The Institute of Mathematical Sciences, Chennai

Quarterly Report

October - December 2022

Highlights: research and events

Algebraic Number Theory

Sanoli Gun, Olivier Ramare and Jyothsnaa Sivaraman prove a fully explicit generalized Brun-Titchmarsh theorem for an imaginary quadratic field . More precisely, for any finite family of linearly independent linear forms with coefficients in Ø, the authors count the number of integers at which all these linear forms take prime values in Ø.

Let **K** be a number field and ***q*** an integral ideal in Ø. A result of Tatuzawa from 1973, computes the asymptotic (with an error term) for the number of ideals with norm at most x in a class of the narrow ray class group of **K** modulo ***q***. This result bounds the error term with a constant whose dependence on ***q*** is explicit but dependence on **K** is not explicit. The authors, using geometry of numbers, prove a fully explicit bound for the error term in the asymptotic.

Modular forms

Let τ denote the Ramanujan tau function. In a recent work, Bennett, Gherga, Patel and Siksek proved that for any prime p and integer m ≥ 2, the largest prime factor $P(\tau(p^m))$ of $\tau(p^m)$ satisfies

$$P(\tau(p^m)) > \alpha \cdot \frac{\log\log(p^m)}{\log\log\log(p^m)}$$

provided τ (p) is not equal to 0. Here α is an absolute positive constant. In [G4], the authors prove that for any r > 0 and integer m ≥ 1,

$$P(\tau(p^m)) > (\log p)^{1/8}(\log\log p)^{3/8-r}$$

for almost all primes p. The above results are also valid for any non-CM normalized Hecke eigenforms with integer Fourier coefficients.

Y. Bilu , S. Gun, and S. L. Naik investigate a non-Archimedean analogue of a question of Atkin and Serre. More precisely, they derive lower bounds for the largest prime factor of non-zero Fourier coefficients of non-CM normalized Hecke eigen cusp forms of weight k, level N with integer Fourier coefficients. In particular, the authors show that for such a form f and for any real number r > 0, the largest prime factor of the p-th Fourier coefficient $a_f(p)$ of f, denoted by $P(a_f(p))$, satisfies

$$P(a_f(p)) > (\log p)^{1/8}(\log\log p)^{3/8-r}$$

for almost all primes p. This improves on earlier bounds. The authors also investigate a number field analogue of a recent result of Bennett, Gherga, Patel and Siksek about the largest prime factor of $a_f(p^m)$ for $m \geq 2$.

Pila and Tsimerman proved in 2017 that for every k there exists at most finitely many k-tuples $(x_1, \ldots, x_k)$ of distinct non-zero singular moduli with the property "$x_1, \ldots, x_k$ are multiplicatively dependent, but any proper subset of them is multiplicatively independent". The proof was non-effective, using Siegel's lower bound for the class number. In 2019 Riffaut obtained an effective version of this result for k = 2. Moreover, he determined all the instances of $x^m y^n \in^{\times}$, where x, y are distinct singular moduli and m, n non-zero integers. Y. Bilu , S. Gun, and E. Tron obtain a similar result for k = 3. Authors show that $x^m y^n z^r \in^{\times}$ (where x, y, z are distinct singular moduli and m, n, r non-zero integers) implies that the discriminants of x, y, z do not exceed 1010.

Transcendental number theory

The study of linear independence of L(k, χ) for a fixed integer k > 1 and varying χ depends critically on the parity of k vis-a-vis χ. This has been investigated by a number of authors for Dirichlet characters χ of a fixed modulus and having the same parity as k. The focal point of [S. Gun, N. Kandhil, and P. Philippon∗ . On linear independence of dirichlet l values. 2022] is to extend this investigation to families of Dirichlet characters modulo distinct pairwise co-prime natural numbers. The interplay between the resulting ambient number fields brings in new technical issues and complications hitherto absent in the context of a fixed modulus (consequently a single number field lurking in the background). This entails a very careful and hands-on dealing with the arithmetic of compositum of number fields which the authors undertake in this work. Results of this article extend earlier works of the first author with Murty-Rath as well as the works of Okada, Murty-Saradha and Hamahata.

Condensed Matter Physics

Given below is a summary of the research work [Ajit C. Balram, Zhao Liu, Andrey Gromov, and Zlatko Papić, "Very-High-Energy Collective States of Partons in Fractional Quantum Hall Liquids", Phys. Rev. X 12, 021008]
https://journals.aps.org/prx/abstract/10.1103/PhysRevX.12.021008

A complex system of many interacting particles can be more than the sum of its parts. A classic example that has captured the imagination of physicists for four decades is the

fractional quantum Hall (FQH) effect: a phenomenon where electrons confined to two dimensions and in strong magnetic fields condense into unusual quantum fluids. In many FQH fluids, by binding together quantized vortices of the magnetic field, electrons transform their identity to that of new particles called composite fermions. Here, we theoretically show that when some FQH fluids are taken far out of their equilibrium states, they manifest surprising dynamical behavior that can no longer be described by composite fermions.

At sufficiently high energies, the properties of these FQH phases reveal the emergence of a new kind of particle called a parton, which behaves as a fraction of the electron. Similar to quarks in the nucleus, partons can be seen only when a lot of energy is injected into the system. Based on this idea, we propose an experiment to reveal the presence of partons, which we describe using methods of effective field theory and numerical simulations.

The identification of dynamical signatures of partons illustrates the need for better understanding of nonequilibrium properties of FQH phases. While direct observation of partons remains the key goal for future experiments, on the theory side, the methods developed in this paper may lead to a better understanding of the more fragile FQH phases, including some enigmatic states with properties applicable to quantum information technology.

Participation in conferences

Coimbatore Balram, Ajit

Participated in Young Investigator's Meet on Quantum Condensed Matter Theory - 2022 held at NISER, Bhubaneshwar during Oct 29 – Nov 1, 2022, and gave an invited talk

Participated in 88th Annual Meeting of the Indian Academy of Sciences held at SRM University, AP during Nov 4 – Nov 6, 2022. https://am2022.ias.ac.in/

Gun, S.

Visited Universite Paris Cite, France during Oct 4 – Nov 5, 2022. Invited Professor

Visited Universit´e of Bordeaux, France during Oct 12 – Oct 15, 2022. Invited speaker in number theory seminar

Participated in Analytic Number Theory held at Oberwolfach, Germany during Nov 6 – Nov 12, 2022. Invited speaker

Visited Oberwolfach, Germany during Nov 6 – Nov 12, 2022. Invited Speaker

Visited Sorbonne University, France during Nov 12 – Nov 15, 2022. Invited speaker in Paris number theory seminar

Participated in Conference on Class Groups of Number Fields and Related Topics held at KSOM during Nov 21 – Nov 24, 2022. Invited Speaker

Participated in Collaborative Research Group in L-functions and Analytic Number Theory Seminar held at Online mode on Nov 24, 2022. Invited speaker

Participated in 37th Annual Conference of Ramanujan Mathematical Society held at Sri Sivasubramaniya Nadar College of Engineering, Chennai during Dec 6 – Dec 8, 2022. Invited speaker in the Number Theory Symposium

Mahajan, Meena B.

Visited Friedrich-Schiller University, Jena, Germany during Sep 17 – Oct 8, 2022. Research Collaboration.

Participated in Theory and Practice of SAT and Combinatorial Solving held at Schloss Dagstuhl, Leibniz Zentrum for Informatics during Oct 9 – Oct 14, 2022. Gave a talk titled ”Quantified Boolean Formulas: (Solving and) Proof Complexity”.

Participated in the 88th Annual Meeting of the Indian Academy of Sciences held at SRM AP, Amaravati. during Nov 4 – Nov 6, 2022. Gave a talk titled ”What is hard to prove, and why”.

Participated in IARCS Winter School on SAT+SMT held at IIT Madras during Dec 15 – Dec 17, 2022. Gave an invited talk on Proof Complexity for QBFs

IMSc Outreach

Azadi Ka Amrit Mahotsav

Title **:** Forces of nature: Padmanabhan and gravitation

Speaker **:** Prof. L. Sriramkumar, Department of Physics, Indian Institute of Technology Madras, Chennai

Date & Time **:** 04 November 2022, 16:00 - 17:00

Venue **:** Ramanujan Auditorium, IMSc Chennai

Abstract : In this talk, after a brief outline of the academic history of Prof. Thanu Padmanabhan, including a few personal reminiscences, I will highlight some of the scientific contributions of Prof. Padmanabhan in the fields of astrophysics, gravitation and cosmology.

Link : https://www.youtube.com/watch?v=gBKFjAXHFHM

Title : Microbial and plant bio-factories for sustainable production of high-value low-volume Phytochemicals: Need for a rational and integrated approach

Speaker : Prof. Smita Srivastava, Department of Biotechnology, Indian Institute of Technology Madras, Chennai

Date : 15 November 2022

Time : 16:00 - 17:00

Venue : Ramanujan Auditorium, IMSc Chennai

Link : https://youtu.be/2a-o8fj46KU

IMSc60:

Title : Quantum Technologies A.D. 2022
Speaker : Prof. Rafal Demkowicz-Dobrzanski, Faculty of Physics, University of Warsaw
Date & Time : Friday, October 7, 2022 16:00 - 17:00
Venue : screened live at Ramanujan Auditorium

Abstract:

Quantum computing, quantum communication and quantum metrology can be regarded as the three main pillars of the whole endeavour which goes under the name of quantum technologies. I will describe the most spectacular recent achievements in the field, discuss the prospects as well as confront expectations and promises with the reality and difficulties that the field faces.

Title : Geometric models of cell fate specification
Speaker : Dr. Archishman Raju, National Center For Biological Sciences, Bengaluru
Date & Time : Wednesday, October 12, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

Cell fate decisions emerge as a consequence of a complex set of gene regulatory networks. Detailed models of these networks are known to suffer from over-parameterization. We will describe recent work formalizing an alternative approach first presented by Waddington, which likens differentiation of different cell types to flow through a landscape in which valleys represent alternative fates. This allows the construction of minimally parameterized models consistent with cell behaviour. We will describe how this construction leads to intuitive models that are well adapted to biological data. We will also describe how to think about models of spatial pattern formation in a geometric manner. This leads to a more unified description of cell fate specification and we will end with some remarks on differences and similarities with universality in physics

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Title : Turbulence, Kolmogorov spectrum and its different variations
Speaker : Prof. Jayanta K Bhattacharjee, School of Physical Sciences, IACS, Kolkata
Date & Time : Date: Wednesday, October 26, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

Fully developed turbulence is known to be a rather difficult problem to handle. But Kolmogorov in 1941 found a result which was strikingly simple. He considered the energy spectrum of maintained turbulence, defining it by where k is the wave-number and E is the total kinetic energy - a quantity conserved in the inviscid, unforced limit. Physical reasoning and dimensional analysis led to . More than eighty years later, the goal of many turbulence researchers is to find something as elegant as that. We will discuss a number of related situations and also the difficulties still encountered in making the proportionality an equality.

Title : Certification of quantum technologies
Speaker : Prof. Dr. Antonio Acin, ICFO-The Institute of Photonic Sciences, Barcelona

Date & Time : Monday, November 14, 2022 16:00 - 17:00
Venue : screened live at Ramanujan Auditorium

Abstract:

Quantum information technologies promise applications with no classical counterpart: more powerful computers or novel forms of encryption become possible when information is encoded on quantum systems. Given a quantum system, how can one ensure that it (i) is entangled (ii) random (iii) secure (iv) and performs a computation correctly? The concept of quantum certification embraces all these questions. It is crucial for the future development of quantum information technologies, but also goes at the heart of the fundamental question of what distinguishes quantum from classical physics. The talk first provides an introduction to the general problem of quantum certification and then focuses on methods for the certification of quantum cryptography protocols and quantum optimisers.

Title : Quantum mechanics with patterns of light
Speaker : Prof. Andrew Forbes, School of Physics, University of the Witwatersrand
Date & Time : Wednesday, November 16, 2022 16:00 - 17:00
Venue : screened live at Ramanujan Auditorium

Abstract:

Structured light is an exploding topic, giving rise to new applications, from classical to quantum. The structuring can be done with classical beams, single photons or entangled states, offering access to the infinite alphabet of patterns of light for high-dimension quantum states. In this talk I will review the recent progress in quantum entanglement of photons in their spatial degree of freedom. I will explain how to create high-dimensional quantum states in the laboratory, how to measure them, and what the present state of the art is in terms of applications. In particular, I will outline the advantages and disadvantages of using such entangled states as a means to encode information for secure quantum communication channels, and highlight the open challenges that remain.

Title : Precision physics and perturbative calculation
Speaker : Prof. Narayan Rana, Indian Institute of Technology Kanpur
Date & Time : Friday, November 18, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

After successful runs at 7, 8 and 13 TeV and the discovery of the Higgs boson, there is still no clear evidence of BSM physics at the LHC. Although the high-luminosity phase will collect a large amount of data and exciting discoveries are still well possible, it is by now clear that an alternative path to uncover possible new physics is the search for small deviations from the predictions of the Standard Model, and that precision is the key for this path. In this talk, we will discuss the challenges and how we overcome them to obtain a per-mille level theoretical prediction in perturbation theory for Drell-Yan production, a precision benchmark process at the LHC. This prediction will have a direct impact on the precise measurement of the W-boson mass.

Title : The finite part of infinity
Speaker : Prof. Joseph Samuel, ICTS-TIFR & Raman Research Institute, Bengaluru
Date & Time : Monday, November 21, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

Ramanujan's notebooks contain the equation 1 + 2 + 3 + ... = -1/12. While this seems to go against common sense, there is deep mathematics behind this assertion. I will try to explain in what sense this must be understood. Such mathematical interpretations of divergent series have had an impact in physics, leading for example to a quantitative understanding of the Casimir effect, which has been measured in the laboratory. I will try to bring out how higher mathematics meets physics. Finally, I will describe a connection with infinities in quantum field theory and its connection to the cosmological constant problem.

Title : New perspectives on the relativistic binary problem
Speaker : Prof. Jan Steinhoff, Max Planck Institute for Gravitational Physics, Potsdam
Date & Time : Wednesday, November 23, 2022 16:00 - 17:00
Venue : screened live at Ramanujan Auditorium

Abstract:

Continuing the success of gravitational wave observations requires a large effort on improving their theoretical predictions in the next decade, in order to keep their accuracy on par with improvements of the detectors. This requires innovations on the methods by which gravitational waves from compact binaries are calculated. In this talk, we focus on new approaches to analytic, perturbative predictions for relativistic binaries inspired by

high-energy physics. In this area, effective field theories are highly useful and scattering amplitudes (the primary observable) can be calculated very efficiently using novel tools. These methods can indeed be applied to the classical binaries and their gravitational waves. We give a basic introduction to the ideas of these approaches and recent progress.

Title : Quantum steampunk: Quantum information meets thermodynamics
Speaker : Prof. Nicole Yunger Halpern, NIST, QuICS & University of Maryland
Date & Time : Thursday, November 24, 2022 17:00 - 18:00
Venue : screened live at Ramanujan Auditorium

Abstract:

Thermodynamics has shed light on engines, efficiency, and times arrow since the Industrial Revolution. But the steam engines that powered the Industrial Revolution were large and classical. Much of todays technology and experiments are small-scale, quantum, far from equilibrium, and processing information. Nineteenth-century thermodynamics needs re-envisioning for the 21st century. Guidance has come from the mathematical toolkit of quantum information theory. Applying quantum information theory to thermodynamics sheds light on fundamental questions (e.g., how does entanglement spread during quantum thermalization? How can we distinguish quantum heat from quantum work?) and practicalities (e.g., quantum engines and the thermodynamic value of coherences). I will overview how quantum information theory is being used to revolutionize thermodynamics in quantum steampunk, named for the steampunk genre of literature, art, and cinema that juxtaposes futuristic technologies with 19th-century settings.

Title : Dark matter and its indirect detection
Speaker : Prof. Ranjan Laha, CHEP, Indian Institute of Science
Date & Time : Friday, December 2, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

What is the Universe made up of? Surprisingly, we do not know the answer to this basic question. In this talk, I will introduce dark matter, which comprises of 80% of the matter density of the Universe. I will discuss some of the gravitational evidences that we have in favour of dark matter. I will then talk about its non-gravitational search strategy, in particular indirect detection. I will discuss how various messengers can be used to study the indirect detection of dark matter. I will try to convey the various challenges that one encounters in

these searches, and how one can try to overcome them. I will also discuss how discoveries can be made using this search strategy.

Title : Chaos and Noise in the aid of Logic
Speaker : Prof. Sudeshna Sinha, Indian Institute of Science Education & Research Mohali
Date & Time : Monday, December 5, 2022 17:00 - 18:00
Venue : Ramanujan Auditorium

Abstract:

We discuss how understanding the nature of chaotic dynamics allows us to manipulate these complex systems, and such a controlled chaotic system can then serve as a versatile pattern generator that can be used for a range of applications. Specifically we will discuss the application of chaos to the design of reconfigurable logic gates. Further we indicate how one can exploit the interplay of nonlinearity and noise to obtain more consistent and robust logic operations. We also suggest how these concepts may be applied to systems ranging from electronic circuits and synthetic genetic circuits to nanomechanical oscillators.

Title : Universal aspects of quenches in quantum theories
Speaker : Prof. Diptarka Das, Indian Institute of Technology Kanpur
Date & Time : Wednesday, December 7, 2022 16:00 - 17:00
Venue : Ramanujan Auditorium

Abstract:

The colloquium will review various non-equilibrium aspects that emerge in a time-dependent many body quantum system. We will look at universal scalings in certain simple field theories. We shall discuss the interplay of these scalings with thermalization and symmetry. Thereafter we shall look at the effects of quenches in scrambling.