MATSCIENCE

INSTITUTE OF MATHEMATICAL SCIENCES
MADRAS, INDIA.

TENTH ANNIVERSARY

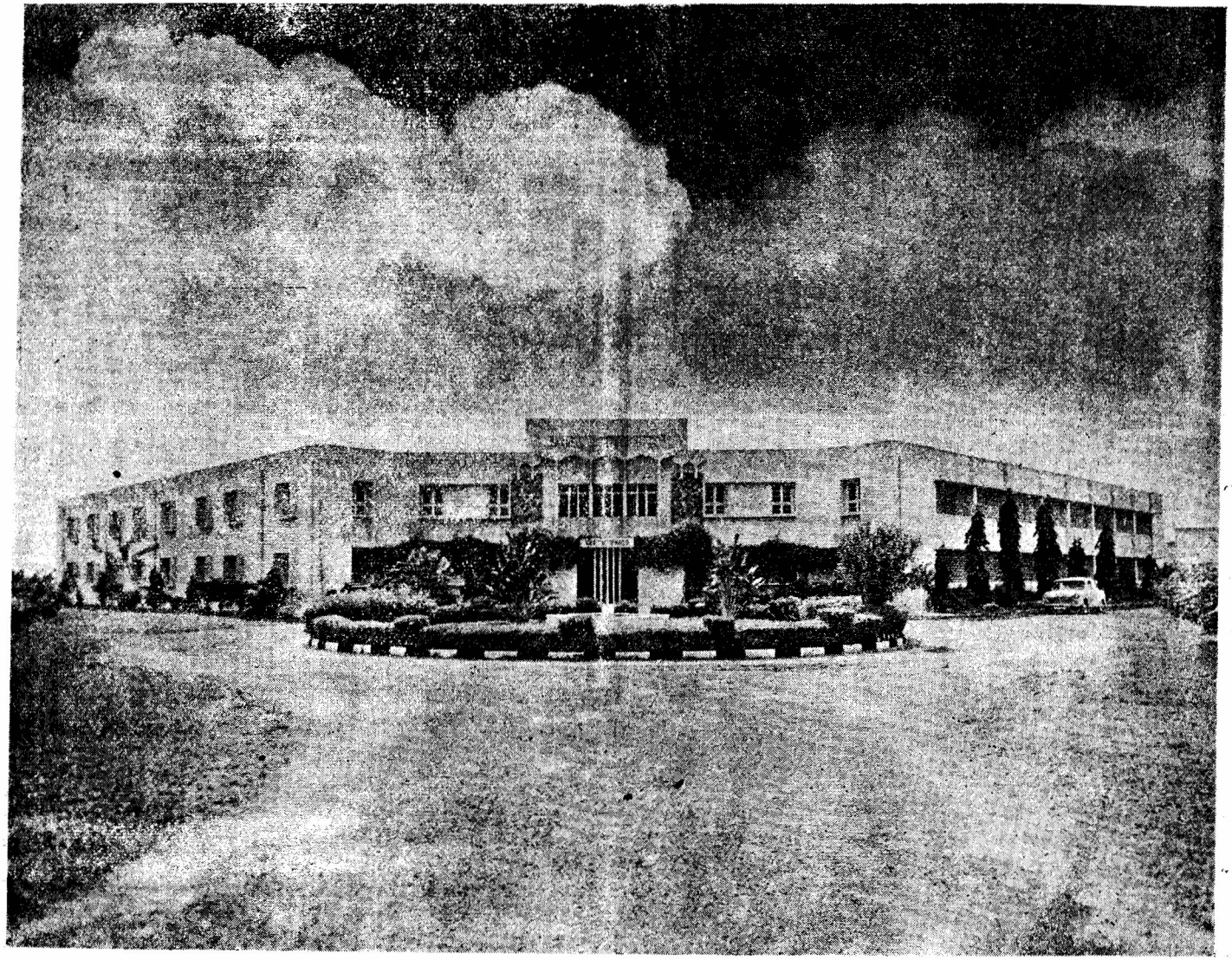
ANNUAL REPORT 1971

The Institute of Mathematical Sciences

Madras

"The pursuit of science is at its best
when it is a part of a way of life"

Annual Report 1971



Patron:

Mr. C. Subramaniam

Union Minister for Planning and Minister for Science and Technology, Government of India

Chairman of the Board of Governors:

Mr. V. R. Nedunchezhiyan

Minister for Education, Government of Tamil Nadu

Director:

Professor Alladi Ramakrishnan

General Information

Aims and Objects

1. To create and provide an atmosphere and environment suitable for creative work and the pursuit of knowledge and advanced learning in the mathematical sciences for their own sake,

2. To promote and conduct research and original investigation on fundamental sciences in general, with particular emphasis on Mathematics, Applied Mathematics, Theoretical Physics and Astrophysics,

3. To foster a rigorous mathematical discipline, to stimulate a zest for creative work and cultivate a spirit of intellectual collaboration among academic workers in pure and applied branches of science,

4. To arrange lectures, meetings, seminars and symposia in pursuance of its academic work for the diffusion of scientific knowledge,

5. To invite scientists in India and abroad actively engaged in creative work to deliver lectures and participate in academic activity.

Academic Activities

The primary activity of the Institute is creative research in Mathematical Sciences. In pursuit of the objectives of the Institute weekly seminars as well as series of lectures on various topics of interest, both by visiting scientists and the academic staff of the Institute are held.

To commemorate the inauguration of the Institute an Anniversary symposium is held in January for which scientists from India and abroad are invited to deliver one hour addresses summarizing their original work on recent advances in various branches of Mathematical Sciences. The Institute also organises a seminar in Analysis in which lectures range from introductory to advanced levels.

Academic Staff

The Academic Staff consists of Professors, Permanent Members and Associate Members of the various Faculties, Visiting Scientists, Temporary Members, Research Fellows and Research Trainees.

Ph.D Programme

Facilities are available for postgraduate students to work for Ph.D. degree under the guidance of the academic staff of the Institute in various faculties. Senior and Junior research fellowships are awarded by the Institute. Besides these, fellowships tenable at the Institute are awarded by the Department of Atomic Energy and the Council of Scientific and Industrial Research.

The Standing Committee of the Inter-University Board of India and Ceylon at its meeting held in February 28, 1967 adopted a resolution recognising the Institute as a suitable centre for research work. In view of the above resolution the Institute is now recognised by the various Indian Universities as a centre for research for the doctorate degree in Theoretical physics and Mathematics.

Mode of Admission

Memberships (temporary and permanent) of the Institute are available on invitation or by application to the Director. Students intending to become research trainees and research fellows of the Institute are expected to apply on prescribed forms which are supplied on request.

Visiting Scientists Programme

Facilities are offered to visiting scientists to spend a considerable time in the Institute and work in collaboration with academic staff of the Institute. The visiting membership programme of the Institute is designed to enable young, active and promising scientists to pursue research and take part in various academic activities of the Institute. Such memberships are available on invitation or by request for such periods as may be fixed in consultation with the visiting members. Scientists intending to be invited under the scheme can contact the Director of the Institute giving particulars of their academic career and indicating the probable period of their stay. Besides the above, scientists from various institutions may be invited to deliver lectures and seminars for which suitable honorarium is offered.

Publications

1. RESEARCH PAPERS (Preprints and reprints are available on request)

2. MATSCIENCE REPORTS based on the lecture courses delivered at the Institute both by visiting scientists and academic staff (Price Rs. 5/- within India or U.S.$ 1·00 outside India)

3. PROCEEDINGS OF THE SEMINAR IN ANALYSIS based on lecture courses delivered at the Institute by visiting scientists or members of the Institute during the Annual Seminar in Analysis (Price Rs. 5/- within India or U.S.$ 1.00 outside India).

4. Proceedings of the Summer School and the Anniversary Symposium published as a series entitled "SYMPOSIA ON THEORETICAL PHYSICS AND MATHEMATICS" by the Plenum Press, New York.

5. MASTECH PROCEEDINGS based on papers presented at the MASTECH Conference under the auspices of the Council of Scientific and Industrial Research.

News of the Institute

Ninth Anniversary Symposium

The NINTH Anniversary Symposium of the Institute was held for a week starting from 12th January. The Symposium entitled "Bhabha's contribution to Modern Physics" was inaugurated by Hon'ble Mr. V. R. Nedunchezhiyan, Minister for Education, Tamil Nadu and Chairman of the Board of Governors of the Institute. Dr. Malcolm S. Adiseshiah, formerly acting Director-General of UNESCO delivered the inaugural address on "Science and National Development".

On this occasion, Dr. A. Lakshmanaswami Mudaliar, formerly Vice Chancellor of the University of Madras, unveiled the portrait of Professor H. J. Bhabha, F.R.S, Professor C. R. Rao, F.R.S., Indian Statistical Institute, released the MATSCIENCE REPORT No. 70 on "The Grammar of Dirac Matrices" by Prof. Alladi Ramakrishnan. Among the scientists who delivered lectures in the symposium were Professors B. S. Madhava Rao, G. Abraham and Surya Prakash besides Prof. Alladi Ramakrishnan, Dr. N. R. Ranganathan, Dr. T. S. Santhanam and Dr. K. H. Mariwalla.

Fourth Seminar in Analysis

The Fourth Seminar in Analysis, conducted for a period of ten days at Ootacamund, from February 1, 1971 was inaugurated by Mr. M. A. S. Rajan, Managing Director, Hindustan Photo Films, Ooty. This Seminar was supported by a special grant from the Department of Atomic Energy, Government of India.

The following three lecturers gave a series of lectures on advanced topics:

Professor H. G. Garnier Liege University, Belgium :	"Topological Vector Spaces- Bornological Spaces"
Professor Ethienne Liege University, Belgium :	"Functional Analysis and its Applications to Control Theory"
Professor K. R. Unni MATSCIENCE, Madras	"On Multiplier Problem"

The participants also presented their original contributions.

Other Conferences

A two day symposium on "Clifford Algebra, its Generalization and Applications", inaugurated by Mr. M. A. S. Rajan, I.A.S., Managing Director, Hindustan Photo Films, was held at Ooty on 30th and 31st of January 1971. Prof. Alladi Ramakrishnan gave the inaugural lecture on "Simplicity and Elegance of Complex and Hypercomplex Numbers".

Among the participants were Professors T. Nono, Fukuoda University and K. Yamazaki, University of Tokyo, of Japan.

Two MATSCIENCE Symposia were held during the year.

The Symposium on "Cosmology, Gravitation and Applications to Particle Theory" was held in Bangalore during Nov. 5-9, 1971. Professor Alladi Ramakrishnan inaugurated the symposium and presented a paper on "Einstein as a Natural Completion of Newton". Participants from various universities and research institutions presented papers on their recent work.

The Symposium on "Fourier Optics, Lasers and Holography" was held at the Institution of Engineers, Mysore between 11-15, Nov. 1971. It was the first symposium in coherent optics in the country. Twenty one participants from various universities and research institutions presented papers on their original work. Professor Alladi Ramakrishnan, inaugurated the symposium and presented a paper on "Light from Newton to Feynman".

A one-day symposium on 'Fifty years of Mathematical Education in India' was held at MATSCIENCE on 29th December, 1971. Dr. Malcom S. Adiseshiah inaugurated the seminar. This seminar was organised by Professor Alladi Ramakrishnan, at the suggestion of Professor S. R. Ranganathan, in collaboration with the Association of Mathematical Teachers of India (AMTI) to commemorate the birth centenary of Professor V. Ramaswamy Iyer, founder of the Indian Mathematical Society. Participants from several institutions and universities, besides member of Matscience, spoke on various aspects of Mathematics and its mode of teaching.

Mastech Conferences

The THIRD of the series of conferences sponsored by the Council of Scientific and Industrial Research, entitled "Statistical Mechanics and its Applications to Science and Technology" was held at National Aeronautical Laboratory,

Bangalore for a week from January 24, 1971. The following papers were adjudged to be the best papers presented at the conference :

R. Sridhar (with R. Pratap);	Body Instabilities in a Magnetoactive Plasma (Paper presented by R. Sridhar)
Dipan K. Ghosh (with A.L. Davalos)	A Functional Integral Approach to Ising-Heisenberg Linear Chain (Paper presented by D. K. Ghosh)

Prof. Alladi Ramakrishnan, Director, MATSCIENCE and convener of the conference presented the MASTECH award to Mr. R. Sridhar of MATSCIENCE and Dr. D. K. Ghosh of Tata Institute of Fundamental Research, Bombay.

The FOURTH of the above series entitled "Functional Analysis and its Applications to Science and Technology" was held at NAL, Bangalore for four days from 14th September 1971. The following papers were adjudged to be the best papers presented at the conference:

K. C. Das :	Extrema of the Rayleigh Quotient and Normal Behaviour of an Operator.
M. R. Subrahmanya :	On Simultaneous Chebyshev Approximation.

Prof. Alladi Ramakrishnan, Director, MATSCIENCE and convener of the conference presented the award to Mr. M. R. Subrahmanya of MATSCIENCE and Mr. K. C. Das of Indian Institute of Technology, Kharagpur.

Academic Staff

Professor Alladi Ramakrishnan — *Director*

Permanent Members:

Dr. R. Vasudevan*
Dr. K. R. Unni

Associate Member

Dr. N. R. Ranganathan

Temporary Members

Dr. V. Radhakrishnan
Dr. T. S. Santhanam
Dr. K. H. Mariwalla
Dr. A. R. Prasanna

Senior Research Fellows:

Mr. K. Srinivasa Rao
Miss. P. K. Geetha
Dr. A. Lakshmi
Mr. R. Sridhar
Dr. T. S. Shankara

Junior Research Fellows:

Mr. M. R. Subrahmanya
Mr. G. N. Keshavamurthy
Miss. Nalini B. Menon
Miss. Vimala Walter‡
Miss. G. Shanthi‡‡
Mr. A. Tekumalla

Research Trainees:

Mr. R. Jagannathan
Mr. V. Krishnaswamy
Mr. P. Jayavelu
Mr. P. Nageswara Rao
Mr. P. V. Subramaniam
Mr. D. Vijayakumar
Mr. R. Swaminathan
Mr. A. K. Chellappa
Miss. Kasturi Nagarajan
Miss. P. R. Aruna
Miss. S. Ramamani
Miss. V. Uma

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* On leave of absence at: University of Southern California, Los Angeles, California, U.S.A.

‡ Junior Research Fellow, C.S.I.R. Scheme

‡‡ C.S.I.R. Junior Research Fellow

Invitations and Delegations

In response to invitations from various research institutions, Prof. Alladi Ramakrishnan gave lectures at the following research centres abroad on his recent work during May-June 1971:

Europe: Univ. of Bonn, Bonn, Germany; Univ. of Liege, Liege, Belgium.

U.S.A.: Yeshiva Univ., N.Y.; Rutgers Univ., N. J.; Naval Res. Lab. Washington D.C.; Howard Univ., Washington D.C.; Univ. of North Carolina, North Carolina; St. Louis Univ. St. Louis; Univ. of Wisconsin, Milwaukee; Iowa State Univ., Ames; Utah State Univ., Logan; Stanford Univ., Calif.; Univ. of Calif., Riverside, Berkeley and Irvine; Univ. of Southern Calif., Los Angeles; North Texas Univ., and Univ. of Texas, Dallas;

Canada: Univ. of Montreal and McGill Univ., both at Montreal.

Prof. Ramakrishnan was invited to participate in the "Rutherford Centennial Symposium on the Structure of Matter", which was conducted at the University of Canterbury, Christchurch, New Zealand during July 1971. He also delivered lectures at the following institutions *in Australia*: Latrob Univ., Melbourne; Univ. of Sydney, Sydney; Australian National Univ., Canberra;

Prof. Ramakrishnan was invited to participate in the "International Conference on Stochastic Point Processes", sponsored by IBM, New York, during August 1971. During this trip he also visited the following research centres in *U.S.A*: Univ. of Rhode Island, Rhode Island; Stanford Univ., Stanford; Univ. of Southern Calif., California.

Dr. T. S. Santhanam has been appointed as an Associate Member at the International Centre for Theoretical Physics, Trieste, Italy. This will enable him to spend a few months at ICTP, every year, during the next three years. He is now at Trieste and he will also be visiting some research centres in Europe. and U.S.A.

Dr. N. R. Ranganathan was invited to participate in the Indo-French Seminar on Optical Behaviour of Materials at IIT, Delhi. He was also invited as a guest speaker for the U.G.C. Summer Institute on Diffraction Physics at the Centre for Advanced Study in Biophysics, Univ. of Madras.

Drs. V. Radhakrishnan, N. R. Ranganathan and K. H. Mariwalla were invited to present their recent work at the Third Conference on Photographic science and technology, conducted by Hindustan Photo Films, Ooty in April 1971.

Dr. V. Radhakrishnan was invited to spend two weeks as a visiting scientist at the Indian Institute of Science, Bangalore in June 1971. He was invited by the Vice-Chancellor of the Univ. of Calicut to join the general inspection commission on Post Graduate course in Physics at the University.

Dr. A. R. Prasanna spent two weeks at the Radio Astronomy Centre (Tata Institute of Fundamental Research) Ootacamund during January-February 1971, at the invitation of Prof, G. Swarup and gave a course of six lectures on "General Relativity and Cosmology". He visited the centre again in December 1971 for discussions on certain cosmological studies.

Mr. K. Srinivasa Rao was invited to spend six weeks as a visiting scientist at Physical Research Laboratory, Ahmedabad, by Prof. S. P. Pandya, in March-April 1971. During his stay, he delivered a series of lectures. He also gave a lecture at Nizam College, Hyderabad.

Special lectures at the Engineering College, Guindy:

One of the happy and welcome developments during the year is the course of special lectures by the staff of MATSCIENCE to various postgraduate departments in the College of Engineering, Guindy. The lectures were sought to focus attention on recent development in various topics and it is felt such an interaction between the scientists and the engineers will be of mutual benefit. The following members of staff of MATSCIENCE participated in the above programme and gave series of lectures on various topics: Professor Alladi Ramakrishnan, Drs. V. Radhakrishnan, N. R. Ranganathan, K. H. Mariwalla and A. R. Prasanna.

Visiting Scientists

Dr. J. M. Anderson
Department of Mathematics
University of London
London, U.K.

Dr. G. V. Gehlen
University of Bonn
Bonn, West Germany

Prof. G. Kartha
Roswell Park Memorial Institute
University of Buffallo
New York, U.S.A.

Prof. D. Lichtenberg
Department of Physics
Indiana University
Bloomington, Indiana
U.S.A.

Dr. Eric A. Lord
King's College
University of London
London, U.K.

Prof. G. S. S. Ludford
Cornell University
U.S.A.

Prof. T. Mathews
University of Calgary
Canada

Dr. J. P. Meyer
Service d'Electronique Physique
Saclay, France

Prof. T. Nono
Fukuoda University of Education
Fukuoda, Japan

Prof .G. Rasche
Institut fur Theoretische Physik
Universitat Zurich
Switzerland

Prof. R.L.E. Schwarzenberger
University of Warwick
Coventry, U.K.

Prof. M. M. Shapiro
Chief Scientist
Laboratory for Cosmic ray physics
Naval Research Laboratory
Washington, U.S.A.

Prof. D. ter Haar
University of Oxford
Oxford, U. K.

Prof. K. Yamazaki
College of Education
University of Tokyo
Tokyo, Japan

Invited Lectures

Dr. R. K. Bansal	:	"Sum rules in single Nucleon transfer reactions"
Dr. Bipin R. Desai	:	"Complex Regge poles: a realistic way to fit experiments"
Dr. V. Devanathan	:	"Polarization of the Neutron in Muon capture"
Prof. G. Kartha	:	"Some recent work on the structure of macromolecules"
Dr. A. Lakshmi	:	1. "Coherent pair of extension of locally eompact groups"
	:	2. "An introduction to minimal sets"
Prof. D. Lichtenberg	:	"Aspects of duality"
Prof. T. Mathews	:	"Some aspects of cosmic ray physics"
Dr. J. P. Meyer	:	1. "Cosmic ray nuclei: General remarks after the HOBART conference"
		2. "Low energy cosmic ray deuterium and sources energy spectra"
Prof. T. Nono	:	"General Canonical transformations" (3 lectures)
Dr. K. R. Parthasarathy	:	"Continuous tensor products and representations of current groups"
Dr. R. Pratap	:	"Solar Bursts"
Dr. P. N. Rangachari	:	"Microbiological transformations of Terpenes"

Prof. G. Rasche	:	1. " History of isotopic spin "
	:	2. " $\pi^- p$ elastic and charge exchange scattering "
Prof. M. M. Shapiro	:	1. " Transformations of cosmic rays in space and the age of cosmic radiation "
	:	2. " Cosmic ray source composition "
Prof. D. ter Haar	:	1. " Pulsars "
	:	2. " Fluctuations in the ground state of Bose systems "
Prof. K. Yamazaki	:	" Projective representation of groups " (3 lectures)

Research Papers

Theoretical Physics

Alladi Ramakrishnan

A matrix decomposition theorem
(J. Math. Anal. Appl., in press)

On the shell structure of an L-matrix
(J. Math. Anal. Appl., in press)

A new approach to matrix theory
(INSDOC, 1971)

Stochastic theory of evolutionary processes (1937-1971)
(to be pub. in Proc. of the Inter. Conf. on Stochastic Point Processes, IBM, New York, 1971)

Generalized Clifford algebra and its applications (or)
A new approach to internal quantum numbers
(To be publ Proc. Rutherford Centennial Symposium on the structure of matter, July 5-7, 1971, Univ. of Canturbury, Christchurch, New Zealand)

Einstein – A natural completion of Newton
(to be pub. in the special volume to be brought out in honour of Salomon Bochner)

The weak interaction Hamiltonian in L-matrix theory
(J. Math. Anal. Appl., in press)

Alladi Ramakrishnan and R. Vasudevan

On generalized idempotent matrices
(J. Math. Anal, Appl., V. 32, p. 414-423, 1970)

Alladi Ramakrishnan, R. Vasudevan and P. S. Chandrasekaran

Para-Fermi operators and special unitary algebras
(J. Math. Anal. Appl., V, 35, p. 249-254, 1971)

Algebras derived from polynomial conditions
(J. Math. Anal. Appl., V. 35, p. 131-134, 1971)

R. Vasudevan, R. Sridhar and R. Pratap

Collective effects in an electron gas – a study in de Hasvan Alphen oscillations (IL Nuo. Cimento, in press)

N. R. Ranganathan and V. Radhakrishnan

On stochastic aspects of latent image formation in silver halides
(to appear in Proceedings of the Indo-French Seminar in optical behaviour of materials, I.I.T.. New Delhi, 1971)

N. R. Ranganathan, V. Radhakrishnan and K. H. Mariwalla

Kinetics of silver growth in print out process
(To appear in proceedings of the third conference on photographic sciences, Ooty, 1971)

K. H. Mariwalla and R. Vasudevan

Does a uniformly accelerated charge radiate ?
(Lettere Nuo, Cim., V. 1, p. 225–228, 1971)

K. H. Mariwalla

On the equivalence of potential and field formulation of Maxwell equations
(J. Math. Phys., in press)

K. H. Mariwalla, N. R. Ranganathan and V. Radhakrishnan

Role of positive-holes in the growth of the latent image speck
(To appear in proceedings of the third conference on photographic sciences, Ooty, 1971)

V. Radhakrishnan, N R. Ranganathan and K. H Mariwalla

Effects of electron tunnelling in the formation of latent-image speck
(To appear in proceedings of the third conference on photographic sciences, Ooty, 1971)

Eric A. Lord

The Clifford algebras and rotation groups
(J. Math. Anal. Appl., in press)

Six dimentional formulation of meson equations
(Int. J. Theor. Phys. V. 5, 1971)

Algebraic properties of relativistic equations for zero rest-mass.
(Int. J. Theor. Phys., V. 5, 1971)

Ramesh Chand

Quark model for double charge-exchange meson-Baryon scattering
(Phy. Rev. D, V. 3, p. 1165–1167, 1971)

Quark model for high energy $\not{p}_N - \bar{\beta}B$ reactions
(Prog. Theor. Phys., in press)

Ramesh Chand and A. Sundaram

High energy photoproduction of pseudoscalar mesons
(Phy. Rev. -D, V. 2, p. 1952-1955, 1970)

T. S. Santhanam and A. R. Tekumalla

Relativistic wave equation describing the neutrinos
(Nuovo Cim. Letters, in press)

T. S. Santhanam

Remarkable connection between multi-Veneziano integrand and the character of special unitary groups (J. Math. Phys., in press)

T. S. Santhanam, P. S. Chandrasekaran and Nalini B. Menon

General involutional transformations and the representations of GL (N).
(J. Math. Phy., V. 12, p. 377, 1971)

On a class of linear relativistic wave equation describing particles and spin $\frac{1}{2}$.
(Prog. Theor. Phys., in press)

S. Mukherjee

Broken conformal invariance and the possible existence of an operator algebra.
(Lettere Nuo. Cim., V. 2, p. 17–19, 1971)

N. K. Sehgal and U. Trivedi

Prescription for meson-trajectory construction
(Lettere Nuo. Cim., V. 2,p. 11–16, 1971)

A. R. Prasanna

On certain space-times having the fourth component of energy momentum complex identically zero.
(Prog. Theor. Phys., V. 45, p. 1330–1335, 1971)

A new invariant for electromagnetic fields in curved spacetime.
(Phys. Letters - A, in press)

K. Srinivasa Rao

Analogs of giant resonances and photoproduction of positive pions from ^{16}O.
(J. Phys., - A in press)

K. Srinivasa Rao (with V. Devanathan and G. N. S. Prasad)

Photoproduction of charged pions from ^{12}C.
(Nuc. Phys., V. 159-A, p. 97–102, 1970)

R. Sridhar and R. Pratap

Statistical mechanics of magneto - active plasma
(IL Nuo. Cimento, in press)

A. Sundaram

Regge cuts and Isobar production
(Lettere Nuo. Cim., V. 1, p. 177–181, 1971)

A. R. Tekumalla and I. V. V. Raghavacharyulu

Solution of the difference equations of generalized Lucas polynomials
(J. Math. Phys., in press)

Pure Mathematics

K. R. Unni and P. K. Geetha

On multiplier transformations
(Tohoku Math. J., V. 23, p. 155-168, 1971)

G. N. Keshavamurthy and K. R. Unni

On a class of functions satisfying the Lipschitz Condition.
(J. Math. Anal. Appl., in press)

M. R. Subrahmanya

A complete solution to a problem of Rivlin
(Revue Roumaine de Math. et appl., in press)

MATSCIENCE REPORT

Report Number	*Author*	*Title*
72	A. R. Prasanna	"Lecture notes on Gravitational collapse and Gravitational radiation"

Library

Books :

During the year under report, 1062 new books, (including bound periodicals) were added to the library bringing the total number of volumes to 10,819. These include many of the recent publications in mathematics and physics.

Periodicals :

The following new Journals were subscribed for during the year under report :

1. Atomic Data
2. Advances in Mathematics
3. Fields and Quanta
4. Annals of Library Science and Documentation
5. Index to Mathematical papers
6. Journal of Institute of Mathematics and its Application
7. Journal of Combinatorial Theory (A & B)
8. Journal of Number Theory
9. Linear Algebra and its Applications
10. Reports on Mathematical Physics

Periodicals obtained on exchange basis from institutions all over the world are being received regularly.

Preprints :

About 150 preprints in the field of theoretical physics and pure mathematics are being received from institutions throughout the world like CERN, NORDITA, Berkely Trieste, Princeton etc.

Lecture Notes :

Lecture Notes of various institutions all over the world are regularly received in exchange to our MATSCIENCE REPORTS

Lists Published :

1. List of Preprints received in the Library (issued Fortnightly)
2. List of Periodicals (issued Yearly)
3. List of MATSCIENCE REPORTS (issued Yearly)
4. List of Reprints (issued Yearly)
5. List of Preprints (issued Yearly)

EINSTEIN — A NATURAL COMPLETION OF NEWTON*

ALLADI RAMAKRISHNAN

MATSCIENCE, The Institute of Mathematical Sciences,

Madras-20, INDIA.

It is generally accepted that the theory of relativity is the greatest contribution to scientific thought that has emanated from a single mind since Newton's formulation of the laws of motion. At the same time Einstein's contributions have been interpreted as a departure from Newtonian ideas and this belief is essentially due to the fact that new concepts like the equivalence of mass and energy and the symmetry of space—time were not envisioned in the Newtonian universe. We shall now present the relativistic theory as a natural continuance and completion of Newtonian ideas. The mansion of relativity has many entrances and the most suitable one for entering it from the Newtonian structure is the *velocity transformation formula.*

We start with the following simple argument which takes us right into the heart of the theory of special relativity. Considering one dimensional motion, if v is a possible velocity of a point particle of mass m, so is $-v$ since it merely implies a reversal in direction. Accepting the definition of relative velocity and the axiomatic principle of no preference for any particular frame we find $2v$ is a realisable velocity. Therefore $2^n v$ is also realisable where n can be chosen as large as we please. If we make the postulate that such a world admitting velocities as large as we please would be 'chaotic', then an upper limit l has to be prescribed for the relative velocity. Thus if v_a and v_b are velocities of two point particles a and b, then the relative velocity is assumed to be

$$v_r = \frac{v_a - v_b}{f(v_a, v_b)} \tag{1}$$

We now require that

$$v_r < l \text{ if } v_a < l \text{ and } v_b < l \tag{2}$$

and that

$$\begin{aligned} &v_r \to v_a - v_b \text{ as } v_a \text{ and } v_b \to o \\ &v_r \to l \text{ as } v_a \text{ and (or) } v_b \to l \end{aligned} \tag{3}$$

The only choice of $f(v_a, v_b)$ turns out to be

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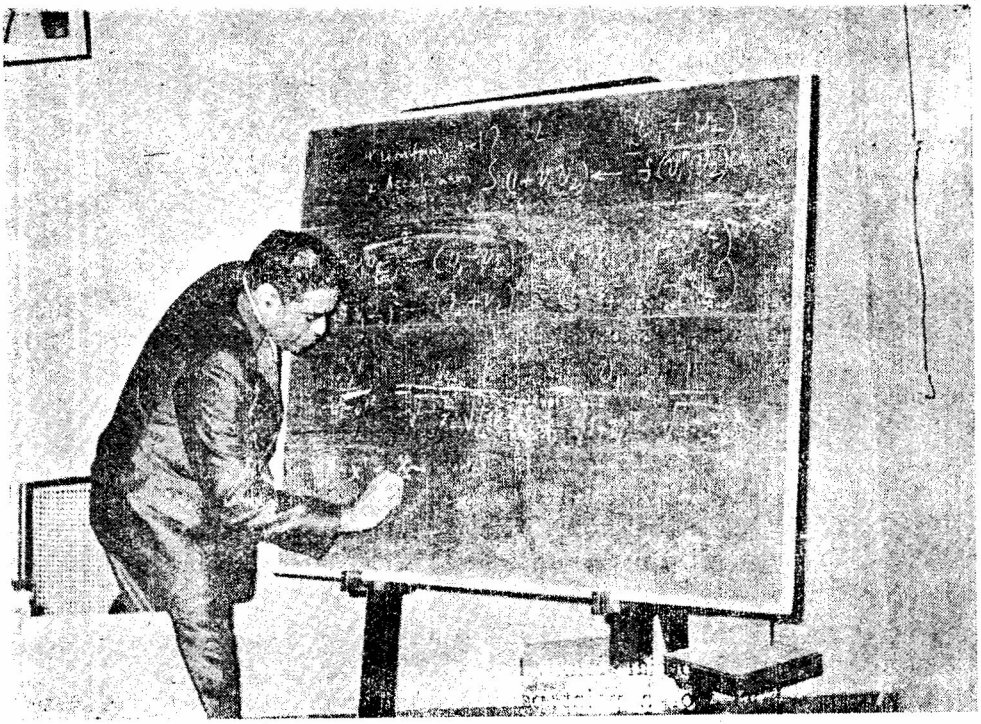
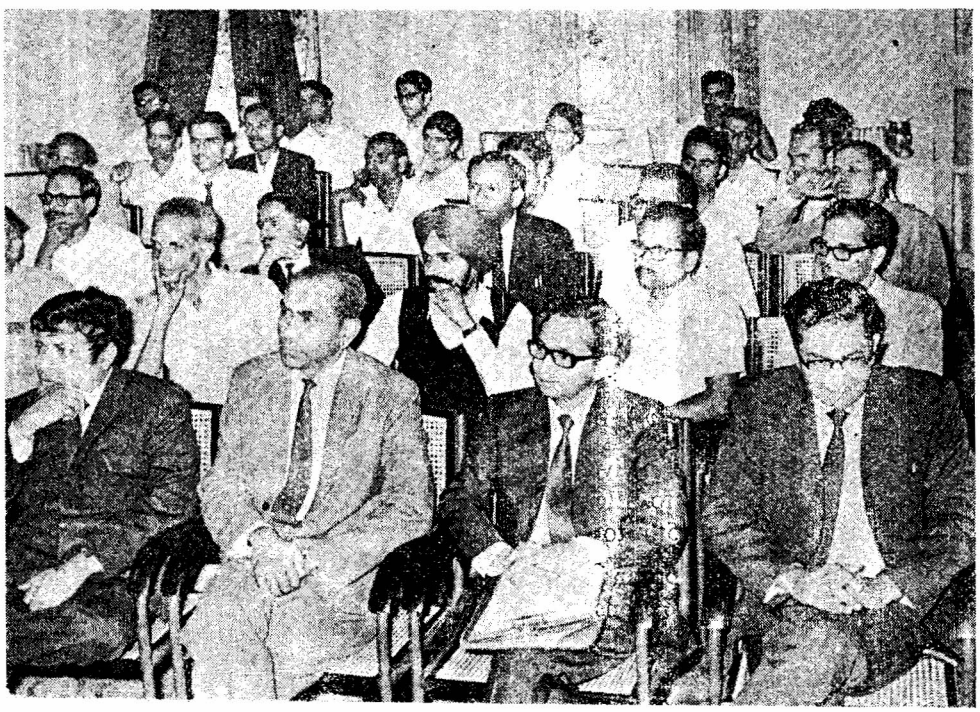
* (Inaugural lecture delivered at the conference on "Cosmology, Gravitation and Application to Elementary Particles" conducted by MATSCIENCE in Bangalore from 5th to 9th November, 1971).

This article is to appear in the special volume of the "Journal of Mathematical Analysis and Applications" (Academic Press N.Y.) in honour of Salomon Bochner.



"The mansion of relativity has many entrances and the most suitable one for entering it from the Newtonian structure is the velocity transformation formula".



(4) $$1 - v_a\, v_b/l^2$$

We find that on setting $l = 1$ without loss of generality the relative velocity formula is given by

(5) $$v_r = \frac{v_a - v_b}{1 - v_a\, v_b}$$

Equivalently

(6) $$v_a = \frac{v_r + v_b}{1 + v_r\, v_b}; \qquad v_b = \frac{v_a - v_r}{1 - v_a\, v_r}$$

v_a and v_b can be considered velocities of particles a and b relative to a particle c. Thus (5) and (6) demonstrate the symmetry among the velocities v_a, v_b, v_r on the axiomatic principle that all velocities are relative.

Let us define the following pairs of quantities

(7) $$\xi_r = \frac{v_r}{\sqrt{1 - v_r^2}}; \qquad \eta_r = \frac{1}{\sqrt{1 - v_r^2}}$$

Substituting ξ_a and η_a in (5) and using the simple identity

(8) $$(1 - v_a\, v_b)^2 - (v_a - v_b)^2 \equiv (1 - v_a^2)\,(1 - v_b^2)$$

We find

(9) $$\begin{pmatrix} \xi_r \\ \eta_r \end{pmatrix} = \begin{pmatrix} \eta_b & -\xi_b \\ -\xi_b & \eta_b \end{pmatrix} \begin{pmatrix} \xi_a \\ \eta_a \end{pmatrix}$$

We immediately recognise this to be the Lorentz transformation for space and time or momentum and energy, if we define

(10) $$\begin{aligned} x_r \text{ or } p_r &= K\,\xi_r; \quad t_r \text{ or } E_r = K\,\eta_r \\ x_a \text{ or } p_a &= K\,\xi_a; \quad t_a \text{ or } E_a = K\,\eta_a \end{aligned}$$

Where K is a constant independent of the velocities. For establishing the correspondence we have assumed that x and t are such that $|\,x/t\,| < 1$, that is we are considering events separated in space and time coordinates in such a manner that they can be connected by a signal with velocity less than unity or equivalently with the motion of a *single particle* with uniform velocity $v = \frac{x}{t} = \frac{p}{E}$. Thus writing

(11) $$t^2 - x^2 = K^2$$

K is identified to be the 'proper time'. If we write

(12) $$E^2 - p^2 = K^2$$

K is the mass of the point particle in a Lorentz transformation.

The transformation laws have hitherto been obtained on the assumption that the three ratios $\frac{\xi_r}{\eta_r}$, $\frac{\xi_a}{\eta_a}$ and $\frac{\xi_b}{\eta_b}$ are less than one. However these relations can be analytically continued to the case when any *two* of these ratios are greater than unity while the third is less than unity. In such a case the *two ratios which are greater than unity cannot be interpreted as velocities* while the third one retains such an interpretation. We now observe that for example if we assume that

(13) $$\left|\frac{x_a}{t_a}\right| = \left|\frac{\xi_a}{\eta_a}\right| > 1, \quad \left|\frac{x_r}{t_r}\right| = \left|\frac{\xi_r}{\eta_r}\right| > 1 \text{ and } \left|\frac{\xi_b}{\eta_b}\right| < 1$$

we can still treat the equation (9) as valid without giving a physical interpretation for the ratios ξ_a/η_a and ξ_r/η_r as velocities. However the equation is not valid if all the three ratios are greater than unity or only one of them is greater and the other two less than unity.

Such a procedure is not unfamiliar to a mathematician and in fact has been the established mode of creating new mathematical structures from existing ones. For example, in number theory, starting with positive integers we introduce the operation of addition. Defining subtraction as the inverse of addition we find that it necessiates the inclusion of negative integers. Similarly the definition of division as the inverse of multiplication leads to fractional numbers. Irrational numbers are obtained as the limits of rational sequences and thus we are able to 'fill' the entire real line.

In much the same spirit we now interpret the variables x and t though $\frac{x}{t} = \frac{\xi}{\eta} > 1$. Since the velocity concept breaks down if $\xi/\eta > 1$, it is quite clear that x and t have to be interpreted as the spatial and temporal separation of events which are not the result of one another *i.e.* have no causal connection between them. In other words, *they refer to two particles which exist independently of one another.* Since a time difference may exist in some frame of reference we cannot speak of the simultaniety of the events but only of independence or acausality. Such an extension of the Lorentz transformation to values of x and t such that $\frac{x}{t} > 1$ thus implies the independent existence of two particles, a concept which has to be *postulated* in a Newtonian theory but which can be *derived* as an analytic continuation in the Einstein approach. If matter itself is interpreted as the collection of independently existing point particles distributed in space, the existence of matter in bulk seems to be a natural consequence of the existence of space like intervals. To identify two points in space we need the independent existence of two particles or in other words *we need matter to identify space.* Though matter may not *fill* the whole of space, it can *span* the whole of it when viewed from moving frames of reference.

A similar interpretation can be carried over to a description in the language of momentum and energy. The existence of two independent particles should not be interpreted as events relating to a single particle moving faster than light. This amounts to the assumption of the spontaneous splitting of one particle into two which is thought to be a test of tachyons. Far from being so it leads to the conceptual difficulties of considering two independently existing particles as one and the same.

Such difficulties arise due to the over—emphasis on the symmetry of space and time coordinates. However the concept of velocity itself is not symmetrical in space and time, since time is preferred as the denominator and the spatial coordinates as numerators.

In the case $\frac{x}{t} > 1$ the vanishing of time interval in a suitably chosen frame would imply that the ratio $\frac{x}{t}$ becomes infinite and therefore cannot be interpreted as a velocity since we have started with the postulate that infinite velocities are not realisable. Such a difficulty does not arise in the case when the numerator or the spatial interval vanishes in suitably chosen frames of reference when considering events for which $\frac{x}{t} < 1$.

Moreover if we wish to study multi-dimensional spaces, the increase in the number of dimensions is possible only in space coordinates, while there can be *one and only one time coordinate.* This conclusion emerges as a natural consequence of the L-matrix theory systematically developed by the author in recent years.

It is a well known panegyric on Newton—

The Lord said " let Newton be and all was light ".

For Einstein's theory, it may read

The Lord said " let there be light, all else is space, time and matter ".

THE WORLD LINE*

We will try to visualize the state of things by the graphic method. Let x, y, z *be rectangular co-ordinates for space, and let* t *denote time. The objects of our perception invariably include places and times in combination. Nobody has ever noticed a place except at a time, or a time except at a place. But I still respect the dogma that both space and time have independent significance. A point of space at a point of time, that is, a system of values* x, y, z, t, *I will call a world-point. The multiplicity of all thinkable* x, y, z, t *systems of values we will christen the world. With this most valiant piece of chalk I might project upon the blackboard four world-axes. Since merely one chalky axis, as it is, consists of molecules all a-thrill, and moreover is taking part in the earth's travels in the universe, it already affords us ample scope for abstraction, the somewhat greater abstraction associated with the number four is for the mathematician no infliction. Not to leave a yawning void anywhere, we will imagine that everywhere and everywhen there is something perceptible. To avoid saying "matter" or "electricity" I will use for this something the word "substance". We fix our attention on the substantial point which is at the world-point* x, y, z, t, *and imagine that we are able to recognize this substantial point at any other time. Let the variations* dx, dy, dz *of the space co-ordinates of this substantial point correspond to a time element* dt. *Then we obtain, as an image, so to speak, of the everlasting career of the substantial point, a curve in the world, a world-line, the points of which can be referred unequivocally to the parameter* t *from* $-\infty$ to $+\infty$. *The whole universe is seen to resolve itself into similar world-lines, and I would fain anticipate myself by saying that in my opinion physical laws might find their most perfect expression as reciprocal relations between these world-lines.*

H. MINKOWSKI (1908)

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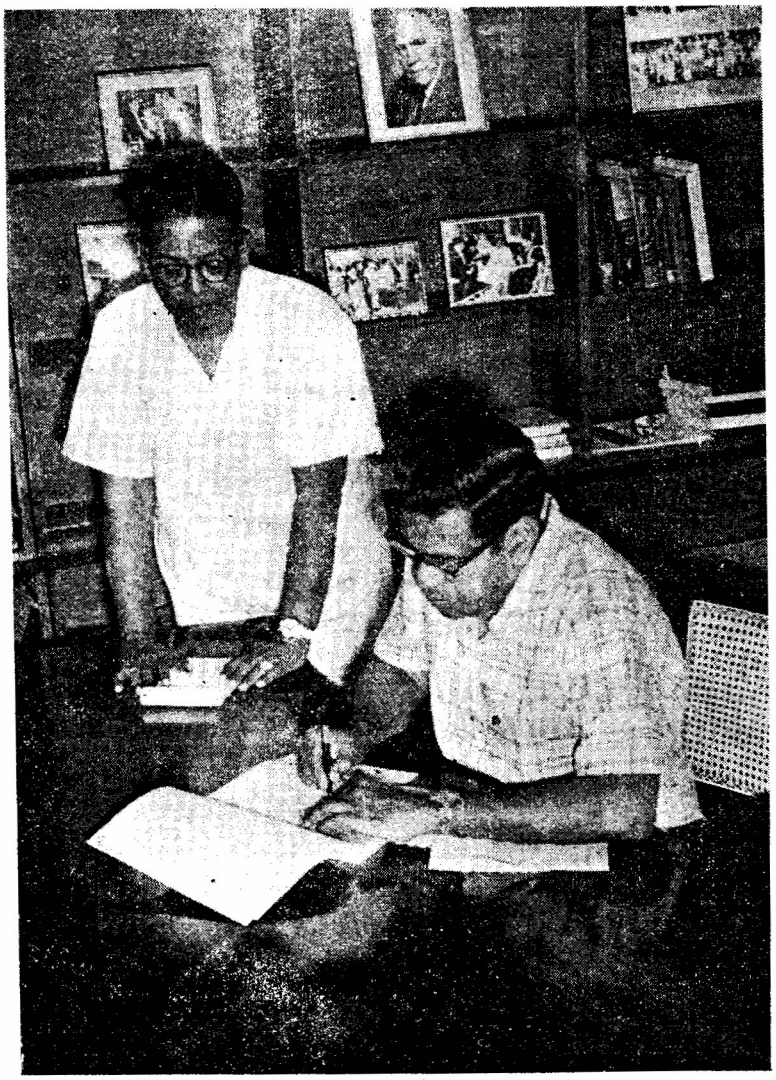
* An abstract from an address on 'Space and Time' delivered by H. Minkowski at the 80th Assembly of German Natural Scientists and Physicians at Cologne, 21st September 1908.



"... participated in the conception, creation, evolution, growth and established prosperity of Matscience."

Nambi - A part of the Matscience Tradition

Our dear Nambi is no more. He was part of ourselves, our hopes, our ambitions, our weaknesses, our twin passions - mathematical sciences and Matscience. Ten hours a day for more than ten years he was the companion friend, follower, asssistant of our Director, ministering to his needs great and small, listening to his conversation, speeches and exhortations.

He lost his liver, his life, in the service of the Institute and his devotion to the cause of advanced research. He participated in the conception, creation, evolution, growth and established prosperity of Matscience.

He protected the interests of Matscience, as if he shared its possessions. Thirty thousand letters to the Director's dictation, notes and jottings ranging from research papers to paint shop bills, communications with friends, scientists and Noble Laureates, arranging the itineraries of Bethe and Dirac and settling accounts with the plumber - in short everything needed his attention provided it was the work of Matscience. He was part of the stuff and texture of our academic and scientific life. He inspired love and affection and above all trust in all those who came in contact with him. His flamelike honesty in financial matters made all of us feel secure that our matters were in responsible hands.

It is hard to imagine a Nambi - less Matscience, and even the healing force of memory cannot compensate for such loss. Nambi was the part of the great tradition which makes Matscience a haven of the finest ideals of the academic way of life. Neither jealousy nor forgetful ingratitude can touch its purity for to us Matscience means just the spirit and substance of such ideals. Nothing else matters and as the future unfolds, the past becomes a reality indeed an inspiration under the magic hand of human memory, the most precious gift of God to mortal man.