ANGULAR ANALAYSIS OF B DECAYING INTO TENSOR,VECTOR AND SCALAR MODES

By

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BONAFIDE CERTIFICATE

Certified that this dissertation titled **ANGULAR ANALAYSIS OF B DECAYING INTO TENSOR,VECTOR AND SCALAR MODES** is the bonafide work of **Mr. Chandradew Sharma** who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other dissertation of the basis of which a degree or award was conferred on an earlier occasion for this or any other candidate.

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Abstract

We study the decays $B \to J/\psi T$, $B \to J/\psi V$ and $B \to J/\psi S$, where T, V and S are tensor ($J^P = 2^+$), vector ($J^P = 1^-$) and scalar ($J^P = 0^+$) mesons respectively. Our study is inspired by the recent observation of the decays $B \to J/\psi K_X^\star(1430)$, where $X = 0, 2$. There exist two mesons, a tensor meson $K_2^\star(1430)$ and a scalar meson $K_0^\star(1430)$ at the same mass of 1430 MeV , making it difficult to measure the branching fraction for $B \to K_2^\star(1430)$, without including contributions from $B \to K_0^\star(1430)$. There also exists a vector meson resonances at 1410 MeV, $K^\star(1410)$, that is close enough to overlap with the wide $K_X^\star(1430)$ resonances. Since, all of these decay modes contribute to the same final state $B \to K\pi\ell^+\ell^-$, contributions from the various decay channels cannot be separated by cuts on the kinematics. We show in detail how angular analysis can be used to separate contributions from each of the decay modes.

We begin by writing the most general effective matrix elements for each of the decay channels using Lorenz invariance and current conservation. The decay spectrum for the final state $K\pi\ell^+\ell^-$ is calculated. The $K_X^\star(1430)$ and $K^\star(1410)$ are considered to decay to $K\pi$ and J/ψ to $\ell^+\ell^-$. We study the angular distribution of the K in the $K\pi$ center of mass (c.m.) frame and ℓ^- in the $\ell^+\ell^-$ c.m. frame. We also study the correlation between the $K\pi$ decay plane and the $\ell^+\ell^-$ decay frame. The formulation developed here can be applied to include any scalar, vector or tensor meson, in addition to $K_X^\star(1430)$ and $K^\star(1410)$. The study performed in this thesis finds immediate application in the analysis of data collected by the B factories running at KEK (Japan) and SLAC (U.S.A.).

பணிச்சுருக்கம்

இவ்வாய்வில் $B \to J/\psi T$, $B \to J/\psi V$ மற்றும் $B \to J/\psi S$ ஆகியவற்றை பற்றி பார்க்கலாம். சமீபத்தில் கண்ணுற்ப்பெற்ற $B \to J/\psi K_X^*(1430)$, $X = 0, 2$ நிகழ்வுகளின் அடிப்படையில் இவ்வாய்வு மேற்கொள்ளப்பட்டது. மேலே டென்சர் மிசான், $K_2^*(1430)$ மற்றும் ஸ்கேலார் மிசான் $K_0^*(1430)$ ஆகிய இரு மிசான்களும் ஒரே நிரையுடன்(1430 MeV) உண்டாகின்றன. இவை $B \to K_0^*(1430)$ என்பத்னைப்பற்றி அறியாமல் $B \to K_2^*(1430)$ என்பதன் அளவிட்டை காணமுடியாமல் செய்து விடுகின்றன. மேலும் $B \to K_2^*(1430)$ என்ற நிகழ்வில் வெக்டார் மிசான் (1410 MeV), $K^*(1410)$ தோன்றுகிறது. இது $K_X^*(1430)$ பற்றி அறியவிடாமல் செய்கிறது.

மேற்கண்ட எல்லா மிசான்கலும் டிக்கே $B \to K\pi l^+ l^-$ என்பதனை அடைவதால் பல்வேறு டிக்கே சேனல்கள் இயங்குவியலின் மூலம் பிரிக்க முடியாமல் போகிற்து. நாம் ஆங்குலர் பகுப்பாய்வின் மூலம் இவை ஒவ்வொன்றின் விளைவினைக்காண்போம்.

ஒவ்வொரு டிக்கே சேனலுக்கும் லொரண்ஸ் முறையை பயண்படுத்தி பெறப்படும் அணியினைப்பெறுவோம். இதன்மூலம் இறுதிநிலை $K\pi l^+ l^-$ க்கான் டிக்கே ஸ்பெக்ட்ரம் காண்ப்படுகிறது. $K_2^*(1430), K^*(1410)$ ஆகியவை $K\pi$-யையும், J/ψ என்பது $l^+ l^-$-யையும் அடைவதாகக்கொள்வோம். நாம் $K\pi$ டிக்கே தளம், $l^+ l^-$ டிக்கே பிரேம் இரண்டுக்கும் இடைப்பட்ட தொடர்பினை காண்போம்.

மேற்சொல்லப்பட்ட வழிமுறையில் எந்த ஒரு வெக்டார், ஸ்கேலார் அல்லது டென்சார் மிசான் மற்றும் $K_X^*(1430), K^*(1410)$ ஆகியவற்றை உள்ளடக்கலாம். KEK(ஜப்பான்) மற்றும் SLAC(U.S.A) ஆகியவற்றின் B பேக்ட்ரிகளின் டெட்டாக்களை பற்றிய் பகுப்பாய்வினை நிகழ்த்த மேல் சொன்ன வழிமுறை மிகுந்த பயன் அளிக்கிறது.

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Chandradew Sharma

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Chapter 1

INTRODUCTION

The purpose of this thesis is to investigate the angular distribution of the decays $B \to K\pi\ell^+\ell^-$, where the $K\pi$ are produced via the intermediate decays of $K_2^\star(1430), K^\star(1410), K_0^\star(1430)$ and $\ell^+\ell^-$ are the decay products of J/ψ. The aim is to derive the matrix elements by combining the three decay modes $B \to J/\psi K_2^\star(1430)$ (tensor meson), $B \to J/\psi K^\star(1410)$(vector meson), $B \to J/\psi K_0^\star(1430)$(scalar meson).The processes $B \to K\pi\ell^+\ell^-$ is described in terms of the kinematical variable s_l (the invariant mass of the lepton pair), s_k (the invariant mass of the $K\pi$ pair) and $\cos\theta_l$ (the angular distribution ℓ^- in the $\ell^+\ell^-$ c.m.system). The additional information is the distribution in $\sin\theta_k$ and $\cos\theta_k$,where θ_k is the angle of the K in the $K\pi$ c.m. frame, and in ϕ,the angle between the the planes formed by $K\pi$ and $\ell^+\ell^-$ respectively [1]. The information is sensitive to the polarization state of the tensor meson $K_2^\star(1430)$,vector meson $K^\star(1410)$ and scalar meson $K_0^\star(1430)$,and thus provide a new probe using the effective theory to study in CP violation [1].

We begin by writing the most general effective matrix elements for each of the decay channels using Lorenz invariance and current conservation [2]. The decay spectrum for the final state $K\pi\ell^+\ell^-$ is calculated. The $K_X^\star(1430)$,where $X = 0, 2$ and $K^\star(1410)$ are considered to decay to $K\pi$ and J/ψ to $\ell^+\ell^-$. We study the angular distribution of the K in the $K\pi$ center of mass (c.m.) frame and ℓ^- in the $\ell^+\ell^-$ c.m. frame. We also study the correlation between the $K\pi$ decay plane and the $\ell^+\ell^-$ decay frame. The formulation developed here can be applied to include any scalar, vector or tensor meson, in addition to $K_X^\star(1430)$ and $K^\star(1410)$.

It is interesting that angular parts can be separated out cleanly from the total matrix elements.Although the angular parts are separated ,it is not possible to separate all partial waves out cleanly for the $J/\psi K_2^\star(1430)$ final state. We can separate out only between CP even and CP odd partial waves because it is not possible to distinguish between S and D partial waves in this mode.However, it is sufficient to study CP violation in these modes [3]. The interference terms will give CP violation in the mode .Recently large samples of data have collected on the decay $B \to J/\psi K_2^\star(1430)$ by the B factories running at KEK(Japan) and SLAC(U.S.A). These results will be useful to perform a detail analysis of the data collected so far.

Chapter 2

MATRIX ELEMENTS

We consider decays of B meson into J/ψ with a momentum q and a either $K_0^\star(1430)$, $K^\star(1410)$ or $K_2^\star(1430)$ with momentum k. In section 2.1 , the square of matrix elements in which B decays into $J/\psi K_0^\star(1430)$,with $J/\psi \to l^+l^-$ and $K_0^\star(1430) \to K\pi$ is calculated. In section 2.2 we evaluate the square of matrix elements in which B decays into $J/\psi K^\star(1410)$,with $J/\psi \to l^+l^-$ and $K^\star(1410) \to K\pi$. In section 2.3 , the square of matrix elements in which B decays into $J/\psi K_2^\star(1430)$,with $J/\psi \to l^+l^-$ and $K_2^\star(1430) \to K\pi$ is calculated.In section 2.4 ,interference between the two decays (B decays into $J/\psi K_0^\star(1430)$,with $J/\psi \to l^+l^-$ and $K_0^\star(1430) \to K\pi$ and B decays into $J/\psi K^\star(1410)$, $J/\psi \to l^+l^-$ and $K^\star(1410) \to K\pi$) is calculated .In section 2.5 ,interference between the two decays (B decays into $J/\psi K_0^\star(1430)$,with $J/\psi \to l^+l^-$, $K_0^\star(1430) \to K\pi$ and B decays into $J/\psi K_2^\star(1430)$, with $J/\psi \to l^+l^-$ and $K_2^\star(1430) \to K\pi$) is calculated .In section 2.6 ,interference between the two decays (B decays into $J/\psi K^\star(1410)$,with $J/\psi \to l^+l^-$ and $K^\star(1410) \to K\pi$ and B decays into $J/\psi K_2^\star(1430)$,with $J/\psi \to l^+l^-$ and $K_2^\star(1430) \to K\pi$) is calculated . In section 2.7 , the square of the total matrix elements in which B decays into the three modes ($B \to J/\psi K_0^\star(1430)$,with $J/\psi \to l^-l^+$ and $K_0^\star(1430) \to K\pi$ and ,$B \to J/\psi K^\star(1410)$,with $J/\psi \to l^-l^+$ and $K^\star(1410) \to K\pi$ and $B \to J/\psi K_2^\star(1430)$,with $J/\psi \to l^-l^+$ and $K_2^\star(1430) \to K\pi$) is calculated.

2.1 Scalar Mode

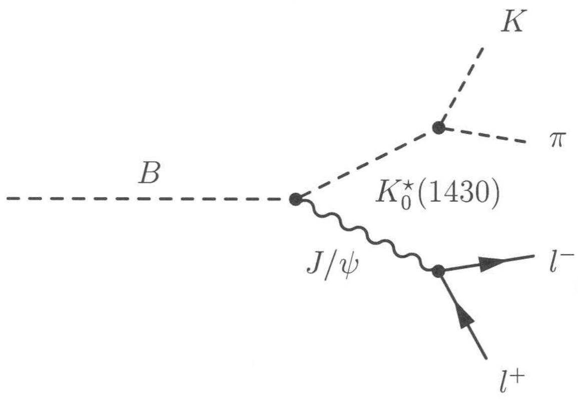

Figure 2.1: **Feynman diagram for the process $B \to J/\psi K_0^\star(1430)$,with $J/\psi \to l^- l^+$ and $K_0^\star(1430) \to K\pi$**

The vector particle J/ψ carries the four momentum q and decays into lepton and antilepton with momentum q_1 and q_2 respectively.
Hence, the matrix element for leptonic part is [4]

$$L_\sigma = -ie \sum \overline{u}\gamma_\sigma v \tag{2.1}$$

Therefore, leptonic tensor is

$$\begin{aligned} L_{\sigma\sigma'} &= e^2 \sum\sum (\overline{u}\gamma_\sigma v)(\overline{v}\gamma_\sigma u) \\ &= e^2 \mathrm{tr}(q_1^\alpha \gamma_\alpha \gamma_\sigma q_2^\beta \gamma_\beta \gamma_{\sigma'}) \\ &= 4e^2(q_{1\sigma}q_{2\sigma'} + q_{2\sigma}q_{1\sigma'} - g_{\sigma\sigma'}q_1.q_2) \\ &= 2e^2(q_\sigma q_{\sigma'} - Q_\sigma Q_{\sigma'} - g_{\sigma\sigma'}q^2) \\ &= 2e^2 L'_{\sigma\sigma'} \quad \text{, where } L'_{\sigma\sigma'} = q_\sigma q_{\sigma'} - Q_\sigma Q_{\sigma'} - g_{\sigma\sigma'}q^2 \end{aligned} \tag{2.2}$$

The B (pseudoscalar) particle decays into vector particle J/ψ with momentum q and scalar particle $K_0^\star(1430)$ with momentum k .On using the general Lorenz structure and conservation of current, it is possible to write the most general form of the tensor of rank one for the particle decaying into the constituents particles.
The most general form of the tensor of rank 1, which is a function of the momenta k and q is [2]

$$W_\rho = a_1 k_\rho + a_2 q_\rho. \tag{2.3}$$

Using the conservation of current

$$q^\rho W_\rho = 0, \tag{2.4}$$

from eqns(2.3) and (2.4) , On solving we get

$$W_\rho = aF_{\rho(k,q)}, \text{ where } \quad F_{\rho(k,q)} = -k_\rho + \frac{k.q}{q^2} q_\rho. \tag{2.5}$$

Similarly the $K_0^\star(1430)$ particle decays into pseudoscalar particles K with momentum k_1 and π with momentum k_2.So, the tensor structure is just constant.
The matrix element for hadronic part is

$$\begin{aligned} H_S^\sigma &= -\frac{aF_{\rho(k,q)}\theta^{\rho\sigma}}{(k^2 - m_k^2 + i\epsilon)(q^2 - m_q^2 + i\epsilon)}, \text{ where } \quad \theta^{\rho\sigma} = -g^{\rho\sigma} + \frac{q^\rho q^\sigma}{q^2} \\ &= \frac{aF^\sigma_{(k,q)}}{\Delta}, \text{ where } \quad \Delta = (k^2 - m_k^2 + i\epsilon)(q^2 - m_q^2 + i\epsilon) \end{aligned} \tag{2.6}$$

Hence, the hadronic tensor is

$$\begin{aligned} H_S^{\sigma\sigma'} &= H_S^\sigma H_S^{\sigma'\star} \\ &= \frac{|a|^2}{\Delta^2} F^\sigma_{(k,q)} F^{\sigma'}_{(k,q)} \end{aligned} \tag{2.7}$$

Therefore, the square of matrix element is

$$\begin{aligned} |M_S|^2 &= H_S^{\sigma\sigma'} L_{\sigma\sigma'} \\ &= \frac{|a|^2}{\Delta^2} F^\sigma_{(k,q)} F^{\sigma'}_{(k,q)} (2e^2 L'_{\sigma\sigma'}) \end{aligned} \tag{2.8}$$

It is shown in eqn (5.76) in Appendix, $F^{\sigma}_{(k,q)}F^{\sigma'}_{(k,q)}L'_{\sigma\sigma'} = X^2(1-\cos^2\theta_\ell)$. Therefore, the square of the matrix elements is

$$\begin{aligned} |M_S|^2 &= \frac{2|a|^2e^2X^2}{\Delta^2}(1-\cos^2\theta_\ell) \\ &= \frac{2|a|^2e^2(x^2-1)s_\ell s_k}{\Delta^2}[1-\cos^2\theta_\ell] \end{aligned} \tag{2.9}$$

2.2 Vector Mode

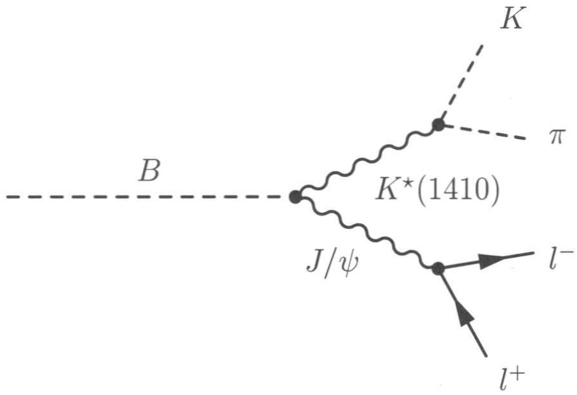

Figure 2.2: **Feynman diagram for the process $B \to J/\psi K^\star(1410)$, with $J/\psi \to l^- l^+$ and $K^\star(1410) \to K\pi$**

Here the B(pseudoscalar) particle decays into vector particle J/ψ with momentum q and vector particle $K^\star(1410)$ with momentum k.On using the general Lorenz structure and conservation of currents, it is possible to write the most general form

of the tensor of rank two ,from the given two independent momentum k and q ,for the particle decaying into the constituents particles.
The most general form of the tensor of rank 2 ,which is a function of the momenta q and k is [2]

$$\begin{aligned} W_{\mu\rho} &= bg_{\mu\rho} + c_1 k_\mu k_\rho + c_2 k_\mu q_\rho + c_3 k_\rho q_\mu + c_4 q_\mu q_\rho + \\ & id_1 \epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta + id_2 \epsilon_{\mu\rho\alpha\beta} q^\alpha k^\beta \end{aligned} \quad (2.10)$$

On using the conservation of currents

$$q^\rho W_{\mu\rho} = 0, \quad (2.11)$$

$$k^\mu W_{\mu\rho} = 0. \quad (2.12)$$

From eqns (2.10), (2.11) and (2.12), the tensor structure is

$$W_{\mu\rho} = bg_{\mu\rho} + ck_\rho q_\mu + id\epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta \ [5,6]. \quad (2.13)$$

Similarly , $K^\star(1410)$ particle decays into two pseudoscalar particles K with momentum k_1 and π with momentum k_2 .Using the same process as did before, the tensor structure is

$$\begin{aligned} M_\nu &= k_1 - k_2 \\ &= K_\nu \text{ where } \quad K_\nu = k_1 - k_2 \end{aligned} \quad (2.14)$$

So, the matrix element for hadronic part is

$$H_V^\sigma = W_{\mu\rho} \frac{i}{k^2 - m_k^2 + i\epsilon} \theta^{\mu\nu} K_\nu \frac{i}{q^2 - m_q^2 + i\epsilon} \theta^{\rho\sigma} \quad (2.15)$$

,where $\theta^{\mu\nu} = -g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}$, $\theta^{\rho\sigma} = -g^{\rho\sigma} + \frac{q^\rho q^\sigma}{q^2}$ and
$\Delta = (k^2 - m_k^2 + i\epsilon)(q^2 - m_q^2 + i\epsilon)$

On simplifying the eqn (2.15), the matrix elements for hadronic part is

$$H_V^\sigma = -\frac{1}{\Delta}(bE^\sigma + cF_{(K,k)}.qF^\sigma_{(k,q)} + id\epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta F^\mu_{(K,k)} \theta^{\rho\sigma}) \quad (2.16)$$

,where $E^\sigma = K^\sigma - \frac{k.K}{k^2} k^\sigma - \frac{q.K}{q^2} q^\sigma + \frac{k.Kk.q}{k^2 q^2} q^\sigma$

Hence, the hadronic tensor is

$$\begin{aligned}
H_V^{\sigma\sigma'} &= H_V^{\sigma} H_V^{\sigma'\star} \\
&= \{-\frac{1}{\Delta}(bE^{\sigma} + cF_{(K,k)}.qF^{\sigma}_{(k,q)} + id\epsilon_{\mu\rho\alpha\beta}k^{\alpha}q^{\beta}F^{\mu}_{(K,k)}\theta^{\rho\sigma})\} \\
&\quad \{-\frac{1}{\Delta}(b^{\star}E^{\sigma'} + c^{\star}F_{(K,k)}.qF^{\sigma'}_{(k,q)} - id^{\star}\epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}F^{\mu'}_{(K,k)}\theta^{\rho'\sigma'})\} \\
&= \frac{1}{\Delta^2}[|b|^2E^{\sigma}E^{\sigma'} + |c|^2(F_{(K,k)}.q)^2F^{\sigma}_{(k,q)}F^{\sigma'}_{(k,q)} + \\
&\quad |d|^2\epsilon_{\mu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}k^{\alpha}q^{\beta}k_{\alpha'}q_{\beta'}F^{\mu}_{(K,k)}\ F^{\mu'}_{(K,k)}\theta^{\rho\sigma}\theta^{\sigma'}_{\rho'} + \\
&\quad 2\mathrm{Re}(bc^{\star})F_{(K,k)}.qE^{\sigma}F^{\sigma'}_{(k,q)} + 2\mathrm{Im}(bd^{\star})E^{\sigma}\epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}F^{\mu'}_{(K,k)}\theta^{\rho'\sigma'} - \\
&\quad 2\mathrm{Im}(dc^{\star})F_{(K,k)}.q\epsilon_{\mu\rho\alpha\beta}k^{\alpha}q^{\beta}F^{\mu}_{(K,k)}\theta^{\rho\sigma}F^{\sigma'}_{(k,q)}]
\end{aligned} \tag{2.17}$$

The leptonic part is remain same as in scalar case. Therefore, the square of the matrix elements is

$$\begin{aligned}
|M_V|^2 &= H_V^{\sigma\sigma'}L_{\sigma\sigma'} \\
&= [\frac{1}{\Delta^2}\{|b|^2E^{\sigma}E^{\sigma'} + |c|^2(F_{(K,k)}.q)^2F^{\sigma}_{(k,q)}F^{\sigma'}_{(k,q)} + \\
&\quad |d|^2\epsilon_{\mu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}k^{\alpha}q^{\beta}k_{\alpha'}q_{\beta'}F^{\mu}_{(K,k)}F^{\mu'}_{(K,k)}\theta^{\rho\sigma}\theta^{\sigma'}_{\rho'} + \\
&\quad 2\mathrm{Re}(bc^{\star})(F_{(K,k)}.q)E^{\sigma}F^{\sigma'}_{(k,q)} + 2\mathrm{Im}(bd^{\star})E^{\sigma}\epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}F^{\mu'}_{(K,k)}\theta^{\rho'\sigma'} - \\
&\quad 2\mathrm{Im}(dc^{\star})(F_{(K,k)}.q)\epsilon_{\mu\rho\alpha\beta}k^{\alpha}q^{\beta}F^{\mu}_{(K,k)}\theta^{\rho\sigma}F^{\sigma'}_{(k,q)})\}e^2L'_{\sigma\sigma'}]
\end{aligned} \tag{2.18}$$

From eqns(5.76),(5.77),(5.78),(5.80),(5.81) and (5.82), the square of matrix elements is

$$\begin{aligned}
|M_V|^2 &= \frac{2e^2\beta^2}{\Delta^2}[-|b|^2\{(k.q)^2\cos^2\theta_\ell\cos^2\theta_k + s_\ell s_k\sin^2\theta_\ell\sin^2\theta_k\cos^2\phi - \\
&\quad \frac{1}{2}\sqrt{s_\ell s_k}(k.q)\sin 2\theta_\ell\sin 2\theta_k\cos\phi - (s_\ell s_k + X^2\cos^2\theta_k)\} + \\
&\quad |c|^2X^4\sin^2\theta_\ell\cos^2\theta_k + |d|^2X^2s_\ell s_k\sin^2\theta_k(\cos^2\theta_\ell + \sin^2\theta_\ell\cos^2\phi) + \\
&\quad 2\mathrm{Re}(bc^{\star})X^2\{k.q\sin^2\theta_\ell\cos^2\theta_k + \frac{1}{4}\sqrt{s_\ell s_k}\sin 2\theta_\ell\sin 2\theta_k\cos\phi\} - \\
&\quad 2\mathrm{Im}(bd^{\star})X\{\frac{1}{4}k.q\sqrt{s_\ell s_k}\sin 2\theta_\ell\sin 2\theta_k\sin\phi - \\
&\quad \frac{s_\ell s_k}{2}\sin^2\theta_\ell\sin^2\theta_k\sin 2\phi\} + \\
&\quad \frac{1}{2}\mathrm{Im}(dc^{\star})X^3\sqrt{s_\ell s_k}\sin 2\theta_\ell\sin 2\theta_k\sin\phi]
\end{aligned} \tag{2.19}$$

On substituting of $X^2 = (x^2-1)s_\ell s_k$ and $k.q = x\sqrt{s_\ell s_k}$ in eqn(2.19) and collecting the terms without ϕ, co-efficient of $\cos\phi$, co-efficient of $\cos^2\phi$, co-efficient of $\sin\phi$, co-efficient of $\sin^2\phi$,we have the square of matrix elements as

$$\begin{aligned}
|M_V|^2 = & \frac{2e^2\beta^2 s_\ell s_k}{\Delta^2}[|b|^2 + |d|^2(x^2-1)s_\ell s_k \cos^2\theta_\ell + \\
& \{|b|^2 + |c|^2(x^2-1)s_\ell s_k + 2\mathrm{Re}(bc^\star)x\sqrt{s_\ell s_k}\}(x^2-1)\cos^2\theta_k - \\
& \{|b|^2x^2 + |c|^2(x^2-1)^2 s_\ell s_k + |d|^2(x^2-1)s_\ell s_k + \\
& 2\mathrm{Re}(bc^\star)x(x^2-1)\sqrt{s_\ell s_k}\}\cos^2\theta_\ell\cos^2\theta_k + \\
& 2\{|b|^2x + \mathrm{Re}(bc^\star)(x^2-1)\sqrt{s_\ell s_k}\}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\theta_k\cos\phi + \\
& (|d|^2(x^2-1)s_\ell s_k - |b|^2)\{1-\cos^2\theta_\ell - \cos^2\theta_k + \cos^2\theta_\ell\cos^2\theta_k\}\cos^2\phi + \\
& 2\{\mathrm{Im}(dc^\star)(x^2-1)\sqrt{s_\ell s_k} - \mathrm{Im}(bd^\star)x\}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\theta_k\sin\phi + \\
& \mathrm{Im}(bd^\star)\sqrt{(x^2-1)}\sqrt{s_\ell s_k}(1-\cos^2\theta_\ell - \cos^2\theta_k + \cos^2\theta_\ell\cos^2\theta_k) \\
& \sin 2\phi]
\end{aligned} \tag{2.20}$$

2.3 Tensor Mode

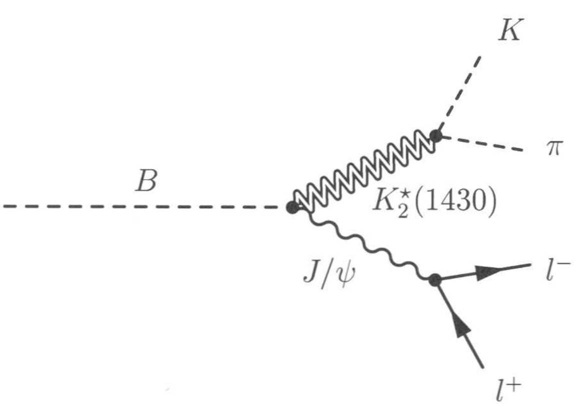

Figure 2.3: **Feynman diagram for the process $B \to J/\psi K_2^\star(1430)$,with $J/\psi \to l^- l^+$ and $K_2^\star(1430) \to K\pi$**

Here the B(pseudoscalar) particle decays into vector particle J/ψ with momentum q and tensor particle $K_2^\star(1430)$ with momentum k .And on using the general Lorenz structure and conservation of currents, it is possible to write the most general form of the tensor of rank three ,from the given two independent momentum k and q and metric tensor,for the particle decaying into the constituents particles.
The most general form of the tensor of rank 3, which is a function of momenta q and k is [2,7]

$$\begin{aligned} W_{\mu\nu\rho} &= f_1 g_{\mu\nu} k_\rho + f_2 g_{\mu\nu} q_\rho + f_3 g_{\mu\rho} k_\nu + f_4 g_{\mu\rho} q_\nu + f_5 g_{\nu\rho} k_\mu + f_6 g_{\nu\rho} q_\mu + \\ & g_1 k_\mu k_\nu k_\rho + g_2 k_\mu k_\nu q_\rho + g_3 k_\mu q_\nu k_\rho + g_4 k_\mu q_\nu q_\rho + g_5 q_\mu k_\nu k_\rho + g_6 q_\mu k_\nu q_\rho \\ & + g_7 q_\mu q_\nu k_\rho + g_8 q_\mu q_\nu k_\rho + i d_1 \epsilon_{\mu\rho\alpha\beta} q_\nu k^\alpha q^\beta + i d_2 \epsilon_{\mu\rho\alpha\beta} q_\nu q^\alpha k^\beta + \\ & i d_3 \epsilon_{\mu\rho\alpha\beta} k_\nu k^\alpha q^\beta + i d_4 \epsilon_{\mu\rho\alpha\beta} k_\nu q^\alpha k^\beta + i d_5 \epsilon_{\nu\rho\alpha\beta} q_\mu k^\alpha q^\beta + \\ & i d_6 \epsilon_{\nu\rho\alpha\beta} q_\mu q^\alpha k^\beta + i d_7 \epsilon_{\nu\rho\alpha\beta} k_\mu k^\alpha q^\beta + i d_8 \epsilon_{\nu\rho\alpha\beta} k_\mu q^\alpha k^\beta + \\ & i d_9 \epsilon_{\mu\nu\alpha\beta} k_\rho k^\alpha q^\beta + i d_{10} \epsilon_{\mu\nu\alpha\beta} q_\rho k^\alpha q^\beta + i d_{11} \epsilon_{\mu\nu\alpha\beta} k_\rho q^\alpha k^\beta + \\ & i d_{12} \epsilon_{\mu\nu\alpha\beta} q_\rho q^\alpha k^\beta \end{aligned} \tag{2.21}$$

Using the conservation of currents

$$q^\rho W_{\mu\nu\rho} = 0, \tag{2.22}$$

$$k^\mu k^\nu W_{\mu\nu\rho} = 0, \tag{2.23}$$

$$g^{\mu\nu} W_{\mu\nu\rho} = 0. \tag{2.24}$$

On solving the eqns (2.22), (2.23) and (2.24), the tensor structure is

$$W_{\mu\nu\rho} = h(g_{\mu\rho} q_\nu - \frac{q_\mu q_\nu k\rho}{k.q}) + i f(\epsilon_{\mu\rho\alpha\beta} q_\nu + \epsilon_{\nu\rho\alpha\beta} q_\mu) k^\alpha q^\beta \tag{2.25}$$

Similarly $K_2^\star(1430)$ particle decays into two pseudoscalar particles which momentums are k_1 and k_2 respectively.And using the same process as did before, the tensor structure is

$$T_{\alpha'\beta'} = k_{1\alpha'} k_{2\beta'} \tag{2.26}$$

$$= \frac{1}{4}(k_{\alpha'} k_{\beta'} + k_{\beta'} K_{\alpha'} - k_{\alpha'} K_{\beta'} - K_{\alpha'} K_{\beta'}) \tag{2.27}$$

The matrix elements for hadronic part is

$$H_T^\sigma = W^{\mu\nu\rho}\frac{i}{k^2-m_k^2+i\epsilon}\Theta^{\mu\nu\alpha'\beta'}T_{\alpha'\beta'}\frac{i}{q^2-m_q^2+i\epsilon}\theta^{\rho\sigma} \tag{2.28}$$

,where $\Theta^{\mu\nu\alpha'\beta'} = \frac{1}{2}(\theta^{\alpha'\mu}\theta^{\beta'\nu}+\theta^{\beta'\mu}\theta^{\alpha'\nu}) - \frac{1}{3}\theta^{\alpha'\beta'}\theta^{\mu\nu}$

On simplifying the eqn(2.28), the matrix elements of the hadronic part becomes

$$H_T^\sigma = -\frac{1}{4\Delta}(hS^\sigma + ifR^\sigma) \tag{2.29}$$

,where $S^\sigma = F_{(K,k)}.qF^\sigma_{(K,k)} - \frac{F_{(K,k)}.K}{3}F^\sigma_{(q,k)} - \frac{(F_{(K,k)}.q)^2}{k.q}k^\sigma + \frac{F_{(K,k)}.KF_{(q,k)}.q}{3k.q}k^\sigma$

and $R^\sigma = (\epsilon_{\mu\rho\alpha\beta}q_\nu + \epsilon_{\nu\rho\alpha\beta}q_\mu)k^\alpha q^\beta(-F^\mu_{(K,k)}F^\nu_{(K,k)} + \frac{F_{(K,k)}.K}{3}\theta^{\mu\nu})\theta^{\rho\sigma}$

Hence, the hadronic tensor is

$$\begin{aligned} H_T^{\sigma\sigma'} &= H_T^\sigma H_T^{\sigma'\star} \\ &= (-\frac{1}{4\Delta}\{hS^\sigma + ifR^\sigma))(-\frac{1}{4\Delta}(h^\star S^{\sigma'} - if^\star R^{\sigma'})) \\ &= \frac{1}{16\Delta^2}\{|h|^2 S^\sigma S^{\sigma'} + |f|^2 R^\sigma R^{\sigma'} - 2\text{Im}(hf^\star)R^\sigma S^{\sigma'}\} \end{aligned} \tag{2.30}$$

The leptonic part is remain the same as in scalar case.Therefore, the square of the matrix elements is

$$\begin{aligned} |M_T|^2 &= H_T^{\sigma\sigma'}L_{\sigma\sigma'} \\ &= \frac{1}{16\Delta^2}\{|h|^2 S^\sigma S^{\sigma'} + |f|^2 R^\sigma R^{\sigma'} - 2\text{Im}(hf^\star)R^\sigma S^{\sigma'}\}(2e^2 L'_{\sigma\sigma'}) \end{aligned} \tag{2.31}$$

From eqns (5.83), (5.84) and (5.85) , the square of the matrix elements is

$$\begin{aligned} |M_T|^2 = \; & \frac{e^2\beta^4}{8\Delta^2}[-|h|^2X^2\; s_l s_k\{\frac{X^2}{(k.q)^2}\cos^4\theta_k - \frac{((k.q)^2 - \frac{2}{3}s_\ell s_k)}{(k.q)^2}\cos^2\theta_k - \\ & \frac{s_\ell s_k}{9(k.q)^2} + \frac{s_\ell s_k}{(k.q)^2}(\cos^4\theta_k - \frac{2}{3}\cos^2\theta_k + \frac{1}{9})\cos^2\theta_\ell + \\ & \frac{1}{4}\sin^2 2\theta_k \sin^2\theta_\ell \cos^2\phi - \frac{\sqrt{s_\ell s_k}}{2k.q}(\cos^2\theta_k - \frac{1}{3})\sin 2\theta_k \sin 2\theta_\ell \cos\phi\} \\ & +|f|^2 X^4 s_\ell s_k \sin^2 2\theta_k(\cos^2\theta_\ell + \cos^2\phi\sin^2\theta_\ell) + \end{aligned}$$

$$2\text{Im}(hf^\star)X^3 s_\ell s_k\{\frac{\sqrt{s_\ell s_k}}{k.q}(\cos^2\theta_k - \frac{1}{3})\sin 2\theta_k \sin 2\theta_\ell \sin\phi -$$
$$\frac{1}{2}\sin^2 2\theta_k \sin^2\theta_\ell \sin 2\phi\}] \tag{2.32}$$

On substituting of $X^2 = (x^2-1)s_\ell s_k$ and $k.q = x\sqrt{s_\ell s_k}$ in eqn (2.32) and collecting the terms without ϕ, co-efficient of $\cos\phi$, co-efficient of $\cos^2\phi$, co-efficient of $\sin\phi$, co-efficient of $\sin^2\phi$,we have the square of matrix elements as

$$\begin{aligned}
|M_T|^2 &= \frac{e^2\beta^4(x^2-1)s_l^2 s_k^2}{8\Delta^2 x^2}[\frac{1}{9}|h|^2 + |h|^2(x^2-\frac{2}{3})\cos^2\theta_k + \\
&\{|f|^2x^2(x^2-1)s_l s_k - \frac{1}{9}|h|^2\}\cos^2\theta_l + \\
&\{\frac{2}{3}|h|^2 - |f|^2x^2(x^2-1)s_l s_k\}\cos^2\theta_k\cos^2\theta_\ell - \\
&|h|^2(x^2-1)\cos^4\theta_k - |h|^2\cos^4\theta_k\cos^2\theta_l + \\
&2|h|^2 x\sin\theta_k\sin\theta_l(\cos^3\theta_k\cos\theta_l - \frac{1}{3}\cos\theta_k\cos\theta_\ell)\cos\phi + \\
&x^2\{|f|^2(x^2-1)s_l s_k - (|f|^2(x^2-1)s_l s_k + |h|^2)\cos^2\theta_k - \\
&|f|^2(x^2-1)s_l s_k\cos^2\theta_l + (|f|^2(x^2-1)s_l s_k + \\
&|h|^2)\cos^2\theta_k\cos^2\theta_l + |h|^2\cos^4\theta_k - |h|^2\cos^4\theta_k\cos^2\theta_\ell\}\cos^2\phi + \\
&8\text{Im}(hf^\star)x\sqrt{x^2-1}\sqrt{s_l s_k}\sin\theta_k\sin\theta_l\{\cos^3\theta_k\cos\theta_l - \\
&\frac{1}{3}\cos\theta_k\cos\theta_l\}\sin\phi + 4\text{Im}(hf^\star)x^2\sqrt{x^2-1}\sqrt{s_l s_k}\{-\cos^2\theta_k + \\
&\cos^2\theta_k\cos^2\theta_l + \cos^4\theta_k - \cos^4\theta_k\cos^2\theta_l\}\sin 2\phi]
\end{aligned} \tag{2.33}$$

2.4 Scalar-Vector Interferences

On using the hadronic matrix elements of scalar $K_0^\star(1430)$ and vector $K^\star(1410)$, the scalar-vector interference hadronic tensor is

$$\begin{aligned}
H_{SV}^{\sigma\sigma'} &= H_S^\sigma H_V^{\sigma'\star} + H_V^\sigma H_S^{\sigma'\star} \\
&= (-\frac{aF_{(k,q)}^\sigma}{\Delta})(-\frac{1}{\Delta}(b^\star E^{\sigma'} + c^\star F_{(K,k)}.qF_{(k,q)}^{\sigma'} - id^\star \epsilon_{\mu'\rho'\alpha'\beta'} k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'})) + \\
&(-\frac{1}{\Delta}(bE^\sigma + cF_{(K,k)}.qF_{(k,q)}^\sigma + id\epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta F_{(K,k)}^\mu \theta^{\rho\sigma}))(-\frac{a^\star F_{(k,q)}^{\sigma'}}{\Delta}) \\
&= \frac{2}{\Delta^2}[\mathrm{Re}(ba^\star)E^\sigma F_{(k,q)}^{\sigma'} + \mathrm{Re}(cd^\star)F_{(K,k)}.qF_{(k,q)}^\sigma F_{(k,q)}^{\sigma'} - \\
&\mathrm{Im}(da^\star)\epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta F_{(K,k)}^\mu \theta^{\rho\sigma} F_{(k,q)}^{\sigma'}] \qquad (2.34)
\end{aligned}$$

Therefore, the square of the matrix elements of scalar-vector interference part is

$$\begin{aligned}
|M_{SV}|^2 &= H_{SV}^{\sigma\sigma'} L_{\sigma\sigma'} \\
&= \frac{2}{\Delta^2}[\mathrm{Re}(ba^\star)E^\sigma F_{k,q}^{\sigma'} + \mathrm{Re}(cd^\star)F_{K,k}.qF_{k,q}^\sigma F_{k,q}^{\sigma'} - \\
&\mathrm{Im}(da^\star)\epsilon_{\mu\rho\alpha\beta} k^\alpha q^\beta F_{K,k}^\mu \theta^{\rho\sigma} F_{k,q}^{\sigma'}]2e^2 L'_{\sigma\sigma'} \qquad (2.35)
\end{aligned}$$

From eqns (5.76), (5.78) and (5.81), the square of the matrix elements is

$$\begin{aligned}
|M_{SV}|^2 &= \frac{4e^2\beta X}{\Delta^2}[\mathrm{Re}(ba^\star)\{k.q\cos\theta_k\cos^2\theta_\ell - \sqrt{s_\ell s_k}\sin\theta_k\sin 2\theta_\ell\cos\phi - \\
&k.q\cos\theta_k\} + \mathrm{Re}(ca^\star)X^2(\cos\theta_k\cos^2\theta_\ell - \cos\theta_k) - \\
&\frac{1}{2}\mathrm{Im}(da^\star)X\sqrt{s_\ell s_k}\sin\theta_k\sin 2\theta_\ell\cos\phi] \qquad (2.36)
\end{aligned}$$

On substituting of $X^2 = (x^2-1)s_\ell s_k$ and $k.q = x\sqrt{s_\ell s_k}$ in eqn(2.36) and collecting the terms without ϕ, co-efficient of $\cos\phi$, co-efficient of $\cos^2\phi$, co-efficient of $\sin\phi$, co-efficient of $\sin^2\phi$,we have the square of matrix elements is

$$\begin{aligned}
|M_{SV}|^2 &= \frac{4e^2\beta\sqrt{(x^2-1)s_\ell s_k}}{\Delta^2}[-\{\mathrm{Re}(ba^\star)x + \mathrm{Re}(ca^\star)(x^2-1)\sqrt{s_\ell s_k}\}\cos\theta_k + \\
&\{\mathrm{Re}(ba^\star)x + \mathrm{Re}(ca^\star)(x^2-1)\sqrt{s_\ell s_k}\}\cos^2\theta_\ell\cos\theta_k - \\
&\{2\mathrm{Re}(ba^\star) + \mathrm{Im}(da^\star)\sqrt{x^2-1}\sqrt{s_\ell s_k}\}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\phi] \qquad (2.37)
\end{aligned}$$

2.5 Scalar-Tensor Interferences

On using the hadronic matrix elements of scalar $K_0^\star(1430)$ and tensor $K_2^\star(1430)$, the scalar-tensor interference hadronic tensor is

$$\begin{aligned} H_{ST}^{\sigma\sigma'} &= H_S^\sigma H_T^{\sigma'\star} + H_T^\sigma H_S^{\sigma'\star} \\ &= (-\frac{aF^\sigma_{(k,q)}}{\Delta})(-\frac{1}{4\Delta}(h^\star S^{\sigma'} - if^\star R^{\sigma'})) + (-\frac{1}{4\Delta}(hS^\sigma + ifR^\sigma))(-\frac{a^\star F^{\sigma'}_{(k,q)}}{\Delta}) \\ &= \frac{1}{2\Delta^2}[\mathrm{Re}(ha^\star)S^\sigma F^{\sigma'}_{(k,q)} - \mathrm{Im}(fa^\star)R^\sigma F^{\sigma'}_{(k,q)}] \end{aligned} \tag{2.38}$$

Therefore, the square of the matrix elements of scalar-tensor interference part is

$$\begin{aligned} |M_{ST}|^2 &= H_{ST}^{\sigma\sigma'} L_{\sigma\sigma'} \\ &= \frac{1}{2\Delta^2}[\mathrm{Re}(ha^\star)S^\sigma F^{\sigma'}_{(k,q)} - \mathrm{Im}(fa^\star)R^\sigma F^{\sigma'}_{(k,q)}]2e^2 L'_{\sigma\sigma'} \end{aligned} \tag{2.39}$$

From eqns (5.87) and (5.91), the square of the matrix elements is

$$\begin{aligned} |M_{ST}|^2 &= \frac{e^2\beta^2 X^2\sqrt{s_\ell s_k}}{\Delta^2}[-\mathrm{Re}(ha^\star)\sqrt{s_\ell s_k}\{\frac{\sqrt{s_\ell s_k}}{k.q}(\cos^2\theta_k - \frac{1}{3})\sin^2\theta_\ell + \\ &\quad \frac{1}{4}\sin 2\theta_k \sin 2\theta_\ell \cos\phi\} + \frac{1}{2}\mathrm{Im}(fa^\star)X\sin 2\theta_k \sin 2\theta_\ell \sin\phi] \end{aligned} \tag{2.40}$$

On substituting of $X^2 = (x^2-1)s_\ell s_k$ and $k.q = x\sqrt{s_\ell s_k}$ in eqn (2.40) and collecting the terms without ϕ, co-efficient of $\cos\phi$, co-efficient of $\cos^2\phi$, co-efficient of $\sin\phi$, co-efficient of $\sin^2\phi$,we have the square of matrix elements is

$$\begin{aligned} |M_{ST}|^2 &= \frac{e^2\beta^2(x^2-1)s_\ell s_k}{\Delta^2}[-\frac{1}{3}\mathrm{Re}(ha^\star) + \frac{1}{3}\mathrm{Re}(ha^\star)\cos^2\theta_\ell + \mathrm{Re}(ha^\star)\cos^2\theta_k \\ &\quad -\mathrm{Re}(ha^\star)\cos^2\theta_\ell\cos^2\theta_k + \mathrm{Re}(ha^\star)\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\theta_k\cos\phi + \\ &\quad 2\mathrm{Im}(fa^\star)\sqrt{(x^2-1)}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\theta_k\sin\phi] \end{aligned} \tag{2.41}$$

2.6 Vector-Tensor Interferences

On using the hadronic matrix elements of vector $K^\star(1410)$ and tensor $K_2^\star(1430)$, the vector-tensor interference hadronic tensor is

$$\begin{aligned}
H_{VT}^{\sigma\sigma'} &= H_V^\sigma H_T^{\sigma'\star} + H_T^\sigma H_V^{\sigma'\star} \\
&= (-\frac{1}{\Delta}(bE^\sigma + cF_{(K,k)}.qF_{(k,q)}^\sigma + id\epsilon_{\mu\rho\alpha\beta}k^\alpha q^\beta F_{(K,k)}^\mu \theta^{\rho\sigma})) \\
&\quad (-\frac{1}{4\Delta}(h^\star S^{\sigma'} - if^\star R^{\sigma'})) + (-\frac{1}{4\Delta}(hS^\sigma + ifR^\sigma)) \\
&\quad (-\frac{1}{\Delta}(b^\star E^{\sigma'} + c^\star F_{(K,k)}.qF_{(k,q)}^{\sigma'} - id^\star \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'})) \\
&= \frac{1}{2\Delta^2}[\mathrm{Re}(hb^\star)S^\sigma E^{\sigma'} + \mathrm{Re}(hc^\star)F_{(K,k)}.qS^\sigma F_{(k,q)}^{\sigma'} + \\
&\quad \mathrm{Re}(fd^\star)R^\sigma \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'} + \\
&\quad \mathrm{Im}(hd^\star)S^\sigma \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'} - \\
&\quad \mathrm{Im}(fb^\star)R^\sigma E^{\sigma'} - \mathrm{Im}(fc^\star)F_{(K,k)}.qR^\sigma F_{(k,q)}^{\sigma'}] \qquad (2.42)
\end{aligned}$$

Therefore, the square of the matrix elements of vector-tensor interference part is

$$\begin{aligned}
|M_{VT}|^2 &= H_{VT}^{\sigma\sigma'} L_{\sigma\sigma'} \\
&= \frac{1}{2\Delta^2}[\mathrm{Re}(hb^\star)S^\sigma E^{\sigma'} + \mathrm{Re}(hc^\star)F_{(K,k)}.qS^\sigma F_{(k,q)}^{\sigma'} + \\
&\quad \mathrm{Re}(fd^\star)R^\sigma \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'} + \\
&\quad \mathrm{Im}(hd^\star)S^\sigma \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'} q^{\beta'} F_{(K,k)}^{\mu'} \theta^{\rho'\sigma'} - \\
&\quad \mathrm{Im}(fb^\star)R^\sigma E^{\sigma'} - \mathrm{Im}(fc^\star)F_{(K,k)}.qR^\sigma F_{(k,q)}^{\sigma'}]2e^2 L'_{\sigma\sigma'} \qquad (2.43)
\end{aligned}$$

From eqns (5.86), (5.87),(5.88),(5.89),(5.90) and (5.91), the square of the matrix elements is

$$\begin{aligned}
|M_{VT}|^2 &= \frac{e^2}{\Delta^2}[\mathrm{Re}(hb^\star)\beta^3 X\sqrt{s_\ell s_k}\{(k.q\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos^2\theta_k + \\
&\quad \frac{s_\ell s_k}{k.q}\sin\theta_\ell\sin\theta_k\cos\theta_\ell(\cos^2\theta_k - \frac{1}{3}))\cos\phi - \\
&\quad \sqrt{s_\ell s_k}\cos^2\theta_\ell\cos\theta_k(\cos^2\theta_k - \frac{1}{3}) - \sqrt{s_\ell s_k}(1-\cos^2\theta_\ell -
\end{aligned}$$

$$\cos^2\theta_k + \cos^2\theta_\ell\cos^2\theta_k)\cos\theta_k\cos^2\phi + \frac{2}{3}\sqrt{s_\ell s_k}\cos\theta_k\} +$$
$$\mathrm{Re}(hc^\star)\beta^3X^3\sqrt{s_\ell s_k}\{-\frac{\sqrt{s_\ell s_k}}{k.q}\cos^2\theta_\ell(\cos^3\theta_k - \frac{1}{3}\cos\theta_k) +$$
$$\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos^2\theta_k\cos\phi\} + 2\mathrm{Re}(fd^\star)\beta^3X^3s_\ell s_k\cos\theta_k\{\cos^2\theta_\ell$$
$$-\cos^2\theta_\ell\cos^2\theta_k - (1-\cos^2\theta_\ell - \cos^2\theta_k + \cos^2\theta_\ell\cos^2\theta_k)\cos^2\phi\}$$
$$-\mathrm{Im}(hd^\star)\beta^3X^2s_\ell s_k\{\frac{1}{2}\sin^2\theta_\ell\sin^2\theta_k\cos\theta_k\sin 2\phi +$$
$$\frac{\sqrt{s_\ell s_k}}{k.q}(\frac{1}{3}\sin\theta_\ell\sin\theta_k\cos\theta_\ell - \sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos^2\theta_k)\sin\phi\} +$$
$$\mathrm{Im}(fb^\star)\beta^3X^2s_\ell s_k\{\cos\theta_k - \cos^3\theta_k - \cos^2\theta_\ell\cos\theta_k +$$
$$\cos^2\theta_\ell\cos^3\theta_k\}\sin 2\phi - 2\mathrm{Im}(fc^\star)\beta^3X^4\sqrt{s_\ell s_k}\sin\theta_\ell\sin\theta_k\cos\theta_\ell$$
$$\cos^2\theta_k\sin\phi] \qquad (2.44)$$

On substituting of $X^2 = (x^2-1)s_\ell s_k$ and $k.q = x\sqrt{s_\ell s_k}$ in eqn (2.44) and collecting the terms without ϕ, co-efficient of $\cos\phi$, co-efficient of $\cos^2\phi$, co-efficient of $\sin\phi$, co-efficient of $\sin^2\phi$,we have the square of matrix elements is

$$|M_{VT}|^2 = \frac{e^2\beta^3(x^2-1)^{1/2}(s_\ell s_k)^{3/2}}{\Delta^2}[\frac{2}{3}\mathrm{Re}(hb^\star)\cos\theta_k + \{\frac{1}{3}\mathrm{Re}(hb^\star) +$$
$$\frac{1}{3}\frac{(x^2-1)}{x}\sqrt{s_\ell s_k}\mathrm{Re}(hc^\star) + (x^2-1)s_\ell s_k\mathrm{Re}(hd^\star)\}\cos^2\theta_\ell\cos\theta_k +$$
$$\{\frac{(x^2+1)}{x}\mathrm{Re}(hb^\star) + (x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(hc^\star)\cos^2\theta_k - \frac{1}{3x}\mathrm{Re}(hb^\star)\}$$
$$\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\phi - (\mathrm{Re}(hb^\star) + 2(x^2-1)s_\ell s_k\mathrm{Re}(fd^\star))$$
$$\{\cos\theta_k - \cos^2\theta_\ell\cos\theta_k - \cos^3\theta_k + \cos^2\theta_\ell\cos^3\theta_k\}\cos^2\phi$$
$$\{-\frac{\sqrt{x^2-1}}{3x}\sqrt{s_\ell s_k}\mathrm{Im}(hd^\star)\sin\theta_\ell\sin\theta_k\cos\theta_\ell -$$
$$\sqrt{x^2-1}\sqrt{s_\ell s_k}(2(x^2-1)\sqrt{s_\ell s_k}\mathrm{Im}(fc^\star) + 2x\mathrm{Im}(fb^\star) -$$
$$\frac{1}{x}\mathrm{Im}(hd^\star))\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos^2\theta_k\}\sin\phi +$$
$$\sqrt{x^2-1}\sqrt{s_\ell s_k}(\mathrm{Im}(fb^\star) - \frac{1}{2}\mathrm{Im}(hd^\star))\{\cos\theta_k -$$
$$\cos^3\theta_k - \cos^2\theta_\ell\cos\theta_k + \cos^2\theta_\ell\cos^3\theta_k\}\sin 2\phi] \qquad (2.45)$$

2.7 Total Matrix Elements

Here it is considering that B particle decays into there possible channels .One possibility is that B decays into a scalar particle and a vector particle then the scalar particle decays into two pseudoscalar particles and the vector particle decays into a lepton and an antilepton.And second possibility is that B decays into two vector particles then one vector particle decays into two pseudoscalar particles and another vector particle decays into a lepton and an antilepton.And third possibility is that B decays into a tensor particle and a vector particle then the tensor particle decays into two pseudoscalar particles and the vector particle decays into a lepton and an antilepton.Therefore on including all the three possibilities, the hadronic matrix elements is

$$H^\sigma = H_S^\sigma + H_V^\sigma + H_T^\sigma \tag{2.46}$$

Therefore, the square of the total matrix elements is

$$\begin{aligned}
|M|^2 &= H^\sigma H^{\sigma'} L_{\sigma\sigma'} \\
&= (H_S^\sigma + H_V^\sigma + H_T^\sigma)(H_S^{\sigma'} + H_V^{\sigma'} + H_T^{\sigma'}) L_{\sigma\sigma'} \\
&= H_S^\sigma H_S^{\sigma'} L_{\sigma\sigma'} + H_V^\sigma H_V^{\sigma'} L_{\sigma\sigma'} + H_T^\sigma H_T^{\sigma'} L_{\sigma\sigma'} \\
&\quad (H_S^\sigma H_V^{\sigma'} + H_V^\sigma H_S^{\sigma'}) L_{\sigma\sigma'} + (H_S^\sigma H_T^{\sigma'} + H_T^\sigma H_S^{\sigma'}) L_{\sigma\sigma'} \\
&\quad (H_T^\sigma H_V^{\sigma'} + H_V^\sigma H_T^{\sigma'}) L_{\sigma\sigma'} \\
&= |M_S|^2 + |M_V|^2 + |M_T|^2 + |M_{SV}|^2 + |M_{ST}|^2 + |M_{VT}|^2
\end{aligned} \tag{2.47}$$

From eqns (2.9),(2.20),(2.33),(2.37),(2.41),(2.45) and (2.47), the square of the total matrix elements is

$$\begin{aligned}
|M|^2 = \; & \frac{e^2}{\Delta^2}[s_\ell s_k\{\beta^2|b|^2 + (x^2-1)(2|a|^2 - \frac{1}{3}\beta^2 \mathrm{Re}(hb^\star) + \frac{1}{72}\beta^4 s_\ell s_k |h|^2)\} + \\
& 2\beta\sqrt{x^2-1}\, s_\ell s_k \{\frac{1}{3}\beta^2\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star) - 2x\mathrm{Re}(ba^\star) - \\
& 2(x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(ca^\star)\}\cos\theta_k + (x^2-1)s_\ell s_k\{-2|a|^2 + \beta^2(2 s_\ell s_k |d|^2 \\
& + \frac{1}{3}\mathrm{Re}(ha^\star)) + \frac{\beta^4 s_\ell s_k}{8x^2}(|f|^2 x^2(x^2-1)s_\ell s_k - \frac{1}{9}|h|^2)\}\cos^2\theta_\ell +
\end{aligned}$$

$$\beta^2(x^2-1)s_\ell s_k\{2|b|^2+\mathrm{Re}(ha^\star)+4x\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star)+2(x^2-1)s_\ell s_k|c|^2$$
$$+\beta^2 s_\ell s_k\frac{(x^2-\frac{2}{3})}{x^2}|h|^2\}\cos^2\theta_k+\beta\sqrt{x^2-1}s_\ell s_k\{\frac{1}{3}\beta^2\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star)+$$
$$4x\mathrm{Re}(ba^\star)+(x^2-1)(4\mathrm{Re}(ca^\star)+\beta^2 s_\ell s_k(\frac{1}{3x}\mathrm{Re}(hc^\star)+\sqrt{s_\ell s_k}\mathrm{Re}(hd^\star)\}$$
$$\cos^2\theta_\ell\cos\theta_k+\beta^2 s_\ell s_k\{\frac{\beta^2 s_\ell s_k}{8}\frac{(x^2-1)}{x^2}(\frac{2}{3}|h|^2-$$
$$x^2(x^2-1)s_\ell s_k|f|^2)-2x^2|b|^2-(x^2-1)(\mathrm{Re}(ha^\star)+2s_\ell s_k|d|^2)-$$
$$4x(x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star)-2(x^2-1)^2 s_\ell s_k|c|^2\}\cos^2\theta_\ell\cos^2\theta_k-$$
$$\frac{1}{8}\beta^4\frac{(x^2-1)}{x^2}(s_\ell s_k)^2|h|^2\cos^2\theta_\ell\cos^4\theta_k+\beta s_\ell s_k\{-\sqrt{x^2-1}(8\mathrm{Re}(ba^\star)+$$
$$4\sqrt{x^2-1}\sqrt{s_\ell s_k}\mathrm{Im}(da^\star)+\frac{\beta}{3x}\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star))+\beta(4x|b|^2+$$
$$(x^2-1)(4\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star)+\mathrm{Re}(ha^\star)-\frac{\beta^2}{12x}s_\ell s_k|h|^2))\cos\theta_k+$$
$$\beta^2\sqrt{x^2-1}\sqrt{s_\ell s_k}(\frac{x^2+1}{x}\mathrm{Re}(hb^\star)+(x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(hc^\star))\cos^2\theta_k+$$
$$\frac{1}{4}\beta^3\frac{(x^2-1)}{x}s_\ell s_k|h|^2\cos^3\theta_k\}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\phi+$$
$$\beta^2 s_\ell s_k\{2(|d|^2(x^2-1)s_\ell s_k-|b|^2+\frac{1}{16}\beta^2(x^2-1)^2(s_\ell s_k)^2|f|^2)-$$
$$\beta\sqrt{x^2-1}\sqrt{s_\ell s_k}(\mathrm{Re}(hb^\star)+2(x^2-1)s_\ell s_k\mathrm{Re}(fd^\star))\cos\theta_k-$$
$$\frac{1}{8}\beta^2(x^2-1)s_\ell s_k|h|^2\cos^2\theta_k\}(1-\cos^2\theta_\ell-\cos^2\theta_k+\cos^2\theta_\ell\cos^2\theta_k)$$
$$\cos^2\phi+\beta^2 s_\ell s_k\{-\beta\frac{(x^2-1)}{3x}s_\ell s_k\mathrm{Im}(hd^\star)+(4(x^2-1)\sqrt{s_\ell s_k}\mathrm{Im}(dc^\star)-$$
$$4x\mathrm{Im}(bd^\star)+(x^2-1)^{3/2}(2\mathrm{Im}(fa^\star)-\frac{\beta^2}{3x}(s_\ell s_k)^{3/2}\mathrm{Im}(hf^\star)))\cos\theta_k-$$
$$\beta(x^2-1)^2 s_\ell s_k(2(x^2-1)\sqrt{s_\ell s_k}\mathrm{Im}(fc^\star)+2x\mathrm{Im}(fb^\star)-\frac{1}{x}\mathrm{Im}(hd^\star))\cos^2\theta_k$$
$$+\beta^2\frac{(x^2-1)^{3/2}}{x}(s_\ell s_k)^{3/2}\mathrm{Im}(hf^\star)\cos^3\theta_k\}\sin\theta_\ell\sin\theta_k\cos\theta_\ell\sin\phi+$$
$$\beta^2\sqrt{(x^2-1)}(s_\ell s_k)^{3/2}\{2\mathrm{Im}(bd^\star)+\beta\sqrt{(x^2-1)}\sqrt{s_\ell s_k}(\mathrm{Im}(fb^\star)-$$
$$\frac{1}{2}\mathrm{Im}(hd^\star)+\mathrm{Im}(hb^\star))\cos\theta_k+\frac{1}{2}\beta^2(x^2-1)s_\ell s_k\mathrm{Im}(hf^\star)\cos^2\theta_k\}$$
$$(1-\cos^2\theta_\ell-\cos^2\theta_k+\cos^2\theta_\ell\cos^2\theta_k)\sin 2\phi] \qquad (2.48)$$

$$\Rightarrow |M|^2 \quad = \quad \frac{e^2}{\Delta^2}(f_0 + f_1 \cos\phi + f_2 \cos^2\phi + f_3 \sin\phi + f_4 \sin 2\phi) \qquad (2.49)$$

,where $f_p = \sum \cos^m \theta_\ell \cos^n \theta_k (a^p_{mn} + b^p_{mn} \sin\theta_\ell + c^p_{mn} \sin\theta_k + d^p_{mn} \sin\theta_\ell \sin\theta_k)$ and all the non vanishing co-efficients are given in the table 5.2 [8]

Chapter 3

ANALYSIS

3.1 Introduction

From eqns(2.48) or (2.49) we see that the angular part is separated out cleanly .From table 5.2 we see that all co-efficients are not nonzero .Some co-efficients are zero(i.e.some informations are missing).So, we can say that though the angular part is separable cleanly,it is not possible to separate all the partial waves out cleanly.However, we can separate all the partial waves out into CP even and CP odd. Consequently, it is not possible to separate the partial waves out within CP even (i.e.we can't distinguish between S and D waves in this mode) .Similarly we can't separate the partial waves out within CP odd ((i.e.we can't distinguish between P and F waves in this mode) [9].Therefore, in the decay modes we can get partial information about all the partial waves.However, it is sufficient information to study CP violation in the decay process.

3.2 Differential Decay Rate

On rewriting the eqn(2.49) as

$$|M|^2 \;=\; a^0_{00} + a^0_{01}\cos\theta_k + a^0_{20}\cos^2\theta_\ell + a^0_{02}\cos^2\theta_k + a^0_{21}\cos^2\theta_\ell\cos\theta_k +$$

$$
\begin{aligned}
&a_{22}^{0}\cos^2\theta_\ell\cos^2\theta_k + a_{24}^{0}\cos^2\theta_\ell\cos^4\theta_k + d_{10}^{1}\sin 2\theta_\ell\sin\theta_k\cos\phi +\\
&d_{11}^{1}\sin 2\theta_\ell\sin 2\theta_k\cos\phi + d_{12}^{1}\sin 2\theta_\ell\sin 2\theta_k\cos\theta_k\cos\phi +\\
&d_{13}^{1}\sin 2\theta_\ell\sin 2\theta_k\cos^2\theta_k\cos\phi + a_{00}^{2}\sin^2\theta_\ell\sin^2\theta_k\cos^2\phi +\\
&a_{01}^{2}\sin^2\theta_\ell\sin^2\theta_k\cos\theta_k\cos^2\phi + a_{24}^{2}\sin^2\theta_\ell\sin^2\theta_k\cos^2\theta_k\cos^2\phi +\\
&d_{10}^{3}\sin 2\theta_\ell\sin\theta_k\sin\phi + d_{11}^{3}\sin 2\theta_\ell\sin 2\theta_k\sin\phi +\\
&d_{12}^{1}\sin 2\theta_\ell\sin 2\theta_k\cos\theta_k\sin\phi + d_{13}^{1}\sin 2\theta_\ell\sin 2\theta_k\cos^2\theta_k\sin\phi +\\
&a_{00}^{4}\sin^2\theta_\ell\sin^2\theta_k\sin 2\phi + a_{01}^{4}\sin^2\theta_\ell\sin^2\theta_k\cos\theta_k\sin 2\phi +\\
&a_{24}^{4}\sin^2\theta_\ell\sin^2\theta_k\cos^2\theta_k\sin 2\phi \qquad (3.1)
\end{aligned}
$$

We know that only for angular distribution, differential decay rate is proportional to the square of matrix elements [9] i.e.

$$\frac{d\Gamma}{d\cos\theta_\ell\ d\cos\theta_k\ d\phi} \quad \propto \quad |M|^2 \qquad (3.2)$$

$$\frac{d\Gamma}{d\cos\theta_\ell\ d\cos\theta_k\ d\phi} \quad = \quad \Omega|M|^2 \qquad (3.3)$$

,where Ω is proportionality constant and independent of θ_ℓ,θ_k and ϕ ,
and θ_ℓ is the angle of the l^- in the l^-l^+ c.m.frame ;θ_k is the angle of the K in the $K\pi$ system; ϕ is the angle between the two planes l^-l^+ and $K\pi$.And the physical regions are

$$0 \le \phi \le 2\pi, -1 \le \cos\theta_k \le 1 \text{ and } -1 \le \cos\theta_\ell \le 1 \qquad (3.4)$$

3.3 Angular Distribution

We derive one-dimensional angular distributions namely $\frac{d\Gamma}{d\cos\theta_\ell}$,$\frac{d\Gamma}{d\cos\theta_k}$, and $\frac{d\Gamma}{d\phi}$ from the differential decay rate .These distributions as well as the observable are calculated ,depend on different combinations of the co-efficients 'a_{ij}^{p}'

3.3.1 Decay rate as a function of $\cos\theta_k$

Integrating the eqn(3.3) over $\cos\theta_\ell$ and ϕ we obtain

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta_k} &= 2\pi\Omega\{a_{00}^0 + \frac{1}{3}(2a_{20}^0 + a_{00}^2) + (2a_{01}^0 + \frac{1}{3}a_{01}^2)\cos\theta_k + \\ &\frac{1}{3}(6a_{02}^0 + 2a_{21}^0 + 2a_{22}^0 - a_{00}^2 + a_{24}^2)\cos^2\theta_k - \frac{1}{3}a_{01}^2\cos^3\theta_k + \\ &\frac{1}{3}(2a_{24}^0 - a_{24}^2)\cos^4\theta_k\} \end{aligned} \quad (3.5)$$

Now we define the forward-backward(FB) asymmetry in $K\pi$ system

$$\begin{aligned} A_{FB}^K &= \frac{\int_0^1 \frac{d\Gamma}{d\cos\theta_k}d\cos\theta_k - \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_k}d\cos\theta_k}{\int_0^1 \frac{d\Gamma}{d\cos\theta_k}d\cos\theta_k + \int_{-1}^0 \frac{d\Gamma}{d\cos\theta_k}d\cos\theta_k} \\ &= \frac{2a_{01}^0 + \frac{1}{6}a_{01}^2}{2a_{00}^0 + \frac{4}{3}(a_{20}^0 + a_{02}^0 + \frac{1}{3}a_{21}^0 + \frac{1}{3}a_{22}^0 + \frac{1}{5}a_{24}^0 + \frac{1}{3}a_{00}^2 + \frac{1}{15}a_{24}^2)} \\ &\neq 0 \end{aligned} \quad (3.6)$$

This is not vanishing due to the presence of $\cos\theta_k$ and $\cos^3\theta_k$; these terms are present due interference between vector and scalar,between vector and tensor .However, it vanishes in each separate decay mode

3.3.2 Decay rate as a function of $\cos\theta_\ell$

Integrating the eqn(3.3) over $\cos\theta_k$ and ϕ we obtain

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta_\ell} &= 4\pi\Omega\{a_{00}^0 + \frac{1}{3}a_{00}^2 + \frac{1}{15}a_{24}^2 + \\ &(a_{01}^0 + \frac{1}{3}a_{22}^0 + \frac{1}{5}a_{24}^0 - \frac{1}{3}a_{00}^0 - \frac{1}{15}a_{24}^2)\cos^2\theta_\ell\} \end{aligned} \quad (3.7)$$

Similarly the forward-backward(FB) asymmetry in l^-l^+ system

$$A_{FB}^l = 0 \quad (3.8)$$

The absence of a term odd in $\cos\theta_\ell$ is connected with the fact that the l^-l^+ system is in a pure $L = 1$ state.As a consequence, the forward-backward(FB) asymmetry in the system vanishes

3.3.3 Decay rate as a function of ϕ

Finally ,the distribution in the angle ϕ between the lepton and meson planes, after integration the eqn(3.3) over other variables takes the form

$$\begin{aligned}\frac{d\Gamma}{d\phi} &= 4\{a^0_{00} + \frac{1}{3}(a^0_{20} + a^0_{02} + a^0_{21} + a^0_{22} + \frac{1}{5}a^0_{24} + \frac{1}{3}a^2_{00} + \frac{1}{9}a^2_{24}) + \\ &\quad \frac{1}{9}(a^2_{00} + \frac{1}{5}a^2_{24})\cos 2\phi + \frac{2}{9}(a^4_{00} + \frac{1}{5}a^4_{24})\sin 2\phi\} \qquad (3.9)\end{aligned}$$

The presence of $\sin 2\phi$ term is a clean signal of CP violation in the ϕ distribution in the decay process.

Chapter 4

CONCLUSION

We used the most general effective matrix elements for each of the decay channels using Lorenz invariance and current conservation for the angular analysis of B decaying into scalar, vector and tensor modes . The decay spectrum for the final state $K\pi\ell^+\ell^-$ is calculated. The $K^\star_X(1430)$, $X = 0, 2$ and $K^\star(1410)$ are considered to decay to $K\pi$ and J/ψ to $\ell^+\ell^-$. We study the angular distribution of the K in the $K\pi$ center of mass (c.m.) frame and ℓ^- in the $\ell^+\ell^-$ c.m. frame. We also study the correlation between the $K\pi$ decay plane and the $\ell^+\ell^-$ decay frame.

We analyzed the angular part and found ,"the angular part is separated out cleanly,however it is not possible to separate all the partial waves out cleanly;Nevertheless all the partial waves can be separated out into CP even and CP odd. "This separation of CP even and CP odd is necessary to study CP violation

We derived one-dimensional angular distributions namely $\frac{d\Gamma}{d\cos\theta_\ell}, \frac{d\Gamma}{d\cos\theta_k}$, and $\frac{d\Gamma}{d\phi}$ from the differential decay rate .These distributions as well as the observable are calculated. And we found that the forward-backward(FB) asymmetry in $K\pi$ system is not vanishing due to the presence of $\cos\theta_k$ and $\cos^3\theta_k$; these terms are present due interference between vector and scalar,as well as between vector and tensor.However, the forward-backward(FB) asymmetry in l^-l^+ system vanishes due to the absence of a term odd in $\cos\theta_\ell$ is connected with the fact that the l^-l^+ system is in a pure $L = 1$ state.Finally ,the distribution in the angle ϕ between the lepton and meson planes has calculated and this will be very useful to study CP violation in these modes

Recently, large samples of data have collected on the decay $B \to K_2^\star(1430)J/\psi$ by the B factories running at KEK(Japan) and SLAC(U.S.A). These results will be useful to perform a detail analysis of the data collected so far.

Chapter 5

APPENDICES

5.1 Kinematics

Table 5.1: **Notation and Defination of Entities**

S.No.	Entities	Notation
1.	The four momentum of J/ψ in C.M. Frame of B	q
2.	The four momentum of $K_X^\star(1430)/K^\star(1410)$ in C.M. Frame of B	k
3.	The four momentum of lepton in C.M. Frame of J/ψ	q_1
4.	The four momentum of antilepton in C.M. Frame of J/ψ	q_2
5.	The four momentum of K in C.M. Frame of $K_X^\star(1430)/K^\star(1410)$,where $X = 0, 2$	k_1
6.	The four momentum of π in C.M. Frame of $K_X^\star(1430)/K^\star(1410)$,where $X = 0, 2$	k_2
7.	B^2	M_B^2
8.	k^2	s_k
9.	q^2	s_ℓ
10.	k_1^2	M_k^2

S.No.	Entities	Notation
11.	k_2^2	$M_{k'}^2$
12.	For massless leptons q_1^2	0
13.	For massless antileptons q_2^2	0
14.	in C.M. Frame of J/ψ $q_1 - q_2$	Q
15.	in C.M. Frame of $K_X^\star(1430)/K^\star(1410)$ $k_1 - k_2$	K
16.	$a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$	$\lambda(a, b, c)$
17.	$\frac{(M_k^2 - M_{k'}^2)}{s_k}$	ξ
18.	$\frac{\lambda^{1/2}(k^2, M_k^2, M_{k'}^2)}{k^2}$	β
19.	$\frac{1}{4}\lambda(M_B^2, s_\ell, s_k)$	X^2
20.	The unit vector along k	$\hat{n}$
21.	The unit vector along J/ψ	$-\hat{n}$
22.	The unit vector normal to $K_X^\star(1430)/K^\star(1410)$ frame ,where $X = 0, 2$	$\overrightarrow{c}$
23.	The unit vector normal to J/ψ frame	$\overrightarrow{d}$
24.	The angle between $K_X^\star(1430)/K^\star(1410)$ plane and $\overrightarrow{k_1}$,where $X = 0, 2$	θ_k
25.	The angle between J/ψ and $\overrightarrow{q_1}$	θ_ℓ
26.	The angle between the planes J/ψ and $K_X^\star(1430)/K^\star(1410)$,where $X = 0, 2$ (i.e. between $\overrightarrow{c}$ and $\overrightarrow{d}$)	ϕ

On using the four momentum conservation in C.M.frame of B

$$B = k + q \tag{5.1}$$

On squaring and putting the value of B^2 , k^2 and q^2 we have

$$\boxed{k.q = \frac{1}{2}(M_B^2 - s_k - s_\ell)} \tag{5.2}$$

In k frame on contracting between $k = k_1 + k_2$ and $K = k_1 - k_2$ and putting the value $k_1^2 = M_k^2$ and $k_2^2 = M_{k'}^2$ we have

$$k.K = k^\mu K_\mu = k_1^2 - k_2^2 = M_k^2 - M_{k'}^2 \tag{5.3}$$

From S.No.17 from table(5.1)

$$\boxed{k.K = \xi s_k} \tag{5.4}$$

Similarly on contracting between K and K and putting the value $k_1^2 = M_k^2$,$k_2^2 = M_{k'}^2$ and $2k_1.k_2 = k^2 - k_1^2 - k_2^2$

$$\begin{aligned} K^2 &= K^\mu K_\mu \\ &= k_1^2 + k_2^2 - 2k_1.k_2 \\ &= 2(k_1^2 + k_2^2) - k^2 \\ &= 2(M_k^2 + M_{k'}^2) - s_k \end{aligned} \tag{5.5}$$

$$\boxed{K^2 = 2(M_k^2 + M_{k'}^2) - s_k} \tag{5.6}$$

And on contracting between Q and Q and putting the value $q_1^2 = 0$,$q_2^2 = 0$ and $2q_1.q_2 = q^2$

$$Q^2 = Q^\mu Q_\mu = q_1^2 + q_2^2 - 2q_1.q_2 = -s_\ell \tag{5.7}$$

$$\boxed{Q^2 = -s_\ell} \tag{5.8}$$

Now we contract between q and Q

$$q.Q = q^\mu Q_\mu = q_1^2 - q_2^2 = 0 \tag{5.9}$$

$$\boxed{q.Q = 0} \tag{5.10}$$

In J/ψ C.M. frame and for massless lepton and antilepton ,the energy of lepton and antilepton are same and momentum of lepton and antilepton are equal in magnitude but opposite in sign. so the components of Q in J/ψ C.M. frame are $(0, 2\overrightarrow{q_1})$ and the unit vector normal to $K^\star_X(1430)/K^\star(1410)$ frame $\overrightarrow{c} = \frac{\overrightarrow{k_1} - \overrightarrow{k_1}.\hat{n}\ \hat{n}}{\sqrt{|\overrightarrow{k_1}|^2 - (\overrightarrow{k_1}.\hat{n})^2}}$ [10]. Similarly, the unit vector normal to J/ψ frame $\overrightarrow{d} = \frac{\overrightarrow{q_1} - \overrightarrow{q_1}.\hat{n}\ \hat{n}}{\sqrt{|\overrightarrow{q_1}|^2 - (\overrightarrow{q_1}.\hat{n})^2}}$. Therefore, on writing the value of $\overrightarrow{k}_1$ and $\overrightarrow{q}_1$ with respect to $\overrightarrow{c}$, $\overrightarrow{d}$ respectively.

$$\overrightarrow{k_1} = \sqrt{|\overrightarrow{k_1}|^2 - (\overrightarrow{k_1}.\hat{n})^2}\ \overrightarrow{c} + (\overrightarrow{k_1}.\hat{n})\hat{n} \tag{5.11}$$

$$\overrightarrow{q_1} = \sqrt{|\overrightarrow{q_1}|^2 - (\overrightarrow{q_1}.\hat{n})^2}\ \overrightarrow{d} + (\overrightarrow{q_1}.\hat{n})\hat{n} \tag{5.12}$$

Now on contracting between $\overrightarrow{k_1}$ and $\overrightarrow{q_1}$

$$\overrightarrow{k_1}.\overrightarrow{q_1} = |k_1||q_1|(\sin\theta_k \sin\theta_\ell \cos\phi - \cos\theta_k \cos\theta_\ell) \tag{5.13}$$

In C.M. frame of B ,the momentum of $K^\star_X(1430)$ or $K^\star(1410)$ and J/ψ frames are equal in magnitudes and opposite in sign .so let the velocity of $K^\star_X(1430)$ or $K^\star(1410)$ frame $\overrightarrow{V} = \frac{|\overrightarrow{k}|}{k_0}\hat{n}$ and the velocity of J/ψ frame $\overrightarrow{V'} = -\frac{|\overrightarrow{q}|}{q_0}\hat{n}$ since $\gamma = \frac{1}{\sqrt{1-V^2}}$ so $\gamma = \frac{k_0}{\sqrt{s_k}}$. similarly $\gamma' = \frac{q_0}{\sqrt{s_\ell}}$

On boosting Q from J/ψ C.M. frame to B C.M. frame

$$\begin{aligned} Q_0 &= \gamma(Q'_0 + \overrightarrow{V}.2\overrightarrow{q'_1}) \\ &= \frac{q_0}{\sqrt{s_\ell}}(0 - \frac{2|\overrightarrow{q}|}{q_0}\hat{n}.\overrightarrow{q'_1}) \\ &= \frac{q_0}{\sqrt{s_\ell}}\frac{2|\overrightarrow{q}|}{q_0}|q'_1|\cos\theta_\ell \\ &= |\overrightarrow{q}|\cos\theta_\ell \\ &= |\overrightarrow{k}|\cos\theta_\ell \end{aligned} \tag{5.14, 5.15}$$

$$\Rightarrow Q_0 = |\overrightarrow{k}|\cos\theta_\ell \tag{5.16}$$

$$\begin{aligned} \overrightarrow{Q} &= 2\overrightarrow{q_1} + (\gamma - 1)\frac{(2\overrightarrow{q}.\overrightarrow{V})}{|V|^2}\overrightarrow{V} \\ &= 2\overrightarrow{q_1} + 2(\gamma - 1)(\overrightarrow{q}.\hat{n})\hat{n} \end{aligned} \tag{5.17}$$

$$\Rightarrow \overrightarrow{Q} \;=\; 2\overrightarrow{q_1'} - 2(\gamma-1)|q_1'|\cos\theta_\ell \hat{n} \tag{5.18}$$

Now, on contracting between k and Q in B C.M. frame

$$\begin{aligned}
k.Q &= k_0|q|\cos\theta_\ell + 2|k||\overrightarrow{q_1'}|\cos\theta_\ell + 2(\gamma-1)|k||\overrightarrow{q_1'}|\cos\theta_\ell \\
&= (k_0 + 2|\overrightarrow{q_1'}|\gamma)|k|\cos\theta_\ell \\
&= (k_0 + q_0)|k|\cos\theta_\ell \\
&= M_B\frac{1}{2M_B}\ \lambda^{1/2}(M_B^2, k^2, q^2)\cos\theta_\ell \\
\Rightarrow k.Q &= X\cos\theta_\ell
\end{aligned} \tag{5.19}$$

$$\boxed{k.Q = X\cos\theta_\ell} \tag{5.20}$$

Similarly, on boosting K' from $K_X^\star(1430)$ or$K^\star(1410)$ C.M.frame to B C.M.frame and on contracting between q and K in B C.M. frame

$$\boxed{q.K = \xi q.k + \beta X\cos\theta_k} \tag{5.21}$$

And contracting between K and Q in B C.M.frame

$$\begin{aligned}
K.Q &= K_0Q_0 - \overrightarrow{K}.\overrightarrow{Q} \\
&= (\xi k_0 + 2\frac{|\overrightarrow{k}||\overrightarrow{k_1}|}{\sqrt{s_k}}\cos\theta_k)|\overrightarrow{k}|\cos\theta_\ell - \\
&\quad (\overrightarrow{2k_1} + 2(\gamma'-1)|\overrightarrow{k_1}|\cos\theta_k\hat{n} + \frac{\gamma'\xi\sqrt{s_k}|\overrightarrow{k}|}{k_0}\hat{n}). \\
&\quad (2\overrightarrow{q_1'} - 2(\gamma-1)|q_1|\cos\theta_\ell\ \hat{n})
\end{aligned} \tag{5.22}$$

$$\Rightarrow K.Q \;=\; \xi k.Q + \beta[k.q\cos\theta_k\cos\theta_\ell - \sqrt{s_\ell s_k}\sin\theta_k\sin\theta_\ell\cos\phi] \tag{5.23}$$

$$\boxed{K.Q = \xi k.Q + \beta[k.q\cos\theta_k\cos\theta_\ell - \sqrt{s_\ell s_k}\sin\theta_k\sin\theta_\ell\cos\phi]} \tag{5.24}$$

$$\begin{aligned}
\epsilon_{\mu\nu\alpha\beta}k^\mu K^\nu q^\alpha Q^\beta &= \epsilon_{\mu\nu\alpha\beta}(B^\mu - q^\mu)K^\nu q^\alpha Q^\beta \\
&= \epsilon_{\mu\nu\alpha\beta}B^\mu K^\nu q^\alpha Q^\beta - \epsilon_{\mu\nu\alpha\beta}q^\mu K^\nu q^\alpha Q^\beta
\end{aligned}$$

$$
\begin{aligned}
&= \epsilon_{0\nu\alpha\beta} M_B K^\nu q^\alpha Q^\beta \\
&= M_B \epsilon_{\nu\alpha\beta} K^\nu q^\alpha Q^\beta \\
&= M_B (\overrightarrow{K} \times \overrightarrow{q}).\overrightarrow{Q} \\
&= -M_B \;|\overrightarrow{q}|\; (\overrightarrow{K} \times \hat{n}).\overrightarrow{Q} \\
&= -M_B \;|\overrightarrow{q}|\; [\{\overrightarrow{2k'_1} + 2(\gamma' - 1)|\overrightarrow{k'_1}| \cos\theta_k \hat{n} + \\
&\quad \frac{\gamma' \xi \sqrt{s_k} |\overrightarrow{k}|}{k_0} \hat{n}\} \times \hat{n}].[2\overrightarrow{q'_1} - 2(\gamma - 1)|q'_1| \cos\theta_\ell \; \hat{n}] \\
&= -4M_B \;|\overrightarrow{k}|\; (\overrightarrow{k'_1} \times \hat{n}).\overrightarrow{q'_1} \\
&= -4M_B \;|\overrightarrow{k}| [\{(\sqrt{|\overrightarrow{k'_1}|^2 - (\overrightarrow{k'_1}.\hat{n})^2} \;\overrightarrow{c} + \\
&\quad (\overrightarrow{k'_1}.\hat{n})\hat{n}) \times \hat{n}\}.(\sqrt{|\overrightarrow{q'_1}|^2 - (\overrightarrow{q'_1}.\hat{n})^2} \;\overrightarrow{d} + (\overrightarrow{q'_1}.\hat{n})\hat{n})] \\
&= -4M_B \;|\overrightarrow{k}||\overrightarrow{k'_1}||\overrightarrow{q'_1}| \sin\theta_k \sin\theta_\ell \sin\phi
\end{aligned} \tag{5.25}
$$

$$
\boxed{\epsilon_{\mu\nu\alpha\beta} k^\mu K^\nu q^\alpha Q^\beta = -\sqrt{s_\ell s_k} \beta X \sin\theta_k \sin\theta_\ell \sin\phi} \tag{5.26}
$$

5.2 Detailed Calculation of Matrix Elements

$$
\theta^{\mu\nu} k_\nu = (-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}) k_\nu = -k^\mu + k^\mu = 0 \tag{5.27}
$$

$$
\boxed{\theta^{\mu\nu} k_\nu = 0} \tag{5.28}
$$

Similarly, on contracting between θ and four momentum we have

$$
\boxed{\theta^{\rho\sigma} q_\rho = 0} \tag{5.29}
$$

$$
\boxed{\theta^{\rho\sigma} Q_\rho = -Q^\sigma} \tag{5.30}
$$

$$
\theta^{\rho\sigma} g_{\sigma\sigma'} = (-g^{\rho\sigma} + \frac{q^\rho q^\sigma}{q^2}) g_{\sigma\sigma'} = -\delta^\rho_{\sigma'} + \frac{q^\rho q_{\sigma'}}{q^2} \tag{5.31}
$$

$$
\boxed{\theta^{\rho\sigma} g_{\sigma\sigma'} = -\delta^\rho_{\sigma'} + \tfrac{q^\rho q_{\sigma'}}{q^2}} \tag{5.32}
$$

$$\theta^{\rho\sigma}k_\rho \;=\; (-g^{\rho\sigma} + \frac{q^\rho q^\sigma}{q^2})k_\rho = -k^\sigma + \frac{k.q}{q^2}q^\sigma = F^\sigma_{(k,q)} \tag{5.33}$$

$$\boxed{\theta^{\rho\sigma}k_\rho = F^\sigma_{(k,q)}} \tag{5.34}$$

$$\boxed{\theta^{\mu\nu}q_\nu = F^\mu_{(q,k)}} \tag{5.35}$$

$$\boxed{\theta^{\mu\nu}K_\nu = F^\mu_{(K,k)}} \tag{5.36}$$

On contraction between F's and momentum and using the results from kinematics we have

$$\begin{aligned} F_{(k,q)}.k &= (-k^\sigma + \frac{k.q}{q^2}q^\sigma)k_\sigma \\ &= -k^2 + \frac{(k.q)^2}{q^2} = \frac{X^2}{s_\ell} \end{aligned} \tag{5.37}$$

$$\boxed{F_{(k,q)}.k = \tfrac{X^2}{s_\ell}} \tag{5.38}$$

Similarly, we get the other relations

$$\boxed{F_{(k,q)}.q = 0} \tag{5.39}$$

$$\boxed{F_{(k,q)}.K = \tfrac{X}{s_\ell}(\xi X + \beta k.q\cos\theta_k)} \tag{5.40}$$

$$\boxed{F_{(k,q)}.Q = -X\cos\theta_\ell} \tag{5.41}$$

$$\boxed{F^2_{(k,q)}q^2 = -X^2} \tag{5.42}$$

$$\boxed{F_{(K,k)}.k = 0} \tag{5.43}$$

$$\boxed{F_{(K,k)}.q = F_{(q,k)}.K = -\beta X\cos\theta_k} \tag{5.44}$$

$$\boxed{F_{(K,k)}.K = \beta^2 s_k} \tag{5.45}$$

$$\boxed{F_{(K,k)}.Q = -\beta(k.q\cos\theta_\ell\cos\theta_k - \sqrt{s_\ell s_k}\sin\theta_\ell\sin\theta_k\cos\phi)} \tag{5.46}$$

$$\boxed{F_{(q,k)}.k = 0} \tag{5.47}$$

$$\boxed{F_{(q,k)}.q = \tfrac{X^2}{s_k}} \tag{5.48}$$

$$\boxed{F_{(q,k)}.Q = \tfrac{k.qX\cos\theta_\ell}{s_k}} \tag{5.49}$$

$$\boxed{F_{(K,k)}.F_{(k,q)} = -\tfrac{\beta X k.q}{s_\ell}\cos\theta_k} \tag{5.50}$$

$$\boxed{F_{(q,k)}.F_{(k,q)} = \tfrac{k.qX^2}{s_\ell s_k}} \tag{5.51}$$

Again, on contraction between E and q and using the results from kinematics we get

$$\begin{aligned} E.q &= (K^\sigma - \frac{(k.K)}{k^2}k^\sigma - \frac{(q.K)}{q^2}q^\sigma + \frac{(k.K)(k.q)}{k^2q^2}q^\sigma)q_\sigma \\ &= q.K - \frac{(k.K)}{k^2}k.q - q.K + \frac{(k.K)}{k^2}k.q = 0 \end{aligned} \tag{5.52}$$

$$\boxed{E.q = 0} \tag{5.53}$$

Similarly, on contracting between E and k and using the results from kinematics we get

$$\boxed{E.k = -\tfrac{\beta X(k.q)}{s_\ell}\cos\theta_k} \tag{5.54}$$

$$\begin{aligned} E.Q &= (K^\sigma - \frac{(k.K)}{k^2}k^\sigma - \frac{(q.K)}{q^2}q^\sigma + \frac{(k.K)(k.q)}{k^2q^2}q^\sigma)Q_\sigma \\ &= K.Q - \frac{(k.K)(k.Q)}{k^2} \\ &= \beta(k.q\cos\theta_\ell\cos\theta_k - \sqrt{s_\ell s_k}\sin\theta_\ell\sin\theta_k\cos\phi) \end{aligned} \tag{5.55}$$

$$\boxed{E.Q = \beta(k.q\cos\theta_\ell\cos\theta_k - \sqrt{s_\ell s_k}\sin\theta_\ell\sin\theta_k\cos\phi)} \tag{5.56}$$

Similarly the other relations are

$$\boxed{E^2q^2 = -\beta^2(s_\ell s_k + X^2\cos^2\theta_k)} \tag{5.57}$$

$$\boxed{E.F_{(k,q)}q^2 = \beta X k.q\cos\theta_k} \tag{5.58}$$

$$\boxed{E.F_{(q,k)}q^2 = -\tfrac{\beta X(k.q)^2}{s_k}\cos\theta_k} \tag{5.59}$$

$$\boxed{E.F_{(K,k)} = \tfrac{\beta^2}{s_\ell}(s_\ell s_k + X^2\cos^2\theta_k)} \tag{5.60}$$

Now again on contracting between S and four momentums and using the relations from kinematics we can get

$$\begin{aligned} S.q &= (F_{(K,k)}.qF^\sigma_{(K,k)} - \frac{F_{(K,k)}.K}{3}F^\sigma_{(q,k)} - \frac{(F_{(K,k)}.q)^2}{k.q}k^\sigma + \frac{F_{(K,k)}.KF_{(q,k)}.q}{3k.q}k^\sigma)q_\sigma \\ &= (F_{(K,k)}.q)^2 - \frac{F_{(K,k)}.K}{3}F_{(q,k)}.q - (F_{(K,k)}.q)^2 + \frac{F_{(K,k)}.K}{3}F_{(q,k)}.q \\ &= 0 \end{aligned} \tag{5.61}$$

$$\boxed{S.q = 0} \tag{5.62}$$

Similarly, on contracting between S and Q

$$\boxed{S.Q = \tfrac{\beta^2 X s_\ell s_k}{k.q}\cos\theta_\ell(\cos^2\theta_k - \tfrac{1}{3}) - \tfrac{1}{2}\beta^2 X\sqrt{s_\ell s_k}\sin\theta_\ell\sin 2\theta_k\cos\phi} \tag{5.63}$$

And the value of S^2q^2 is

$$\boxed{S^2q^2 = \tfrac{\beta^4X^2s_\ell s_k}{(k.q)^2}\{X^2\cos^4\theta_k - ((k.q)^2 - \tfrac{2}{3}s_\ell s_k)\cos^2\theta_k - \tfrac{1}{9}s_\ell s_k\}} \tag{5.64}$$

Now again on contracting between R and four momenta
So, the value $R.q$ is

$$\boxed{R.q = 0} \tag{5.65}$$

The value of $R.Q$ is

$$\boxed{R.Q = \beta^2 X^2 \sqrt{s_\ell s_k} \sin 2\theta_k \sin\theta_\ell \sin\phi} \tag{5.66}$$

$$\boxed{R^2 q^2 = 0} \tag{5.67}$$

$$\boxed{R.Eq^2 = 0} \tag{5.68}$$

$$\boxed{R.F_{(k,q)} q^2 = 0} \tag{5.69}$$

$$\boxed{R.Sq^2 = 0} \tag{5.70}$$

$$\boxed{S.F_{(k,q)} q^2 = \tfrac{\beta^2 X^2 s_\ell s_k}{k.q}(\cos^2\theta_k - \tfrac{1}{3})} \tag{5.71}$$

$$\boxed{S.Eq^2 = -\tfrac{2}{3}\beta^3 X s_\ell s_k \cos\theta_k} \tag{5.72}$$

On contracting between $L'_{\sigma\sigma'}$ and other entities and using the previous relations The value of $\theta^{\rho'\sigma'} L'_{\sigma\sigma'}$ is

$$\theta^{\rho'\sigma'} L'_{\sigma\sigma'} = \theta^{\rho'\sigma'}(q_\sigma q_{\sigma'} - Q_\sigma Q_{\sigma'} - g_{\sigma\sigma'} q^2) \tag{5.73}$$

$$\boxed{\theta^{\rho'\sigma'} L'_{\sigma\sigma'} = Q_\sigma Q^{\rho'} + \delta^{\rho'}_\sigma q^2 - q^{\rho'} q_\sigma} \tag{5.74}$$

Similarly, the value of $\theta^{\rho\sigma}\theta^{\rho'\sigma'} L'_{\sigma\sigma'}$ is

$$\boxed{\theta^{\rho\sigma}\theta^{\rho'\sigma'} L'_{\sigma\sigma'} = -Q^\rho Q^{\rho'} + q^2\theta^{\rho\rho'}} \tag{5.75}$$

$$\boxed{F^\sigma_{(k,q)} F^{\sigma'}_{(k,q)} L'_{\sigma\sigma'} = X^2(1 - \cos^2\theta_\ell)} \tag{5.76}$$

Using the previous results the value of $E^\sigma E^{\sigma'} L'_{\sigma\sigma'}$ is

$$\begin{aligned} E^\sigma E^{\sigma'} L'_{\sigma\sigma'} &= \beta^2\{s_\ell s_k + X^2\cos^2\theta_k - (k.q)^2\cos^2\theta_\ell\cos^2\theta_k - \\ &\quad s_\ell s_k \sin^2\theta_\ell \sin^2\theta_k \cos^2\phi + \frac{1}{2}\} \\ &\quad \sqrt{s_\ell s_k} k.q \sin 2\theta_\ell \sin 2\theta_k \cos\phi \end{aligned} \tag{5.77}$$

Similarly the contracting between E,F and $L'_{\sigma\sigma'}$

$$\begin{aligned} E^{\sigma}F^{\sigma'}_{(k,q)}L'_{\sigma\sigma'} &= -\beta X\{k.q\sin^2\theta_\ell\cos\theta_k + \\ &\quad \frac{1}{2}\sqrt{s_\ell s_k}\sin 2\theta_\ell\sin\theta_k\cos\phi\} \end{aligned} \tag{5.78}$$

On contracting between $\Gamma^{\sigma'}$,$L'_{\sigma\sigma'}$ and E^{σ}

$$\begin{aligned} E^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} &= \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}F^{\mu'}_{(K,k)}E^{\sigma}\theta^{\rho'\sigma'}L'_{\sigma\sigma'} \\ &= \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}(-K^{\mu}+\frac{(k.K)}{k^2}k^{\mu})(E.QQ^{\rho'}+E^{\rho'}q^2) \\ &= -E.Q\ \epsilon_{\mu'\rho'\alpha'\beta'}k^{\alpha'}q^{\beta'}K^{\mu}Q^{\rho'} \\ &= -E.Q\ \epsilon_{\rho'\beta'\mu'\alpha'}k^{\mu'}K^{\rho'}q^{\alpha'}Q^{\beta'} \\ &= E.Q\epsilon_{\mu'\rho'\alpha'\beta'}k^{\mu'}K^{\rho'}q^{\alpha'}Q^{\beta'} \\ &= \beta\{k.q\cos\theta_\ell\cos\theta_k - \sqrt{s_\ell s_k}\sin\theta_\ell\sin\theta_k\cos\phi\} \\ &\quad (-\sqrt{s_\ell s_k}\beta X\sin\theta_k\sin\theta_\ell\sin\phi) \end{aligned} \tag{5.79}$$

So, the value of $E^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'}$ is

$$\begin{aligned} \Rightarrow E^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} &= -\beta^2 X\{\frac{(k.q)\sqrt{s_\ell s_k}}{4}\sin 2\theta_\ell\sin 2\theta_k\sin\phi - \\ &\quad \frac{s_\ell s_k}{2}\sin^2\theta_\ell\sin^2\theta_k\sin 2\phi\} \end{aligned} \tag{5.80}$$

Doing in the same way the value of the following relations are

$$F^{\sigma}_{(k,q)}\Gamma^{\sigma'}L'_{\sigma\sigma'} = \frac{1}{2}\beta X^2\sqrt{s_\ell s_k}\sin 2\theta_\ell\sin\theta_k\sin\phi \tag{5.81}$$

$$\Gamma^{\sigma\sigma'}L'_{\sigma\sigma'} = \beta^2 X^2 s_\ell s_k(\cos^2\theta_\ell + \sin^2\theta_\ell\cos^2\phi)\sin^2\theta_k[12] \tag{5.82}$$

$$\begin{aligned} S^{\sigma}S^{\sigma'}L'_{\sigma\sigma'} &= -\beta^4 X^2 s_l s_k\{\frac{X^2}{(k.q)^2}\cos^4\theta_k - \\ &\quad \frac{((k.q)^2-\frac{2}{3}s_\ell s_k)}{(k.q)^2}\cos^2\theta_k - \frac{s_\ell s_k}{9(k.q)^2} + \\ &\quad \frac{s_\ell s_k}{(k.q)^2}(\cos^4\theta_k - \frac{2}{3}\cos^2\theta_k + \frac{1}{9})\cos^2\theta_\ell + \\ &\quad \frac{1}{4}\sin^2 2\theta_k\sin^2\theta_\ell\cos^2\phi - \\ &\quad \frac{\sqrt{s_\ell s_k}}{2k.q}(\cos^2\theta_k - \frac{1}{3})\sin 2\theta_k\sin 2\theta_\ell\cos\phi\} \end{aligned} \tag{5.83}$$

$$R^{\sigma}R^{\sigma'}L'_{\sigma\sigma'} = \beta^4X^4 \; s_l s_k \sin^2 2\theta_k(cos^2\theta_\ell + \sin^2\theta_\ell\cos^2\phi) \tag{5.84}$$

$$\begin{aligned} R^{\sigma}S^{\sigma'}L'_{\sigma\sigma'} &= -\beta^4X^3s_\ell s_k\{\frac{\sqrt{s_\ell s_k}}{k.q}(\cos^2\theta_k - \frac{1}{3})\sin 2\theta_k\sin 2\theta_\ell\sin\phi \\ &\quad -\frac{1}{2}\sin^2 2\theta_k\sin^2\theta_\ell\sin 2\phi\} \end{aligned} \tag{5.85}$$

$$\begin{aligned} S^{\sigma}E^{\sigma'}L'_{\sigma\sigma'} &= \beta^3X\sqrt{s_\ell s_k}\{k.q\sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos^2\theta_k\cos\phi + \\ &\quad \frac{s_\ell s_k}{k.q}\sin\theta_\ell\sin\theta_k\cos\theta_\ell(\cos^2\theta_k - \frac{1}{3})\cos\phi - \\ &\quad \sqrt{s_\ell s_k}\cos^2\theta_\ell\cos\theta_k(\cos^2\theta_k - \frac{1}{3}) - \\ &\quad \sqrt{s_\ell s_k}(1 - \cos^2\theta_\ell - \cos^2\theta_k + \\ &\quad \cos^2\theta_\ell\cos^2\theta_k)\cos\theta_k\cos^2\phi + \frac{2}{3}\sqrt{s_\ell s_k}\cos\theta_k\} \end{aligned} \tag{5.86}$$

$$\begin{aligned} S^{\sigma}F^{\sigma'}_{(k,q)}L'_{\sigma\sigma'} &= \beta^2X^2\sqrt{s_\ell s_k}\{\frac{\sqrt{s_\ell s_k}}{k.q}\cos^2\theta_\ell(\cos^2\theta_k - \frac{1}{3}) - \\ &\quad \sin\theta_\ell\sin\theta_k\cos\theta_\ell\cos\theta_k\cos\phi \end{aligned} \tag{5.87}$$

Similarly ,on contracting between R,L' and Γ so the value of $R^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'}$ is

$$\begin{aligned} R^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} &= (\epsilon_{\mu\rho\alpha\beta}q_\nu + \epsilon_{\nu\rho\alpha\beta}q_\mu)k^\alpha q^\beta(-F^\mu_{K,k}F^\nu_{K,k} + \\ &\quad \frac{F_{K,k}.K}{3}\theta^{\mu\nu})\theta^{\rho\sigma}\Gamma' L'_{\sigma\sigma'} \\ &= (-2q.F_{K,k}\epsilon_{\mu\rho\alpha\beta}F^\mu_{K,k} + \frac{2}{3}K.F_{K,k}\epsilon_{\mu\rho\alpha\beta}F^\mu_{q,k}) \\ &\quad k^\alpha q^\beta\theta^{\rho\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} \\ &= -2q.F_{K,k}\epsilon_{\mu\rho\alpha\beta}F^\mu_{K,k}k^\alpha q^\beta\theta^{\rho\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} + 0 \\ &= -2q.F_{K,k}\Gamma^{\sigma\sigma'}L'_{\sigma\sigma'} \\ &= 2\beta^3X^3s_\ell s_k(\cos^2\theta_\ell + \sin^2\theta_\ell\cos^2\phi)\sin^2\theta_k\cos\theta_k \\ \Rightarrow R^{\sigma}\Gamma^{\sigma'}L'_{\sigma\sigma'} &= 2\beta^3X^3s_\ell s_k\{\cos^2\theta_\ell\cos\theta_k - \cos^2\theta_\ell\cos^3\theta_k - \\ &\quad (\cos\theta_k - \cos^2\theta_\ell\cos\theta_k - \cos^3\theta_k + \\ &\quad \cos^2\theta_\ell\cos^3\theta_k)\cos^2\phi\} \end{aligned} \tag{5.88}$$

$$S^\sigma \Gamma^{\sigma'} L'_{\sigma\sigma'} = \beta^3 X^2 s_\ell s_k \{ \frac{1}{2} \sin^2\theta_\ell \sin^2\theta_k \cos\theta_k \sin 2\phi + \frac{\sqrt{s_\ell s_k}}{k.q} (\sin\theta_\ell \sin\theta_k \cos\theta_\ell - \sin\theta_\ell \sin\theta_k \cos\theta_\ell \cos^2\theta_k) \sin\phi \} \quad (5.89)$$

$$R^\sigma E^{\sigma'} L'_{\sigma\sigma'} = \beta^3 X^2 \{ s_\ell s_k (\cos\theta_k - \cos^3\theta_k - \cos^2\theta_\ell \cos\theta_k + \cos^2\theta_\ell \cos^3\theta_k) \sin 2\phi - 2k.q\sqrt{s_\ell s_k} \sin\theta_\ell \sin\theta_k \cos\theta_\ell \cos^2\theta_k \sin\phi \} \quad (5.90)$$

$$R^\sigma F^{\sigma'}_{(k,q)} L'_{\sigma\sigma'} = 2\beta^2 X^3 \sqrt{s_\ell s_k} \sin\theta_\ell \sin\theta_k \cos\theta_\ell \cos\theta_k \sin\phi \quad (5.91)$$

5.3 Nonvanishing Co-efficients of Spherical Harmonics of the Matrix Elements

Table 5.2: **Nonvanishing Co-efficients of Spherical Harmonics of the Matrix Elements**

S.No.	co-efficients	expression
1.	a^0_{00}	$e^2 s_\ell s_k\{\beta^2\|b\|^2 + (x^2-1)(2\|a\|^2 - \frac{1}{3}\beta^2\mathrm{Re}(hb^\star) + \frac{1}{72}\beta^4 s_\ell s_k\|h\|^2)\}$
2.	a^0_{01}	$e^2\beta\sqrt{x^2-1}s_\ell s_k\{\frac{1}{3}\beta^2\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star) - 2x\mathrm{Re}(ba^\star) - 2(x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(ca^\star)\}$
3.	a^0_{02}	$e^2\beta^2(x^2-1)s_\ell s_k\{2\|b\|^2 + \mathrm{Re}(ha^\star) + 4x\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star) + 2(x^2-1)s_\ell s_k\|c\|^2 + \beta^2 s_\ell s_k\frac{(x^2-\frac{2}{3})}{x^2}\|h\|^2\}$
4.	a^0_{20}	$e^2(x^2-1)s_\ell s_k\{-2\|a\|^2 + \beta^2(2s_\ell s_k\|d\|^2 + \frac{1}{3}\mathrm{Re}(ha^\star)) + \frac{\beta^4 s_\ell s_k}{8x^2}(\|f\|^2 x^2(x^2-1)s_\ell s_k - \frac{1}{9}\|h\|^2)\}$
5.	a^0_{21}	$e^2\beta\sqrt{x^2-1}s_\ell s_k\{\frac{1}{3}\beta^2\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star) + 4x\mathrm{Re}(ba^\star) + (x^2-1)(4\mathrm{Re}(ca^\star) + \beta^2 s_\ell s_k(\frac{1}{3x}\mathrm{Re}(hc^\star) + \sqrt{s_\ell s_k}\mathrm{Re}(hd^\star)\}$
6.	a^0_{22}	$e^2\beta^2 s_\ell s_k\{\frac{\beta^2 s_\ell s_k}{8}\frac{(x^2-1)}{x^2}(\frac{2}{3}\|h\|^2 - x^2(x^2-1)s_\ell s_k\|f\|^2) - 2x^2\|b\|^2 - (x^2-1)(\mathrm{Re}(ha^\star) + 2s_\ell s_k\|d\|^2) - 4x(x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star) - 2(x^2-1)^2 s_\ell s_k\|c\|^2\}$
7.	a^0_{24}	$e^2\frac{1}{8}\beta^4\frac{(x^2-1)}{x^2}(s_\ell s_k)^2\|h\|^2$
8.	d^1_{10}	$-e^2\beta s_\ell s_k\sqrt{x^2-1}\{8\mathrm{Re}(ba^\star) + 4\sqrt{x^2-1}\sqrt{s_\ell s_k}\mathrm{Im}(da^\star) + \frac{\beta}{3x}\sqrt{s_\ell s_k}\mathrm{Re}(hb^\star)\}$
9.	d^1_{11}	$e^2\beta^2 s_\ell s_k\{4x\|b\|^2 + (x^2-1)(4\sqrt{s_\ell s_k}\mathrm{Re}(bc^\star) + \mathrm{Re}(ha^\star) - \frac{\beta^2}{12x}s_\ell s_k\|h\|^2)\}$
10.	d^1_{12}	$e^2\beta^2\sqrt{x^2-1}(s_\ell s_k)^{3/2}\{\frac{x^2+1}{x}\mathrm{Re}(hb^\star) + (x^2-1)\sqrt{s_\ell s_k}\mathrm{Re}(hc^\star)\}$

S.No.	co-efficients	expression
11.	d^1_{13}	$e^2\beta^4\frac{(x^2-1)}{4x}(s_\ell s_k)^2\|h\|^2$
12.	a^2_{00}	$2e^2\beta^2 s_\ell s_k\{\|d\|^2(x^2-1)s_\ell s_k - \|b\|^2 +$ $\frac{1}{16}\beta^2(x^2-1)^2(s_\ell s_k)^2\|f\|^2\}$
13.	a^2_{01}	$-e^2\beta^3\sqrt{x^2-1}(s_\ell s_k)^{3/2}\{\mathrm{Re}(hb^\star)+$ $2(x^2-1)s_\ell s_k\mathrm{Re}(fd^\star)\}$
14.	a^2_{20}	$-a^2_{00}$
15.	a^2_{02}	$-(a^2_{00}+a^2_{04})$
16.	a^2_{21}	$-a^2_{01}$
17.	a^2_{03}	$-a^2_{01}$
18.	a^2_{22}	$a^2_{00}+a^2_{04}$
19.	a^2_{04}	$\frac{1}{8}e^2\beta^4(x^2-1)(s_\ell s_k)^2\|h\|^2$
20.	a^2_{23}	a^2_{01}
21.	a^2_{24}	$-a^2_{04}$
22.	d^3_{10}	$-\beta^3(s_\ell s_k)^2\frac{(x^2-1)}{3x}\mathrm{Im}(hd^\star)$
23.	d^3_{11}	$\beta^2 s_\ell s_k\{(4(x^2-1)\sqrt{s_\ell s_k}\mathrm{Im}(dc^\star) - 4x\mathrm{Im}(bd^\star)+$ $(x^2-1)^{3/2}(2\mathrm{Im}(fa^\star) - \frac{\beta^2}{3x}(s_\ell s_k)^{3/2}\mathrm{Im}(hf^\star))\}$
24.	d^3_{12}	$-\beta^3(x^2-1)^2(s_\ell s_k)^2\{2(x^2-1)\sqrt{s_\ell s_k}\mathrm{Im}(fc^\star)+$ $2x\mathrm{Im}(fb^\star) - \frac{1}{x}\mathrm{Im}(hd^\star)\}$
25. .	d^3_{13}	$\beta^4(s_\ell s_k)^{5/2}\frac{(x^2-1)^{3/2}}{x}\mathrm{Im}(hf^\star)$
26.	a^4_{00}	$2\beta^2\sqrt{(x^2-1)}(s_\ell s_k)^{3/2}\mathrm{Im}(bd^\star)$
27.	a^4_{01}	$\beta^3(s_\ell s_k)^2(x^2-1)^2\{\mathrm{Im}(fb^\star) - \frac{1}{2}\mathrm{Im}(hd^\star) + \mathrm{Im}(hb^\star)\}$
28.	a^4_{04}	$-\frac{1}{2}\beta^4(x^2-1)^{3/2}(s_\ell s_k)^{5/2}\mathrm{Im}(hf^\star)$
29.	a^4_{20}	$-a^4_{00}$
30.	a^4_{02}	$-(a^4_{00}+a^4_{04})$
31.	a^4_{03}	$-a^4_{01}$
32.	a^4_{21}	$-a^4_{01}$
33.	a^4_{22}	$a^4_{00}+a^4_{04}$
34.	a^4_{23}	a^4_{01}
35.	a^4_{24}	$-a^4_{04}$

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