



THE INSTITUTE OF MATHEMATICAL SCIENCES,  
TARAMANI, CHENNAI 600 113.

Quarterly Report  
(July - September 2018)

## Research Highlights

### Modular forms

Ramanujan introduced the famous  $\tau$  function as coefficients of the following infinite product;

$$\Delta(z) := \sum_{n \geq 1} \tau(n) q^n = q \prod_{\ell \geq 1} (1 - q^\ell)^{24}.$$

Ramanujan's investigation about the arithmetic properties of this function lead to the theory of modular forms. Development of this theory lead to the solutions of some of the outstanding problems in Mathematics, e.g., Fermat's last theorem, Serre's conjecture, Sato-Tate conjecture and so on. One of most well-known open problems about Ramanujan  $\tau$  function is a conjecture of Lehmer which states that  $\tau(n)$  is non-zero for all  $n$ . This conjecture has been investigated by Deligne, Serre, Rankin, Selberg and other distinguished mathematicians. In joint work with J.M. Deshouillers, Y.F. Bilu and F. Luca, Sanoli Gun of IMSc has shown show that the first  $k$   $\tau$ -values are non-zero if and only if infinitely many blocks of consecutive values of  $\tau$  of length  $2k$  are non-zero. In order to prove this, they use certain techniques of Ramanujan, some recently developed Sieve theoretic tools and the Sato-Tate conjecture which is now a theorem.

### Automata, Logic and Concurrency

Since the 1960s, logic has been related to formal language theory. Fixing words or trees as models, definable sets can be seen as word or tree languages. Many mathematical questions can be stated in first-order logic, making it a natural descriptive formalism. Rabin (1970) showed that a large number of questions can be formulated in first-order logic on trees, and solved the problem whether a given sentence has a model. Meyer and Stockmeyer (1975) showed that the amount of memory used by the algorithm cannot be bounded by a fixed tower of exponentials. The answers connected logic to the theory of finite automata, and for the definability problem on words to algorithms based on the theory of finite algebras with an associative operation, developed by Schützenberger (1965). Kamp showed in his PhD thesis (1976) that every first-order sentence can be expressed using three variables. Meyer and Stockmeyer's lower bound applies to three-variable logic.

This raised the definability problem for two-variable sentences of first-order logic, which was solved by Thérien and Wilke (1998), forming part of Wilke's habilitation thesis (1998). Algebraic techniques developed by Schützenberger (1976) were used. Given a finite automaton description, definability in two-variable logic is decided by an algorithm using a polynomial amount of memory. Given a sentence of two-variable logic, whether it has a model is decided by a nondeterministic algorithm using an exponential number of steps. In practice this means using an exponential amount of memory.

In joint work with Krebs, Pandya and Straubing over two years, Kamal Lodaya of IMSc has proposed a logic on words extending two-variable logic by relations which specify that a letter occurs between two positions on the word. These are typical three-variable properties, the idea goes back to Hilbert (1899). An algebraic condition is found, using operations developed by Schützenberger around his (1976) paper, which solves the definability problem for this intermediate logic, deciding it by an algorithm as in the earlier work. In particular there are (infinitely many) languages in three-variable logic which are not definable in the intermediate logic. Given a sentence of the intermediate logic, whether it has a model is decided using an exponential amount of memory. These computational bounds are shown to be tight.

## **Outreach Programmes:**

### **Facets: (5th - 6th July 2018)**

Mathematics program for college students

This was the 2018 edition of the institute's outreach program for advanced undergraduate (BSc third year) and postgraduate (MSc) students of mathematics.

The speakers were

- Amritanshu Prasad, IMSc
- Aaloka Kanhere, Homi Bhabha Centre for Science Education (Mumbai)
- Balaji K, Adobe Research (Bangalore)
- Nemani Suryanarayana, IMSc
- Rahul Siddharthan, IMSc
- Sivaguru R, TIFR Centre For Applicable Mathematics (Bangalore)
- Sushmita V, IMSc

About 180 students participated in this program.

### **Science, Journalism, Media: Communicating Science in a Changing India** (20th - 21st Aug 2018)

In collaboration with the Indian Academy of Sciences, IMSc organized a two-day workshop on "Science, Journalism, Media: Communicating Science in a Changing India" between August 20 - 21, 2018. The workshop was organized by Rahul Siddharthan and Gautam Menon from the Computational Biology group at IMSc. It brought together about 80 panelists and participants, largely scientists interested in communicating to the public across multiple media and science journalists with an interest in accurately describing Indian science, its breakthroughs as well as its problems. It tried to provide scientists with an idea of "what journalists really want" as well as to provide journalists with an idea of scientist's concerns about how their work was represented. The workshop was attended by a large number of journalists, including from such prominent outlets as the Hindu, the Indian Express, the Eastern Chronicle, Nature India, Anandabazar and the Wire as well as governmental organizations such as Vigyan Prasar. Large-scale science funders such as the DBT-Wellcome India Alliance were represented, as was the Indian Academy of Science along with

scientists from NCBS, TIFR, INSTEM, IITM and JNCASR. Local language sites such as ippodhu.com, as well several independent science writers and individuals involved in science communication participated. The format was based on panel discussions rather than long talks. Each panelist made short presentations before opening the topic to discussion, enabling active participation by all attendees. Prof. K. VijayRaghavan, PSA to the GOI, attended the workshop and was part of a panel. The program was exceptionally successful. Its proceedings were videographed and are available freely from:

<https://www.imsc.res.in/~scimedia/>

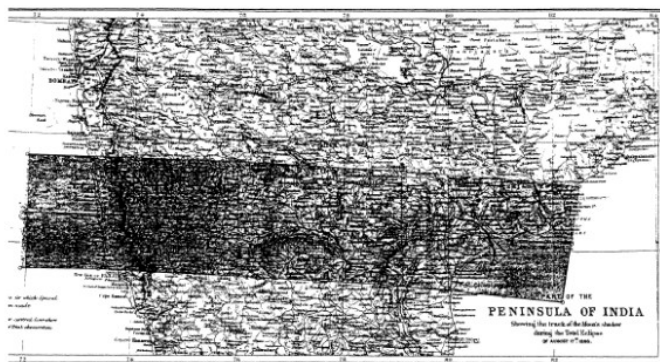


### **Discovery of Helium from Andhra Pradesh: (17th August 2018)**

Lecture celebrating 150th anniversary of fingerprinting the Universe, by Dr. Niruj Mohan Ramanujam.

The 18th of August, 2018 marks the 150th anniversary of the discovery of the element Helium that happened during a Total Solar Eclipse observed by European astronomers from Machilipatnam and Guntur in 1868.

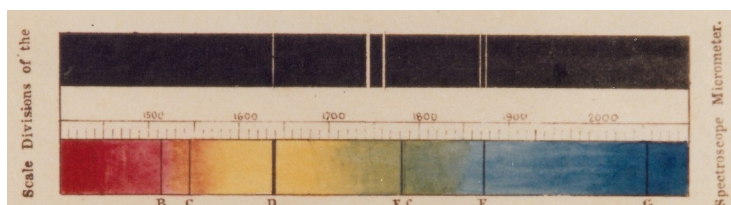
Helium remains the only element to have been discovered first in space, before being found on Earth. The story of this discovery itself is fascinating – the truth behind who among Janssen, Lockyer and Pogson (of Madras Observatory) should get the credit, was cleared up only a few years ago. More importantly, this discovery truly marks the beginning of modern Astrophysics. This beginning is intricately linked with the history of thermodynamics, atomic theory, and chemistry.



Path of totality on 18 Aug 1868 (calculated by Major Tennant)



Norman Pogson (1829-1891)  
Director, Madras Observatory



<https://www.youtube.com/watch?v=eEbSV6HNWGU>

**IMSc Open Day: (15th Sept 2018)**

A day of fun mathematics and science talks and activities for school children.

The program was organized for students from 8th - 10th standards.

The program comprised of lectures and demonstrations in a range of topics by students and professors of the institute. Sharing of the curiosity and excitement that we have for mathematics, science and research to the school students is the focus of this programme.