graphs of the type



(Fig 8)

to ensure unitarity in the s-channel also. But this leads to the following behaviour

$$A_i(s,t) \sim f(t)\, s^{\alpha(t)} + \frac{g(t)}{\log s}\, s^{\alpha_M(t)} \qquad (31)$$

The second term indicates the existence of a cut in the angular momentum variable from $\alpha_M(t) \geq \alpha(t)+\alpha(0)-1$

Further one finds that $\alpha_M(0)=\alpha(0)=1$ and the second term becomes comparable to first for

$$|t| \sim \frac{\log(\log s)}{\log s} \qquad (32)$$

because asymptotically $\alpha_M(t) \approx 2\,\alpha\left(\frac{t}{4}\right) - 1$

(6) This model is applicable only between 0 and $|t|$,

where $$|t| = \log\log s / \log s \quad \text{for } s \to \infty \qquad (33)$$

and then we may escape the "ghost". But then the approach to asymptotic σ is only logrithemic in s. The way out suggested by many people is that complete restoration of unitarity in the t channel (outside the strip region) may remove these cuts. Gell-Mann and Udgoankar[24] have suggested that cuts occur <u>only</u> when the particles participating have anamalous thresholds like nuclei. The need for the cut was recognized by them on the basis of a factoring principle on the coupling of Regge Poles.

This principle is common to all field theoretic model in which there is just one dominant channel for the interaction.

4) Bardacki has given a proof for the existence of Regge poles in relativistic S-matrix theory for $Re\,l > 1$.

Topics of interest to be explored can be now summarized:

1) Scattering of particles by resonances and the definition of Clebsch-Gordon coefficients to complex values of the indices (l).

2) Identification of the interpolation function $f(l,k)$ with partial wave amplitude $f_l(k)$ for all l. Coincidence unto $l = 1$ seems to be alright but for $l = 0$ it is very difficult to prove.

3) Extension of multiperipheral model to three body problems.

4) The question of the existence of Regge cuts remains. This may be the reason for the constancy of the diffraction peak width for πp and $K p$ scattering.

5) Study of Regge poles for complex potentials.

6) Possible connections between symmetries and Regge Poles in high energy scatterings.

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ON COMPOSITE PARTICLES AND REGGE POLES IN PERTURBATION THEORY AND IN ELECTROMAGNETIC AND WEAK INTERACTIONS.

K.Raman.

Perturbation theory gives a more vivid dynamical picture of Regge poles.

In this talk we shall discuss Regge poles in terms of perturbation theory and present some speculation, especially relating to the possible role of Regge poles in weak and electromagnetic interactions.

First we note that in perturbation theory, a resonance or bound state must appear as a sum over a set of Feynman diagrams, and presumably so must a Regge pole. Models for obtaining bound states and resonances from perturbation theory (by summing Feynman diagrams) have been constructed, for instance, by Thirring using the Zachariasen model (which, however, cannot give a Regge pole)[1)]

To begin with, we note the following in connection with Regge poles in perturbation theory.

(i) For a Yukawa potential, it has been proved that perturbation theory gives the same trajectories as the Schrodinger equation treated by other methods.[2)]

(ii) The ladder diagrams in high-energy scattering have been proved to sum to give a Regge behaviour. (Amati, Fubini et al)[3)]

(iii) Gell-Mann and Goldberger showed that the radiative corrections arising from the coupling witha neutral vector meson field will reggeize the nucleon pole in pion-nucleon scattering.[4)]

(iv) Levy has shown that the radiative corrections in electron scattering sum to give a Regge behaviour; we shall come back to this presently.[5]

(v) Polkinghorne has recently shown that a Regge behaviour may be obtained by summing a wide class of Feynman diagrams.[6]

(vi) Blankenbecler et al speculated that the photon may also be on a Regge trajectory and studied the effect of this on the form factors for e-π, e-N and e-α scattering.[7] Whether the electron also lies on a Regge trajectory may be tested from backward Compton scattering, and from pair annihilation, as pointed out by Contogouris[8]

(vii) We may think of Regge poles in perturbation theory as fictitions composite particles, introduced as by Weinberg,[9] which may help to give a finite perturbation theory.

We shall here assume that

1) Field theory is correct in many essentials, and that perturbation theory gives a correct picture of the important properties of the S-matrix.

2) Regge poles are a feature of a correct relativistic theory also.

3) Symmetries must as far as possible be formulated in a way in which it is interwoven with the dynamics of the system.

In particular, we suggest that the various regularities in high-energy cross-sections that would follow from the dominance of different Regge poles may be taken as a basis for introducing higher symmetries into an S-matrix theory. ~~Thus if the heights and slopes of the various Regge~~

trajectories can be obtained from a dynamical theory, e.g. from a composite-particle model (like the Sakata model) assuming baryon number, strangeness and isospin conservation, then we obtain relations between high-energy cross-sections which are analogous to those obtained in higher symmetry schemes (which are expected to be valid for high-energy scattering, where mass differences may be neglected).

Marshak: Higher symmetries give relationship between particles with the same spin, i.e. along horizontal lines on the Chew-Frautschi plot, which may be complementary to information about variation in the vertical direction.

SOME SPECULATION ABOUT THE POSSIBLE ROLE OF REGGE POLES IN WEAK INTERACTIONS

We now consider the possible role of Regge poles in weak interactions.

That the concept of Regge poles may prove useful in weak interactions also is suggested by the following:

(i) Composite-state models have been useful in providing a theory of weak interactions that links these with the strongly interacting particles[10, 11]. In particular, certain observed regularities of weak interactions, e.g. the conservation of the vector current and 'partial conservation' of the axial-vector current seem to follow naturally from such composite-state models.

As a correct description of composite particles seems to be closely related to the idea of Regge poles, it may be of interest to examine the role of the latter in weak interactions.

(ii) Regge poles may help in solving the divergence problem of weak interactions. As is well known, neither the universal Fermi theory nor the charged vector boson theory gives a finite result when considered to all orders; also, the universal Fermi theory violates unitarity as high energies.[12)] Thus a theory that starts with the UFI or the vector boson interaction as the basic interaction requires a damping mechanism that gives a finite result.[13)]

A somewhat similar situation occurs in electrodynamics also where the lowest order term in electron scattering seems to become unreasonably large at higher energies. (This is suggested by the observation of Blankenbecler, Cook and Goldberger that the 1-photon exchange contribution to pp scattering exceeds the strong interaction contribution at a sufficiently high energy.[7)] However Levy has pointed out that in electron scattering, the infra-red radiative corrections provide a damping mechanism; further, the total effect (lowest order + the infra-red corrections) is equivalent to that of the positronium Regge pole in the crossed reaction.[5)] Thus the positronium Regge pole provides a means of correctly summing the set of Feynman diagrams that dominate the high-energy behaviour in electron scattering and gives a damped asymptotic behaviour (as compared with 1-photon exchange.)

It is possible to speculate that such a property is quite general, i.e. that although the weak interaction theories mentioned above are unrenormalizable, one may conjecture that a correct (field) theory of weak interactions would give a high-energy behaviour that is equivalent to the contribution of a few Regge poles in the crossed reaction. (i) and (ii) suggest that we may attempt to use the idea of Regge poles as generalised composite states to incorporate the observed regularities of weak interactions in a finite theory. One may think of the Regge poles as Weinberg's fictitious composite particles[9)] which may help to give a finite perturbation theory of weak interactions and electrodynamics. [Note: It is known that the slope of the trajectories on the Chew-Frautschi plot corresponds to the 'size' or radius characterising the scattering. In particular, a horizontal trajectory would correspond to point particles. There may be a similarity between an attempt to obtain a convergent theory using Regge poles, and older attempts to obtain finite theories by introducting a 'finite length', cf. Heisenberg.]

Current-Current Interactions and Regge poles:-

The current-current picture of an interaction makes possible a factorization of the coupling strength in the lowest order in perturbation theory. When the interaction is iterated and summed to all orders, then the result (if formally obtainable) cannot, in general, be expected to be factorable into a similar form. However, it may be possible to group the terms of different order) (i.e. the different order Feynman diagrams) into various sets, such, that in some limit (e.g.) that of high energy) one of these sets dominates. If this dominant set can be proved to have a Regge behaviour when summed, then we can assume a factorization of the coupling β (t) and think of the Regge trajectory as a generalization of the current-current interaction.

Concerning this Regge trajectory we have different possibilities:

(a) The trajectory may reduce to the original current-current interaction at the point J=1 or $\alpha(t)=1$. For weak interactions this may be looked upon as a 'reggeization of the weak current itself, analogous to the reggeization of the photon considered by Blankenbecler, Cook, and Goldberger.[7]

(b) Alternatively, the point J=1 on the trajectory may not correspond to the original interaction, as for the positronium trajectory summing the dominant infra-red radiative corrections for high energy electron scattering. (The point J=1 on the positronium trajectory does not correspond to the photon). If a similar result obtains in high-energy 'elastic' neutrino reactions, one may regard the particle corresponding to J=1 on the Regge trajectory as a real or fictitious composite particle, e.g., as the vector boson appearing in certain theories.

(c) The Regge trajectory may be obtained as one of those in strong interactions. For instance, in weak interactions involving mesons and baryons, one may take the weak current to different orders (in the sense of perturbation theory) and couple the interaction in each order to the Regge poles in the form factors of the strongly interacting particles (just as McMillan and Predazzi, and Freund have done for the electromagnetic form factors for one-photon and two-photon exchange respectively.[14,15]) This may give a useful improvement over the 'pole approximation', esp. in low-energy weak interactions. In such an approach, summing over all orders in the weak current would seem to give the various strong trajectories; thus these would dominate the weak interactions as well of mesons and baryons.

Consideration of (a) and (b) raises the question whether in strong interactions also the Regge trajectories may be obtained by starting with a current-current interaction such as those in the gauge theories of strong interactions. That this is possible is suggested by Gell-Mann and Goldberger's demonstration that the radiative corrections from the coupling with a neutral vector meson field reggeize the nucleon pole.[4])

An immediate consequence of replacing the current-current interaction by a Regge trajectory is the violation of the "local action of lepton currents", which is consistent with the expectation that the Regge behaviour is in some way obtained by summing higher order terms. [The hypothesis of Regge behaviour prescribes a particular mode of violation of the 'local action' of lepton currents.] The high-energy differential cross-section of a neutrino reaction $\nu + T \rightarrow F + \ell$ is no longer a quadratic function of the neutrino momentum p_ν, the lepton energy ω_ℓ or $\cos\theta_{\nu F}$ (as it should be for a local lepton current (see ref.[16]); it is asymtotically of the form $s^{2\alpha(t)-1}$.

Form factors become a function of the energy as well as the momentum transfer (as for a reggeized photon), their energy dependence is asymptotically mainly in a factor $s^{\alpha(t)-1}$.

Considering leptonic weak interactions, we expect that at low energies and for physical points on the Regge trajectories, the leptons must be coupled locally in pairs; we shall assume that leptons are always coupled in pairs $(\ell\nu)$ to a Regge trajectory. This would imply the absence of Regge poles in backward 'elastic' scattering $\nu + T \rightarrow F + \ell$, etc; thus backward scattering may be expected to be negligible even at moderate energies.

Concerning the number of Regge trajectories required and their properties, the following hypothesis may be made.

We may conjecture that there exists a Regge trajectory $\alpha_W(t)$ (and the corresponding anti-particle trajectory) characteristic of the weak interactions (just as the photon trajectory is characteristic of the electromagnetic interactions.) In order to explain the main features of leptonic weak interactions, viz. the possibility of $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ currents, $\Delta S < 2$, etc., one may, for instance, require the trajectory to have the properties of the sextet of vector bosons recently postulated by T.D. Lee.[17] (On the other hand, these properties may follow from the properties of the strong Regge trajectories that may dominate the weak interactions of baryons and mesons). If we postulate that the trajectory has an <u>even</u> signature, then the vector boson (like the Pomeranchuk) is not a physical particle and will not be observed. The coupling $\beta(t)$ of the trajectory to all physical systems must presumably vanish at the point $t=t_0$ such that $\alpha(t_0) = 0$. Regarding the value $t=t_1$, where $\alpha(t_1) = 1$, the usual arguments for a heavy vector boson do not apply when the trajectory has an even signature. As the weak current seems to be a vector (and axial vector) current in low-momentum transfer reactions at low energy (such as neutron decay, etc.), we expect that t_1 is small. Noting that both the Pomeranchuk trajectory and the photon trajectory have $\alpha(0)=1$, we may speculate that $\alpha_W(0)=1$ also, and that strong, electromagnetic and weak interactionseach has a characteristic Regge trajectory with $\alpha(0)=1$ and perhaps the same slope. The Pomeranchuk presumably has even signature and definite isospin I, the photon trajectory would have an odd signature and would be a schizon with I = 0 or 1, while the W trajectory would have an even signature and the schizoid behaviour of Lee's vector bosons.

Chew and Frautschi's conjecture that the highest trajectory must have all quantum numbers zero may apply only to strong interactions. Also, not only the strong, but also the electromagnetic and weak interactions may saturate unitarity. (A possible objection against this would be that one would then expect the photon trajectory to determine high-energy electron scattering while conventional electrodynamics seems to indicate that it should be the positronium trajectory.)

If the W trajectory corresponds to the alternative (b) above, its slope may have a different order of magnitude from those of the Pomeranchuk and positronium trajectories (which themselves differ in order of magnitude.[7]). One may conjecture a relation between the strength of an interaction and the slope of the trajectory that dominates at high energies. A difficult question is that of the parity of the W trajectory. We must presumably require that the 'coupling' of the trajectory to leptons does not conserve parity (all along the trajectory).

An an alternative to the above scheme, we may assume that at least for weak interactions involving baryons and mesons, the Regge trajectories occurring are the strong ones. (The occurrance of $\Delta S=0$ and $\Delta S=1$ interactions and the absence of $\Delta S \geq 2$ interactions is reflected in the fact that any $S \geq 2$ mesonic states that may exist must presumably have a large mass and hence a low trajectory.)

Some consequences of this would be the following:

(i) One expects that all the Regge trajectories that appear in strong interactions would appear in weak reactions also.

(Note: This is already expected of the physical particles on the trajectories in the different 'pole approximation' pictures.)

The Pomeranchuk trajectory would appear only in 'inelastic' neutrino reactions of the type

$$\nu + T \rightarrow (\ell^- + F^+) + T',$$

where T and T' are the initial and final target particles, and F^+ is a group of mesons emitted in the forward direction together with the final lepton. If the Pomeranchuk has both Δ S=0 and ΔS=1 couplings with leptons, this would result in the production of π's and K's respectively in the forward cone. Dominance of the Pomeranchuk trajectory would mean that the dominant high energy ν-reactions would have a forward cone with total charge zero.

(ii) Reactions like

$$\bar{\nu} + p \rightarrow \begin{array}{l} n + \ell^+ \\ N^{*0} + \ell^+ \end{array}$$

would receive contributions from I$\geq$1, S=0 trajectories alone, reactions producing hyperons or hyperon isobars only from S = ±1 trajectories, and reactions of the form $\bar{\nu} + p \rightarrow \begin{array}{l} (\ell^+ \pi^-) + p \\ (\ell^+ K^-) + p \end{array}$ from the Pomeranchuk as well as I$\geq$1, S=0 trajectories. A factorisation of the residue at the poles would lead to obvious relations of the form

$$\frac{\sigma(\bar{\nu} p \rightarrow n + \ell^+)}{\sigma(\bar{\nu} p \rightarrow N^{*0} + \ell^+)} = \frac{\sigma(np \rightarrow pn)}{\sigma(np \rightarrow p N^{*0})},$$

$$\frac{\sigma(\bar{\nu} p \rightarrow Y^0 \ell^+)}{\sigma(\bar{\nu} p \rightarrow Y^* \ell^+)} = \frac{\sigma(\pi p \rightarrow Y^0 K^*)}{\sigma(\pi p \rightarrow Y^* K^*)}, \text{ etc.,}$$

if we assume that the same Regge pole dominate in the weak and the corresponding strong interactions.

(iii) Variation of the Cross-section at high energies:

For the reactions $\bar{\nu} + p \to n + l^{+}$ and $\nu + n \to p + l^{-}$, the high energy behaviour of the cross-section has been estimated by Cabibe and Gatto using the form factors obtained from the conserved current hypothesis.[18] They find that the cross-sections tend to constant limits at very high energies. However, if a Regge trajectory dominates the cross-section at high energies, the cross-sections must slowly decrease with increasing energy at high energies.

(iv) For inelastic processes like $\nu + N \to N + l + \pi$, a single-pion exchange, with the final pion being produced at the weak vertex, gives a cross-section that increases rapidly at high energies.[19]

It is not known how the corresponding Regge pole contribution can be calculated, but an extension of the conjecture for $2 \to 2$ reactions would lead us to expect an asymptotic decrease in the cross-section.

Also, reactions like

$$\bar{\nu} + N \to (l^{+}\pi^{-}) + N$$
$$\nu + p \to (l^{-}\pi^{+}) + p.$$

will presumably be dominated by the Pomeranchuk pole, and the reactions

$$\bar{\nu} + p \to (l^{+}\pi^{0}) + n,$$
$$\nu + n \to (l^{-}\pi^{0}) + p$$

would be comparatively suppressed as they will receive contributions only from lower Regge trajectories.

Similar results may be expected for strangness-changing weak reactions.

With any hypothesis about type of Regge trajectories occurring, we may compare the predictions of the UFI theory and the Regge pole hypothesis.

For the reactions

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_\mu + \mu^- \qquad (a)$$

and

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \qquad (b)$$

UFI would give differential cross-sections that become equal in the forward directions.[13] The allowed Regge trajectories are the same for the two reactions, as they are determined by the quantum numbers of the t-reaction; thus we expect the forward differential cross-sections for the two reactions to behave similarly here also. Concerning the backward scattering, we note that (a) and (b) are the crossed reactions of each other. If ν_μ and ν_e have the same leptonic number L, then each side of (a) has $L = 0$, while (b) has $L = 2$. Also, the muon number n_μ of (a) is 0 while that of (b) is 1, assuming additive muon number conservation. Thus Regge trajectories contributing to (a) must have $L = 2, n_\mu = 1$, while those contributing to (b) must have $L = 0, n_\mu = 0$; hence trajectories similar to the π, K, etc. could contribute to (b) but not to (a). Thus the backward scattering would be quite different for the two reactions.

Regge poles in low-energy weak interactions:

There is no reason to assume that the Regge pole contributions should dominate at low energies. However, as mentioned above, replacing the amplitude by a Regge pole may be regarded as an improved 'pole approximation', in that the pole now includes the exchange of a set of particles with the allowed quantum numbers. Looked at slightly differently, one may say that if all poles of the S-matrix are to be regarded as moving poles, the poles at low energy also should be so treated.

As any deviation from local action seems to be negligible at low energies, we assume an effective local vector or axial vector current coupled to the Regge poles in the 'strong' form factors in meson or baryon decays. The contribution of a Regge trajectory to the weak form factor will be of the form $\frac{\beta(t)}{1-\alpha(t)}$, as for the electromagnetic form factors.[14] Approximating (t) by a linear form gives back the 'pole approximation' of dispersion theory; taking quadratic and higher terms gives deviations from the pole approximation. Application of this to meson & baryon decays is being studie. A difference between this and the electromagnetic case is that here there is an axial vector current also; thus trajectories corresponding to axial-vector mesons (like the one suggested in Ref.20) and possible 1^+ strange mesonic states will contribute . As in Freund's treatment of the 2-photon exchange[15], one may also take the coupling of the weak current (acting twice) to the Pomeranchuk trajectory.

Conserved Currents and Regge Trajectories

If we assume the existence of the photon Regge trajectory and a weak vector trajectory, then the conservation of corresponding currents implies that the coupling strength $\beta(t)$ is universal at the point t_1 such that $\alpha(t_1) = 1$.
A possible generalisation that immediately suggests itself is that such a universality may hold all along the 'conserved' Regge trajectory which would then be characterised not only by a definite shape $\alpha(t)$ but also by a definite residue $\beta(t)$.

If the photon trajectory has this property, this may be interpreted as a generalisation of current conservation to an S-matrix theory of electrodynamics. (However, this is still a symmetry imposed ad hoc and not derived). One result that such a conjecture would imply is that the electromagnetic form factors of all particles are the same. It would be of interest to test this experimentally. A similar assumption that the weak trajectory with negative parity is 'conserved' would lead to the universality of all weak vector form factors. This would give a definite relation between K decay and Λ, Σ decay, π decay and n decay, etc.

Partial conservation of the axial vector current, if assumed to be true, may find its analogue in the smoothness properties of $\alpha(t)$ and $\beta(t)$.

Composite Models and Regge Trajectories.

The fact that summing over a set of Feynman diagrams can give a Regge behaviour suggests an equivalence between conventional field theory and S-matrix theory with Regge poles. It may be useful to attempt to get this equivalence explicitly in a composite model theory.

We may start with the 'minimal model' of Okun and take, for instance, the Λ, n and p as the basic fields. One may think of all the Regge trajectories appearing in strong interactions as composite states formed from Λ, n, and p. The fact that ideas like conserved and partially conserved currents follow naturally from a Sakata model suggests, as pointed out above that the same composite states may determine weak interactions also. To get the actual composite states, one may follow Weinberg's suggestion, and introduce more and more fictitious composite particles that make theory converge. These composite states may be the Regge trajectories. Apart from the Λ, n and p fields it is presumably necessary to introduce the μ, e, ν_μ and ν_e fields also in order to include the weak and electromagnetic interactions as well.

Regge Poles from perturbation Theory in Weak Interactions:

Finally, there is the question of obtaining a Regge behaviour explicitly by summing suitable sets of Feynman diagrams. Owing to the non-renormalizability of both the UFI and the vector boson interactions, it is not clear how this can be done explicitly

One may conjecture that the ladder diagrams may add here also to give a Regge behaviour. If vector bosons exist, their emission may give an exponential factor at high energies (as with the infra-red corrections in electron scattering). The emission of neutral vector mesons presumably gives an exponential factor; the question of proving this for charged vector bosons also is being studied.

One may also think of the leptons as being reggeized by the vector boson radiative corrections (as the nucleon would be); however, this would presumably be much smaller than the reggeizing effect of the electromagnetic radiative corrections (for the charged leptons;) and the idea of regarding the neutrino as a Regge pole does not seem to be of direct utility.

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Note added:

Feynman has observed that the principle of minimal electromagnetic interaction (which excludes terms like Pauli moments in the interaction) restricts the high-energy behaviour of cross-sections; the presence of a Paulimoment, for example, would mean more subtractions. (cf. Feynman: Report to the Solvay Congress, 1961.) He has also suggested that one may try to replace the principle of minimal electromagnetic interaction by a statement that high-energy electrodynamic cross-sections are limited in some particular way.

We note that an appropriate replacement of the principle of minimal electromagnetic interaction may be the requirement that the high-energy cross-sections have a Regge behaviour. We may conjecture a similar principle for weak cross-sections to replace the idea of a minimal weak interaction.

Restrictions on the high-energy behaviour of cross-sections may also be expressed by imposing γ_5-invariance; this may also be related to a Regge type of behaviour, as it limits the number of subtractions (e.g. it excludes Pauli-moment terms).

DETERMINATION OF SPIN-PARITY OF RESONANCES.

I do not presume to present in this talk a comprehensive survey of the methods that have been used or suggested for the spin-parity determination but shall restrict the scope of my talk to a brief description of the two principal methods, viz.,

(1) the Adair method

and (2) the Dalitz method

that have been successfully utilised in this connection.

The Adair method was first suggested in 1955 for the determination of the Λ^0-spin from

$$\pi^- + p \rightarrow \underset{\underset{2\pi}{\downarrow}}{K^0} + \underset{\underset{N + \pi}{\downarrow}}{\Lambda^0}$$

Let us denote by A, B the spins of the particle K^0 and Λ^0 respectively and let the spins add up to form a total spin S. Let L denote the relative orbital angular momentum of the two particles and j, m the total angular momentum and projection quantum numbers.

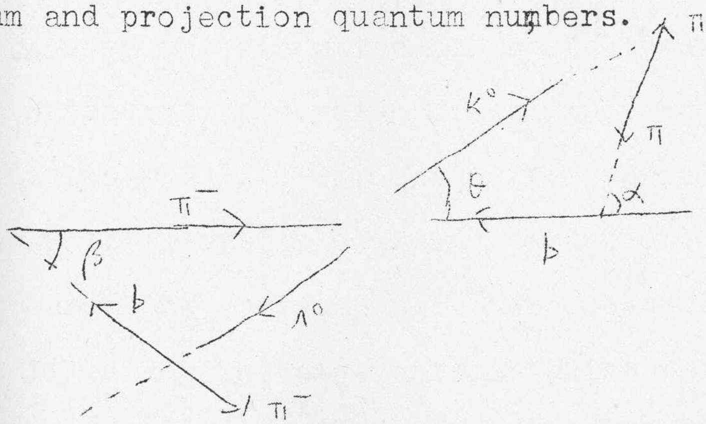


Fig. 1. The reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$ in the centre of mass frame.

If the Λ^0 decays into a nucleon and pion with relative orbital angular momentum ℓ (i.e. (in the rest frame of the Λ^0), ℓ and the nucleon spin 1/2 combine to form B. Since the pions do not carry intrinsic spin K^0 decays into two pions with relative orbital angular momentum, A. We can therefore write the amplitude for the process as

$$f = \sum_{\lambda} T_{\lambda}\, \mathcal{X}(m)\, (2j+1)^{-1/2} \sum_{m_L} C(L\,S\,j;\, m_L\, m_S\, m)\, Y_{L,m_L}(\theta,\phi)$$

$$\times \sum_{m_A} C(A\,B\,S;\, m_A\, m_B\, m_S)\, Y_{A,m_A}(\alpha,\phi_\alpha) \sum_{m_\ell} C(\ell\,\tfrac{1}{2}\,B;\, m_\ell\, \mu\, m_B)$$

$$\times\, Y_{\ell,m_\ell}(\beta,\phi_\beta)\; {}^{1/2}\chi_{\mu} \qquad \text{-------------- (1)}$$

where λ denotes the angular momentum and parity channel quantum numbers $\lambda = \pi, L, S, j, m$ where π denotes the parity $\pi = (-1)^{L+\ell+A+1}$ and $\mathcal{X}(m)$ are unit vectors such that $\mathcal{X}(m)\,\mathcal{X}(m') = \delta_{mm'}$ the initial system being unpolarised. If we choose the direction of the incident beam as the axis of quantization, the value of m must be equal to the spin orientation ($\pm$ 1/2) of the initial proton the projection of the initial orbital angular momentum on the beam direction being zero; the various angles are shown in Fig. 1.

If we now select events in which the Λ^0 and K^0 are produced parallel to the beam, m_L has to be zero and if we further restrict the study to the angular distribution of the Λ^0 decay products (in the Λ^0 rest frame) in conjunction with the K^0 decaying along or opposite the beam when $m_\Lambda = 0$ only contributes to (1) and $m_B = m$ the amplitude of interest reduces to the form

$$\sum_{m_\ell} C(\ell \tfrac{1}{2} B; m_\ell - \mu\, m)\, Y_{\ell, m_\ell}(\beta, \phi_\beta)\, {}^{1/2}\chi_\mu \qquad (2)$$

which results in angular distributions dependent on the Λ^0 spin, B as shown in Table 1.

Spin value B	Angular distribution
1/2	Isotropic
3/2	$1/2 + 3/2 \cos^2 \beta$
5/2	$3/4 - 3/2 \cos^2 \beta + 15/4 \cos^4 \beta$

etc.

On the other hand, selecting again the events with production along the beam and studying the angular distribution of the K^0 decay products (in the K^0 rest frame with respect to the incident beam direction) in conjunction with the Λ^0 decaying along (or opposite) the beam, it is readily seen that we have now $m_\Lambda = m - \mu$ and consequently

$$Y_{\Lambda, m+1/2} \quad \text{and} \quad Y_{\Lambda, m-1/2} \qquad (3)$$

contribute to the amplitude resulting in an angular distribution of the form

$$a \cos^{2\Lambda} \alpha + a' \cos^{2(\Lambda-1)} \alpha \tag{4}$$

so that a study of the distribution determines Λ (or if $a = 0$) at least $\Lambda - 1$ since a and a' cannot vanish simultaneously.

The application of the method to the determination of the K^* spin from

$$\begin{array}{ccc} K^+ + p \rightarrow & K^{*0} & + \; N^{*\,++}_{33} \\ & \downarrow & \downarrow \\ & K^+ + \pi & p + \pi^+ \end{array}$$

is straight forward theoretically, the K^{*0} with spin Λ decaying into 2 spinless particles and $N^{*\,++}_{33}$ (with spin B) decaying into a nucleon and a pion. Recent experimental data (Phys. Rev. Lett. 9, 330 (1962)) gave a good fit with a pure $\cos^2\alpha$ distribution, thus indicating that the K^* spin $\Lambda \geq 1$ which combined with earlier evidence $\Lambda < 2$ determines the spin-parity of the K^* as 1^-. The experimental observations also favoured the K^* spin to be strongly aligned in the m = 0 state with reference to the incident beam direction.

The second principal method viz., the Dalitz method is concerned with three particle decays; for example, the decay into 3 pions

$$\omega \rightarrow \pi^+ + \pi^- + \pi^0$$

It is clear that in the rest frame of the ω^0 the three pions are coplanar and

$$m_\omega = T_0 + T_+ + T_- + 3\, m_\pi \tag{5}$$

where the T's denote the K.E.s and m's the masses. If we now describe the decay events, by plotting a point x for each event inside an equilateral triangle ABC such that the projections xP, xQ, xR are proportional to the K.E.'s T_0, T_+ and T_- respectively, the requirement that these energies allow momentum conservation confines the points X to the inscribed circle within the triangle in the non-relativistic case, the boundary getting distorted approaching a triangular shape in the extreme relativistic cases. The density distribution of points inside the boundary are characteristic of the spin-parity and other attributes of the decaying state.

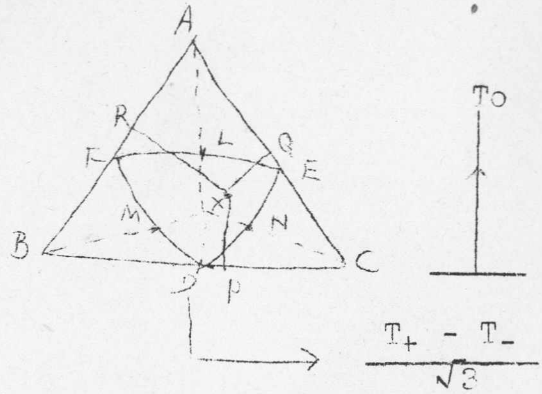


Fig. 2. The Dalitz plot

For example, since the isotopic spin of ω is zero, we argue that the isotopic spins of any two pions must add up to 1 and consequently the isotopic spin state of the system must be odd under exchange, and since the pions are bosons the state must be antisymmetric also under space exchange. Therefore, for example, if the ω spin-parity is 0^- and if $\vec{q}$ denotes the relative momentum between any pair of pions and $\vec{p}$ the momentum of the third pion in the ω^0 rest frame the matrix element for the ω^0 decay is limited to scalars of the form

$$\vec{p}.\ \vec{q}\ \cdot\ F\left((\vec{p}.\ \vec{q})^2\right)$$

which vanishes when $\vec{p}$ is perpendicular to $\vec{q}$ or when the energies of any two of the pions are the same that is, the density of points in the Dalitz plot must vanish along the lines A D, B E, and C F. In fact, since the space wave function is antisymmetric the density of points should be unchanged by a reflection across any of the three lines AD, BE, CF so that the distribution is essentially the same in each of the six sectors (irrespective of the spin-parity of the ω^o). The type of matrix elements A for various choices of spin-parity for the ω meson and the corresponding features of the Dalitz plot are listed in table 2 and the

TABLE. 2

Spin-parity of ω^o	Decay matrix element A	Density vanishes at
1^-	$(\vec{p}_o \times \vec{p}_+) + (\vec{p}_+ \times \vec{p}_-) + (\vec{p}_- \times \vec{p}_o)$	whole boundary
0^-	$(E_- - E_o)(E_o - E_+)(E_+ - E_-)$	The straight lines AD, BE, CF of symmetry
1^+	$E_- (\vec{p}_o - \vec{p}_+) + E_o (\vec{p}_+ - \vec{p}_-) + E_+ (\vec{p}_- - \vec{p}_o)$	Centre of the Dalitz plot

dependence of the six folded $|A|^2$ on the distance from the centre of the Dalitz plot is shown in Fig. 3. Experimental observations favour the assignment 1^- and since the simplest matrix elements for spin-parity assignments $2^\pm$ also vanish at

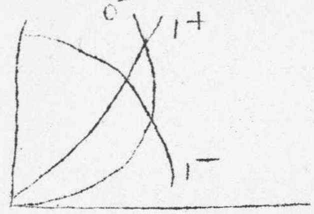


Fig. 3. Density of events vs. Distance from the Centre of DALITZ PLOT.

the centre of the plot one concludes that the spin-parity of the ω meson is 1^-. The assignment 0^+ is incompatible with 3π decay since if we divide the 3π system into a system consisting of 2π's and a third pion and if ℓ denotes the relative angular momentum between the 2 pions the relative angular momentum between the two pion and single pion systems must also be ℓ so that the parity of the system must necessarily be odd as the pions are pseudo scalar particles.

G-PARITY AND ELECTROMAGNETIC EFFECTS IN THE RESONANCES

K.Raman.

In this talk we shall discuss the following topics:

A) The role of G-parity in determining the properties of the resonances.

B) Various electromagnetic effects relating to the resonances, and the electromagnetic properties of the resonances, in particular, the following:

i) Electromagnetic mixing effects and isospin violation;

ii) The electromagnetic interaction in radiative and non-radiative decays;

iii) Mass differences within isomultiplets made up of the resonances

iv) Form factors and electromagnetic moments of the resonances; and

v) Role of the resonances in various photo-processes, especially bransstrahlung.

Owing to limitations of time, we shall mainly confine ourselves to mentioning various questions in relation to the above.

A) <u>The role of G-parity.</u>

The resonances are generally characterised by the various quantum numbers which are conserved by the strong interactions, viz. $\mathcal{J}$, Π, B, S, I, etc. An important quantum number that is not respected by the electromagnetic interaction is the isospin $\vec{I}$.

Consider a neutral system with zero baryon number and zero strangeness; for strong interactions, the conserved quantum numbers are C, Π, S, I (Π is the parity).

Conservation of I means rotational invariance in isospace, together with the conservation of C, this leads to reflection invariance in isospace also. The G operation, which is the inversion operation in isospace, may be defined as

$$G = C \exp(i\pi I_x) \quad \underline{\text{OR}} \text{ as } \quad G = C \exp(i\pi I_y)$$

according as one adopts the convention

$$C|\pi^{\pm}\rangle = +|\pi^{\pm}\rangle \quad \underline{\text{OR}} \quad C|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle$$

respectively.

Only ~~natural~~ neutral states (with B = 0, S = 0) are eigenstates of C, whereas the complete isomultiplet is an eigenstate of G. The eigenvalue for the G-operation is called the G-parity.

Denoting eigenvalues by primed letters, we have the following relations:

i) $G' = C'(-1)^{I'}$ for systems with S=0, B=0.

ii) $G|n\pi\rangle = (-1)^n|n\pi\rangle$ for a system of n-pions.

iii) For particle-antiparticle pairs,

$$C' = (-1)^{\ell + s}$$

where ℓ and s are the orbital angular momentum and the total spin respectively.

Thus

$$G' = (-1)^{\ell + s + I} \quad \text{for particle-antiparticle pairs.}$$

Consequences of G Conservation:

For strong interactions, the selection rules are

$$|\Delta \vec{I}| = 0 \, , \quad \Delta G = \text{NO} .$$

Consequences of this for decays of multi-pion states have been noted by different authors. We give below a few decays that are forbidden by G-conservation but allowed by all other conservation laws (C, Π, etc.)

G-forbidden Transitions

I	J^{PG}	C	Example of particle	G-Forbidden transitions (in addition to e.m. transitions)
0	0^{++}	+		
1	0^{+-}	+	ζ	$2\pi, 4\pi, \ldots$
1	0^{--}	+	π	$4\pi, 6\pi, \ldots$
0	0^{-+}	+	η^0	$3\pi, 5\pi, \ldots$
1	1^{-+}	−	ρ	$3\pi, 5\pi, \ldots$
0	1^{--}	−	ω^0	$2\pi, 4\pi, \ldots$

Ref. D.B.Lichtenberg and Summerfield: Phys. Rev. 127, p.1806 (1962)

Lichtenberg and Summerfield have also pointed out that the various quantum numbers of a meson determine whether it can appear as a pole term in NN scattering. The same holds for a Regge trajectory also.

Thus in order that a meson may give rise to a pole term in the NN scattering amplitude, it must have

$$I \leq 1 , \qquad G = C(-1)^{I} , \qquad \text{and}$$

$$\begin{array}{lll} J = 0 , & \Pi = \pm & \text{implying } C = + \\ J \geq 1 , & \Pi = (-1)^{J} & \text{implying } C = (-1)^{J} \\ J \geq 1 , & \Pi = (-1)^{J+1} & \text{implying } C = \pm \end{array}$$

Thus the ζ meson could be the I = 1, 1^{+-} particle that would fit nucleon-nucleon scattering if its decay into 2π violated G-conservation.

<u>G-Violating Interactions</u>:

The isospin may change in an interaction involving photons. However, the change obeys $|\Delta \vec{I}| = 0$ or 1 for interactions involving a single photon. This, together with the fact that the isovector and isoscalar electromagnetic currents have definite G-parity, leads to certain regularities in G-violating decays. Feinberg and Pais[1)] have shown that the following results obtain, when the electromagnetic interaction is treated to the lowest order:

a) Radiative decays $A \to B + \gamma$:

A, B are both G-eigenstates; A is an unstable meson and B a group of mesons.

The electromagnetic current J_μ may be split into isovector and isoscalar parts J_μ^V, J_μ^S with positive and negative G-parity respectively. In the decay $A \to B + \gamma$, the electromagnetic current must have a definite G-parity and can therefore involve only J_μ^V or J_μ^S.

(i) If $G_A = G_B$, only J_μ^V contributes ; thus $|\vec{I}_A - \vec{I}_B| = 1$.

(ii) If $G_A = -G_B$, only J_μ^S contributes; $\therefore \vec{I}_A = \vec{I}_B$.

Feinberg and Pais have shown that application of this gives the branching ratios for the decays $\rho \to \pi + \gamma$, $\omega \to 2\pi + \gamma$, and $\eta \to 2\pi + \gamma$.

We have pointed out that the result above leads to selection rules for $N\bar{N}$ and $K\bar{K}$ annihilation into multi-pion states and one photon.

b) Non-radiative decays $A \to B$,

A B, (which are suppressed, as either G and/or I changes. e.g. $\eta(0^{-+}) \to 3\pi$, $\omega \to 2\pi$.

If we assume that such decays proceed through the emission and reabsorption of single photon, then the matrix element must be proportional to $\langle B | J_\mu J_\nu | A \rangle$

Since either G and/or I must change in the type of decays being considered, terms of the type $J_\mu^S J_\nu^S$ cannot contribute. Then we have (-Feinberg & Pais)

(i) If $G_A = G_B$, then $\langle A | J_\mu J_\nu | B \rangle = \langle A | J_\mu^V J_\nu^V | B \rangle$, which is an isotensor of rank 2 ; $\therefore$ $I_A \neq I_B$

(ii) If $G_A = -G_B$, then $\langle A | J_\mu J_\nu | B \rangle = \langle A | J_\mu^S J_\nu^V + J_\mu^V J_\nu^S | B \rangle$

$\therefore$ $|\vec{I}_A - \vec{I}_B| = 1$.

Feinberg and Pais show how these results can again predict the branching ratios into charge-states, and how a) and b) together seem to rule out the assignment 0^{--} for the η meson.

Regarding G-parity, we briefly mention the following also

1) Lee and Yang have suggested a generalization of G-parity to systems other than those with S = 0 , B = 0. [2]

2) Even the usual G-parity leads to some selection rules for decays of systems other than those with S = 0, B = 0. These may be obtained by considering partial systems. For instance, one may think of a $(K+\eta)$ bound state, which can decay into $(K+\pi)$ only by violation of G-conservation.

The restrictions considered above follow from isospin and G-parity. Further restrictions follow if we assume higher symmetries than isospin.

B) ELECTROMAGNETIC EFFECTS RELATING TO THE RESONANCES:

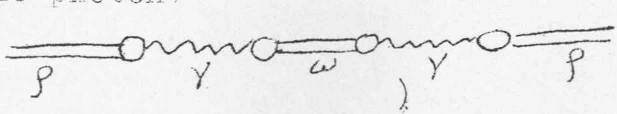
The stable strongly interacting particles are coupled to the electromagnetic field; this coupling provides a convenient probe into their structure. The resonances are also coupled to the electromagnetic field, as shown by their production in electromagnetic interactions and their radiative decays.

(i) Electromagnetic mixing effects and Isospin violation:

From the observation that the ω decays into 2π about 5% of the time, that the masses of the ρ and ω are quite close, and that the 2π from ω decay must be in a 1^- and hence $I = 1$, state, one may expect certain mixing effects between the ρ^0 and the ω^0.

Starting with a source that is an equal mixture of ρ^0 and ω^0, the states get mixed as time passes (just as K^0 and $\bar{K}^0$ pass into each other) and the $\frac{\rho}{\omega}$ ratio varies with time, leading to certain effects. Feinberg and Bernstein[3] have given a theory of the mixing effect in terms of coupled time-dependent equations; they conclude that the main effect of the ω^0 on the ρ^0 mass spectrum is the appearance of a sharp peak (the 'needle') next to the broad a spectrum (the 'haystack').

The mixing effects have also been considered by J.C. Taylor who points out that they may lead to a splitting of the ρ^0.[4)] [The simplest mechanism for the mixing effect would be via a single photon:



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It is not clear whether this mixing is of a magnitude that would lead to observable effects.

(ii) <u>The Electromagnetic interaction in radiative decays:</u>

Some of the allowed and forbidden radiative decays of multipion states are as follows:

I	J^{PG}	C	Particle	Allowed radiative decays (for neutral member)	Forbidden radiative decays
0	0^{++}	$+$		$2\gamma,\ \pi^0+2\gamma,\ \pi^+\pi^-\gamma$	$\pi^0+\gamma$
0	0^{+-}	$-$		$2\pi+\gamma$	$\pi^0+\gamma,\ 2\gamma$
1	0^{+-}	$+$	ζ	2γ	$\pi^0+\gamma,\ 2\pi^0+\gamma$
1	0^{--}	$+$	π	2γ	$\pi^0+\gamma,\ 2\pi^0+\gamma$
0	0^{-+}	$+$	η	2γ	$\pi^0+\gamma,\ 2\pi^0+\gamma$
1	1^{-+}	$-$	ρ	$\pi^0+\gamma$	$2\gamma,\ \pi^0+2\gamma$
0	1^{--}	$-$	ω	$\pi^0+\gamma$	$2\gamma,\ \pi^0+2\gamma$

As is well known, relations between the decay amplitudes of different mesons may be obtained if one assumes some higher symmetry; e.g. the 8-fold way would give

$$M(\rho\to\eta\gamma) = M(\omega\to\pi^0\gamma) = -\frac{1}{\sqrt{3}}\,M(\omega\to\eta\gamma) = \frac{1}{3}\,M(\rho\to\pi^0\gamma)\ ;$$

$$M(\eta^0\to 2\gamma) = \frac{1}{\sqrt{3}}\,M(\pi^0\to 2\gamma)\ ,\ \text{etc.} \qquad 5)$$

Information about the structure of 3- and 4-body radiative decays may be obtained by examining the spectra and angular correlations of the decay products.

In the decay $A\to B+\gamma$, when the group of particles B have all quantum numbers the same as the initial particle A, there will be a peak in the spectra at zero photon energy (for in such a decay, the photon energy can actually go to zero, giving the non-radiative decay); as an example we have $\rho^+ \to \pi^0 + \pi^+ + \gamma$. In a decay mode like $\omega^0 \to \pi^+\pi^-\gamma$, the final pions may differ from the ω^0 only in G-parity, and there may be a suppressed peak at zero photon energy corresponding to the (G-violating) non-radiative decay $\omega \to \pi^+\pi^-$. Similarly for $\eta \to \pi^+\pi^-\pi^0 + \gamma$.

When the group of final particles have all quantum numbers the same as the initial particle, then the final particles can be correlated into the initial particle itself. The decay may be pictured as the particle A emitting a photon and going away as a virtual particle A, which then decays into the particles B. It is known that in such a case, the radiative decay may be expressed in terms of the non-radiative decay and the electromagnetic moments of the particles involved. Some information about the magnetic moments of the resonances may be thus obtained. For instance, it may be possible to use the photon spectra in $\rho^\pm \to \pi^\pm + \pi^0 + \gamma$ and $\rho^0 \to \pi^+ + \pi^- + \gamma$ to estimate the magnetic moment of the ρ meson.

(iii) Mass differences within isomultiplets:

Just as attempts have been made to derive the np mass difference and the $\pi^\pm\pi^0$ and $K^0K^\pm$ mass differences theoretically, we may also try to calculate the mass differences between the different resonant states forming a multiplet, perhaps relating them to the form factors of the resonances. (As pointed out by Prof. Marshak, however, all the <u>narrow</u>, pionic resonances seem to be isosinglets; the multiplets have a width that is comparable to the mass differences.)

The mass differences will also be a sensitive test of the model used in describing the resonances. Consider, for instance, the ρ meson, forgetting for a moment its large width. The $\rho^{\pm}$ masses seem to be about 750-770 Mev, while the ρ^{0} mass is about 725 Mev.

The ρ is commonly thought of as a (2π) resonance, for instance, Schiff has put forward a model in which a ρ is constituted of two pions.[6] However, in such a model, the ρ^{0} must be considered a ($\pi^{+}\pi^{-}$) system and the $\rho^{\pm}$ as ($\pi^{\pm}\pi^{0}$) systems; the mass difference and the assumption of charge independence of the $\pi\pi$ forces (apart from the small Coulomb forces) then predict that the ρ^{0} should be <u>heavier</u> than the $\rho^{\pm}$ Thus the simple (2π) picture of the ρ gives even the sign of the mass difference wrong. Note, however, that a) the width of the ρ is larger than the mass differences, as pointed out by Prof. Marshak, and the argument may not have significance, and b) we have not considered the effect of the ω^{0}, which may affect the position of the ρ^{0} meson,

However, we believe that the effect of the ω^{0} on the position of the ρ^{0} will be small. If the masses assumed above are correct, one may take it is as an indication that a ( ) model of the ρ

is quite inadequate. We may think of the wave function of the ρ as receiving contributions from (2π), (4π), (6π), - - states,

$$\psi_\rho = A_2\,\psi(2\pi) + A_4\,\psi(4\pi) + A_6\,\psi(6\pi) + \cdots$$

With (4π) states, it is possible to obtain a mass difference of the correct sign, of a magnitude $<$ 5Mev., while (6π) stats can give a mass difference of the correct sign and $<$ 15Mev. The ratio of (4π) to (2π) states may be estimated from the branching ratios for the decays with and , taking into account the phase space suppression of the 4π mode.

One may also take into account states like $(\omega+\pi)$

In general, it would be inadequate to think of a resonance as made up of the products of its dominant decay mode; many-particle states may play a more important role than hitherto considered. In dispersion theory, this may mean that many-particle intermediate states and their singularities play a decisive role. A different method would be to treat a resonance as a particle and relate the mass differences to form factors which may presumably be calculated.

The considerations above may be applied to other isomultiplets; also higher symmetry schemes like unitary symmetry lead to relations between the mass differences within different isomultiplets and the widths.

(iv) Form factors and electromagnetic moments of the resonances:

The relation between the spin of a particle and its allowed electromagnetic moments is well known.

e.g. A spin-zero particle can have only a charge form-factor, a spin-half particle also a magnetic moment and a spin-one particle a quadrupole moment in addition. Many of the resonances have a high-spin; the ones of most interest are probably the vector mesons and the spin $3/2$ baryons.

We mention a few questions relating to the electromagnetic moments of the resonances.

1) How does one define electromagnetic moments for unstable particles?

J.S. Bell has shown recently how charge and current distributions maybe defind for unstable particle; they are complex quantities which oscillate with increasing amplitude at large distances. 7)

(Bell has also shown that even an unstable particle cannot have an electric dipole moment, contrary to a statement by Zeldovich.)

2) What happens to the electromagnetic and form factors when a resonance is regarded as a Regge pole?

The spin $\alpha(t)$ of a Regge pole varies with the energy. Consider a reaction in which the trajectory $\alpha(t)$ is exchanged. At the point t_1 such that $\alpha(t_1) = 1$, the exchanged particle has a quadrupole moment and a magnetic moment as well as a charge (for particles with non-zero isospin). At the point $t_{1/2}$ where $\alpha(t_{1/2}) = \frac{1}{2}$, the quadrupole moment disappears, while at the point t_0 such that $\alpha(t_0) = 0$, the magnetic moment also vanishes. At the intermediate unphysical points, the trajectory will have projections on all J values and can have electromagnetic couplings of any rank (depending on the spins of the external particles).

It is interesting to speculate how such effects, if real, could be observed.

3) Electromagnetic moments of the resonances and higher symmetry schemes

Relations between the electromagnetic moments of the resonances may be obtained from unitary symmetry. For instance, in the octet model, one has for the members of a baryon-resonance octet the well-known relations between the magnetic moments.[8] For meson octets, etc., the relations are not very useful; they lead to the vanishing of the form factors of all neutral particles, and the equalities

$$\mu(\rho^+) = \mu(K^{*+}) = -\mu(\rho^-) = -\mu(K^{*-}).$$

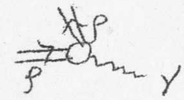
4) Possible methods for determining the electromagnetic moments of resonances.

(i) We have already mentioned how information about the magnetic moments may be obtained from the I.B. peak in radiative decays.

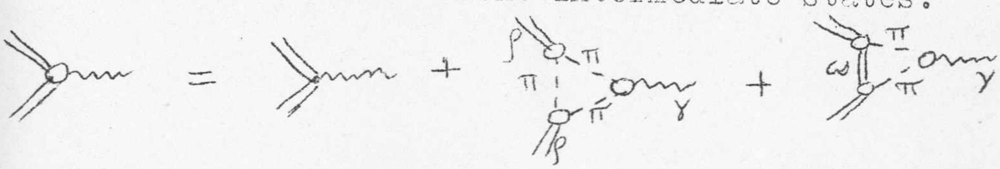
(ii) It may be possible to apply some of the methods used for stable particles to the narrowresonances.

(iii) Production of pairs of unstable vector mesons by a photon or in higher energy electron-position collisions may give some information about their magnetic moments.

5) Calculation of the form factors of the resonances from dispersion theory:

We consider vertices like that shown below: 

Owing to the istability of the ρ, anomalous threshold will be important. If dispersion theory is applicable, one may take the contribution of different intermediate states.



The pion form factor, itself dominated by the ρ will probably make the dominant contributions to the ρ form factors.

v) Role of the resonances in various photoprocesses.

The resonances play a role in various photoprocesses. We shall here briefly consider their role in brensstrahlung, noting some results obtained by Yennie and Feshbach. They point out in general how information about structure of high-energy interactions may be obtained by comparing a scattering process with the corresponding brem sstrahlung process.

In particular, when there is a resonance in the scattering, it will lead to the following features in the bremsstrahlung (ref. Yennie and Feshbach)[9])

1) Near a resonance, the scattering amplitude undergoes a large phase change; the bremsstrahlung spectrum should reflect the effects of this phase change, in the form of a deviation from the $\frac{dk}{k}$ law at energies comparable to the width of the resonance, Yennie and Feshbach have also suggested that this may be used to measure the width of a narrow resonance.

2) To test whether an observ d fairly broad bump is a true resonace or not, one may observe the brensstrahlung cross-section width the initial particles' energy being fixed [at $E_R + \frac{1}{2}\Gamma$

and the final particles' energy at $E_R - i\frac{\Gamma}{2}$, where E_R and Γ are the position and width of the bump. If the bump is a resonance, the two amplitudes will be almost 90° out of phase, else they are almost in phase; this will imply a considerable difference in the bremsstrahlung cross-sections.

Questions relating to these are being studied.

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THE SINGLE-PARTICLE EXCHANGE MODEL AND THE RESONANCES

K.Raman.

In this talk we shall discuss the single-particle exchange model and its generalisations in relation (both experimentally and theoretically) to the resonances.

In field theory, an interaction between 2 particles may be described as mediated by the quanta of some particular field. In perturbation theory, one may think of the interaction as the sum of 1 quantum exchange, 2 quantum exchange etc. The 1 quantum exchange approximation is called the one-particle-exchange (OPE) model.

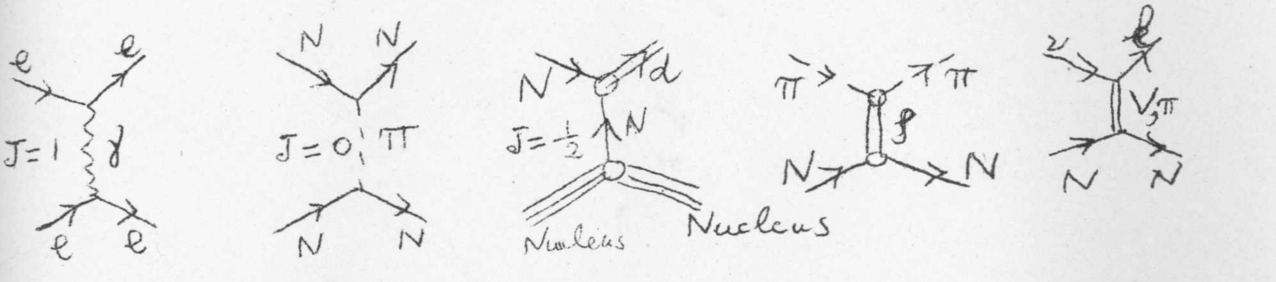
We may immediately ask two questions:

1) Given the nature of the quanta and the strength of the interaction, when is OPE a good approximation?

2) Since quanta are to a certain extent fictitious, is it possible to invent quanta such that OPE is a good approximation?

The second question is intimately related to the resonances and bound states and Regge poles; the introduction of new particles is one of the central features of Weinberg's recent work on "The Elementary - Particle Theory of Composite Particles"[1].

As examples of 1-quantum exchange, we note the following:

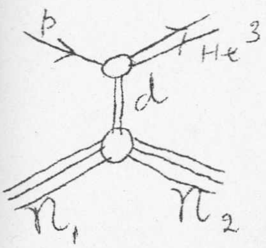


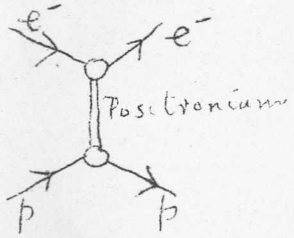
One -photon exchange and Yukawa's single-meson exchange are perhaps the best-known examples.

In electrodynamics and in weak interactions, the coupling constant is small and OPE may be a good approximation. However, in nucleon-nucleon scattering, this is not so; OPE does not give the main contribution to the NN potential. However, it has been pointed out that in certain regions of phase space, the OPE contribution may be expected to dominate. This region is that of high (but not too high) energies and low-momentum transfers (that is, close to the forward direction at such high energie

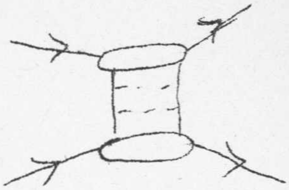
The argument is, loosely, that the matrix element for the OPE graph has a factor $\frac{1}{t-m^2}$, where m is the mass of the exchanged particle. The pole $t = m^2$ lies in the unphysical region. As $s \to \infty$ and $\theta = 0^\circ$, $t \to 0^-$ for elastic scattering and we can get quite close to the pole. (In some inelastic processes, one can have $t > 0$). One expects that the proximity of the pole may help OPE to dominate.

However, this argument may not be quite correct. For at best, the proximity of the pole can only give a large enhancement and also, sets of quite complicated Feynman diagrams can give quite pronounced peaks. e.g. Consider a reaction in which a bound state is being exchanged.





In dispersion theory, such an exchange is often represent by a double line. However, the same bound state may be represented in a different picture, e.g. the Bethe-Salpeter pictur by a much more complicated graph - a ladder diagram. It has



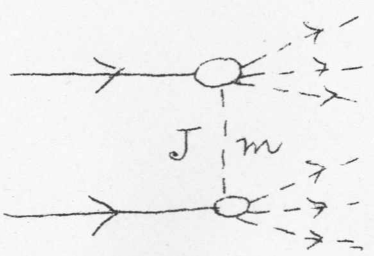
been shown by Fubini et al that the sum over ladder diagrams gives, asymptotically at least, the same behaviour as a bound state pole. Quite generally, the exchange of a composite particle is expected to be equivalent to a set of Feynman diagrams, whose sum thus simulates a pole.

Thus one may not conclude merely from the proximity of a pole that OPE should dominate. However, when the exchanged particle has a very small mass, e.g. the photon or the pion, OPE may be a good approximation in some small domain.

THE PROPERTIES OF THE OPE CONTRIBUTION

We shall examine whether the different properties depend on the nature of the exchanged particle, viz. on its spin and mass, and on the elasticity of each of the two vertices, and how these properties may be made use of in testing the validity of the model.

Consider a general OPE process:



1) THE RANGE OF THE INTERACTION AND THE PARTIAL WAVES INVOLVED

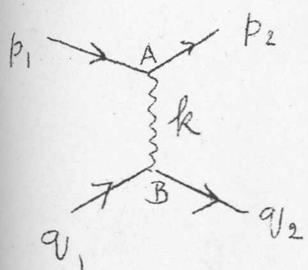
When the exchanged particle is light, like a pion or photon, the interaction is of a long range. At high energies, therefore, the OPE contribution should be dominated by the higher partial waves, as these are weighted by a factor $(2J+1)$. The angular distributions, asymmetries, etc. should be characteristic of the higher partial waves.

This should be true irrespective of the spin of the exchanged particle.

In general, for light exchanged particles, one expects pronounced peaking effects.

2) THE FACTORIZATION PROPERTY:

<u>When the exchanged particle has zero spin</u>, the matrix element can be factorized into two different parts, one characteristic of each vertex (apart from a propagator). We may stress that this factorisation property sterns from the zero spin of the exchanged particle and not merely from the fact that there is just one line joining the two vertices. e.g. In electron-electron scattering via 1-photon exchange, the matrix element is not directly factorable:



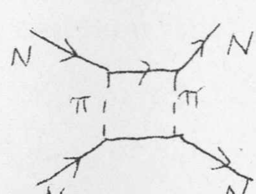
$$\bar{u}(p_2)\,\gamma_\mu^{(A)}\,u(p_1)\,\frac{1}{k^2}\,\bar{u}(q_2)\,\gamma_\mu^{(B)}\,u(q_1)$$

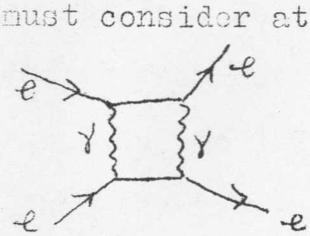
This factorisation property continues to hold when the vertices are made inelastic and can in fact be used to test the model (— the Treiman-Yang test) [9]

3) ZERO IMAGINARY PART:

When each vertex is 'elastic', i.e. there is only 1 particle coming out of each vertex, the matrix element for the OPE diagram has zero imaginary part. Thus all physical quantities proportional to this imaginary part should vanish. Examples of such quantities are various quantities depending on the spin, e.g. (i) for πN scattering, NN scattering, $\pi + N \to K + Y$, etc. the polarization of the final fermion (for initially unpolarized particles) depends on the imaginary part and $= 0$ for the OPE contribution.

Another such quantity is the spin momentum correlation in fermion-fermion scattering, i.e. dependence of the matrix element on quantities like $\vec{s} \cdot (\vec{p}_1 \times \vec{q}_1)$, which are given in terms of the imaginary part of the matrix element integrals. Thus it is well known that 1-photon exchange in e-e scattering or 1-pion-exchange in NN scattering will not give rise to any momentum-spin correlation. One must consider at least 4th order diagrams:





When either vertex is made inelastic, the matrix element can have an imaginary part, for the inelastic vertex may have an imaginary part. However, the matrix element will still continue to be almost real if the emergent particles at a vertex are produced as an isobar.

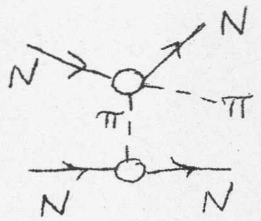
Marshak: This effect will depend on the width of the isobar.

This property holds independently of the spin of the exchanged particle; even if the exchanged particle is replaced by a Regge trajectory it continues to be true.

4) Branching ratios into Charge-states:

An OPE diagram with both vertices elastic makes a definite contribution to each isospin amplitude (as well as to each spin amplitude). For inelastic vertices, when an isobar is produced at each vertex, the same is true

Even when isobar production is not the dominant mechanism, the branching ratios may be



estimated, e.g. for the diagram above by using observed πN scattering cross-sections. This may give a useful experimental check of the model.

5) Kinematics of the OPE Contribution:

This has been studied by Chew and Low,[3] Salzman and Salzman[4] etc. We just mention the interesting fact noted by Salzman and Salzman that when one of the vertices in the OPE diagram is 'excited' and the other 'de-excited', than the four-momentum transfer may be time-like, i.e. t can be greater than zero. (For elastic reactions, t is always less than or equal to zero)

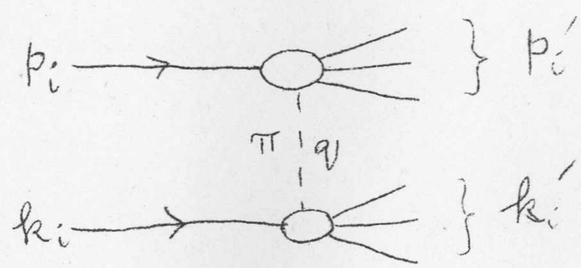
In this connection we note that when the exchanged particle is replaced by a Regge trajectory, the conjecture made by Contogouris, Frautschi and Wong[5] that the "rules" for elastic processes hold here also will not be true in general. For we can have processes in which t can be > 0 in which the Pomeranchuk can be exchanged, leading to a violation of unitarity.

Examples are processes like $\omega + N \rightarrow \gamma + N^*$, and $\rho + N \rightarrow \gamma + N^*$, in which t can be > 0 and the Pomeranchuk can be exchanged. (These are given in analogy with the reaction $\pi + N \rightarrow \gamma + N^*$ considered by Salzman and Salzman, which however cannot proceed through the exchange of a Pomeranchuk.) Thus the conjectures can be carried over to inelastic processes only under certain restrictions.

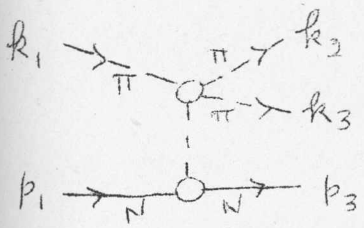
Tests of the OPE Model:

a) The Treiman-Yang Test

As already mentioned, this depends on the spinless nature of the pion.



In the frame where p is at rest, the differential cross-section must be invariant under simultaneous rotation of all the $\vec{p}_i{}'$ about $\vec{q}$ defined by $\vec{q} = \vec{k} - \Sigma \vec{k}_i = \Sigma \vec{p}_i$.
In particular, consider the reaction $\pi + N \rightarrow \pi + \pi + N$ proceeding through single-pion exchange.



In the frame in which $\vec{k}_1 = 0$, the differential cross-section must be invariant under rotation of the plane $(\vec{k}_2 \times \vec{k}_3)$ about the momentum vector $(\vec{p}_1 - \vec{p}_2)$.

Analysis of experiments on $\pi + N \rightarrow 2\pi + N$ showed that except at very low momentum transfers, there was appreciable correlation. The experiments suggested two types of causes for the correlation:

(i) In some of tho experiments, production of 2π from the electromagnetic decay of ω's produced along with the ρ's would contribute to the correlation, as ω's could not be produced by 1π exchange.

(ii) Final-state interaction between one of the pions and the nucleon could contribute to the correlation.

b) <u>Alignment of Final-state particles.</u>

Chinowsky, Goldhaber et al[6] have mentioned in connection with their recent experiment determining the spin of the K^*, that in their reaction $K^- + p \rightarrow K^{*0} + N^{*++}$, the observed distribution in the K^* - decay angle α is almost pure $\cos^2\alpha$, indicating that the spin projection $m_s(K^*)$ is always $= 0$.

They have pointed out that such an alignment wouldbe expected from OPE; this would be a good test for the validity of the OPE model.

c) Observation of the dominant partial waves:

R.J.N. Phillips[1] has pointed out that in the reactions $K + N \rightarrow K^* + N$; $K + N \rightarrow K^* + N^*$, the observed cross-sections can be interpreted as mostly coming from S and P waves, whereas OPE would favour the higher partial waves. This seems to be evidence against OPE.

Marshak: Interference between higher partial waves could lead to this. For instance, the near-isotropy in pp scattering at 300 Mev does not mean pure s-wave scattering

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The tests indicate that OPE is a valid approximation for some reactions but not for others.

Causes for Deviation from OPE.

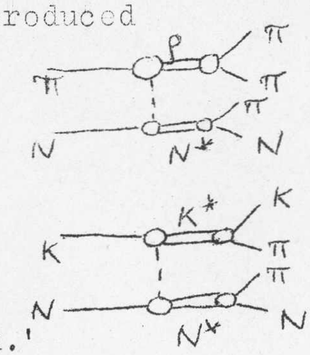
Deviation from OPE may arise from

(i) Multi-particle exchange e.g. the 2-photon exchange in electron-scattering.

(ii) Final-state interactions between particles emerging from the different vertices in the OPE diagram.

Evidence for this is given by the observation that in the reaction $\pi + N \rightarrow \pi + \pi + N$, deviations from OPE have been correlated with production of nucleon isobars.

It is interesting to note that although a study of the reaction $K + N \rightarrow K^* + N$ by MacDowell indicated that other contributions than OPE were important, OPE seems to dominate the reaction $K + N \rightarrow K^* + N^*$. One may speculate that when isobars are produced at both vertices, there is less final-state interaction between a pion at the upper vertex and the nucleon isobar, owing to some kind of 'saturation.'



MARSHAK: Concerning saturation, note that pions are bosons.

(iii) Regge behaviour of the exchanged particle:

If we assume that a Regge behaviour is a result of higher-order corrections, then considering the exchanged particle to be a Regge pole is equivalent to taking into account certain contributions other than OPE.

It has been pointed out by Contogouris, Frautschi and Wong[5] that when the exchanged pion is reggeized, the cross-section would fall off at high energies owing to a factor $s^{2[\alpha_\pi(t) - 1]}$, $\alpha_\pi(0) < 1$, as compared to the exchange of a normal pion.

Criticism of the OPE Model:

The main defect of the model is that its domain of validity is not defined clearly.

Generalisation of the model:

1) Gourdin and Martin[8] have pointed out that the OPE model is a special case of analysing the cross-section of a reaction in terms of the partial waves of the crossed reaction. They have generalised this to the case of arbitrary spin in the crossed reaction.

2) The OPE model may be taken as the starting point for different approximations. An example is the multiperipheral model considered by Amati Fubini et al,

3) We may allow the exchanged particle to be more general, e.g. a composite particle or a Regge pole. The introduction of new composite particles is of interest in connection with Weinberg's recent work.

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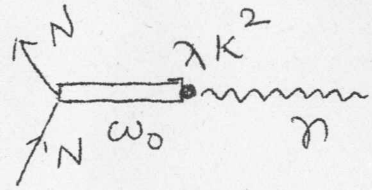
PION RESONANCES By T.S.Santhanam.

As early in 1957, Nambu[1] remarked that it was rather difficult to understand such a large radius as had been observed for the isoscalar part of the nucleon form factor and suggested the possibility that there might exist a meson coupled strongly with the nucleon which have the same quantum numbers as the photon. The simplest gauge invariant coupling for this interaction being of the form

$$\lambda\, S_\mu \frac{\partial}{\partial x_\mu}(F_{\mu\nu}) = \lambda\, S_\mu \Box^2 A_\mu ,$$ using

the Lorentz condition $\frac{\partial A_\nu}{\partial x_\nu} = 0$. As shown in figure, this interaction leads to the contribution

$\frac{-\lambda G K^2}{K^2 - m_S^2}$ to the nucleon form factor, G is related with the strength of appropriate nucleon coupling and m_S is the mass of the meson.



If the remaining contribution to this form factor is denoted by C (approximately independent of K^2 since they do not account for the large r.m.s. radius oberved). The form factor is given by

$$F_S(K^2) = C - \frac{K^2}{K^2 - m_S^2} = F_S(0)\left[1 + \alpha - \frac{\alpha m_S^2}{m_S^2 - K^2}\right],$$

with $\alpha = \frac{\lambda G}{C}$.

1. Y. Nambu, Phys. Rev. 106 (1957) 1366.

F is given by

$$F = F^{Vec} + F^{Sc}$$

The mean square radius is given by

$$r^2 = (r^V)^2 + (r^S)^2 = .8f \quad \text{for proton}$$

$$= (r^V)^2 - (r^S)^2 = 0.0f \quad \text{for neutron}$$

Comparison of this with experimental data(2) for the charge and magnetic form factors requires the value.

$$m_S \approx 4.8\ m_\pi \approx 670\ MeV$$

It was on this basis that Nambu suggested the existence of a massive $I = 0$ vector meson and discussed the decay process to be expected for this ω^0 state as

$$\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$$
$$\rightarrow \pi^0 + \gamma$$

through which it might be detected.

A similar possibility also existed for the interpretation of the isovector part of the nucleon form factor. that the observed isovector radius might be due to the existance of a massive $I = 1$ vector particle. The quantum number of such a particle allow it to decay rapidly into 2 pions so that it would appear as a resonance in the wave pion-pion interaction. The existance of such a pion-pion resonance was suggested by number of

2) Hofstadter et al. Phys. Rev. 110 (1958) 552, Phys. Rev. 111 (1958) 934.

workers and the details of its connection with the structure of the nuclear magnetic moment was first worked by Frazer and Fulco (3).

Also, from Hofstadter experiments one concludes that

$$r^V \approx r^S \quad \text{and since} \quad r^2 = \frac{6}{m_{res}^2},$$

$$m_{res}^v \approx m_{res}^S$$

Thus the isoscalar particle of Nambu was expected to have nearly the same mass as the ρ predicted by Frazer and Fulco. Sudarshan predicted that

$$m_\rho < m_\omega < m_\rho + m_\pi .$$

II. THE VARIOUS $\pi - \pi$ RESONANCES

a) The ρ meson:- It was studied in the peripheral pion-nucleon collisions $\pi + N \rightarrow 2\pi + N$ and the photoproduction process $\gamma + N \rightarrow 2\pi + N$ (Pick up group, Anderson group, Ervin group) Its mass and width were found to be

$$m_\rho \gtrsim 765\ MeV, \quad \Gamma/2 \simeq 50\ MeV .$$

It appears as a resonance in the pion-pion system in the $I = 1$, $J = 1^{-+}$ state. The ρ meson appears as a resonance in the pion-pion system; it is easy to see that the exchange of a ρ meson between two pions yields an attractive force in the J = 1, I = 1 state. The dominent decay mode is

$$\rho^0 \rightarrow \pi^+ + \pi^0, \quad \rho^\pm \rightarrow \pi^\pm + \pi^0.$$

3) W. R. Frazer and J. R. Fulco. Phys. Rev. Lett. 2 (1959) 369

Some recent experiments seem to indicate a fine structure of ρ_0 meson and the original broad resonances seem to be ρ_1 and ρ_2 each of width $<$ 10 Mev

b) The mass of the ω particle is determined to be 787 Mev and $\Gamma/2 \leq$ 15 Mev.

$$I = 0, \quad J = 1^{--}$$

Nambu and Sakurai give the following with regard to the decay mode

$$\Gamma\omega \rightarrow l^+ + l^- = 5 \times 10 \text{ Mev}, \qquad l = e, \mu, \nu$$

$$\Gamma\omega \rightarrow \pi^+ + \pi^- = 5 \times 10 \text{ Mev}$$

$$\Gamma\omega \rightarrow \pi^+ + \pi^- + \pi^0 = 5 \times 10 \text{ Mev}$$

c) Persner et al observed two peaks in the histogram of the effective mass of the 3π system in the reactions

$$\pi^+ + d \rightarrow \underbrace{\pi^+ + \pi^- + \pi^0} + p + p \quad \text{at 550 Mev}$$

and 770 Mev. The larger peak near 770 Mev is clearly identifiable as ω_0. The other peak was identified as η meson. The mass and width were found to be m_η = 546 Mev. $\Gamma/2 \leq$ 25 Mev.

I = 0, Bastein et al observed in the reactions

$$K^- + p \rightarrow \Lambda + \eta^0$$
$$\hookrightarrow \pi^+ + \pi^- + \pi^0$$
$$\hookrightarrow \text{neutrals.}$$

They deduce the ratio

$$\frac{\eta^0_{char}}{\eta_{neu}} \text{ at 760 Mev/c} = .31 \pm 0.11$$

The assignment 0^{-+} is largely favoured. The study of the decay mode of η perhaps covers many pages of P.R.L. in recent days.

Pickup et all observe that

$$\sigma(\eta \rightarrow \pi^+ + \pi^- + \pi^0) = 57 \pm 10 \ \mu b .$$

It seems interesting and necessary to assume 3π decay mode (G forbidden for η) occur through virtual electromagnetic transitions

$$\sigma(\eta \rightarrow \text{neutrals}) = 140 \pm 55 \ \mu b$$

$$\therefore \quad \frac{\Gamma \ \eta \rightarrow \text{neutrals}}{\Gamma \ \eta \rightarrow \pi^+ + \pi^- + \pi^0} = \frac{140 \pm 55}{57 \pm 10} \approx 2.5 \pm 1$$

Since 3π decay mode could occur only through electromagnetic transitions, $\eta \rightarrow \pi^+ + \pi^- + \gamma$ should be relatively abundent.

But

$$\frac{\Gamma \ \eta \rightarrow + - \gamma}{\Gamma \ \eta \rightarrow + - 0} < 9\% \ \left(\frac{5 \ \mu b}{55 \ \mu b}\right)$$

This result seems difficult to consilewith G = 1 for η

Nambu and Sakurai calculations give

$$\Gamma \ \eta \rightarrow + - 0 = 1 \times 10^{-2} \ MeV,$$

$$\Gamma \ \eta \rightarrow \pi^0 + \gamma = 3 \times 10^{-2} \ MeV,$$

$$\Gamma \ \eta \rightarrow l^+ + l^- = 3 \times 10^{-4} \ MeV,$$

$$\Gamma \ \eta \rightarrow \pi^+ + \pi^- = .5 \times 10^{-4} \ MeV.$$

Also

$$\eta \rightarrow \pi^+ + \pi^- + \pi^0 \qquad \sim \alpha^2$$

$$\eta \rightarrow \pi^+ + \pi^- + \gamma \qquad \sim \alpha$$

$$\eta \rightarrow 2\gamma \qquad \sim \alpha^2$$

A possible motivation was given on the basis of R invariance by Prof. Marshak.

d) ξ meson:-

The ξ meson was obtained in the reaction

$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$ by the French group at saclay and at Michigan in the reaction.

$p + p \rightarrow D + \pi^+ + \pi^0$ by Zorm at Brookhaven

$M_\xi = 575 \pm 15$ Mev. $\Gamma = 70$ Mev. I = 1.

Feld indicates that ξ could be a 0^{++} meson ad its dominnt decay mode $\xi \rightarrow \pi^+ + \pi^0$ will also violate G parity. But the existance of this particle has been questioned.

e) The A, B, C particles:-

Abashion et al observed these only in two events and is expected to be 0^+ with I = 0 if it exists.

f) Pickup et al observed a peak in the reaction

$p + p \rightarrow p + n + \pi^+ + \pi^+ + \pi^-$ in addition to those in

$p + p \rightarrow 2p + \pi^+ + \pi^- + \pi^0$.

But in the first reaction resonance at 625 Mev. I = 1, Γ = 20 Mev. 3π resonance. It is called α meson and is expected to be 1^{+}. It was predicted by primakoff.

g) Kuong and Lynch observed a peak in the 4π effective mass analysis in the reaction

$$p+\bar{p} \rightarrow 3\pi^{+}+3\pi^{-}$$
$$\rightarrow 3\pi^{+}+3\pi^{-}+3\pi^{0} \text{ at } 1.05\,\text{BeV}$$

It is called χ meson

h) Birge et al observed a peak again at 1 Bev in the reaction

$$\pi^{-}+p \rightarrow n+\pi^{+}+\pi^{-}$$

This may be the spin 2 particle of Chew's theory.

i) At M_{β} =420 Mev, Tripp and Ferro Luzzi observed a peak in the reaction $\pi^{-}+p \rightarrow n+\pi^{+}+\pi^{-}$ called β meson.

DETERMINATION OF QUANTUM NUMBERS

The determination of mass, width and I are wellknown. The spin, parity and G parity are determined by two methods.

a) Dalitz Plot:-

When the resonance is observed to decay into three particles, this method is used. The observed decays are plotted on a travangular Dalitz plot and the distribution of points observed is compared with that expected from different spin assignments. A requisite for this method is that the resonance should decay purely through strong interactions.

b) Adair Method:-

Here spin of a resonance is deduced from its 2 body decay (into spinless particles) by observing the distribution of decay products with respect to the momentum of the incident particle in the reaction in which resonance is produced.

DECAY MODES and selection rules

$I = 0$	2π	3π	4π	$I = 1$	2π	3π	4π	N-sta
0^{++}	yes	P, G	yes	0^{+-}	G	P	G	3^P
0^{-+}	P	G	yes	0^{--}	P, G	yes	G	^1S
1^{+-}	P	yes	G	1^{++}	P	G	yes	1^P
1^{++}	P	G	yes	1^{+-}	P	yes	G	3

[T. Feld. PRL. 8 (1962) 181]

Only S and p waves are considered. Yes $\rightarrow$ Allowed

P $\rightarrow$ transition forbidden ~~ly~~ by P conservation

i.e. only allowed via weak interactions.

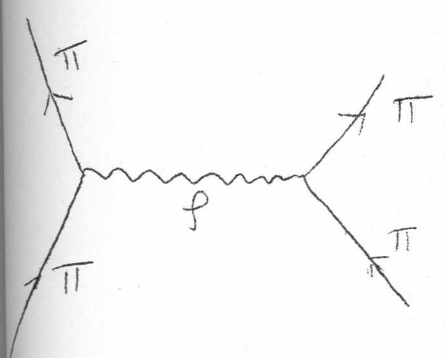
G $\rightarrow$ Electromagnetic allowed decays forbidden in the presence of strong interactions

Last column $\rightarrow$ Nuclear state with the same quantum numbers as the resonance state

Mechanism:-

Assuming the resonance to be dynamical attempts were made to calculate the mass and coupling constant of the meson. But such attempts have not been successful because they involve the solution of a very complicated set of coupled integral equations on a computing machine.= But approximately the existance of the resonance seems to follow from the operation of the "boot-strap mechanism" in which the strong force between two pions in a p state which is needed to produce the resonance is provided by the exchange of a pair of resonating pions. Because of the inherent nature of this Bootstrap mechanism one would expect to be able to obtain the ρ meson's properties as the result of self-consistent calculation with no parameters.

The mechanism is applied on a simple scale with the following approximation



A ρ meson of mass m_ρ is coupled to the pion with a coupling constant $\gamma_{\rho\pi\pi}$ is exchanged between the two pions.

The force thereby produced in the J = 1, I = 1 channel is attractive and depends on m_ρ and $\gamma_{\rho\pi\pi}$. In other words, the ρ meson appears as a resonance in the $\pi - \pi$ system in J = 1, I = 1 state.

It is easy to see that the exchange of a ρ meson between two pion yields an attractive force in the J = 1, I = 1 state. If the parameters of the ρ meson are judiciously chosen, the attraction gives rise to a resonance whose mass and width are those assigned to the ρ meson. The meson has therefore produced itself, so to speak.

Thus we obtain two relations if m_ρ and $\gamma_{\rho\pi\pi}$ are adjusted suitably; this force may be made to produce a resonance at m_ρ with a coupling constant $\gamma_{\rho\pi\pi}$. From these two relations between these quantities we can determine both.

For the I = 1, J = 1 channel we have

$$D\left(\frac{s}{m_\pi^2}, \gamma_{\rho\pi\pi}, \frac{m_\rho^2}{m_\pi^2}\right)$$

$$= 1 + \frac{s - s_0}{\pi} \int_{4\mu^2}^{\infty} \frac{\operatorname{Im} D}{(s'-s_0)\,(s'-s-i\varepsilon)}\, ds' ,$$

where s is the total centre of mass energy squaired

$$\operatorname{Im} D = -\left(\sin\delta\, e^{i\delta}\right)$$

Then we require

$$\operatorname{Re} D \left(\frac{m_\rho^2}{m_\pi^2}, \gamma_{\rho\pi\pi}\right) = 0 \tag{3}$$

and

$$\frac{\operatorname{Im} D\left(\frac{m_\rho^2}{m_\pi^2}, \gamma_{\rho\pi\pi}\right)}{\frac{d}{ds} \operatorname{Re} D \Big|_{s = m_\rho^2}} = \frac{1}{3} \frac{\gamma_{\rho\pi\pi}^2}{4\pi} \left[\frac{(m_\rho^2 - 4\mu^2)^3}{m_\rho^2}\right]. \tag{4}$$

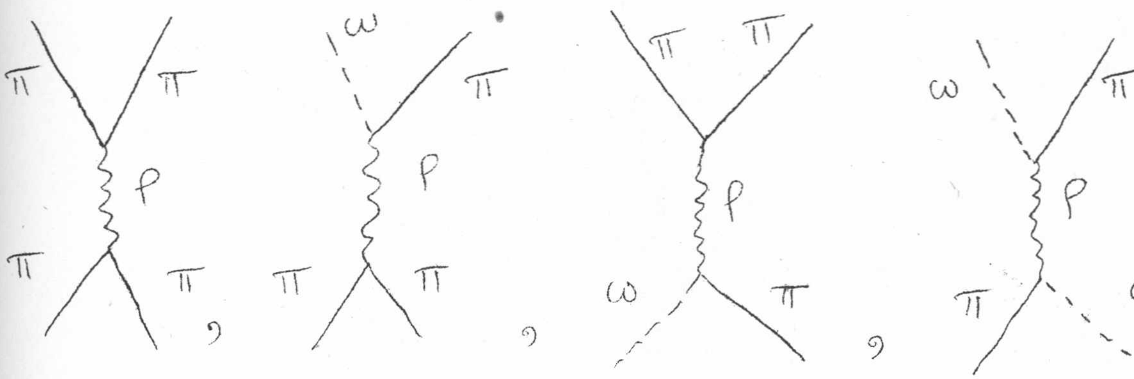
From (3) and (4) fix m_ρ and $\gamma_{\rho\pi\pi}$. An approximate numerical evaluation yields m_ρ = 950 Mev ; $\dfrac{\gamma^2_{\rho\pi\pi}}{4\pi} \approx 2.8$

These are in fair agreement with the present experimental data which indicate ~~me~~ m_ρ = 765 Mev; $\gamma^2_{\rho\pi\pi}/4\pi = 1$

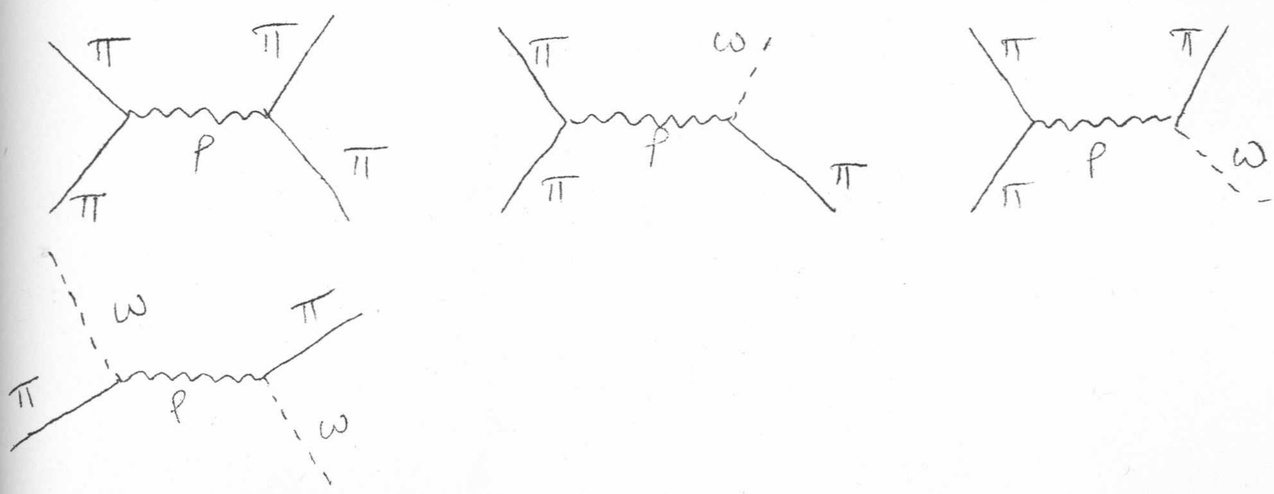
The most important deficiensy of this calculation like this lies in the violation of crossing symmetry.

Zachariasen:-

For a one channel problem the sc amplitude t(s) for a particular partial wave is a function of s, the total centre of mass energy squared. It has a right hand cut in the s plane coming from the unitarity and from direct graphs.



and a left hand cut due to the exchange graphs



If there are any stable one particle states in this channel $t(s)$ will have poles at the masses squared of such states. For s above threshold the unitary condition on $t(s)$ states that

$$\frac{t(s) - t^*(s)}{2i} = t^*(s)\, t(s) \tag{1}$$

(or)

$$\mathrm{Im}\; t^{-1}(s) = -1$$

This analyticity and unitarity are automatically satisfied if it is represented by

$$t(s) = -\frac{N(s)}{D(s)} \tag{2}$$

where $D(s)$ has the same right hand cut at t but no left hand cut, while $N(s)$ has the same left hand cut and with no right hand cut and if

$$D(s) = 1 + \frac{1}{\pi} \int_{\text{threshold}}^{\infty} \frac{N(s')}{s'-s}\, ds' , \tag{3}$$

From the above three, N & D may be determined from their respective dispersion relations and the following expressions for absorptive parts

$$\mathrm{Im}\, D(s) = N(s) ,$$
$$\mathrm{Im}\; N(s) = -D(s)\, \mathrm{Im}\; t_L(s) ,$$

where $t_L(s)$ is the discontinuity of t(s) across its left hand cut. In the lowest order one would use D = 1 obtaining

$N = - t_L$. Then D is obtained as an integral over the input t_L so that finally we get

$$t(s) = t_L(s) \left[1 - \frac{1}{\pi} \int \frac{t_L(s')}{s' - s} ds' \right]^{-1}$$

For our case, the one channel problem consists in forgetting $\pi\omega$ channel and keeping only π-π channel. The input is taken to be exchange of ρ meson between the two pions.

One may then compute the SC amplitude t(s) in J = 1, I = 1 channel, in the simple approximation and ask that the ρ meson itself appear as a resonance with the same parameters as those used in the input diagram. It turns out that there is a self-consistent solution and the resulting ρ meson parameters are found to be m_ρ = 350 Mev. $\gamma^2_{\rho\pi\pi}/4\pi = .6$
Taking into account both the $\pi\pi$ and $\pi\omega$ channels Zachariasen has worked out the mechanism.

b) <u>Particle theory approach to 2π + 3π systems</u>:-

We assume that 2π resonance arises from a potential and ask what effect this potential has on the 3π system. The potential method used here differes from field theory, in three aspects.(1) Number of particles is fixed at 2π or 3π with no nucleon pairs. (2) Partially relativistic Schrodinger-type of ω. equation is used, to describe the motion of these particles. (3) The interaction is represented by a static potential. In the development of point (2) the wave function is assumed to depend on the average time of the several particles not on the time differences. This means that the centre of mass moves relativistically, but retardation effects are neglected in the internal motion. The justification is that retardation is less important in a strongly resonant or bound system in which the particles are close together most of the time.

A single free pion satisfies K.G equation *

$$\left[-\nabla^2 + m^2\right]\phi = -\frac{\partial^2 \phi}{\partial t^2},$$

$\phi = \phi(\vec{r}, t)$; [illegible] , m is the rest mass of pion

For two non-interacting pions, omitting isotopic spin dependence an unsymmetrised wave function may be written as

$$\Psi(r_1, t_1; r_2, t_2) = \phi_1(r_1, t_1)\,\phi(r_2, t_2).$$

where each ϕ satisfies the K.G equation

Ψ satisfies the wave equation

$$\left[-\nabla_1^2 - \nabla_2^2 + 2m^2\right]\Psi = \left[-\frac{\partial^2}{\partial t_1^2} - \frac{\partial^2}{\partial t_2^2}\right]\Psi.$$

Put

$$\text{Average time } t = \tfrac{1}{2}(t_1 + t_2),$$

$$\text{Relative time } \tau = t_1 - t_2,$$

$$\left.\begin{array}{l}\text{Centre of mass}\\ \text{coordinate}\end{array}\right\}\vec{R} = \tfrac{1}{2}(\vec{r}_1 + \vec{r}_2),$$

Relative Coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$. Then

$$\left[-\tfrac{1}{2}\nabla_R^2 - 2\nabla_r^2 + 2m^2\right]\Psi = \left[-\frac{1}{2}\frac{\partial^2}{\partial t^2} - 2\frac{\partial^2}{\partial \tau^2}\right]\Psi$$

[* Unit $\hbar = c = 1$ is assumed]

We neglect the dependence of Ψ on τ and introduce a static pot. V_2 through which two pions interact

$$\left[-\frac{1}{2}\nabla_R^2 - 2\nabla_r^2 + V_2(\vec{r}) + 2m^2\right]\Psi = -\frac{1}{2}\frac{\partial^2\Psi}{\partial t^2}.$$

Let

$$\Psi = \chi(\vec{r})\exp i\left[\vec{P}\cdot\vec{R} - Et\right]$$

E = total energy and P total momentum. Then,

$$\left[-2\nabla_r^2 + V_2(\vec{r}) + 2m^2\right]\chi(\vec{r}) = \frac{1}{2}(E^2-P^2)^{\frac{1}{2}\,2}\chi(\vec{r}).$$

This shows that C.M. moves relativistically like a particle with rest mass $(E^2-P^2)^{1/2} = M_{\Sigma 2}$. Thus M_{2} is the total internal energy of the system. The wave equation for the internal motion is written as

$$\left[-2\nabla_r^2 + V_2(\vec{r})\right]\chi(\vec{r}) = \left[\frac{1}{2}M_{\Sigma 2}^2 - 2m^2\right]\chi(\vec{r}),$$

which is of Schrödinger type.

The foregoing discussion can be extended to any number of pions.

For the 3π system,

$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3),$$

$$\vec{r}_1 = \vec{r}_1 - \vec{r}_2, \quad \rho = \vec{r}_3 - \frac{1}{2}(\vec{r}_1 + \vec{r}_2),$$

$$\left[-2\nabla_r^2 - \tfrac{3}{2}\nabla_\rho^2 + V_3(\vec{r},\rho)\right]\chi(\vec{r},\rho) = \left(\frac{1}{3}M_3^2 - 3m^2\right)\chi(\vec{r},\rho),$$

$$V_3 = \frac{1}{2}V_2(\vec{r}_{12}) + V_2(\vec{r}_{23}) + V_3(\vec{r}_{31}).$$

Recent suggestions:

η_0 and ξ^+ both have the same mass $\sim$ 550 Mev. But $I_{\eta_0} = 0$ and $I_{\xi^+} = 1$. Why should their masses be equal and close to 4π mass? A suggestion is offered by interpreting η_0 and ξ^+ as closely bound nuclei made up of four pions with quantum numbers which forbid 2π and 3π decays in the absence of electromagnetic interactions

(11) if ξ does exist then

$\omega \rightarrow \xi + \pi$ can occur.

- x -

QUANTIZATION OF A SELF-COUPLED BOSON FIELD

(A Discussion of L. I. Schiff's work.)*

R.K. Umerjee.

Some theoretical attempts have been made recently to study pion resonances. In fact we have the method of boot-strap mechanism, formulated in Dispersion theoretic language. (For instance, $\pi\ \pi \rightarrow \pi\ \pi$ producing or exchanging a ρ). Recently, in the 1962 CERN conference, Prof. Schiff has outlined a method for such a study.

The method is intended to explore the extent to which the recently observed multi-pion resonances can be accounted for without explicit introduction of a π-N interaction. In fact the peripheral π-N collisions form one of the best possible sources of information on multi-pion resonances. This is perhaps partly due to the availability of pion beams at different energies. In fact, Chew-Low extrapolation method is used in the case of double-pion production. But the present method is to study the poly-pion resonances in terms of a self-coupled pion field. The variational principle is applied to approximate the wave function in a given representation. The present discussion is with reference to a neutral, spin zero, Boson field and it is hoped that the same methods can be extended to the pion field.

The Hamiltonian chosen consists of a free field Hamiltonian together with a ϕ^4 term to denote the self-coupled nature of the field.

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*Presented at 1962 CERN conference.

$$H = \int \left[1/2\ \pi^2 + 1/2\ (\nabla \cdot \phi)^2 + 1/2\ \mu_0^2\ \phi^2 + 1/4\ \lambda_0 \phi^4 \right] d^3 x \quad (1)$$

with the commutation relations

$$\left[\phi\ (x\ ,\ t),\ \pi(\ x'\ ,\ t) \right] = i\,\delta(\vec{x} - \vec{x}') \quad (2)$$

μ_0 and λ_0 are the unrenormalized rest mass and coupling constant. The integration volume V in (i) consists of a rectangular box with periodic boundary conditions at the edges and the volume is later made arbitrarily large. In addition to the translation invariance associated with V, equations (1) and (2) are also invariant under the substitution $\phi \rightarrow -\phi$; $\pi \rightarrow -\pi$. Thus a new quantum number ''Amplitude Parity'' is assigned.

We have $\phi = V^{-1/2}\ \Sigma\ q_k \exp[i\vec{k}\cdot\vec{x}]$ and $\pi = V^{-1/2}\ \Sigma\ p_k \exp[-i\vec{k}\cdot\vec{x}]$ from which it follows that

$[q_k,\ p_{k'}] = i\delta_{k,k'}$ with other pairs commuting

and also $q_k^* = q_{-k}$; $p_k^* = p_{-k}$ qus and p's are expressed in terms of new Hermitian operators as

$$q_k = 2^{-1/2}\ (\ x_k + i y_k\ );\ p_k = 2^{-1/2}(\ X_k + i Y_k\ )$$

Hence, $[y_k, X_{k'}] = [y_k, Y_{k'}] = i\delta_{k,k'}$ with other pairs commuting. Hence we can write

$$X_k = -i\frac{\partial}{\partial xk};\quad Y_k = -i\frac{\partial}{\partial yk}\ .$$

Hence with the introduction of cylindrical co-ordinates

$x_k = Z_k \cos\theta_k$; $y_k = Z_k \sin\theta_k$., we can write

$$H = \sum{}' \left\{ -\left[\frac{1}{z_k}\frac{\partial}{\partial z_k}\left(z_k \frac{\partial}{\partial z_k}\right) + \frac{1}{z_k^2}\frac{\partial^2}{\partial \theta_k^2} \right] + w_k^2 z_k^2 \right\}$$
$$+ \frac{\lambda_0}{V^2}\int \left[\sum{}' z_k \cos(\vec{k}\cdot\vec{x} + \theta_k) \right]^4 d^3x \qquad (3)$$

the prime on the summation indicating that it extends over half the $\vec{K}$ - space and $w_k^2 = \vec{k}^2 + \mu_0^2$

To obtain the approximate wave function, the variational principle is applied to the equation $H\psi = E\psi$. For the lowest state (the physical vacuum), we choose the trial form

$$\psi_0 = \pi' \; f_k \; (Z_k \;\; \theta_k) \qquad (4)$$

where the f's are normalised.

The expectation value of the Hamiltonian (3) for the trial function (4) is

$$< H > = \Sigma' \int f_k H_k^0 f_k \, dT_k + \frac{3\lambda_0}{4V} \left[\Sigma' \int Z_k^2 f_k^2 \, dt_k \right]^2 \qquad (5)$$

where H_k^0 is one-half the curly bracket in equation (3).

Thus $< H >$ depends on a particular f_k only through, where $\int f_k H_k f_k \, dt_k$, where $H_k = H_k^0 + (\frac{3\lambda_0 A}{2V}) Z_k^2$; $A = \Sigma' \int Z_k^2 f_k^2 \, dt_k$ (6)

It follows that $< H >$ is stationery with respect to variations of the f's when they satisfy the equations $H_k f_k = \epsilon_k f_k$.

From the form of (6), it is clear that f's are two-dimensional harmonic oscillator functions with the lowest eigen-values given by

$$\epsilon_k^2 = \omega_k^2 + \left(\frac{3\lambda_0 A}{V}\right) \quad ; \quad A = \sum{}' \left(\frac{1}{\epsilon_k}\right) \qquad (7)$$

The first excited states of the Hamiltonian (3) may be found by using a trial function of the form (4); but allowing one of the f's to be the first exicted eigen-function of the operator (6). By applying variational principle, it can be shown that the first excited states correspond to relativistic particles of momentum $\vec{K}$ and renormalised res mass. μ.

The second excited states are based on functions of the form (4) in which two of the f's are first excited eigen-functions of (6). The only θ-dependence is through a factor $\exp i(\theta_k - \theta_{k+k})$ so that the state has total momentum $\vec{K}$ and even amplitude parity. A linear combination of such terms is chosen where each term is normalised by itself and the co-efficient is a_k.

Hence we have $\sum{}' |a_k|^2 = 1.$ ——— (8)

It can be shown that the expectation value of H for such a two-particle state is found to exceed the vacuum energy by

$$\sum{}' |a_k|^2 \left(\epsilon_k + \epsilon_{k+k}\right) + \frac{3\lambda}{2V} \left| \sum{}' a_k / \sqrt{\epsilon_k \epsilon_{k+k}} \right|^2 \qquad (9)$$

Setting $\vec{K} = 0$. we can work in the centre of mass system. Variation of the a's to make (9) stationery subject to the normalization restriction (8) is carried out by using Lagrange's multiplier E which can be shown to be the energy of the 2 particle system.

E is determined from

$$\sum{}' \frac{1}{\epsilon_k^2 (E-2\epsilon_k)} = \frac{2V}{3\lambda_0}.$$

and the a's are obtained from

$$(E-2\epsilon_k)\, a_k = \left(\frac{3\lambda_0}{2V\epsilon_k}\right) \sum{}' \frac{a_{k'}}{\epsilon_{k'}}$$

The S-wave scattering phase-shift δ can be expressed in terms of the energy shift $E-2\epsilon_k$ that corresponds to the spherically symmetric part of a_k and is given by

$$\frac{\pi k}{\epsilon} \cot\delta = -\frac{16\pi^2}{3\lambda_0} - P\int_{\mu}^{\infty} \frac{k'\, d\epsilon'}{\epsilon'(\epsilon'-\epsilon)} \qquad \text{———} (10).$$

To remove the logarithmic divergence in (10), we renormalise the coupling constant to absorb the factor

$\int_{\mu}^{\infty} \frac{k\, d\epsilon}{\epsilon^2}$ and we have

$$\frac{\pi k}{\epsilon} \cot\delta = -\frac{16\pi^2}{3\lambda} - \epsilon P\int_{\mu}^{\infty} \frac{k'\, d\epsilon'}{\epsilon'^2(\epsilon'-\epsilon)} \qquad \text{———} (11).$$

(11) may be rewritten in the form

$$e^{i\delta}\sin\delta = \pi \frac{n(\epsilon)}{D(\epsilon)}; \quad n(\epsilon) = -\frac{3\lambda}{16\pi^2} \cdot \frac{k}{\epsilon},$$

$$D(\epsilon) = 1 - \epsilon\int_{\mu}^{\infty} \frac{n(\epsilon')\, d\epsilon'}{\epsilon'(\epsilon'-\epsilon-i\eta)}$$

The scattering cross-section is a monotonically decreasing function of the total energy $2E$ which shows a resonance at zero kinetic energy when

$$\lambda = \lambda_\pi = -\left[32\,\pi^2/3\,(\pi+2)\right] = -\,20.5.$$

There is also a single bound state $(E < \mu)$ for $\lambda < \lambda_\pi$.

Apart from extension to the pion field, it is expected that the present methods can be applied to the scattering and bound states of more than two particles.

K-meson-pion Resonances

K.Raman.

In this talk we shall discuss mesonic states with strangeness, their role in symmetry schemes and their dynamics.

The first piece of evidence for a strange meson other than the K-meson was probably the sharp backward peaking observed in the associated production process $\pi^- + p \to K^0 + \Lambda^0$. Tiermno pointed out that this could be explained as the result of the exchange of a K' meson coupled strongly to the K and π ($K' \rightleftarrows K + \pi$)[1)]

Such a particle had also been suggested on purely theoretical grounds (based on the theory of weak interactions).

(i) It would allow us to construct a highly symmetrical theory of strangeness-changing weak currents, as pointed out by Gell-Mann.[2)] (See also Bernstein and Weinberg)[3)].

(ii) Bernstein and Weinberg[3)] pointed out another related reason for expecting a K' meson, The divergences of the ΔS = 1 currents ($\mathcal{J}_\lambda^V, \mathcal{J}_\lambda^A$) are non-zero; also, by analogy with the situation for ΔS = 0, one may expect these divergences to have certain 'Smoothness' properties. As we can relate the π and K fields to the divergences of J_λ^A and $\mathcal{J}_\lambda^A$, we may also expect the existence of a scalar K' field that is similarly related to $\mathcal{J}_\lambda^V$, i.e. $\mathrm{div}\, \mathcal{J}_\lambda^V \propto K'$.

(iii) In the octet model of unitary symmetry,[4] we expect the existence of a vector meson with $I = \frac{1}{2}$ (and $\therefore S = \pm 1$) to complete the vector octet ($\rho, \omega, K^*, \bar{K}^*$). More generally, higher symmetry schemes require the existence of strange mesons with $S = \pm 1, \pm 2; I = \frac{1}{2}, 3/2$; etc. We shall return to this later.

Here we note that in contrast to the suggestion from weak interactions, the octet model requires a vector K^*. Also, it was noted that the pole contribution from a scalar K' did not give the correct spectra in K decay and it was suggested that a vector K^* may give a better agreement.

Thus when a ($K\pi$) resonance (at 885 Mev) was observed by Alston et al[5] in the reaction $K + N \rightarrow K^* + N$; $K^* \rightarrow K + \pi$), a crucial question was what was the spin of the resonane. The experiment only showed that $J < 2$. (The isospin of the resonanee was shown to be $\frac{1}{2}$ from the decay branching ratios).

Possible methods for determining the spin of the K^*:

1) Observation of the decay modes $K^* \rightarrow K + \gamma$ or $K^* \rightarrow K + 2\pi$ (in addition to $K^* \rightarrow K + \pi$) would mean that the spin $J > 0$ and therefore must be 1 (if $J < 2$).

2) In the original experiment, the K^* seemed to have a small width ($\lesssim$ 16 Mev); this was interpreted as evidence for spin 1.[6] However, later experiments[7] showed that the width was of the order of 40 Mev. This would not give any information about the spin.

3) Schwartz[8] suggested an ingenious method for determining the spin of the K^* by observing its production in $\bar{p}p$ annihilation, in the reactions

$$\bar{p} + p \rightarrow K_1^0 + (K^*)^0 ,$$

assuming that S-state capture dominates, which could be tested by noting that S-state capture allows the reaction $\bar{p} + p \rightarrow K_1^0 + K_2^0$ but not $\bar{p} + p \rightarrow K_1^0 + K_1^0$.

(i) If J (K*) = 0, than in the reaction considered, viz.

$\bar{p} + p \rightarrow K_1^0 + (K^*)^0$, where the K_1^0 is visible,

we must have

$$\frac{K_1^0(V)\ \left(K_1^0(V)\,\pi^0\right)^*}{K_1^0(V)\ \left(K_1^0(I)\,\pi^0\right)^*} \approx 2 , \qquad \text{(a)}$$

obtained by noting that a 'visible' (v) K_1^0 means the decay mode $K_1^0 \rightarrow \pi^+ + \pi^-$ is observed, while an invisible (I) K_1^0 means that the K_1^0 decays according to $K_1^0 \rightarrow 2\pi^0$; and the ratio $K_1^0(V) / K_1^0(I) \approx 2/1$.

(ii) If J (K^*) = 1, the ratio (a) is not predictable, but is expected to be less than 1.

The experiment has been carried out by Armenteros et al at CERN and the results strongly favour J (K^*) = 1

4) Feld and Lichtenberg have shown that the reaction $\bar{p} + p \rightarrow K^* + \bar{K}^*$ can also give information about the K^* spin, provided certain assumptions are justified.[9]

5) Caldwell[10] has noted that if the process $K + He^4 \rightarrow K^* + He^4$ were observed, it would proved that $J(K^*) = 1$, as $J = 0^+$ would violate the conservation of J^P.

However, the reaction above may be much inhibited. Caldwell has shown that the K^* spin may be still obtained from the K^* decay distribution in the reaction $K + He^4 \rightarrow K^* + He^4 + \pi$.

6) Chinowsky, Goldhaber et al[11] have carried out a conclusive experiment that determines the K^* spin to be non-zero from an Adair analysis of the reaction $K^+ + p \rightarrow K^{*0} + N_{33}^{*++}$ Combined with Alston et al's result $J < 2$, this give $J(K^*) = 1$.

Recently, another $(K\pi)$ resonance seems to have been observed at about 730 Mev in an experiment by Kalbfleisch et al[7]. If it exists, it is presumably a narrow resonance (of width $<$ 10 Mev). The small width suggests that it is a vector resonance; however the Q-value for decay into $(K + \pi)$ is less than 100 Mev, and a width of about 15 Mev may be obtained even for a scalar resonance.

Recent experiments by V.Belyakov et al also indicate higher-mass resonances K^{**} and K^{***} at 1150 and 1650 Mev respectively.[12]

The peak at 1650 Mev is obtained in the effective mass distributions for $(K^0\pi^+\pi^-\pi^\pm)$: the possible decay channels observed seem to be

$$\begin{aligned} K^{***} \rightarrow\ & K^* + 2\pi \\ & K^* + \rho \\ & K^* + \pi + \rho . \end{aligned}$$

(A peak may also occur at about 1.4 GeV.)

The peak at 1150 Mev is observed in the ($K^0\pi^\pm\pi^\pm$) effective mass distributions but not in the ($K^0\pi^+\pi^-$) distributions. Thus this peak probably occurs in a state with isospin I = 3/2, or 5/2. Assuming it to have I = 3/2, this may be a member of one of the higher representations of the unitary group. e.g. the 10 and $\overline{10}$ representations or the 27-representation.

K^* resonances and Symmetry schemes:

First we consider what are the strange mesons that may be expected if all bosons are assumed to belong to one or other of the various representations of the unitary group.

It is known that for bosons, the R-conjugate is also the anti-particle and thus R-invariance is ensured by C invariance. (As Prof. Marshak has observed, this means that assuming merely unitary symmetry gives all the unpleasant consequences of (unitary symmetry + R-invariance), e.g. the vanishing of the charge form factors of all neutral bosons.)

The 8 representation requires a strange meson with $S = +1$, $I = \frac{1}{2}$. The 10 and $\overline{10}$ representations require (i) a $S = \pm 1$, $I = \frac{1}{2}$ mesons ($K^*_{1/2}$) (ii) a $S = \pm 1$, $I = 3/2$ meson ($K^*_{3/2}$) and (iii) a $S = \pm 2$, $I = 0$ meson ($(KK)_0$), in addition to a $S = 0$, $I = 1$ meson (π_1^*)

The 27 representation requires

(i) a $S = \pm$, $I = \frac{1}{2}$ meson, (ii) a $S = \pm 1$, $I = 3/2$ meson, and (iii) a $S = \pm 2$, $I = 1$ meson (besides $S = 0$ mesons with $I = 0, 1, 2$.)

It is commonly believed that in the 8 representation, the π, K, $\bar{K}$ and η should constitute the pseudoscalar octet, while the ρ, ω, K^* and $\bar{K}^*$ should form a vector octet. It is well known that the PS octet fits the mass formula (with m^2) very well, whereas putting the ρ and ω into a vector octet predicts a vector K^* at 780 Mev, considerably lower than the observed 885 Mev. (If the $(K\pi)$ resonance at about 730 Mev is confirmed, one may think of identifying this rather than the K^* with the $S = \pm$, $I = \frac{1}{2}$ vector. Even then there is a considerable discrepancy.)

In general, unitary symmetry seems to work much better for baryons (and baryon resonances) than for bosons. A possible reason may be seen by considering the bosons as $(\bar{N}N)$ and $(\bar{\Lambda}N)$ bound states. We may speculate that the very short-range forces involved in binding these boson states do not respect unitary symmetry whereas the longer-range forces that largely determine the dynamics of baryon resonances do respect unitary symmetry.

Then the most tightly bound states should show the worst violation of unitary symmetry; the pion-kaon mass difference may be taken as an example. It seems difficult to reconcile the large pion-kaon mass difference with the remarkable fit to the mass formula of the PS octet (π, η, K, $\bar{K}$). It would seem that either this fit to the mass formula is quite fortuitous, or that the η does not really belong to the same octet as the pion and kaon. This would imply that (a) there is a lower mass $S = 0$, $I = 0$, 0^- meson that completes the $\pi, K, \bar{K}$ octet, and (b) the η must belong to some other representation, perhaps it is a unitary singlet, We conjecture that this is in fact the case.

We assume that it is correct to group the $K^*, \bar{K}^*, \rho$ and ω into an octet. If the 730 Mev $(K\pi)$ resonance exists and turns out to be a scalar resonance, then the ζ (assumed to be 0^{+-}) may be taken to be its $I = 1$ companion in a scalar octet; we would further expect an $I = 0$, <u>scalar</u> pion resonance σ_0 (with a mass not too far from 500 Mev).
The assumption that the members of the octet should all be taken as possible $(N\bar{N})$ and $(\Lambda\bar{N})$ states tells us that the postulated $I = 0$ σ_0 -meson should be 0^{++} (as an $I = 0, 0^{+-}$ meson cannot be an $N\bar{N}$ state, by C invariance). (The reason for assuming the $I = 1$ ζ-meson to be 0^{+-} is the same, an $I = 1$, 0^{++} meson ~~meson~~ cannot be coupled to an $N\bar{N}$ pair, by C invariance).

We assume that the K^{**} at 1150 Mev exists and has $I = 3/2$. This must belong to one of the higher representations. If it belongs to the 10 representation, then it must have companion resonances with $S = 0$, $I = 1$; $S = -1$, $I = \frac{1}{2}$; and $S = -2$, $I = 0$. No such resonances with masses close to 1150 Mev have yet been confirmed. (However, there is a suggestion of a peak at about

1400 Mev; also, a straight line K Regge trajectory would cross J = 2 at about 1470 Mev.) It would be possible to have the following set of resonances fitting the 10 representation:

$Y = 1, \ I = 3/2$	$K^*_{3/2}$	1150 Mev
$Y = 0, \ I = 1$	π_1^*	1280
$Y = -1, \ I = 1/2$	$K^*_{1/2}$	1400
$Y = -2, \ I = 0$	$(\bar{K}\bar{K})$	1505

The actual masses could be considerably different from the values quoted.)

The occurrence of I = 0 non-strange resonances at 1020 and 1250 Mev suggests that a 27-fold shell may be beginning to fill at these energies. As we have noted earlier, the 27 representation requires the following mesons:

		Mass Formula	Possible mass (in Mev)	
$S=0, \ I=2$	π_2^*	$m^2 = m_0^2(1+6b)$	1234	1061
$S=0, \ I=1$	π_1^*	$m^2 = m_0^2(1+2b)$	1096	1193
$S=0, \ I=0$	π_0^*	$m^2 = m_0^2$	1020	1250
$S=1, \ I=3/2$	$K^*_{3/2}$	$m^2 = m_0^2(1+a+\frac{7}{2}b)$	1150	1150
$S=1, \ I=1/2$	$K^*_{1/2}$	$m^2 = m_0^2(1+a+\frac{1}{2}b)$	1016	1236
$S=2, \ I=1$	$(KK)_1$	$m^2 = m_0^2(1+2a+b)$	1012	1222

Identifying the K^* 3/2 with the 1150 Mev and taking the 2 alternatives 1020 Mev and 1250 Mev for the π_0^* suggests the resonances given in the last 2 columns.

An interesting feature of the 10 and 27 representations is the prediction of a $S = \pm 2$ (KK) resonance. We here merely note that such resonances may also be expected on other grounds.

(P.T.O)

The Dynamics of $(K\pi)$ Systems:

Various questions may be raised in relation to the dynamics of $(K\pi)$ systems.

1) There are mainly two ways of looking at the dynamical origin of the $(K\pi)$ resonances.:

A) They may be regarded as built of pions and kaons as the basic building bricks. In a potential picture this means obtaining the $(K\pi)$ resonances by assuming $(K\pi)$ and $(\pi\pi)$ potentials, while in dispersion theory one must also take into account the $K\bar{K}$ interaction. One may alternatively take a field-theoretic coupling, e.g. of the form $\varphi_K^2 \varphi_\pi^2$, and try to obtain the resonances.

B) They may be regarded as bound states of the $(\bar{\Lambda} N)$ system which decay owing to their coupling with the $(K\pi)$ channels. (This is similar to the Fermi-Yang $(\bar{N}N)$ model for pions and some multi-pion states.)

(i) In the first viewpoint, the simplest model would be one in which the parameters of the $(K\pi)$ resonances are obtained by a self-consistent treatment of $(K\pi)$ scattering. Such a calculation with various simplifying approximations has been made, for instance, by A.P.Balachandran[13], considering only the single channel $K + \pi \rightarrow K + \pi$ (and the crossed reactions.) An improvement over this simple bootstrap picture would be one in which more channels are taken into account.

In a potential picture, one may take a K-meson and add pions successively to get various states with $S = \pm 1$. We here give the quantum members of some of these states and note a few simple consequences. (We note that the situation here is more complicated than the multi-pion case in the sense that two-pion and three-pion states are orthogonal while $(K\pi)$ and $(K + 2\pi)$ states are not orthogonal when the total angular momentum is greater than zero. Thus for K-meson-pion systems, considering a fixed number of particles bound to one another by potentials is a much worse approximation than for two-pion and three-pion systems. This is being studied.)

I) First consider states made up of a single kaon and a single pion. Both the K and the pion have spin-parity 0^-; thus different values of the relative orbital angular momentum ℓ would give the following spin-parities J^π. (where $\pi = (-1)^\ell$

$\ell =$	0	1	2	3	4
$J^\pi =$	0^+	1^-	2^+	3^-	4^+

The total isospin may be $\frac{1}{2}$ or 3/2. The I = 3/2 states would have the following projections:

$$|3/2, 3/2\rangle^{++} = |K^+\pi^+\rangle$$
$$|3/2, 1/2\rangle^{+} = \sqrt{2/3}\,|K^+\pi^\circ\rangle + \sqrt{1/3}\,|K^\circ\pi^+\rangle$$
$$|3/2, -1/2\rangle^{\circ} = \sqrt{1/3}\,|K^+\pi^-\rangle + \sqrt{2/3}\,|K^\circ\pi^\circ\rangle$$
$$|3/2, -3/2\rangle^{-} = |K^\circ\pi^-\rangle \quad ;$$

and the $I = \frac{1}{2}$ states would have the projections

$$|\tfrac{1}{2}, \tfrac{1}{2}\rangle^{+} = -\sqrt{1/3}\,|K^{+}\pi^{0}\rangle + \sqrt{2/3}\,|K^{0}\pi^{+}\rangle$$

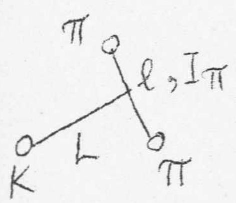
$$|\tfrac{1}{2}, -\tfrac{1}{2}\rangle^{0} = -\sqrt{2/3}\,|K^{+}\pi^{-}\rangle + \sqrt{1/3}\,|K^{0}\pi^{0}\rangle$$

The branching ration into different charge-states are

$$\frac{R(K^{*+}_{1/2} \rightarrow K^{+}\pi^{0})}{R(K^{*+}_{1/2} \rightarrow K^{0}\pi^{+})} = \frac{1}{2}\,, \quad \frac{R(K^{*+}_{3/2} \rightarrow K^{+}\pi^{0})}{R(K^{*+}_{3/2} \rightarrow K^{0}\pi^{+})} = 2, \text{ etc.}$$

II) We next consider states made of a kaon and 2 pions. The possible isospin values are $I = \frac{1}{2}$, 3/2, and 5/2. Considering the pions separately as a partial system, we note that for an $I = 5/2$ particle the pions must be in a pure isospin state with isospin $I_{\pi} = 2$ (and hence even relative angular momentum). For an $I = 3/2$ particle, the partial two-pion system may be in a mixture of $I_{\pi} = 2$ and $I_{\pi} = 1$ states, while for an $I = \frac{1}{2}$ particle, the pions may be in a mixture of $I_{\pi} = 1$ and $I_{\pi} = 0$ states. This would show itself in the energy spectra and final state correlations of the decay products.

To describe the angular momentum of the system, we use the following notation:



ℓ is the relative angular momentum of the two pions. (which have a total isospin I_{π}), while L is the relative orbital angular momentum of the K and the centre-of-mass of the two pions.

Such a system would have a parity ~~II = (1-1) l + L + 1~~ $\Pi = (-1)^{l+L+1}$. The table below gives the total spin-parities J^{Π} for different values of l, L, I_{π} . We denote the spin-parity of the partial (2π) system by $j^{p}(2\pi)$; $\Pi = \{(-1)^{L+1}\}p$.

(a) $I_{\pi} = 0,\ 2$.

$j^p(2\pi) \rightarrow$ / $L \downarrow$	0^+	2^+	
0	0^-	2^-	
1	1^+	$1^+, 2^+, 3^+$	
2	2^-	$0^-, 1^-, 2^-, 3^-, 4^-$	

(b) $I_{\pi} = 1$.

$j^p(2\pi)$ / $L \downarrow$	1^-	3^-	
0	1^+	3^+	
1	$0^-, 1^-, 2^-$	$2^-, 3^-, 4^-$	
2	$1^+, 2^+, 3^+$	$1^+, 2^+, 3^+, 4^+, 5^+$	

We note the following:

(i) In the decay of a 0^- ($I = \frac{1}{2}$) resonance into $K + 2\pi$, the final pions would be preferentially emitted in a state with total isospin $I_\pi = 0$, because of the double angular-momentum barrier that occurs if $I_\pi = 1$. This effect would be more pronounced if the $I = 0$ pion-pion interaction were strong (e.g. enhanced by the presence of a low-energy $I = 0$ two-pion resonance.) We may write the isospin wave function of the final state approximately as

$$|K+2\pi\rangle^+ = \sqrt{1/3}\,|K^+\pi^+\pi^-\rangle + \sqrt{1/3}\,|K^+\pi^-\pi^+\rangle - \sqrt{1/3}\,|K^+\pi^0\pi^0\rangle.$$

We would then expect the following relations for the decay of a $Q = +1$, $I = \frac{1}{2}$ resonance:

$$\frac{R(K^+\pi^0\pi^0)}{R(K^+\pi^+\pi^-)} \simeq 1/2 \;; \quad \frac{R(K^0\pi^+\pi^0)}{R(K^+\pi^0\pi^0)} \simeq 0, \text{ etc.}$$

($R(K^+\pi^0\pi^0)$, etc. are the decay rates.)

Decay into the $K^0\pi^+\pi^0$ mode would be much suppressed. The occurrence of a low-mass $I = 1$ pion-pion resonance would tend to give a significant admixture of $I = 1$ states; this may be important when the initial particle has a large mass (e.g. if it is the 1650 Mev resonance). The ratio $\dfrac{R(K^0\pi^+\pi^0)}{R(K^+\pi^0\pi^0)}$ would be a measure of this admixture.

For an initial 0^-($I = 3/2$) particle, the final pions in a ($K+2\pi$) decay mode would be preferentially emitted in states with $I_{\pi} = 2$, the occurrence of an I = 2 low-mass pion-pion resonance(for which some evidence seems to have been obtained at 625 Mev) would again enhance this effect. The final-state isospin wave function would be approximately

$$|K+2\pi\,;\,0^-;\,I=3/2\rangle^{+} = -\sqrt{\tfrac{1}{15}}\,|K^+\pi^+\pi^-\rangle - \sqrt{\tfrac{1}{15}}\,|K^+\pi^-\pi^+\rangle -$$

$$-\sqrt{\tfrac{2}{15}}\,|K^+\pi^0\pi^0\rangle + \sqrt{\tfrac{3}{10}}\,|K^0\pi^+\pi^0\rangle + \sqrt{\tfrac{3}{10}}\,|K^0\pi^0\pi^+\rangle,$$

leading to decay branching ratios

$$\frac{R(K^+\pi^0\pi^0)}{R(K^+\pi^+\pi^-)} \approx 2\;;\quad \frac{R(K^0\pi^+\pi^0)}{R(K^+\pi^0\pi^0)} \approx \frac{9}{4},\ \text{etc.}$$

(ii) In the decay mode $(K+2\pi)$ of a 1^- particle with I = ½ or 3/2, the two pions will be predominantly in a state of total isospin $I_{\pi} = 1$, this could be enhanced by a resonant final-state interaction via the ρ-meson. For an initial I = ½ particle, the final-state isospin wave function will be predominantly

$$|K+2\pi\,;\,1^-;\,I=1/2\rangle^{+} = -\tfrac{1}{\sqrt{6}}\,|K^+\pi^+\pi^-\rangle + \tfrac{1}{\sqrt{6}}\,|K^+\pi^+\pi^-\rangle +$$

$$+\sqrt{1/3}\,|K^0\pi^+\pi^0\rangle - \sqrt{1/3}\,|K^0\pi^0\pi^+\rangle,$$

giving the branching ratios

$$\frac{R(K^+\pi^0\pi^0)}{R(K^+\pi^+\pi^-)} \approx 0\;;\quad \frac{R(K^0\pi^+\pi^0)}{R(K^+\pi^+\pi^-)} \approx 2.$$

Similarly, for an initial I = 3/2 particle, we have a final-state isospin function

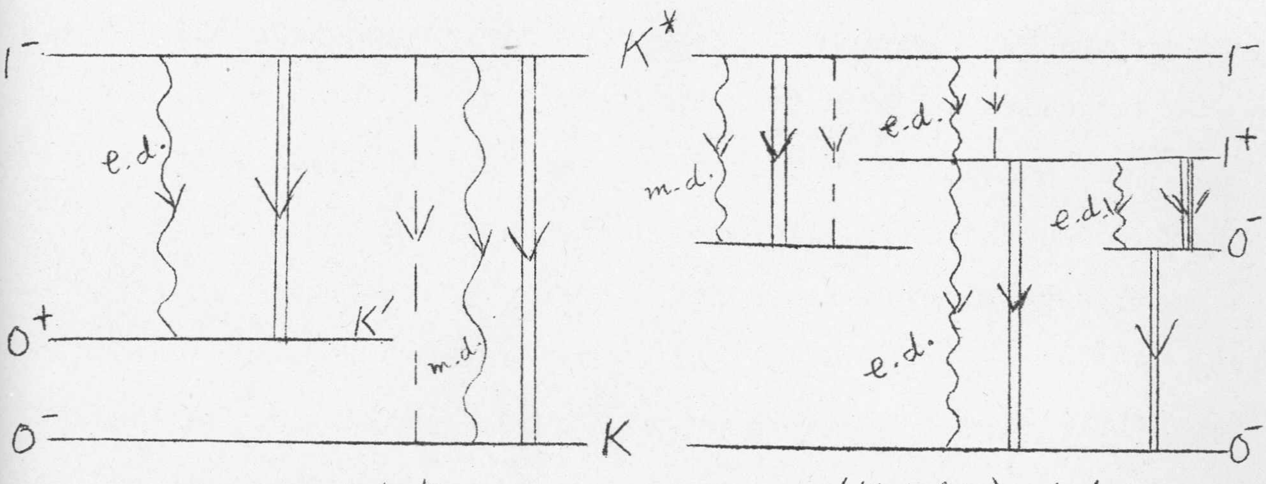
$$|K+2\pi;\,1^-;\,I=3/2\rangle^+ = \sqrt{1/3}\,|K^+\pi^+\pi^-\rangle - \sqrt{1/3}\,|K^+\pi^-\pi^+\rangle + $$
$$+\sqrt{1/6}\,|K^0\pi^+\pi^0\rangle - \sqrt{1/6}\,|K^0\pi^0\pi^+\rangle,$$

and the ratios

$$\frac{R(K^+\pi^+\pi^0)}{R(K^+\pi^+\pi^-)} \approx 0 \quad ; \quad \frac{R(K^0\pi^+\pi^0)}{R(K^+\pi^+\pi^-)} \approx \frac{1}{2}.$$

(iii) For a 1^+ meson decaying into ($K+\pi+\pi$), the final pions would be in a mixture of I = 0 and I = 1 (or I = 1 and I = 2) states; this would again show itself in the branching ratios.

(iv) The only state that is coupled to ($K + \pi$) but not to ($K + 2\pi$) is a 0^+ state; while 0^-, 1^+ , 2^- , 3^+ , states are coupled to ($K + 2\pi$) but not to ($K+\pi$). Transitions between 0^+ and 0^- states can occur by single-pion emission, between 1^- and 0^+ by emission of a γ- ray or of 2 pions, between 0^- and 0^+ by the emission of 2 pions, etc., as shown in the diagram.



(K+π) states (K + 2π) states

Notation: γ π 2π

e.d. = electric dipole. m.d. = magnetic dipole

(v) In the considerations of (i) and (ii) above, only interactions that conserve I and G have been considered. When G-violating effects are considered, there may be significant deviations from the isospin wave functions given above. For instance, we have stated in (i) above that the 2 pions in the decay of a 0^- ($I = \frac{1}{2}$) resonance into ($K + 2\pi$) would be preferentially in a state with $I = 0$ because of the angular momentum barriers that would occur if $I = 1$. However, the 2 pions may be emitted from a 0^{+-} state with $I = 1$ (by G-violating interactions) ; this effect could be enhanced by the existence of the ζ-meson (assumed to be 0^{+-} with $I = 1$). This may give an appreciable decay rate into the mode ($K^0 + \pi^+ + \pi^0$).

Such effects are likely to be important in states with ($K + n\pi$) for $n = 3, 4, \cdots$; for instance the state just considered (K+(2π in 0^{+-}) could be thought of as a (K+5π) state if we assumed conservation of G and I. (Another example would be a (K+4π) bound state with the 4 pions in a 0^{-+} state; this could decay into (K+π) by G-violation).

(B) The approach in which the different K-mesonic states are thought of as ($\bar{\Lambda} N$) states is reminiscent of hyperfragments; we may think of ($\bar{\Lambda} N$) states as obtaind by replacing the $\bar{N}$ in a ($\bar{N}N$) state by a $\bar{\Lambda}$ and compare the different "corresponding" K-mesonic and multi-pion states. The similarity of hyperfragments to ordinary nuclei indicates that the long-range part of the (ΛN) interaction is similar to the long-range part of the (NN) interaction; the same similarity may also be expected between the long-range parts of the $(\bar{\Lambda}N)$ and $(\bar{N}N)$ interactions. However, in very tightly bound states like K, K^*, ... ; π, η, ω, ρ, ... the short-range parts of the interaction are expected to be important. Loosely speaking, we may expect that the most tightly-bound states (i.e. those with lowest mass) should show the greatest deviation from the $(\Lambda$-$N)$ symmetry.

For the more loosely bound states, we may expect that the difference in the mass of corresponding states in $(\bar{\Lambda}N)$ and $(\bar{N}N)$ systems would arise mainly from the difference (Δm) in the reduced masses, which is about 40 Mev; the K-mesonic states are expected to be heavier than the corresponding pionic states. In the octet $(\rho, \omega, K^*, \bar{K}^*)$, the K^* is heavier than the ρ and ω ~~by about 100 to 150 Mev.~~ The mass difference between the ρ meson and the 730 Mev $(K\pi)$ resonance (if both these resonances exist) is again about 150 to 160 Mev; we may also have an I = 0, 0^{++} resonance σ^0 that would complete the octet (as noted earlier). In the lowest octet $(\pi, \eta, K, \bar{K})$, the π-K mass difference seems to abnormally large, and the K is lighter than the η. If these are the correct 'corresponding' particles, one must presumably assume that these anomalies appear because for the most tightly bound states, the masses would be the most sensitive to the differences between the (short-range) $(\bar{\Lambda}N)$ and $(\bar{N}N)$ interactions. (As we have suggested earlier, the η may not belong to the same octet as the pion and kaon). If these differences are considerable, it may mean that the short-range interactions break higher symmetries like global symmetry or unitary symmetry. (As the short-range interactions seem to be very strong, the correlation between the strength of an interaction and its symmetry does not seem to hold here.)

All this speculation, however, is based on a picture in which pions, kaons, etc. are regarded as baryon-antibaryon bound states.

4) K^* resonances and Regge poles:

Assuming that the K and the K^* lie on Regge trajectories leads to certain consequences. The K^* Regge trajectory would appear in reactions like $\bar{K}+N \rightarrow \pi + Y$ and $\pi + N \rightarrow K + Y$, whereas the K trajectory would contribute to (ΛN) exchange scattering.

We note the following:

(i) Wagner and Sharp have suggested that the existence of the 'M-meson' should give rise to a new symmetry at high energies, viz. (when the energy is high enough and the two reactions $\pi + N \rightarrow \Lambda + K$ and $\bar{K} + N \rightarrow \Lambda + \pi$ are dominated by the same pole, the forward amplitude for associated production should be the negative of the forward amplitude for $\bar{K} + N \rightarrow \Lambda + \pi$. (One may say that just as the dominance of the Pomeranchuk trajectory in high-energy elastic scattering leads to the equality of all elastic cross-sections, so also the dominance of the

M-meson trajectory, which we may identify with the K^* trajectory, leads to the symmetry relation above.) Similar relations would be obtained from the dominance of other trajectories.

(ii) On the K trajectory, we may expect a spin 2^-, $I = \frac{1}{2}$ particle at about 1450 Mev if we assume a slope of about $1/50\, m_\pi^2$

(iii) If the K^{**} at 1150 Mev with $I = 3/2$ is a spin 2 particle, then we expect a 'ghost' at $J = 0$ on this trajectory.

(iv) One may attempt to calculate the parameters of the K^* trajectory from $(K\pi)$ scattering. This is being attempted.

(v) It is possible that the Regge trajectories of the various mesons and mesonic resonances will be correctly obtained by considering them as baryon-antibaryon composite states (for mesons that can be so obtained). For instance, it may be that the ρ resonance will be obtained by considering it not merely as a $(\pi\pi)$ resonance but primarily as a $(\bar{N}N)$ state, even though in dispersion theory such high-mass intermediate states would correspond to far-away singularities: Such considerations are being studied.

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PION-NUCLEON RESONANCES

K.Venkatesan

The various pion-nucleon resonances known experimentally at present with relevent data are given in the Table I. In addition to these a step in the $\pi^{+}p$ cross-section at $\sim$ 800 Mev kinetic energy is known and a resonance in the $P_{\frac{1}{2}}, I=\frac{1}{2}$ state has been considered possible on the basis of the angular distribution at 900 Mev.

TABLE I

Resonance	Lab K.E.	Mass	Γ	J	I	P
N^{*}	190	1238	90	3/2	3/2	+
N^{**}	600	1510	160	3/2	1/2	−
N^{***}	900	1680	100	5/2	1/2	+
N^{****}	1300	1900	200	?	3/2	?

We shall study the mechanism of these resonances and the theories proposed in this connection. The earliest to predict a series of isobars in the pion-nucleon system (before the pion was enperimentally discovered) was the strong ampling theory of Heisenberg, Wentzel and others. One of the main features of this theory is that it takes into account the reaction of the meson field produce by the nucleon on the nucleon itself. This is a self-energy effect and since the renormalization procedure was not known then a finite size had to be given to the nucleon which (as also the pion) was treated classically since many virtual pions were expected to be closely associated with the nucleon. The Hamiltonian of interaction used is the same as the one used later by Chew, viz., $H(\text{int}) = -f \int u(\vec{r})\, \vec{\sigma}\cdot\nabla \varphi\, d^3\vec{r}$ (1)

where $U(\vec{r})$ is the shape function of the nucleon. The most important result of the theory was that the nucleon will have excited or isobaric states with $j = \frac{3}{2}, \frac{5}{2}, \ldots$ etc. The excitation energy is

$$E = 2\epsilon\left(j^2 - \frac{1}{4}\right); \quad \epsilon = \frac{6\pi a}{f^2} = \frac{6am^2}{g^2} \qquad (2)$$

$$1/a = \frac{2}{\pi}\int |v(k)|^2\, dk$$

where $v(k)$ is the Fourier transform of $U(r)$. A value $\epsilon \simeq 2.2$ fits the $j = \frac{3}{2}$ resonance but the same value does not predict the higher resonances.

Chew's theory which has in common with the above the idea of a fixed extended source and hence a pseudovector interaction of the pion-nucleon system, also gives a good account of the $\frac{3}{2}, \frac{3}{2}$ resonance. Renormalization effects are incorporated. In the Chew-Low version of the theory, the T-matrix elements for pion-nucleon scattering which obeys the Low's equation is written in terms of the phase shifts by the relation.

$$t_{21}(z) = -\frac{4\pi\, v(q_1)\, v(q_2)}{\sqrt{4\omega_1\omega_2}} \sum_{\alpha=1}^{4} P_\alpha(2,1)\, h_\alpha(z)$$

$$\lim_{\varepsilon \to 0} t_{21}(\omega_2 + i\varepsilon) = T_1(2) \qquad (3)$$

where P_α are projection operators and

$$\lim_{\varepsilon \to 0} h_\alpha(\omega_q + i\varepsilon) = \frac{e^{i\delta_\alpha(\omega_q)} \sin \delta_\alpha(\omega_q)}{q^3 v^2(q)} \quad (4)$$

has the same properties as t_α itself, viz., it has a pole at the single nucleon intermediate state and branch points at higher particle intermediate states. It has the crossing symmetry.

$$h_\alpha(z) = \sum_{\beta=1}^{4} A_{\alpha\beta} h_\beta(-z) \quad (5)$$

Defining a new function $\quad 1/g_\alpha(z) = \frac{z}{\lambda_\alpha} h_\alpha(z) \quad (6)$

($\lambda_\alpha = \frac{4}{3} f^2$ for the 33 states) and making the one meson approximation (i.e. retaining upto only one meson plus one nucleon intermediate state) in the unitarity condition, the analytic properties of $g_\alpha(z)$, the unitarity condition

$$\operatorname{Im} g_\alpha(\omega) = -\frac{\lambda_\alpha}{\omega} q^3 v^2(q), \; 1 < \omega < 2 \quad (7)$$

and the crossing relation,

$$\frac{1}{g_\alpha(z)} = \sum_\beta B_{\alpha\beta} \frac{1}{g_\beta(-z)} \quad (8)$$

enable us to write

$$g_\alpha(z) = 1 - \frac{z}{\pi} \int_1^\infty d\omega_p \frac{p^3 v^2(p)}{\omega_p^2} \left\{ \frac{\lambda_\alpha}{\omega_p - z} + \frac{H_\alpha(\omega_p)}{\omega_p + z} \right\} \quad (9)$$

where H_α is connected to the discontinuity across the left-hand cut. Modifyingthe matrix $B_{\alpha\beta}$ Chew and Low got a solution $g'_\alpha(z)$ which at the origin can be expanded in a Taylor series.

$$g'_\alpha(z) = 1 - r'_\alpha(z) + P'_\alpha(z^2) + O(z^3) \quad (10)$$

where $r'_\alpha = \left(\begin{smallmatrix} -\frac{1}{2} \\ \vdots \end{smallmatrix}\right) r'_3$ which shows that if the effective range for the (33) state is positive, the effective range for the (11) state is negative and zero for the (13) or (31) state. Assuming the integral in (9) to be a slowly varying function of the energy ω_p, the real part of $g_\alpha(z)$ for the 33 state can be written as

$$Re\, g_\alpha(\omega_q) = \frac{\lambda_\alpha}{\omega_q} q^3 v^2(q) \cot\delta_\alpha(\omega_q) = 1 - r_3\omega_q \quad (11)$$

which develops a resonance where $r_3 = \frac{1}{\omega_q}$ With the same parameters viz, the coupling constant f and a cut off $\simeq 6\mu$ Chew's theory also explained the resonance in photoproduction. Application of one-dimensional relativistic dispersion relations can be made to account for them too.

The explanation of the higher resonances is rendered difficult by the inelastic channels which open up and compete with elastic scattering. The work of Peierls, Goebel and Schnitzer, Carruthers and Ball and Frazer shows how the opening of a production channel excites the elastic channel by unitarity, thereby giving rise to a resonance in the scattering process. "Cusps" or "rounded steps" appear in the S -matrix element for a process at the threshold for a new reaction. According to Pais and Nanenberg, there is a possibility of these cusps becoming "wooly". The effects are associated with the opening of the production channel consisting of a nucleon and an unstable vector meson which subsequently decays strongly into two pions.

Peierls assumes formation of the N^* isobar as a possible explanation of the large rise in the production cross-sections which can in turn influence the elastic scattering cross-section. If one of the two pions in the final state of pion production in pion nucleon collisions is in a P- state and the other in an S - state relative to the nucleon we may have a situation corresponding to N^{**} resonance. If, however, both mesons are in a P-state with respect to the nucleon (of course, this statement has meaning only if, the nucleon is static), then they must be necessarily be parallel to each other so that the total angular momentum of the state is $5/2$, corresponding to N^{***} . But the same argument would give an isotopic spin of $5/2$ for the state.

If we consider the pion-pion resonance in the I =1, J = 1 state as being the cause of enhancement in the inelastic (= 4 3 mb at 610 Mev) and hence the elastic cross-sections, this difficulty is avoided, Itabashi et al assume a $\rho - \pi\pi$ interaction of the form

$$H_{\rho\pi\pi} = F \sum_{\alpha,\beta,\gamma} \chi_i^{\gamma} \partial_i \varphi^{\alpha} \varphi^{\beta} \epsilon_{\alpha\beta\gamma} \qquad (12)$$

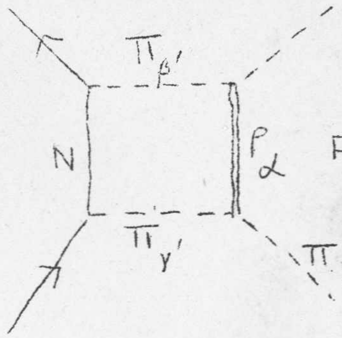
α, β, γ refer to the isospin indices. χ and φ are the ρ and π field operators respectively. Similarly the interaction among the ρ, π and the elctromagnetic fields is written

$$H_{\rho\pi\pi\gamma} = i E \sum_{i,j,k} \epsilon_{ijkl} (\partial_i A_j - \partial_j A_i) \partial_k \varphi^\alpha \chi_l^\alpha \tag{13}$$

The magnitude of the coupling constants ($F^2/4\pi \approx 5$) is shown to be responsible for the second and third resonance. The observed second resonance in photoproduction is supposed to be due to the excitation of the resonant $\rho - N$ state through a ρ- particle production by the incoming γ ray in the pion field of the trarget nucleon, they show that the strong $\rho_{\pi\pi}$ coupling the and interaction $H_{\rho\pi\pi}$ given as strong attractive potential of the $\pi - N$ system in $I = 1/2$ state for energy $\simeq m_\rho$

At these high energies the large observed cross-sections can be expected only through a long-range pion-nucleon interaction. The longest range potential between a pion and a nucleon is the 2-pion exchange potential. The scattering amplitude corresponding to Fig. 1 becomes very large for $W \simeq m_\rho$ since energies of the intermediate ρN states can be very near to the initial energy W of the colliding system. Further the pion-nucleon interaction becomes attractive since the intermediate states have an energy larger than W .

The potential in the lowest order perturbation theory is given by



Fig(1)

$$V=\sum_n \frac{\langle f|H|n\rangle\langle n|H|i\rangle}{W_i-W_n}$$

$$=\sum_n \frac{|\langle n|H|i\rangle|^2}{W_i-W_n} \qquad 14)$$

~~(14)~~

if we take $i=f$ and the sign of $W_i - W_n$ determines the sign of V. The initial energy W_i is ~~les~~ less than the intermediate energy W_n for $\pi^{\pm}p$ scattering and $W_i > W_n$ for $\pi^- p$ scattering in the p-wave state. Therefore the former is attractive and the latter repulsive. Similarly the π-N force through the ρ-N intermediate states is attractive for $W_i < m_\rho$ since $W_n \geq m_\rho > W_i$ always. Even when $W_i > m_\rho$ as in the case of N^{***}, the potential is attractive since most of the intermediate ρ-N states have $W_n > W_i$ when W_i is near to m_ρ.

The isotopic spin dependence of the π-N potential is given by

$$\langle \pi_\beta|V|\pi_\gamma\rangle \sim \sum_{\alpha,\beta',\gamma'} \epsilon_{\alpha\beta\beta'}\tau_{\beta'}\epsilon_{\alpha\gamma\gamma'}\tau_{\gamma'}$$

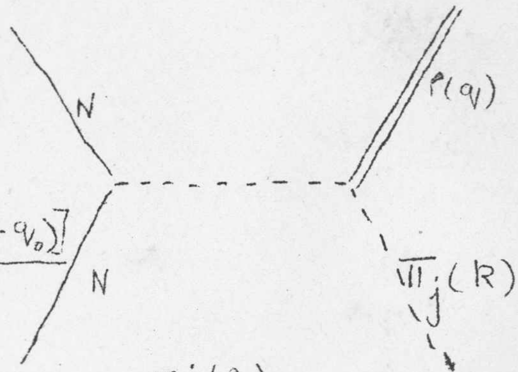
$$=2\delta_{\beta\gamma}+\frac{1}{2}[\tau_\beta,\tau_\gamma]=4P_{1/2}+P_{3/2} \qquad (15)$$

where P_I is the projection operator for isospin state I. The attractive π-N potential is 4 times as large in $I=\frac{1}{2}$ state as in the $I=3/2$ state.

The angular momentum and parity of the resonances are fixed by considering the $\pi N \rightarrow \rho N$ transition (Fig. 2), the matrix element in the lowest order perturbation is

$$\langle \rho^{\alpha} q | T | \gamma, k \rangle = \sum_{\gamma'} \epsilon_{\alpha \gamma \gamma'} T_{\gamma'} F f$$

$$\times \frac{(\vec{\sigma} \cdot \vec{k} - \vec{q})\left[\vec{\rho} \cdot (2\vec{k} - \vec{q}) - \rho_0 (2k_0 - q_0)\right]}{\left[(\vec{k} - \vec{q})^2 - (k_0 - q_0)^2 + 1\right]} \qquad (16)$$



Fig(2)

where ($\vec{\rho}$, ρ_0) is the polarization vector of the ρ particle ($\vec{k}$, k_0) and ($\vec{q}$, q_0) , the energy-momentum 4 vectors of the pion and ρ-meson respectively. For $q = 0$ the matrix element is proportional to

$$2Ff \frac{(\vec{\sigma} \cdot \vec{k})(\vec{\rho} \cdot \vec{k})}{m^2 \rho} \approx -\frac{4\pi}{\sqrt{3}} 2Ff$$

$$\times \left[P(S_{1/2} \rightarrow S_{1/2}) + P(d_{3/2} \rightarrow S_{3/2}) \right] \qquad (17)$$

Similarly retaining the part of the amplitude linear $\vec{q}$ the ratio for $f_{5/2} : p_{3/2} : p_{1/2} = 1 : 3/8 : 3.75$

which shows that in addition to the known $f_{5/2}$ resonance at 900 Mev, there is the possibility of astrong resonance in the

$p_{1/2}$ state.

The ratio of the amplitudes in the $I = 0$ and $I = 2$ can be argued out even with an effective s-wave $\pi - \pi$ interaction, $\frac{1}{4}\lambda_s\,(\varphi_\alpha\ \varphi_\alpha)^2$; The ratio is 5:2 which leads to a ratio of the pion production cross section in the $I = 1/2, I = 3/2$ states of 5:2, If one of the final pions and the nucleon resonances in a relative $I = 3/2$ state, the ratio becomes 10:1 thus favouring the $I = 1/2$ state heavily.

Carruthers and Goebel and Schnibzer use the Chew-Low method to study the pion production process in pion-nucleon collision. They consider the $\pi - \pi$ interaction to be the dominant one, the $\pi - N$, p-wave interaction entering only as a final state interaction. The chief graphs they consider are therefore Figures (3) and (4)

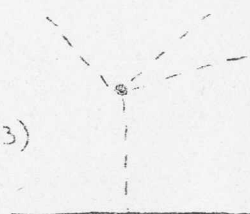


Fig (3)

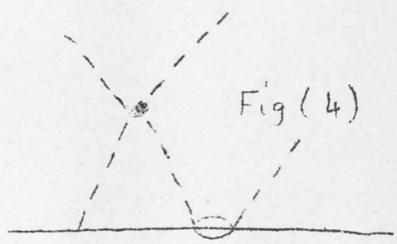


Fig (4)

This leads to an integral equation the kernel of which is $\pi - N$ scattering matrix element which is replaced by the resonant $(3/2, 3/2)$ interaction, the inhomogeneous term correspondingly to Fig. (3). They find a strong enhancement in the partial transition $(D_{3/2} \longrightarrow s\, p_{3/2})$ in the $I = 1/2$ state which is connected to the $I = 1/2$, $D_{3/2}$ pion-nucleon resonance.

Carruthers suggests that the step observed at 850 Mev in the $\pi^{+}-p$ system may possibly be a new resonance in the 850-950 Mev Region, the parity and total angular momentum consignment being $D_{3/2}$. The isotopic spin is of course, $3/2$. Since the direct and charge exchange cross-sections in the $\pi^{-}-p$ system are given in terms of the amplitudes for the $I = 3/2$ and $I = 1/2$ states by

$$\sigma_{el} \cdot \propto |f_{3/2}|^2 + 4|f_{1/2}|^2 + 4\,Re(f_{1/2}^{*} f_{3/2})$$

$$\sigma_{c.e} \propto 2|f_{3/2}|^2 + 2|f_{1/2}|^2 - 4\,Re(f_{1/2}^{*} f_{3/2})$$

and since $f_{1/2}$ is known to vary in a resonant manner, the smallness and decrease in the 500-800 Mev energy range requires that the interference term $Re(f_{1/2}^{*} f_{3/2})$ be large large and positive. If these $I = 3/2$ states were small and non-resonant; we should expect $\sigma_{c.e}$ to rise abruptly on the high-energy side of the resonance.

Since $D_{3/2}, 1/2$ is the most important state in this energy internal, the simplest way to prevent both the occurrence of a peak in $\sigma_{c.e}$ and an abrupt rise above the second resonance is to have the $D_{3/2}, 3/2$ phase shift positive and growing rapidly above 600 Mev.

Finally we shall mention that a study of the photoproduction of pion pairs using the Chew-Low method has led us to conclude that the increase in the total cross-section in the 500-600 Mev, region can be explained on the basis of the ($^3/_2$, $^3/_2$)isobar alone. The π-π interaction has a negligible role to play upto considerable values of the initial photon energy.

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ELASTIC SCATTERING OF CHARGED PIONS BY DEUTERONS AT 300 Mev.

V. Devanathan

The only experiment that is available at this high energy is that of Dul'kova et al[1] and all other experiments[2],[3],[4] reported in the literature are at lower energies, viz., 60, 85 and 140 Mev. The theoretical investigation[5],[6],[7] based on the impulse approximation fits accurately the experimental data at 60 and 85 Mev but the agreement between theory and experiment seems to be poor at 140 Mev and the large theoretical cross-sections at backward angles cannot be explained away by the on-the-energy shell multiple scattering effects and the inclusion of s-wave phase shifts and D-state wave function. As a result of this discrepancy at 140 Mev, Green[5] has made some sceptical remarks about the validity of the impulse approximation in the energy range 140 Mev and above. Recently, Pendleton[8] has made on exhaustive study of the elastic scattering of charged pions at 142 Mev and the effect of the various corrections using the form factor approximation and his final result doesn't seem to differ much from the numerical values we have obtained earlier[7] by a simple approach using the Chew-Low amplitude for the scattering of pions by free nucleons.

Here we present the cross section for the elastic scattering of charged pions at 300 Mev by deuterons. The details of the calculation have been outlined in reference 7). In table 1, the results of the present calculation along with two other sets of values obtained by others[1),9)] are given for the purpose of comparison with the experimental results. We obtain good agreement not only with regard to the angular distribution but also with the integrated cross section (Table II). In the present calculation, we do not distinguish between the scattering of π^+ and that of π^- by deuterons and the results will be the same for both.

It is to be emphasized that the experiments on the elastic scattering are the most important ones for testing the validity of the impulse approximation since a reliable calculation can be made in this case, the final state of the system being well defined. Our investigation seems to indicate that the impulse approximation is a valid approximation in the low (85 Mev) as well as high energies (300 Mev) but fails in the neighbourhood of the pion-nucleon resonance. This conclusion is based on the discordant results that we have obtained at 140 Mev. The continuance of experimental investigation at energies in the close neighbourhood of resonance (200 Mev) is strongly suggested for it is hoped that these experiments will clearly decide the issue and, if our conjecture is confirmed, will stimulate the theoretical investigation of the influence of the pion-nucleon resonance on the impulse approximation. One plausible explanation is that in the resonance region, the pion-nucleon forms a quasibound state and consequently the interaction time is much longer, thus invalidating the impulse assumption that the other nucleon plays just the role of a spectator.

The author is deeply indebted to Professor Alladi Ramakrishnan for constant encouragement and guidance and to Mr. K.Venkatesan for useful discussions.

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TABLE I.

The differential cross section $\frac{d\sigma}{d\Omega}$ in laboratory system for the elastic scattering of charged pions from deuterons at 300 Mev in nnits of mb/sterad.

Laboratory angle	0°	30°	60°	90°	120°	150°	180°
B.M.	10.44	2.72	0.25	0.03	0.02	0.03	0.03
D.S.S. heoretical)	--	18.00	4.00	0	0	0	0
D.S.S. xperimental)	--	7.5 ± 3	1 ± 0.5	1 ± 0.5	0.5 ± 0.25	0.25 ± 0.1	--
P.C.	22.79	7.954	0.3543	0.2007	0.2426	0.1785	--

B.M. = The theoretical values of Bransden and Moorhouse given in ref. 9 at energy 298 Mev.

D.S.S. heoretical) = The theoretical values at 300 Mev obtained on the impulse approximation as reported in ref. 1. The values are taken from the curve given for the purpose of comparison with their experimental results.

D.S.S. xperimental) = The experimental values at 300 Mev taken from the experimental curve given in ref. 1.

P.C. = The present calculation at 300 Mev based on the impulse approximation using the Chew-Low amplitude for the scattering of pions from free nucleons.

TABLE II.

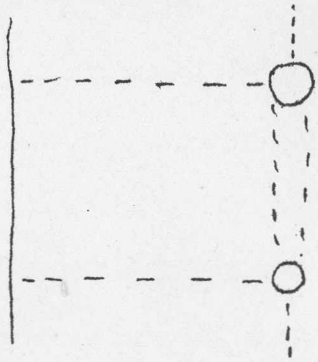
The intergrated cross section for the elastic scattering of charged pions by deuterons at 300 Mev. in the angular range 15° - 170° in the laboratory system in units of mb.

Experimental $\pi^{+} + D \rightarrow \pi^{+} + D$	Experimental $\pi^{-} + D \rightarrow \pi^{-} + D$	Theoretical
21 ± 6	14 ± 4	14

PION-NUCLEON RESONANCES AND THEIR ROLE IN SCATTERINGS AND PRODUCTION PROCESSES

K. Venkatesan.

Ball and Frazer calculated the inelastic contribution to the higher partial waves by means of the strip approximation in which the principal mechanism is the production of the $I = 1$, $J = 1$, $\pi - \pi$ resonance. Fig.1 should give according to the strip approximation a correct estimate of the inelastic contribution for low momentum transfer (high orbital angular momentum) scattering. Assigning to



the $\pi - \pi$ resonance a position and width in accordance with experiment they find strong inelastic scattering in angular momentum and isospin states in which, phenomenological analyses have suggested, higher resonances in elastic scattering occur. The scattering amplitude in the physical region is $f(\nu) = \frac{e^{i\delta(\nu)}-1}{2i}$ where above the inelastic threshold, $\delta = \delta_R + i\delta_I$ where $\delta_I > 0$ according to unitarity.

The inelastic cross-section can be calculated. This is done, by assuming the "strip" approximation and projecting out partial waves from the matrix element in the strip approximation. The inelastic cross-section, which is given by

$$\sigma_I\left(3/2\right) = \frac{4\pi(J+1/2)}{k^2}\;\frac{(1-\eta^2)}{4}$$

rises rapidly in the region of the threshold for production of a $\pi - \pi$ resonance but rises to a hight exceeding the unitarity limit by an order of magnitude. This is because the contribution of processes like $\pi + N \rightarrow \pi + \pi + N$ and $\pi + \pi + N \rightarrow \pi + \pi + N$ to the unitarity condition have not been taken into account. Chew et al conjecture that the required unitarity damping in the inelastic part which comes about through the 4π, 6π .... contributions may be due to the fact that such contributions appear as repulsive forces. In any case the study of the multi-particle channels which is a preliminary to their inclusion in the unitarity condition can be done within the fold of the Mandelstam representation only if some pairs of resonating particles are considered as single particles. In the $\pi - N$ problem there can be the $\pi - \pi$ resonant state simulated by the ρ - meson and the 3π state by the ω - meson and the π-N p-wave resonance by N^*. Conversely a study of interactions in which an unstable particle is present, such as $\pi + N \rightarrow \rho + N$ if ρ is considered an unstable particle can be made only by considering processes such as $\pi + N \rightarrow \pi + \pi + N$ using the existence of the $\pi + \pi$ or $\pi - N$ resonance to reduce the complexity of the three-body state.

In the isobar approximation of Mandelstam et al, if p_1, p_2, $-p_4$, $-p_5$ and $-p_6$ are the momenta of the particles in

$$1 + 2 \qquad 4 + 5 + 6$$

we can choose $s_{12}, s_{14}, s_{16}, s_{45}, s_{24}$ as the independent invariants, s_{14} and s_{24} determine the direction of p_4 (or p_5). If particles 4 and 5 are resonating in an s-state, the corresponding amplitude is independent of s_{14} and s_{24}.

Writing

$$\langle 5'4'|T|45\rangle = \frac{\beta}{M^2 - i\Delta - s_I}, \quad s_I = s_{45}$$

$$\beta = \frac{8\pi M\Delta}{q}$$

where M is the mass in the isobar, Δ is connected with the width of and q the momentum corresponding to the resonance. We can write the 3-body amplitude as

$$\langle 654|T|12\rangle = F(s,t,s_I)\frac{\beta}{M^2 - i\Delta - s_I} + f_0(s_{12} \cdots s_{45});$$

$$s = s_{12}; \quad t = s_{16}$$

We can approximate $F(s,t,s_I) = F(s,t,M^2)$

and neglect f_0. In the model of Steynheimer and Lindenbaum F is independent of t also. For multiple resonances

$$\langle 654|T|12\rangle = \sum_{k=1} F_k(s_k, t_k, M_k^2)\frac{\beta_k}{M_k^2 - i\Delta_k - s_k} + f_0$$

Now $1 + 2 \rightarrow 1' + 2'$ will involve.

$$\int \langle 1'2'|T^*|456\rangle\langle 654|T|12\rangle \, d\tau$$

where $d\tau$ is over the phase space of 4, 5, 6. and this can be evaluated by substituting the above isobar approximation for the three-particle amplitude and assuming that the region of phase space where two isobars can be formed simultaneously is amll

i.e $\int F_k^{1*} F_l \, d\tau_{kl}$ is neglected compared to

$$\int F_k^{1*} F_k \, d\tau$$

Thus the above procedure yields the contribution of the three-particle channels to the two particle amplitude. Similar results to those of Ball and Frazer have also been arrived at by Cock and Lee (who however conclude that the contribution from the inelastic channels can cause a resonance in the elastic channel even below the threshold of the inelastic channels, , if there is a strong coupling between the elastic and inelastic channels

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PION-HYPERON RESONANCES.

We will make a study of the family of Hyperon Isobars characterised by $B = 1$; $S = -1$. The present situation is given in the following table.

State	Mass (Mev.)	Width (Mev.)	I-spin	Spin-parity
Y_1^*	1385	50	1	(3/2 +) ?
Y_0^*	1405	20	0	?
Y_0^{**}	1520	16	0	(3/2 -)
Y_0^{***}	1815	> 120	0	> 5/2 ?
Y_2^*	1550	?	(2) ?	?

Even from the table, it is clear that the intrinsic parameters of the Resonant states such as spin, parity and Isotopic spin are not yet determined with any finality. These resonances appear most directly as resonances in the π-Y system or in the $\overline{K}$-N system or in both.

a) The first such resonance to become established was the π-Λ or Y_1^* resonance found by Alston et al in their study of the reaction

$$K^- + p \rightarrow \pi^+ + \pi^- + \Lambda \qquad (1)$$

for K^- laboratory momentum 1150 Mev/c. The reaction is a two-step process, involving a $\pi - \Lambda$ resonant state as an intermediate step, thus

$$K^- + p \to \left\{ \begin{matrix} Y_1^{*+} + \pi^- \\ Y_1^{*-} + \pi^+ \end{matrix} \right\} \to \Lambda + \pi^+ + \pi \qquad (2)$$

The isotopic spin of this excited hyperon must be one, since it breaks up into a Λ and a π.

The Yale group have observed Y_1^* in

$$K_2^0 + p \to \Lambda + \pi^+ + \pi^0$$

Dahl et al have presented evidence that Y_1^* plays a significiant role in

$$K^- + d \to p + \pi^- + \Lambda \text{ through the sequence}$$

$$K^- + d \to p + Y_1^* \to p + \pi^- + \Lambda$$

Block et al have shown that Y_1^* production plays a similar role in the capture reaction $K^- + He^4 \to \Lambda + \pi^- + He^3$.

b) The $I = 0$, Y_0^* resonance at 1405 Mev was reported by Alston et al from K^--p reactions at $P_{K^-} = 1150$ Mev/c. Alston et al studied the reactions

$$\begin{aligned} K^- + p &\to \Sigma^{\pm} + \pi^{\mp} + \pi^+ + \pi^- \\ &\to \Sigma^0 + \pi^0 + \pi^+ + \pi^- \\ &\to \Lambda + \pi^0 + \pi^+ + \pi^- \end{aligned} \qquad (3)$$

May, 1962:- The Alvarez group at Berkeley has now observed the Y_0^* in $K^- + p \rightarrow Y_0^* + \pi^0$ for $p_{k^-} = 1.22$ Bev/c. The occurence of this resonance in (3) at 1.22 and 1.53 Bev/c has also been confirmed by this group and the observed width is 40 Mev.

C) The next $I = 0$ resonance Y_0^{**} at 1520 Mev has become established through K^-p collisions in the momentum range 300 - 500 Mev/c. (As reported by Ferro-Luzzi et al, Berkeley).

d) A third $I = 0$, Y_0^{***} resonance has been predicted from a study of the total cross-section for $K^- - p$ and $K^- - n$ collisions as functions of the K-N total Barycentric energy. The bump at 1810 Mev has been interpreted as an $I = 0$ resonance, Y_0^{***}.

e) Lundby et al have recently reported that a further Y^* resonance may exist in the mass region of Y_0^{***}. In the reaction

$\pi^- + p \rightarrow K^+ + (X)^-$, by plotting the K^+ production intensity as a function of the incident π^- momentum, they observed a third lump whose position corresponds to a mass value of 1550 Mev. (The first two peaks correspond to Σ^- production and Y_1^{*-} production.) If this does correspond to a Y^* resonance, its I-spin can only be 1 or 2. The absence of any evidence for an $I = 1$ resonance in K^--p interaction for this mass value suggests that this bump might represent a Y_2^*

resonant state. But in CERN, 1962, Alston et al have observed that there is no ~~xnff~~ significant peak in the 1550 Mev region for the interaction

$$K^- + p \rightarrow \Sigma^{\pm} + \pi^{\pm} + \pi^{\mp} + \pi^{\mp}$$

Hence further evidence seems to be needed to establish this Y_2^* resonance.

f) A further Y^* at 1685 has been predicted by observing the following

i) A Bump at 1685 Mev in the $\Lambda\pi$ effective mass distribution in $K^- + p \rightarrow Y + \pi$ at 1.5 Gev/c.

ii) A Bump at 1660 Mev in the $\Sigma\pi$ effective mass distribution in $\pi^- + p \rightarrow Y + K + \pi$ at 2.2 Bev/c.

But further evidence is <u>needed to confirm</u> this resonance.

SPIN - PARITY

We will first analyse the difficulties encountered in the determination of the intrinsic parameters of the resonant states. The most difficult situation is that where the resonance is relatively broad and is accessible experimentally only as a final state interaction, as is the case with the Y^* resonances. The non-availability of a directly accessible entrance channels creates difficulties in the evaluation of the data about Y^* resonances, because the parameters can be studied only from its effects in the final state interaction. In fact, even in the case of availability of an entrance channel as in the case of the N^* resonance, these are found to be many ambiguities in its interpretation until the data available become complete.

There are certain interference effects which make it difficult to deduce the Y* spin from the available data of the reaction

$$K^- + P \rightarrow Y_1^* + \pi \rightarrow (\Lambda + \pi) + \pi$$

Interference Effects.*

a) Dynamical interference:- The properties of the $\Lambda\pi$ resonance may be modified by interference with the primary pion in the above reaction. This would be particularly important close to threshold for Y_1^* production where the primary pion moves slowly and is close to the Y_1^* system when it undergoes decay. At higher energies, the effect is less.

b) Interference between parallel channels:-

We have

$$K^- + p \rightarrow \left\{ \begin{array}{c} Y_1^{*+} + \pi^- \\ Y_1^{*-} + \pi^+ \\ \Lambda + \rho^0 \end{array} \right\} \rightarrow \Lambda + \pi^+ + \pi^- \qquad (4)$$

Even in the absence of dynamical interference, the amplitudes for the first two sequences must be taken coherently, as their interference gives rise to distortions of the angular distributions and resonance shape for the Y_1^* decay.

C) Interference with background production:-

This refers to interference between the amplitudes describing the Y_1^* sequences (4) and non-resonant amplitudes which lead to the same final state $\Lambda\,\pi^+\,\pi^-$. It should be noted

*We have closely followed Prof. Dalitz's lectures at Brookhaven National Laboratory.

that the intensity of thus non-resonant production need not be very large for this interference to produce quite strong distortions from the distributions appropriate to the resonant production alone.

For the isolated model (which neglects the interference a) Between Y^*_1 of opposite charge, b) Between Y_1^* and $(\Lambda \pi^+ \pi^-)$ produced in non-resonant background states and c) final state interaction between π^+ and π^- and the influence of Bose statistics on this system.), the Adair distributions gives a 5 : 1 odds against J=3/2. But this is irrelevent because of the over-simplified nature of the model.

Y^* decay angular distribution with respect to the production normal:- If its spin were 1/2, and in the absence of interfering background, the Y^* must necessarily decay isotropically w.r.t. production normal $n = \frac{(\hat{K} \times \hat{Y}^*)}{|\hat{K} \times \hat{Y}^*|}$. Ely et al have shown that the Y^* decay distribution is not isotopic, but fits the form $1 + a \cos^2\theta$ with $a = 1.5 \pm 0.4$. This shows evidence for $J > 3/2$.

Further even from the point of view of isotopic distributions it is difficult to distinguish between J = 1/2 and J = 3/2 since Dalitz and Miller have shown that the requirements of a Bose statistics on the final pions can cause a Y^* with J= 3/2 to have a relatively isotropic angular distribution in the Adair analysis.

The most probable interpretations of the observed angular correlations in the Y_1^* decay requires that the spin of Y_1^* must be greater than 1/2. If the spin of Y_1^* is then assumed to be 3/2, the relative parity of the Y_1^* and Λ^0 must be even to account for the Λ^0 polarization. However if the Y_1^* has spin 1/2, the parity must be odd. From Helium Bubble Chamber experiments, Block et al have strong evidence for the assignment $J = 3/2$ for Y_1^*. If $J = 3/2$, it is surmised Y_1^* should be a $P\,3/2$ state, from the copious yield of Y_1^*, as the Angular momentum barrier should be lower for $L = 1$. But still, the Spark Chamber data seem to favour spin 1/2 for this resonance.

Beall et al have obtained preliminary results on K^--p elastic scattering cross-section for $p_{K^-} = 700$ to 1400 Mev/c from spark Chamber Analysis. For Y_0^{***}, $\sigma\,K^-$-p (θ) elastic in the neighbourhood of this resonance requires terms of $\cos\theta$ to the 5th power. This is consistent with the assignment $F\,5/2$ to this resonant state. But data and analysis are too preliminary.

Interpretation:-

Attempts have been made to enquire into the predictions of global symmetry regarding Hyperon Resonances. This affords a direct interpretation of the Y^* resonances as the analogues of N^*. Following the prediction of Gell-Mann that there are 2 resonances in the p 3/2 state with I-values 1 and 2 corresponding to N_3^* resonances, Amati et al have estimated the location of these resonances taking into account the $\Sigma - \Lambda$ mass difference.

Kerth and Pais have discussed a phenomenological extension of this analogy to the higher N^* resonances, on the basis of the Global symmetry scheme. It is expected that an $I = 0$ and $I = 1$ Y^* resonance correspond to each $I = 1/2$ $\pi - N$ resonance and an $I = 1, I=2$ Y^* resonance correspond to each $I= 3/2$ π-N resonance.

Another familiar interpretation of Y_1^* is that it represents a virtual $\overline{K}N$ bound-state. (vide Dalitz.) But the recent analysis of Ross and Humphrey as detailed by Hwa and Feldman gives a madification of thus interpretation.

The analyses by Ross and Humphrey of the low energy K^--p data render impossible the virtual $\overline{K}N$ bound-state interpretations of Y_1^*, though it is not ruled out for Y_0^*. By studying the resonances as manifestations of the dynamic structure of the closed pion-hyperon system, it is shown that a consistant interpretation of the observed resonances can be given.

<u>Y_0</u>

a) Taking Y_0^*, since its mass is below the $\overline{K}N$ threshold, we need deal only with the $I = 0$ $\pi\Sigma$ channel. Upon examining the Born terms of the scattering amplitudes, it is readily seen that a resonance is possible in $P_{3/2}$ state. For thus state, the calculated half-width is 8.2 Mev, which can be compared with the experimental value of about 10 Mev.

Evidently it is possible to accommodate Y_0^* as a dynamical $\pi\Sigma$ resonance. With this interpretations for Y_0^* it is unlikely that Y_0^{**} is another resonance in the $P_{3/2}$ state having the same origin. By analogy with the π-N case, we may expect it to be a higher resonance in the $D_{3/2}$ state of the $\pi\Sigma$ system.

b) A possible alternative to the above interpretation of the two I=0 resonances is that we consider Y_0^* as a $\bar{K}N$ state (permitted according to solution II of Ross and Humphrey) and Y_0^{**} as a $\pi\Sigma$ dynamical resonance. In this scheme Y_0^* is in the $S_{1/2}$ state of the $\pi\Sigma$ system and is unrelated to $P_{3/2}$ resonance which can now accommodate Y_0^{**}.

The proper choice of these interpretations can be made only after more definite experimental information is available. considering Y_2^*, we have a simple one - channel problem in the $I = 2$ state of the $\pi\Sigma$ system (ignoring 3 particle channels). Further calculation then indicates that a resonance can exist in the $P_{3/2}$ state with a theoretically predicted half-width of 70 Mev. Verification of this resonance width for Y_2^* should be of great interest.

ERRATA

Page 16. In the table read masses as

$\Lambda\ (Y^{***})$	1115 (1820)
$N\ (N^{***})$	940 (1688)

Page 17. Read 4th line as

'have spin 3/2 (and positive parity), further..

Page 21. Read in eqn. (42) :

$$(10;\ Y=-1,\ T=1/2)^- = \frac{1}{2}(\Lambda K^- - \Xi^-\chi) + \frac{1}{\sqrt{12}}\Big[\Xi^-\pi^0 - \Sigma^0 K^- + \sqrt{2}(\Xi^0\pi^- - \Sigma^-\bar{K}^0)\Big]$$

and in equn. (43)

$$(10;\ Y=1,\ T=1/2)^+ = \frac{1}{2}(\Lambda K^+ - p\chi) + \frac{1}{\sqrt{12}}\left(p\pi^0 + \sqrt{2}\,n\pi^+ - \Sigma^0 K^+ - \sqrt{2}\,\Sigma^+ K^0\right)$$

Page 25. 2nd line after eqn. (50) :

replace 'above' by 'alone'

Page 26. 11th line:

Replace ''For this we have'' by "Further we have with unitary symmetry ."

Page 27. Read eqn. (57) as

$$U = e^{i(u_1 U_1 + u_2 U_2 + u_3 U_3)}$$

Page 39. 1st line after eqn. (19):

Read 'form' as 'four resonances'

Page 48. Read 5th line as

'the N_1^*, Y_1^*, and Ξ^* '

and in 6th line replace y by Y and read $I = 1 + \frac{1}{2}Y$

In the table the decays read as

$N + \pi$

$\Sigma + \pi \sim 16 \cdot 1$

$\Lambda + \pi$

$\Xi\ \pi$

Stable ,

and the 7th line as

"'where the masses of the Ξ^* and Ω have....'"

Page 59a Reference (1) : Prog. Theor. Phys. 27, 949 (1962).

Page 61. 1st line after eqn. (3): Read as

'We can define the scattering amplitude $f(E, \theta)$...'

In eqn. (5) replace $f(\ell, E)$ by $f_\ell(E)$.

Page 63. 1st line: Replace $S(\lambda, k)$ by $S(\ell, k)$ and in 6th line replace $P_{\lambda - 1/2}$ by P_ℓ .

Page 74. Read 2nd and 3rd lines as "signs of the different terms are as follows: the diffraction channel represents attraction; ω and ρ (vector) represent repulsion for identical particles and attraction between particle and antiparticle. The remaining signs are from isotopics."

Page 138. 11th line: Read 'Pevsner et al..'

Page 141. 4th line: Read 'Xuong and .....'

Page 163. 4th line: Read 'S = 0, I = 0, 0^- meson...'

Page 98. 8th line from bottom: Read 'Nuovo Cim. 25, ...'

Page 166. 9th line from bottom: Read '...spin ~~particles~~ parities J^Π (where $\Pi = (-1)^\ell$)

Page 166. 10th line from bottom: Read 'This is.......'

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