PROTON DECAY IN HIGHER DIMENSIONS

By

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BONAFIDE CERTIFICATE

Certified that this dissertation titled **Proton Decay In Higher Dimensions** is the bonafide work of **Mr. Turbasu Biswas** who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other dissertation on the basis of which a degree or award was conferred on an earlier occasion on this or any other candidate.

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Abstract

Proton decay is used as a guiding principle for the search of a Grand unified theory. As a starting point minimal nonsupersymmetric SU(5) model is taken as it is the simplest possible grand unified model. Then modifications are discussed which ultimately leads to a supersymmetric GUT in 5 dimension.The extra dimension in this theory is compactified on a $S^1/(Z_2 \times Z'_2)$ orbifold. A new $U(1)_R$ symmetry is discussed which forces all 4 dimensional and 5 dimensional proton decay operators to be absent.But when supersymmetry breaks, $U(1)_R$ symmetry breaks to R-parity and hence makes the 5 dimensional operators nonzero. An estimate of proton lifetime based on this 5 dimensional operator is calculated which constraints the magnitude of the radius of compactification of the extra dimension.

பணிச்சுருக்கம்

மேலான ஒருங்கிணைப்புக் கோட்பாட்டுக்(GUT)-கானத் தேடுதலில் புரோட்டான் சிதைவு ஒரு வழிகாட்டு நெறிமுறையாக பயன்படுகிறது. இதன் ஆரம்பமாக SU(5)-மாதிரி, ஒரு எளிய மேலான ஒருங்கிணைப்புக் கோட்பாட்டு மாதிரியாக எடுத்துக் கொள்ளப்படுகிறது. பின்னர் திருத்தங்கள்(மாற்றங்கள்) விவாதிக்கப்பட்டு, சூப்பர்சிம்மட்ரிக் GUT-மாதிரி ஐந்து பரிமாணத்தில் பெறப்படுகிறது. இந்த அதிகபட்ச பரிமாணம் ஆர்பிபோல்ட் $S^1/(Z_2 \times Z_2')$-ல் நெருக்கப்படுகிறது. ஒரு புதிய $U(1)_R$ சிம்மட்ரி பெறப்பட்டு அதன் மூலம் அனைத்து 4 மற்றும் 5 பரிமாண புரோட்டான் சிதைவு ஆப்ரேட்டர்கள் நீக்கப்படுகின்றன. ஆனால் சூப்பர் சிம்மட்ரி பிரிக்கப்படும் போது $U(1)_R$ சிம்மட்ரி பிரிந்து R-பாரிட்டி ஆகிறது. மேலும் இது 5 பரிமாண ஆப்ரேட்டரை சுழியாகாமல் செய்கிறது. இந்த ஐந்து பரிமாண ஆப்பரேட்டரினை அடிப்படையாகக் கொண்டு கணக்கிடப்பட்ட புரோட்டானின் ஆயுட்காலம் அந்த அதிகபட்ச பரிமாணத்தின் நெருக்குதலின் ஆரத்தின் மதிப்பினைக் கட்டுபடுத்துகிறது.

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Turbasu Biswas

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Chapter 1

INTRODUCTION

There are four forces that are needed to explain the present universe. Gravitational, strong , weak and electromagnetic.The present believe is that there is a more fundamental theory which will unify at least last three forces in a single theory. Standard model(SM) is by far the most successful theory towards this end. It unifies electromagnetic($U(1)_{EM}$) and weak ($SU(2)_W$) force into electroweak ($SU(2)_L \times U(1)_Y$) force and gives a description of strong ($SU(3)$) and electroweak interaction which is so far in agreement with experiment. The coupling constants of different forces are widely different at present energy. But these constants are functions of energy. The coupling constants according to SM become of comparable strength at some high energy. The running of the constants are shown in the Figure1.1[1]. Experimentally it is also found that the coupling constants are variable with energy. The possibility that the strength of different forces are comparable at some energy strengthens the idea of grand unification. In a grand unified model all the couplings converges into one coupling after some energy scale . But SM predicts that the couplings will diverge again, which may be taken as an indication of SM's invalidity beyond that energy scale.

Except the possibility of unification SM has some other disadvantages to be the

[1] Q in the figure is the energy squared.The line below is the $U(1)_Y$ coupling. Middle one is the $SU(2)_L$ coupling. and the top one is the $SU(3)$ color coupling.The dashed lines shows the region of uncertainty.

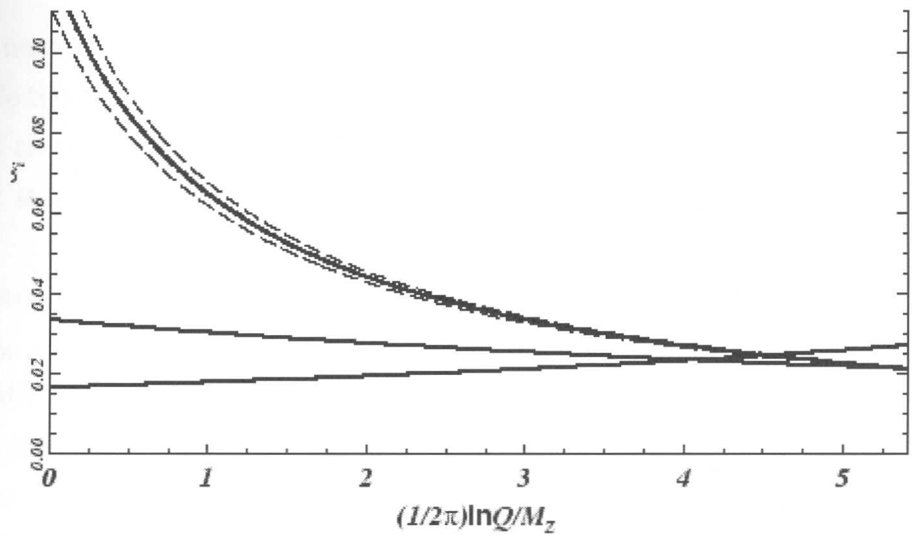


Figure 1.1: Running coupling constants predicted by SM

final theory of nature.

(1) There are too many parameters needed to describe the standard model. It has total nineteen parameters , six quark and three lepton masses, 3 mixing angles and a phase parametrising CP violation, three gauge couplings and two boson mass scale, electroweak and Higgs. In addition there is a further parameter θ in QCD.

(2) Charge quantisation is not understood. The relation of quark to lepton charges is also not understood.

(3) There is no reason why the matter multiplet structure chosen for SM should be as it is.

(4) There is no explanation of even the qualitative features of the mass spectrum. Why are quarks and leptons much lighter than W and Z ? Why families are different in masses? And what is the relation between quark and lepton masses? Why the neutrinos are massless?

These limitations suggest that SM is an effective theory which is valid up to an energy scale. One possibility to go beyond SM is to postulate extra symmetries which are broken at some high mass scale.

The next step toward unification is to unify strong and electroweak theory into one single theory which is customarily called Grand Unified Theory(GUT). The gauge group of this theory should contain SM gauge groups. In spite of the fact that it is the next step towards unification GUT has some extra theoretical advantages over SM.

(1) Electric charge is automatically quantized.

(2) sinθ_W (the weak mixing angle in SM) which is a free parameter in SM becomes a calculable quantity.

(3) The presence of quarks and leptons in the same multiplet may give rise to some simple relations among quark and lepton masses.

(4) All the coupling constants converge to a single contant beyond a energy , called GUT energy.

At the classical level baryon number (B) and lepton number (L) are exact global symmetries of SM. Since quarks and leptons sit in a single gauge multiplet, we can expect B and L violation mediated by some new gauge boson[2]. The apparent excess of matter over antimatter in our present universe is a clear hint in favor of the presence of B violating physical processes at some early epoch in the evolution of universe.

Proton being the lightest baryon, it is a stable particle in SM. So it's decay to some other particles will be an indication of new physics beyond SM. The experimental lower bound of the proton life time will be a constraint on GUT models. In this report we will try to achieve this end for a specific GUT.

[2]This is not necessary but it is easy to construct GUT with B violation.

Chapter 2

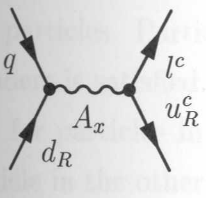
PROTON DECAY IN SU(5) GUT

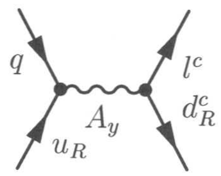
In searching for a GUT we will take proton decay as our guiding principle. We will assume GUT will not change the matter content of SM. It will only introduce new gauge and higgs particles.

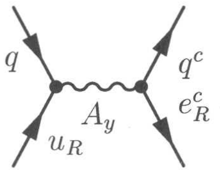
As the first step we will construct all the B violating four fermion operators with SM particle content.

If we assume that the tree level B violation is due to the exchange of a single particle,the only possible tree diagrams are the following.

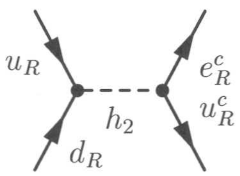
- Gauge boson mediated processes:

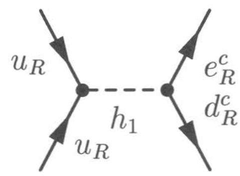


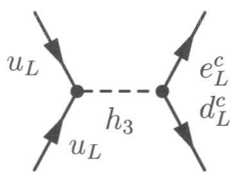


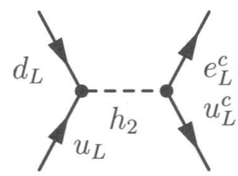


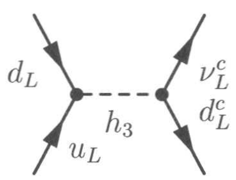
• Higgs scalar mediated processes:

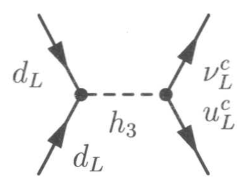












Here $q = (u_L, d_L)$ and $l = (\nu, e_L)$. The q in the diagram represents either of the two particles. Particles are chosen such that conservation of all the gauge quantum numbers is satisfied. Here and wherever not explicitly otherwise indicated, the symbols for particles in the lowest generation are used to represent any corresponding particle in the other generations, that is e stands for e, μ, τ.

The gauge quantum numbers of the new intermediate particles are following

$A_x = (3, 2, 1/3); A_y = (3, 2, -5/3);$

$h_1 = (3, 1, -8/3); h_2 = (3, 1, -2/3);$

$h_3 = (3, 3, -2/3)$; and their hermitian conjugates.

There is one important thing to note in this diagram , that though B and L are violated in the above process , $B - L$ remains a conserved quantity.

So our desired GUT model should contain at least some of these particles.The simplest of such choice is the SU(5) model. The adjoint 24-dimensional representation of SU(5) accommodates the gauge bosons. It decomposes under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
$24 = (8, 1, 0) + (1, 3, 0) + (1, 1, 0) + (3, 2, -5/3) + (\bar{3}, 2, 5/3)$
the bracketed quantities indicate how the gauge bosons transform under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The last two particles represent A_y and it's hermitian conjugate. The fermions can be accommodated in the reducible $\bar{5} + 10$ representation.
where, $5 = (3, 1, -2/3) + (1, 2, 1)$
$\bar{5} = (\bar{3}, 1, 2/3) + (1, 2, -1)$ and
$10 = (\bar{3}, 1, -4/3) + (3, 2, 1/3) + (1, 1, 2)$
So, $\bar{5}$ representation contains d^c_L and l and 10 representation contains u^c_L , q , e^c_L.
The Higgs particle can be represented as a quintet 5 .
5 accommodates SM Higgs doublet h_F as well as it's color-triplet partner h_C. The new particle h_C has the same quantum no. as h_2 .

The upper limit of theoretical calculation of proton life time in $SU(5)$ GUT model is 10^{31} where as the experimental lower bound is 10^{32} (Ross, 1984). So minimal $SU(5)$ GUT model can be discarded as a realistic model. In the next chapter we will discuss possible modification to this model.

Chapter 3

PROTON DECAY IN SUSY GUT

Irrespective of the success of SM in describing all the present day particle phenomena there is a disturbing aspect in SM. The infamous “hierarchy problem”.Supersymmetry elegantly solves this problem (L.Suskind, 1979). Besides supersymmetry is also a consequence of string theory.

Now we will proceed in the same manner as in the case of SM , that is we will construct all the four point interactions that violates B conservation.The particle content is the SM particles together with their superpartners. The supermultiplet containing particles and it's superpartners are symbolised in capital letters shown in Table3.1. The 10 representation T_{10} contains $Q, U^\dagger$ and $E^\dagger$. The $\bar{5}$ representation contains $D^\dagger$ and L. We can not give mass to both up quark and down quark with the same Higgs field as we do in SM. Because the chirality structure of supersymmetric Lagrangian forbids terms involving only scalar superfields of different chirality. So we need to introduce another Higgs superfield $H_{\bar{5}}$. The Yukawa coupling terms are now $T_{10}H_5T_{10}$ and $T_{10}H_{\bar{5}}F_{\bar{5}}$.

Any such interaction can be written as a nonrenormalizable effective interaction of dimensionality $d > g$,with coupling constant suppressed by $d-4$ powers of superheavy mass of the intermediate particle. We will now write a general effective Lagrangian to read off all the interactions. The effective Lagrangian is of the form (Weinberg, 1982)

$$L_{eff} = \mathcal{F}(\Phi)_F + \mathcal{F}(\Phi^\dagger)_F + \mathcal{D}(\Phi^\dagger, \Phi)_D,$$

where $\mathcal{F}$ and $\mathcal{D}$ are arbitrary functions such that all the terms are gauge invariant and Φ and $\Phi^\dagger$ are respectively chiral and antichiral scalar superfield of the theory. The F and D terms are respectively the θ^2 (or $\bar{\theta}^2$) and $\theta^2\bar{\theta}^2$ components.

We take only the F term of $\mathcal{F}$ because if a function of only chiral fields $\mathcal{F}$ can be expressed as $\mathcal{F}(y,\theta) = \phi + \theta\psi + \theta^2 f$,
where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$.
and if a function of only antichiral fields , $\mathcal{F}(z,\bar{\theta}) = \phi^* + \bar{\theta}\bar{\psi} + \bar{\theta}^2 f^*$,
where $z^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$.
When expressed in terms of x, θ, and $\bar{\theta}$ the other powers of θ and $\bar{\theta}$ will come as total derivatives hence neglected. The F term is only supersymmetric[1] here. Similarly only D term of $\mathcal{D}$ is included because being a function of chiral and antichiral fields the D term is the only supersymmetric term.

The mass dimension of θ and $\bar{\theta}$ is -1/2. So the dimensionality of F and D term of any function are equal to the dimensionality of the function plus 1 and 2 , respectively. The dimension of scalar superfield is +1. So the terms in L_{eff} of different dimensionalities are following

$$d = 2 : (\Phi)_F \ ,$$

$$d = 3 : (\Phi\Phi)_F \ ,$$

$$d = 4 : (\Phi\Phi^\dagger)_D, (\Phi\Phi\Phi)_F \ ,$$

[1]under supersymmetry transformation remains invariant or at most transforms to a total derivative

$d = 5 : (\Phi^\dagger\Phi\Phi)_D, (\Phi\Phi\Phi\Phi)_F$,

$d = 6 : (\Phi^\dagger\Phi\Phi\Phi)_D, (\Phi^\dagger\Phi^\dagger\Phi\Phi)_D, (\Phi\Phi\Phi\Phi\Phi)_F$

and so on , plus the Hermitian conjugates.

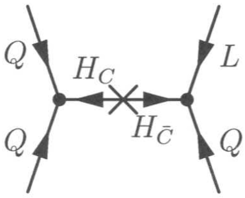
In order to find all the B violating operator we need to know the particle spectrum and gauge symmetries which are given in Table3.1.

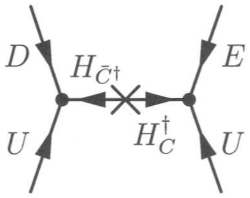
supermultiplets	s=0	s=1/2	Q.no.	B
Q	$\tilde{u_L}$, $\tilde{d_L}$	u_L, d_L	(3,2,1/3)	1/3
U	$\tilde{u_R}$	u_R	(3,1,4/3)	1/3
D	$\tilde{d_R}$	d_R	(3,1,-2/3)	1/3
L	$\tilde{\nu}$, $\tilde{e_L}$	ν, e_L	(1,2,-1)	0
E	$\tilde{e_R}$	e_R	(1,1,-2)	0
H_F	h_u^+, h_u^0	$\tilde{h_u^+}$,$\tilde{h_u^0}$	(1,2,1)	0
$H_{\bar{F}}$	h_d^0, h_d^-	$\tilde{h_d^0}$,$\tilde{h_d^-}$	(1,2,-1)	0
H_C	h_C	$\tilde{h_C}$	(3,1,-2/3)	0
$H_{\bar{C}}$	$h_{\bar{C}}$	$\tilde{h_{\bar{C}}}$	$(\bar{3}, 1, 2/3)$	0

Table 3.1: Quantum Numbers of matter and Higgs superfields in minimal supersymmetric SU(5)

Proton decay is affected by dimension 4, 5 and 6 operators. The 4d operators are given by $(UDD)_F$,$(QLD^\dagger)_F$ and $(LLE^\dagger)_F$. The first two operators can produce proton decay at a catastrophic rate through the exchange of D scalar. But all these operators violates $B - L$. So we can get rid of this operators if we take $B - L$ as a conserved quantity. We will define a multiplicative quantum number called R-parity as $P_R = (-1)^{3(B-L)+2s}$. The product of $(-1)^{2s}$ is of course equal to +1 for the particles involved in any interaction vertex because angular momentum is conserved. The symmetry principle to be enforced is that a term in the Lagrangian is allowed if the product of P_R of all the fields in it is +1. The R-parity odd particles are superparticles(distinguished by a tilde) and parity even particles are ordinary particles. At every interaction vertex there should be even number of sparticles($P_R = -1$).

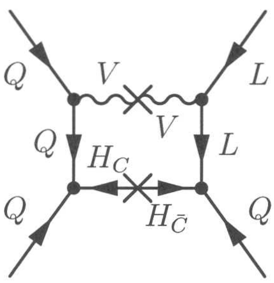
There are only two B violating 5d operators , $(QQQL)_F$ and $(UUDE)_F$ possible if we take MSSM particle spectrum and R-parity as a discrete global symmetry.The intermediate particles are color triplet Higgs field H_C and $H_{\bar{C}}$ connected by a mass term.The cross in the diagrams is the mass insertion necessary to connect two fields with same chirality.

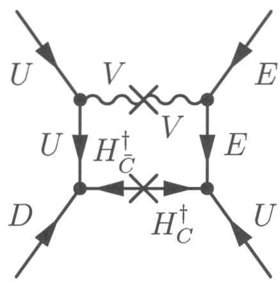




One of the out coming particles is a slepton and other one is a squark. These two particles are heavy compared to the proton , when supersymmetry is broken. So the particles are bound to be internal particles. These two scalar particles will be converted to two fermions by exchange of gauginos or doublet Higgsinos. The process mediated by Higgsino is suppressed by two powers of Yukawa coupling. We will consider only the processes mediated by gauginos. The gluino contribution is the largest but it will vanish completely in the limit where all the squark masses are degenerate (J.Hisano and T.Yanagida, 1992). Since the high degeneracy is required to suppress the unwanted flavor changing neutral current, the gluino contribution turns out to be small. The neutral gaugino exchange contribution is generally small. So the charge wino exchange gives the dominant contribution. Wino exchange is only possible for the $(QQQL)$ operator because $(UUDE)$ operator has all the fields right-handed (weak singlet).

So a typical complete process will be $(QQQL)_F(Q^\dagger QL^\dagger L)_D$, where $Q^\dagger Q$ and $L^\dagger L$ are contracted. The intermediate fermion is the charged wino. Similarly $(UUDE)_F(U^\dagger UD^\dagger$ where the intermediate particle is a neutral gaugino. Here all the external particles are fermions.





Calculations of proton life time in minimal SUSY $SU(5)$ does not contradict with experiments but it needs a large amount of fine tuning (J.Hisano and T.Yanagida, 1992).

Chapter 4

SUSY GUT IN 5D ORBIFOLD SPACETIME

In this chapter we will introduce a higher dimensional model of GUT which is considered in detail by Hall and Nomura (Hall and Nomura, 2001).We only discuss some of the features of the theory. A nonsupersymmetric version was considered by Kawamura (Y.Kawamura, 2000). The motivation that our space-time actually has more than four dimensions came from string theory, where consistency condition determines the dimension of space-time. For simplicity we will discuss the case where we have only one extra dimension (x_4). To mimic the low energy four dimensional universe , the extra dimension is compactified.
$x_4 = x_4 + 2\pi R$, where R is the radius of compactification.
We will take the gauge group as the supersymmetric SU(5). The gauge group $SU(5)$ can not give rise to the R-parity which is present in 4d theories. So we will consider another group $U(1)_R$ such that R-parity automatically results , giving proton stability at dimension 4.

The gauge fields and the Higgs fields propagate in 5 dimension. Now the minimal amount of supersymmetry in 5 dimension corresponds to N=2 supersymmetry in 4 dimension.So using 4 dimensional superfield notation , we can write the vector multiplet as (V, Σ) , where V is a vector multiplet and Σ is a chiral adjoint and the

Higgs hypermultiplet as (H, H^c), where H and H^c are chiral multiplets with opposite gauge transformation.

One way to break the symmetries of this higher dimensional theory ,which we will use here, is by orbifolding one axis and assign non universal parity among the component fields in a multiplet. We define orbifold symmetry on a tensor field ϕ by $\phi(-x_4) = P\phi(x_4)$, where the parity P acts separately on all vector indices of ϕ and an overall sign choice for the parity of the multiplet may also be included in P. There is a relative sign change between the transformation of $V(H)$ and $\Sigma(H^c)$, as required by the P invariance of the 5 dimensional Lagrangian.
By definition that it is a parity operator , P equals 1 or -1. We denote the field with $P = 1 (P = -1)$ by $\phi_+(\phi_-)$. The fields ϕ_+ and (ϕ_-) are Fourier expanded as

$$\phi_+(x_\mu, x_4) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_+^{(n)}(x_\mu) \cos \frac{n x_4}{R},$$

$$\phi_-(x_\mu, x_4) = \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} \phi_-^{(n)}(x_\mu) \sin \frac{n x_4}{R},$$

where n is an integer, and the fields $\phi_\pm^{(n)}(x_\mu)$ acquire a mass n/R upon compactification. But the 4 dimensional massless fields are absent in $\phi_-(x_\mu, x_4)$.

In order to break additional supersymmetry and to give mass to the unwanted superfields (Σ, H^c) we compactify the fifth dimension on an orbifold. We choose a Z_2 orbifold with $x_4 \to -x_4$. The Z_2 breaks half the supersymmetry by acting on the vector multiplet as

$$V(x_\mu, -x_4) = V(x_\mu, x_4)$$

$$\Sigma(x_\mu, -x_4) = -\Sigma(x_\mu, x_4).$$

which allows a massless mode for the 4-d vector but not for the 4-d chiral superfield. We will employ the same mechanism for Higgs hypermultiplet.

In order to break gauge symmetry to strong and electroweak symmetry we have to introduce another orbifold parity. We choose a Z_2' orbifold with $x_4' \rightarrow -x_4'$. Where $x_4' = x_4 - \pi R/2$. We use a parity P' which is $+$ for weak charges and $-$ for strong charges to accomplish doublet-triplet splitting , in the sense that the weak doublets have a zero mode but not the color triplet.

The 5d spacetime now becomes a direct product of 4d Minkowski spacetime M^4 and an extra dimension compactified on the $S^1/(Z_2 \times Z_2')$ orbifold. The $S^1/(Z_2 \times Z_2')$ orbifold can be visualised as a circle of radius R divided by two Z_2 transformations as shown in figure4.1 . Each reflection introduces special points , O and O' , which are fixed points of the transformation.

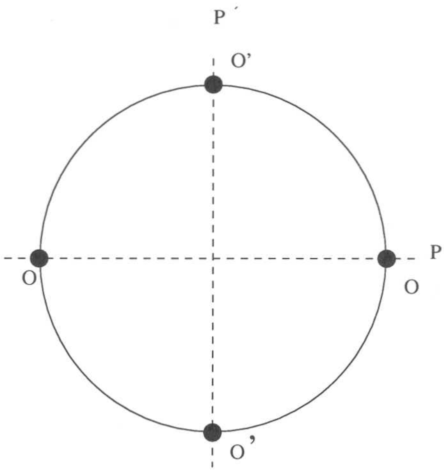


Figure 4.1: $S_1/(Z_2 \times Z_2{'})$ orbifold in the fifth dimension

The physical space is the interval $x_4 : [0, \pi R/2]$.

A generic 5d bulk field has the following mode expansions (R.Barbieri and Y.Nomura, 2001) , depending upon it's P and P' eigen values. Denoting the field with $(P, P') = (\pm 1, \pm 1)$by $\phi_{\pm\pm}$ we get:

(P,P')	4d N=1 superfield	mass
$(+,+)$	V^a , H_F , $H_{\bar{F}}$	2n/R
$(+,-)$	$V^{\hat{a}}$, H_C , $H_{\bar{C}}$	(2n+1)/R
$(-,+)$	$\Sigma^{\hat{a}}$, H^c_C , $H^c_{\bar{C}}$	(2n+1)/R
$(-,-)$	Σ^a , H^c_F , $H^c_{\bar{F}}$	(2n+1)/R

Table 4.1: The (Z_2, Z_2') transformation of gauge and Higgs field.

$$\phi_{++}(x_\mu, x_4) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}\pi R}} \phi_{++}^{(2n)}(x_\mu) \cos\frac{2nx_4}{R},$$

$$\phi_{+-}(x_\mu, x_4) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{+-}^{(2n+1)}(x_\mu) \cos\frac{(2n+1)x_4}{R},$$

$$\phi_{-+}(x_\mu, x_4) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{-+}^{(2n+1)}(x_\mu) \sin\frac{(2n+1)x_4}{R},$$

$$\phi_{--}(x_\mu, x_4) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{--}^{(2n+2)}(x_\mu) \sin\frac{(2n+2)x_4}{R},$$

where 4d fields $\phi_{++}^{(2n)}$, $\phi_{+-}^{(2n+1)}$, $\phi_{-+}^{(2n+1)}$ and $\phi_{--}^{(2n+2)}$ acquire masses $2n/R$, $(2n+1)/R$, $(2n+1)/R$ and $(2n+2)/R$ upon compactification. Parity eigenvalues and masses of the gauge and Higgs fields are listed in Table 4.1.

It is clear from P , P' eigen values of various fields listed in Table4.1 that, on a 4dimensional surface at O all the $N = 1$ $SU(5)$ gauge and Higgs fields are present but not any extra field. We will take this 4d surface localised at point O as our world and call it matter brane. In this model the gauge symmetry breaking is not involved any vacuum expectation value. The doublet-triplet splitting in four dimensions is achieved through compactification , though from 5d viewpoint SU(5) is unbroken. In this type of model grand unification is an inherently extra dimensional symmetry. The 4dimensional brane localised at O' has SM gauge group $SU(3) \times SU(2) \times U(1)$. The quarks and leptons are assumed to be localised on a brane. To keep the desired features of grand unified theory the natural choice of the location of the brane is at O. We call it matter brane. As in the 4d theories $Q, U^\dagger, E^\dagger$ belong to a 10 representation

T_{10} and $D^\dagger, L$ belong to a $\bar{5}$ representation $F_{\bar{5}}$.

The parities P of the matter fields under Z_2 are plus because all the fields are nonzero on the matter brane. The parities P' are constructed from requiring $(-,-,-,+,+)$ on each 5 index. From that we will get $P'(Q,U^\dagger,D^\dagger,L,E^\dagger) = (-,+,-,+,+)$. From the possibility of overall sign changes on T_{10} and $F_{\bar{5}}$ we will get finally $P'(Q,U^\dagger,D^\dagger,L,E^\dagger) = \pm(+,-,-,+,-)$ or $P'(Q,U^\dagger,D^\dagger,L,E^\dagger) = \pm(-,+,-,+,+)$.

	V	Σ	H_5	$H_{\bar{5}}$	H_5^c	$H_{\bar{5}}^c$	T_{10}	F_5
$U(1)_R$	0	0	0	0	2	2	1	1

Table 4.2: $U(1)_R$ charges for vector and chiral superfields.

The $U(1)_R$ charges of all the fields are listed in Table4.2. Any mass term, for colored Higgs field of the form $\sim \int d^2\theta H_C H_{\bar{C}}$ is prohibited under $U(1)_R$. Hence no dimension 5 proton decay operators are generated.

The full 5d $\mathcal{L}$ is

$$\mathcal{L} = \int d^5x[\mathcal{L}_5 + \mathcal{L}_m + \mathcal{L}_s]$$

where $\mathcal{L}_5$ is the bulk Lagrangian for gauge fields and Higgs fields.

$$\mathcal{L}_5 = -\tfrac{1}{2}tr(F_{MN})^2 + tr(\bar{\lambda} i\Gamma^M D_M\lambda) + tr(\tfrac{1}{2}(D_M\phi)^2) + ...$$

$\mathcal{L}_m$ is the Lagrangian for matter fields and their interactions with Higgs fields.

The Yukawa interaction part of $\mathcal{L}_m$ is

$$(\mathcal{L}_m)_{Yukawa} = \int d^2\theta[\frac{1}{2}\{\delta(x_4) - \delta(x_4 - \pi R)\}\sqrt{2\pi R}\lambda_u T_{10}T_{10}H_5 + \frac{1}{2}\{\delta(x_4) \mp \delta(x_4 - \pi R)\}\sqrt{2\pi R}\lambda_d T_{10}F_{\bar{5}}H_{\bar{5}}] + h.c.$$

And $\mathcal{L}_s$ is the Lagrangian on the source brane.

Here we will take the brane at $x_4 = \pm\pi R/2$ as the location for $N = 1$ supersymmetry breaking. We will break SUSY by giving nonzero vacuum expectation value to the F-component of the field S , $(S)_F = \mu\theta^2$ and $(S^\dagger)_F = \mu^*\bar{\theta}^2$. The field S is not a dynamical field in this theory. The vacuum expectation value μ is a parameter. So a nonzero V.E.V does not break supersymmetry spontaneously, it breaks it explicitly. We will take S gauge and $U(1)_R$ singlet on $x_4 = \pm\pi R/2$. The gauge and Higgs field are in the bulk. So they will couple to S .Coupling with the gauge fields will give mass to the gauginos and SUSY will be broken.

$$\mathcal{L}_s = \frac{1}{2}\{\delta(x_4 - \pi R/2) + \delta(x_4 + \pi R/2)\}[\int d^2\theta SWW + \int d^4\theta(S^\dagger H_F H_{\bar{F}} + SS^\dagger H_F H_{\bar{F}} + h.c]$$

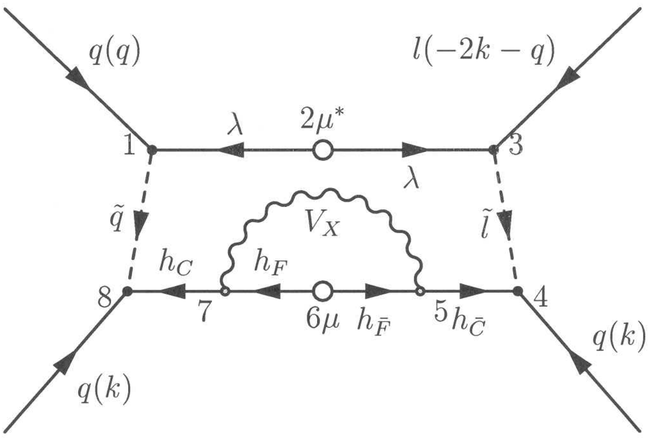
The nonzero vacuum expectation value of S field gives masses also to the Higgs-doublet which breaks $U(1)_R$ symmetry to R-parity and hence allow dimension 5 operators. In the next chapter we will try to estimate proton decay by these operators.

Chapter 5

PROTON DECAY CALCULATION

In chapter3 we wrote the only two 5 dimensional operators that are responsible for proton decay. In this chapter we will give a order of magnitude calculation for the lifetime of proton by these operators. For this we will take various approximations which are not valid if our aim is for a calculation with correct numerical factors.

As we discussed in chapter3 the dominant decay rate is due to $(QQQL)_F$ operator with charged wino exchange. In terms of component fields the feynman diagram looks like



We numbered the vertices as 1,2,3... .The grey vertices(y_5 and y_7) are on the bulk. The white vertices(y_2 and y_6) represents the vacuum expectation value of S (or $S^\dagger$) and are in the source brane. The black vertices are all on matter brane. The momenta of the internal particles are not shown in the diagram. They are listed below.

particle	h_C	h_F	$h_{\bar{F}}$	V	$h_{\bar{C}}$
4 momentum	$-k-q-q''$	q'	$-q'$	$-k-q-q'-q''$	$k+q+q''$

particle	$\tilde{l}$	λ	$\tilde{q}$
4 momentum	$-2k-q-q''$	$\mp q''$	$q+q''$

The incoming particles are two q and the outgoing particles are q^c and l^c (two particles on the top half of the diagram shown as two incoming particles q and l). The q and l are weak doublet but we avoid the weak indices for the sake of simplification of notation. Color indices are also suppressed. Particles at each vertex should be such that to ensure the vertex gauge neutral. It is also understood that we have to take the trace due to the non Abelian structure of the theory. We will compute this 5 dimensional diagram in momentum space in 4 dimension and position space in fifth dimension[1]. The $\mathcal{M}$ matrix is

$$\begin{aligned}\mathcal{M} &= \int d^4q'd^4q''[\langle q\tilde{q}\tilde{h}_C\rangle\langle \tilde{h}_C^\dagger\sigma^\mu V_\mu \tilde{h}_F\rangle\langle \tilde{h}_F^\dagger \mu \tilde{h}_{\bar{F}}^\dagger\rangle\langle \tilde{h}_{\bar{F}}\sigma^\nu V_\nu^\dagger \tilde{h}_{\bar{C}}^\dagger\rangle\langle \tilde{h}_{\bar{C}} q\tilde{l}\rangle] \\ &\times \ [\langle \tilde{q}^\dagger q\lambda\rangle\langle\lambda^\dagger\mu^*\lambda^\dagger\rangle\langle\lambda\tilde{l}^\dagger l\rangle]\end{aligned}$$

There are $\int dy$ for every vertex and for every vertex where $y(=x_4)$ has a particular value there is an additional δ function.

[1]Because translational invariance is broken along 5th direction

The propagators which we calculated in the appendix, at the low momentum limit$(q^2 \ll 1/R^2)^2$.
$\langle\psi^\dagger\psi\rangle \sim \frac{1}{R}\frac{\not q}{q^2-m^2}$
ψ is any fermion of mass 2n/R.
$\langle V^\dagger V\rangle \sim (d\pi/2 - R\pi^2/4)$
where V is a vector field of mass (2n+1)/R.
and $\langle\psi^\dagger\psi\rangle \sim 1$
where ψ is any fermion with mass (2n+1)/R.
So

$$\mathcal{M} = \int d^4q'd^4q''[q(q)\Gamma_2 l(-2k-q)]\frac{1}{(q+q'')^2-m^2}\frac{1}{(2k+q+q'')^2-m^2}[q(k)\Gamma_1 q(k)]$$

where

$$\begin{aligned}\Gamma_1 &= \int dy_4dy_5dy_6dy_7dy_8\delta(y_4)\delta(y_6-\pi R/2)\delta(y_8)(g_4\sqrt{R})^2(\lambda_4\sqrt{R})^2(\mu R)\\ &\times\ (d\pi/2 - R\pi^2/4)(\frac{1}{R}\frac{\not q'}{q'^2-m^2})^2\\ &= (g_4)^2(\lambda_4)^2\mu R^4(\frac{\not q'}{q'^2-m^2})^2\end{aligned}$$

and

$$\begin{aligned}\Gamma_2 &= \int dy_1dy_2dy_3\delta(y_3)\delta(y_2-\pi R/2)\delta(y_1)(g_4\sqrt{R})^2(\frac{1}{R}\frac{\not q''}{q''^2-m^2})^2(\mu^* R)\\ &= g_4^2\mu(\frac{\not q''}{q''^2-m^2})^2\end{aligned}$$

The various 5d coupling coefficients in terms of 4d coupling coefficients are given in the appendix.

[2] both the high momentum and low momentum limits give the same order of magnitude. We choose the low energy limit.

So

$$\begin{aligned}
\mathcal{M} &= g_4^4\lambda_4^2\mu\mu^* R^4 \int_0^{1/R} d^4q' d^4q'' (\frac{\not{q}''}{q''^2 - m^2})^2 (\frac{\not{q}'}{q'^2 - m^2})^2 \frac{1}{(q+q'')^2 - m^2} \frac{1}{(2k+q+q'')^2 -} \\
&\times \langle ql|qq\rangle \\
&= g_4^4\lambda_4^2\mu\mu^* R^4 \int_0^{1/R} d^4q' (\frac{\not{q}'}{q'^2 - m^2})^2 \\
&\times \int_0^{\infty} d^4q'' (\frac{\not{q}''}{q''^2 - m^2})^2 \frac{1}{(q+q'')^2 - m^2} \frac{1}{(2k+q+q'')^2 - m^2} \langle ql|qq\rangle \\
&= g_4^4\lambda_4^2\mu\mu^* R^4 \frac{1}{R^2}\frac{1}{\mu^2}\langle ql|qq\rangle
\end{aligned}$$

here the first integral is a divergent integral and it is quadratically divergent. So the dominating contribution will be $1/R^2$. The second integral is a convergent integral and it's dimension is $[M]^{-2}$. It is thus expected to be come as $1/\mu^2$ where we took the masses of the superparticles as μ (as an estimate).
So $\mathcal{M} = g_4^4\lambda_4^2 R^2\langle ql|qq\rangle$.
Now the final states are free wave . So they each have a normalisation factor $\sqrt{E}$. And the initial state $|qq\rangle$ is a bound state. So the normalisation factor will be $1/r^{3/2}$ for the bound state[3] times $\sqrt{E}$ for the propagation of the center of mass. r is the radius of proton.

If we take the initial and final particle energies to be of the same order of the proton mass then $\mathcal{M} = g_4^4\lambda_4^2 R^2(\frac{m_p}{r})^{3/2}$

As an estimate we can take
$g_4 \sim e \sim \sqrt{\alpha} \sim 10^{-1}$.
λ_4^2 can be estimated from the relation $m_q \sim v_h\lambda_4$ where v_h is the vacuum expectation value of Higgs field. From $m_q \sim 10^{-3}$GeV and $v_h \sim 10^2$GeV the estimate of λ_4 is 10^{-5}.

[3] the bound state amplitude squared $|\psi(0)|^2 \sim \frac{1}{r^3}$. $\psi(0)$ is the amplitude of two interacting quarks to be at the same point. The third quark is treated as a spectator.

$m_p \sim 1$Gev
$r = .87fm$ from that we can get $\frac{1}{r^3} \sim 10^{-2}GeV^{-3}$.

From these $\mathcal{M} = 10^{-16}R^2$GeV.

Now decay rate $d\Gamma$ is $d\Gamma = \frac{1}{2m_p}(\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f})^2|\mathcal{M}|^2(2\pi)^4\delta^4(\sum p)$.
The phase space integration with the delta function has mass dimension 0. So of order 1.Then the total decay rate $\Gamma = 10^{-32}R^4$GeV.
The proton life time is $t = \frac{1}{\Gamma} = \frac{10^{32}}{R^4}GeV^{-1}$.
The experimental lower bound of proton is 10^{33} years $\sim 10^{40}$sec $\sim 10^{65}GeV^{-1}$. From this we can have a constraint on R.

$\frac{1}{R} > 10^8$Gev.
$\Rightarrow R < 10^{-8}GeV^{-1}$
$\Rightarrow R < 10^{-9}fm$.

Chapter 6

CONCLUSION AND DISCUSSION

We have discussed a supersymmetric model of grand unification in higher dimension. Both the features, extra dimension and supersymmetry are well motivated in string theory, which is a likely candidate for a final theory. The supersymmetry and $U(1)_R$ symmetry are broken explicitly in this model. We hope in a more fundamental underlying theory μ will come from a real dynamical field which will break the supersymmetry spontaneously.

The introduction of an extra dimension is not a necessary step to modify $SU(5)$. We can instead change our gauge group to $SO(10)$ or to some nonminimal gauge group. But in that way we will introduce more free parameters.

As a constrain from proton-decay calculation we get an upper bound on the radius of compactification. $R < 10^{-9} fm$. It is interesting that the constraint equation does not involve μ (V.E.V of S field). Actually proton decay rate due to 5-dimensional operators depends upon μ but the dependence is weak. The decay rate is zero when μ is zero but when μ becomes nonzero (more precisely when the value is at or beyond the weak scale) the decay rate no more depends upon μ.

It will be interesting to calculate the propagators numerically to get an exact result. Also stronger constraints may come from other calculations related to F.C.N.C. , CP violation or some other rare decays . We can also use other gauge groups instead of $SU(5)$. The theory we discussed in this report is clearly an effective theory. It will be interesting to construct a fundamental theory which embed this theory.

Appendix 1

APPENDIX

• Gauge coupling in 5 dimension:

We can express various 5 dimensional coupling in terms of 4 dimensional couplings from dimensional analysis.
The various couplings are

$$g_5 = \sqrt{R} g_4$$

$$\lambda_5 = \sqrt{R} \lambda_4$$

And the coupling with the mass term on the source brain is R.

•Propagators in the theory:

As we are interested in propagation between definite positions in the 5th dimension we will write it in Fourier transform in the 5th coordinate.
The scalar or vector propagator is

$$P_s(t) = \sum_{n=0}^{\infty} \frac{e^{ip_n d}}{t-(p_n)^2-m^2}$$

$P_F(t) = \sum_{n=0}^{\infty} \frac{(\not{q} - i\gamma_5 p_n - m)e^{ip_n d}}{t-(p_n)^2-m^2}$

where p_n is the momentum along the 5th direction, equals to $2n/R$ or $(2n+1)/R$ and distance traveled in the 5th direction. t is the 4 momentum.

We can perform the sum exactly (N. Arkani-Hamed and Schmaltz, 1999).

$P_d(t) = \sum_{n=-\infty}^{\infty} \frac{e^{ind/R}}{t-(n/R)^2-m^2} = -\frac{\pi R}{\sqrt{-t+m^2}} \frac{\cosh[(d-\pi R)\sqrt{-t+m^2}]}{\sinh[\pi R\sqrt{-t+m^2}}$
From here we can get
$\sum_{n=-\infty}^{\infty} \frac{e^{i2nd/R}}{t-(2n/R)^2-m^2} = -\frac{\pi R/2}{\sqrt{-t+m^2}} \frac{\cosh[(d-\pi R/2)\sqrt{-t+m^2}]}{\sinh[\pi R/2\sqrt{-t+m^2}}$
and
$\sum_{n=-\infty}^{\infty} \frac{e^{i(2n+1)d/R}}{t-(\frac{2n+1}{R})^2-m^2} = \sum_{n=0}^{\infty} \frac{e^{ind/R}}{t-(n/R)^2-m^2} - \sum_{n=0}^{\infty} \frac{e^{i2nd/R}}{t-(2n/R)^2-m^2}$

It is easy to to work with the two limits $\sqrt{-t} \gg R^{-1}$ and $\sqrt{-t} \ll R^{-1}$. In the former case we obtain

$P_d(t) \simeq -\frac{\pi R}{\sqrt{-t}} e^{-\sqrt{-t}d}$.
In the limit of small momentum limit we obtain

$P_d(t) \simeq -\frac{1}{t-m^2} - R^2(\frac{d^2}{2R^2} - \frac{d\pi}{R} + \frac{\pi^2}{3})$

For $t \ll \frac{1}{R}$

$\sum_{n=-\infty}^{\infty} \frac{e^{i2nd/R}}{t-(2n/R)^2-m^2} = \frac{1}{t-m^2}$

$\sum_{n=-\infty}^{\infty} \frac{e^{i(2n+1)d/R}}{t-(\frac{2n+1}{R})^2-m^2} = R^2(d\pi/2R - \pi^2/4)$

From these expressions we can easily calculate the propagators. For the part in the propagator involving γ_5 we differentiate the above expression with respect to d.

•**Values of different constants used in the calculation:**

$m_{proton} = 1GeV$

$r_{proton} = .87fm$
$m_u = 4.5MeV$

Note : we only use the order of magnitude.

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